ASSIGNMENT

HYPOTHESIS TESTING

Submitted by

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1. Hypothesis Formulation:

A company claims that their new energy drink increases focus and alertness. Formulate the null and alternative hypotheses for testing this claim.

A null hypothesis is an assertion about the value of a population parameter that we hold as true unless we have sufficient statistical evidence to conclude otherwise.

For the above situation, we assume the mean change in focus and alertness after consuming the energy drink as μ. With this assumption we can formulate null and alternate hypothesis as follow:

- Null hypothesis: $H_0 = 0$ (ie, is there is no change in focus and alertness)
- Alternate hypothesis: $H_1 > 0$ (ie, there is an increase in focus and alertness)

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2. Significance Level Selection:

A researcher is conducting a study on the effects of exercise on weight loss. What significance level should they choose for their hypothesis test and why?

Ans:

The significance level, denoted as alpha (α). It is the is the probability of rejecting the null hypothesis when it is actually true. We reject null hypothesis when p-value falls below it.

Commonly used significance levels include 0.05, 0.01, and 0.10.

Finding the optimal value for α is difficult task. We reject null hypothesis when p-value falls below α; therefore, α is the maximum probability of type-I error. As the α increases, the probability of type-I error increases. If we set a low value of α, there is a low probability of type-I error but there is a high probability of type-II error. If we set high value for α, there is a high probability of type-I error but probability of type-II error is low.

We note that selecting a value for α is a question of compromise between probabilities of type-I and type II error. To arrive at a fair compromise, we should know the cost of each type of error.

However, there are the cases where we are not able to determine which type of error is costlier. If the costs are roughly equal, or if we have not much knowledge about the relative cost of two types of errors, then we should keep α = 5% (ie, 0.05).

In the above question, we have no knowledge about the relative costs of type-I and type II errors; therefore, researches can choose a significance level of 5%.

Q 3. In a study investigating the effectiveness of a new teaching method, the calculated p-value is 0.03. What does this p-value indicate about the null hypothesis?

Ans:

p-value is the probability of getting a sample evidence that is equally or more favorable to null hypothesis while the null hypothesis is actually true. In the above scenario, p-value of 0.03 (3%) means that H_0 (ie, the new teaching method is not effective) has only 3% credibility.

- In this case if we select significant level of 0.05 (5%), then the reported p-value is falls below it; therefore, we can reject the null hypothesis and accept the alternate hypothesis H_1 (ie, the new teaching method is effective).
- However, if we select significant level of 0.01 (1%), then the reported p-value is more than the significant level; therefore, we cannot reject null hypothesis.

Q 4. Type I and Type II Errors:

Describe a scenario in which a Type I error could occur in hypothesis testing. How does it differ from a Type II error?

In the context of statistical hypothesis testing, rejecting a true null hypothesis is known as **type I error** and accepting a false null hypothesis is known as a **type II error**.

Instances of Type I and Type II errors:

	H_0 true	H_0 false
Accept H_0	No error	Type II error
Reject H_0	Type I error	No Error

Let's illustrate this with a scenario.

Suppose a pharmaceutical company testing a new drug intended to lower blood pressure.

- The null hypothesis is that the **new drug has no effect** on blood pressure.
- The alternative hypothesis is that **the new drug does have an effect** on lowering blood pressure.

<u>Type I Error (False Positive):</u> *A Type I error would occur if the company concludes that the* ***new drug lowers blood pressure when, in reality, it does not****.*

<u>Type II Error (False Negative):</u> *A Type II error would occur if company conclude* ***that the new drug does not lower blood pressure when it actually does****.*

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5. Right-tailed Hypothesis Testing:

Q. A manufacturer claims that their new light bulb lasts, on average, more than 1000 hours. Conduct a right-tailed hypothesis test with a significance level of 0.05, given a sample mean of 1050 hours and a sample standard deviation of 50 hours.

Sol.

Define the null and alternate hypothesis:

Null hypothesis (H_0): Average lifespan **(μ) ≤ 1000**

As we will use a right tailed hypothesis test; therefore, **alternate hypothesis (H_1)** is **μ > 1000**

As population SD is not known, we will use t-statistic.

