The Maths and Physics Behind the Monkey Launcher

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1 The Basic Principle

1.1 Parameters

Our code takes in many variable parameters that can each be chose by the user. All of these parameters affect in some way the shot that the cannon will generate. All of these parameters are selected on the user interface.

This is the whole list of variables that can be played with :

Physical constants

Mass of the projectile : m (in kg)
Volume of the projectile : V (in m^3)
Length of the rail gun : l (in m)
Angle of the cannon : θ (in rad)
Voltage of the system : U (in V)
Resistance of the metal : R (in Ω)
Space between the branches : d (in m)
Fluid density : μ (in g/ml)
Coefficient of friction : c_d

Program constants

Number of points : nb
Time step between the points : t_{step}

2 The Rail Gun Mechanics

2.1 Intensity

The rail gun takes many variables as inputs (m, l, U, R and d). The first computation that we programmed was the electrical intensity of the rails, simply the Ohm's law :

$$I = \frac{U}{R}$$

2.2 Field

Now that we have a current, we can compute the magnetic field created by the current, given by the integral :

$$2 \times \frac{\mu_0 I}{4\pi} \int_{\theta_1}^{\theta_2} \frac{\sin\theta}{\frac{d^2}{2}} dx = \frac{\mu_0 I}{\pi \frac{d}{2}} \int_{\theta_1}^{\theta_2} \sin\theta d\theta$$

Where θ_1 and θ_2 are two complementary angles that will cancel out to one once integrated by theta. That gives us the value of the field according to the proximity of the bars (d) :

$$B = \frac{\mu_0 I}{\pi \frac{d}{2}}$$

2.3 Force

Once we have the field value in function of I, d and μ_0 (wich is a constant). Now we can calculate the force created by the field and intensity :

$$F = BId$$

2.4 Speed

Now that we have the force, we can use Newton laws to proceed the speed at which the rail gun will shoot the projectile, dictated by :

$$V_{railgun} = \sqrt{2l\frac{F}{m}}$$

The "length" of this speed is the output of the first part of our program

3 Once the Monkey is Launched

3.1 The drag

For this problem, we decided to use the Stokes drag equations to program all the interactions that our monkey will have with the air. The empirical formula for the drag force is :

$$F_{Stokes} = -K_S\vec{V}$$

Where K_S is the constant of drag defined by :

$$K_S = c_d \mu A$$

Where μ is the density of the fluid

3.2 Position

Now that we have a speed and an angle (chosen by the user), we can compute a simple ballistics problem. We also decided to implement air friction in our problem. First, the Stokes air friction formula is given by :

$$-kV_x = m\frac{dV_x}{dt}$$
$$-mg - kV_y = m\frac{dV_y}{dt}$$

in x and y axis respectively.

The speed V_x and V_y are defined by :

$$V_x = V_{railgun}\cos\theta$$
$$V_y = V_{railgun}\sin\theta$$

We can then solve these differential equations. That gives us the two following equations :

For the x component

After solving this separable variables ODE, the function of x according to time is given by :

$$x(t) = \frac{mV_{x0}}{k}(1 - e^{-\frac{kt}{m}})$$

For the y component

This linear ODE can be solved with an integrating factor since it is first order. This resolution gives us :

$$y(t) = -\frac{gm}{k}t + \frac{m}{k}(Vy0 + \frac{gm}{k})(1 - e^{-\frac{kt}{m}})$$

The position in x and in y is now defined according to the the speed that the cannon launched the projectile and the angle that was chosen by the user as well as the aerodynamics parameters that also affect the aerodynamic part of the ballistic of the projectile.

4 The plot

Now that we have an x,y coordinate for every unit of time, we just have to plot a point cloud graph where we iterate these position functions for each time step as long as our projectile does not touch the ground !