First-Order Logic in Knowledge Representation

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A basic language: First-Order Logic

- Propositional logic is very simple:

  John is a man.

  Mark is a man.

  Paul is a man.

- Why not consider "man" a property of some entities? That is, man(John), man(Mark), man(Paul), etc.

Logical variables in STRIPS

```
Initial state: At(A), Level(low), BoxAt(C),
BananasAt(B)
Goal state:     Have(Bananas)
Actions:   _Move(X, Y)_
            //move from X to Y
            Preconditions: At(X), Level(low)
            Postconditions: not At(X), At(Y)
```

Some elements of First-Order Logic

- Constants: Aphrodite and Kronos.
- Predicates / relations: isMan, isSister, buyFrom.
- Logical variables: isMan(X), buyFrom(X,Y).
- Functions: fatherOf(X), sisterOf(Y).

A bit of syntax

- There are symbols for predicates, logical variables, constants, functions.
- A *term* refers to an object (a constant, a logical variable, a function applied to terms): John, X, sisterOf(X).
- An *atom* is a predicate with its "parameters": buyFrom(John, fatherOf(X)).

More syntax

- Boolean connectives amongst atoms:

$$\neg, \wedge, \vee, \rightarrow, \leftrightarrow \ .$$

- Every well-formed "propositional" formula where "propositions" are atoms is ok.
- Example:

$$(\mathrm{buyFrom}(\mathrm{John}, X) \wedge \mathrm{laywer}(X)) \vee \neg \mathrm{buyFrom}(\mathrm{John}, \mathrm{isSister}(\mathrm{Mary})).$$

Quantifiers: some syntax, some (informal) semantics

- Universal $\forall$: "for all"

$$\forall X : \text{person}(X) \to \text{mortal}(X).$$

- Existential $\exists$: "exists"

$$\exists X : \text{intelligent}(X) \wedge \text{computer}(X).$$

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- Any well-formed formula can be used "inside" a quantifer to produce a well-formed formula...

Informal semantics: Some examples

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  Every men loves at least one woman.
- $\exists Y : \forall X : \mathrm{man}(X) \wedge \mathrm{woman}(Y) \rightarrow \mathrm{loves}(X, Y).$
  There is a woman who is loved by every men.

Translation exercises:

- At least one palmerense is a world champion.
- John's brothers are all studying Engineering.
- The father of the father is the grandfather.
- Fish are aquatic craniate animals that lack limbs with digits.

Formal semantics and inference

- There is a formal semantics: the meaning of a set of formulas is given by a set, where each constant corresponds to an element of the set, each function corresponds to a function on the set, etc.

- Inference means determining which formulas hold in every model of a given set of formulas.

Inference?

- Consider:

  All living things need water.
  Roses need water.
  Thus roses are living things.

  32% of people find the last sentence to be a consequence.

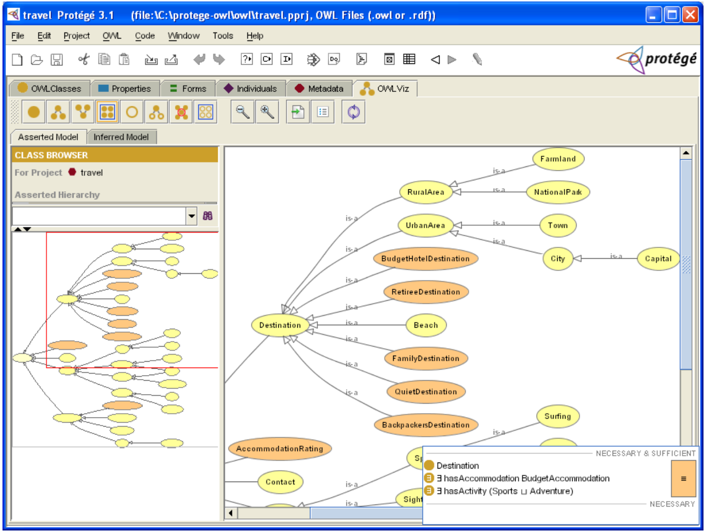
Advantages and disadvantages of first-order logic

- It is a solid basis for the study of representation languages.
- It captures mathematics and many other fields.
- However, it is too complex (inference is very hard),
- and it does not have all that we need (for instance, recursion).

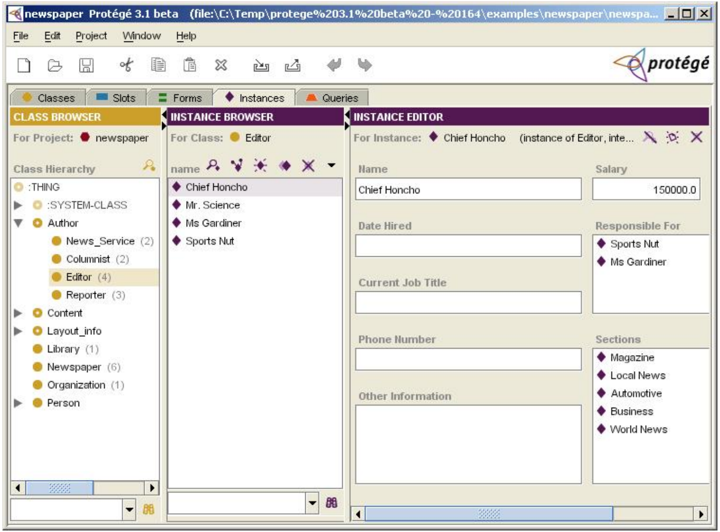
Ontologies

- A set of “axioms” is an ontology (terminology).
- The TBox stores the axioms.
- Assertions are stored in ABox.

Protege system (TBox)



Protege system (ABox)



OWL

- The language OWL, based on description logics, is now the standard for data storage in the *Semantic Web*.
- OWL is based on fragments of $\mathcal{ALC}$; in particular DL-Lite and $\mathcal{EL}$.