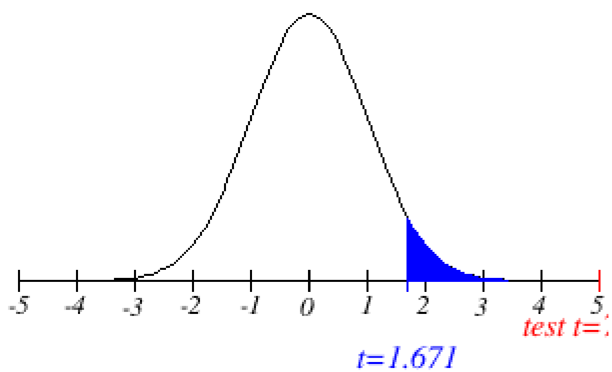
Conduct t-test using:

- Sample size (n) = 50
- Sample mean ($\bar{x}$) = 1050
- Sample standard deviation (s) = 50
- Significance level (α) = 0.05

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$$t = \frac{1050 - 1000}{50/\sqrt{50}}$$

Calculated t-value = 7.071



Degree of freedom (df) = n-1 = 50-1 = 49

Obtain critical value from t-table at df of 49 and significance level of 0.05.

- **Critical t-value = 1.671**

Calculated t-value (7.071) > critical t value (1.671)

*As calculated t-value is more than critical t-value; therefore, we can reject the null hypothesis at significance level of 0.05. Hence, there is **enough evidence** to support the claim that the new light bulb lasts, on average, more than 1000 hours at a 5% significance level, with a sample size of 50.*

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6. Two-Tailed Hypothesis Testing:

Q. A researcher wants to determine if there is a difference in mean exam scores between two groups of students. Formulate the null and alternative hypotheses for this study as a two-tailed test.

- **Null hypothesis (H_0)**: mean difference = 0 (ie, there is no difference in mean exam score between group)
- **Alternate hypothesis (H_1)**: mean difference ≠ 0 (ie, there is a difference in mean exam scores between two groups)

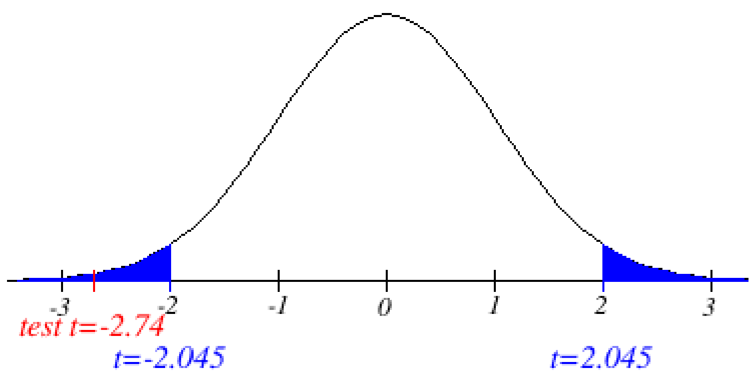
7. One-sample t-test:

Q. A manufacturer claims that the mean weight of their cereal boxes is 500 grams. A sample of 30 cereal boxes has a mean weight of 490 grams and a standard deviation of 20 grams. Conduct a one-sample t-test to determine if there is evidence to support the manufacturer's claim at a significance level of 0.05.

Sol.

Conduct the t-test using:

- Population mean weight (μ) = 500 g
- Number of boxes (n) = 30
- Sample mean ($\bar{x}$) = 490 g
- Sample standard deviation (s) = 20 g



$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

= 490-500 / (20/√30)

t = -2.74

Degree of freedom (df) = 30-1 = 29

Obtain the critical value from t-table at df of 29 and level of significance (α) of 0.05.

- Critical t-value = ±2.045

*As calculated t-value (-2.74) is outside of critical t-value (-2.045); therefore, **there is no enough evidence** to support the manufacturer's claims that the mean weight of their cereal boxes is 500 grams.*

8. Two-sample t-test:

Q. A researcher wants to compare the mean reaction times of two different groups of participants in a driving simulation study. Group A has a mean reaction time of 0.6 seconds with a standard deviation of 0.1 seconds, while Group B has a mean reaction time of 0.55 seconds with a standard deviation of 0.08 seconds. Conduct a two-sample t-test to determine if there is a significant difference in mean reaction times between the groups at a significance level of 0.01.

Sol. ***Sample size is not provided*** *in the question; however, to solve the question* ***we can consider sample size of both the groups as 30****.*

Group A	**Group B**
Sample size (n_1) = 30	Sample size (n_2) = 30
Mean ($\bar{x}_1$) = 0.6 s	Mean ($\bar{x}_2$) = 0.55 s
Standard deviation (S_1) = 0.1 s	Standard deviation (S_2) = 0.08 s

Assume $\boldsymbol{\mu_1 - \mu_2}$ is the difference between population means of group A and group B.

Define the null and alternate hypothesis:

- **Null hypothesis (H_0): $\boldsymbol{\mu_1 - \mu_2 = 0}$** (ie, there is no difference between means of two populations)
- **Alternate hypothesis (H_1): $\boldsymbol{\mu_1 - \mu_2 \neq 0}$** (ie, there is significant difference between means of two populations)

Apply the two-sample t-statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$\frac{(0.6 - 0.55) - 0}{\sqrt{(0.1)^2/30 + (0.08)^2/30}}$$

Calculated t-value = 2.137

Degree of freedom (df) = $n_1 + n_2 - 2$

= 30 + 30 – 2 = 58

Obtain the critical t-value from t table at df of 58 and significance level of 0.01

t critical = 2.390

As calculated t-value is less than critical t-value; therefore, ***we fail to reject the null hypothesis****. This means that* ***there in no significant difference between populations means of group A and group B****.*

9. Process Control Example:

A call center manager implements a new training program aimed at reducing call waiting times. The average waiting time before the training program was 4.5 minutes, and after the program, it is measured to be 4.0 minutes with a standard deviation of 0.8 minutes. Conduct a hypothesis test to determine if there is evidence that the training program has reduced waiting times, using a significance level of 0.05.

Sol.

Again, sample size is not provided *in the question; however, to solve the question* ***we can consider sample size of both the groups as 30****.*

Group A (before training)	**Group B (after training)**
Sample size (n_A) = 30	Sample size (n_B) = 30
Mean ($\bar{x}_1$) = 4.5 min	Mean ($\bar{x}_2$) = 4.0 min
Standard deviation (S_1) = 0.8 min	Standard deviation (S_2) = 0.8 min

Assume $\boldsymbol{\mu_A - \mu_B}$ is the difference between population means of group A and group B.

Define the null and alternate hypothesis:

- **Null hypothesis (H_0): $\mu_A - \mu_B = 0$** (ie, there is no difference between means of two populations)
- **Alternate hypothesis (H_1): $\mu_A - \mu_B \neq 0$** (ie, there is significant difference between means of two populations)

Apply the two-sample t-statistic

$$t = \frac{(\overline{x_A} - \overline{x_B})}{\sqrt{\frac{S_A^2}{n_A} + \frac{S_B^2}{n_B}}}.$$

$$= \frac{(4.5 - 4.0)}{\sqrt{(0.8)^2/30 + (0.8)^2/30}}$$

= 0.5/0.292

= 1.712

Calculated t-value = 1.712

Degree of freedom (df) = $n_A + n_B - 2$

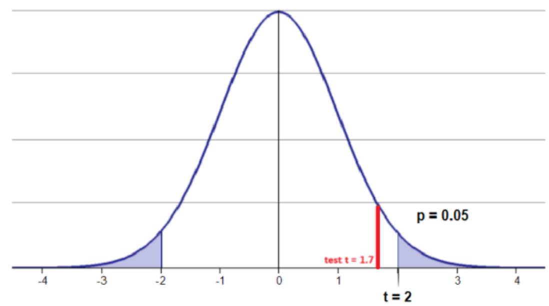
= 30 + 30 – 2 = 58

Obtain the critical t-value from t table at df of 58 and significance level of 0.05

t critical = 2.00

Calculated t-value = 1.712

As calculated t-value is less than critical t-value; therefore, ***we fail to reject the null hypothesis****. This means that* ***there is no significant difference between populations means of group A and group B****.*



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10. Interpreting Results:

After conducting a hypothesis test, the calculated p-value is 0.02. What can you conclude about the null hypothesis based on this result, assuming a significance level of 0.05?

Significance level = 0.05

Calculated p-value = 0.02

In this case, calculated p-value < 0,05 (significance level).

Therefore, we can reject the null hypothesis and accept the alternative hypothesis.
