# Modeling Discrete Elastic Rods – HW2 Report

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***Abstract* — This report details various simulations and experiments conducted on MATLAB to highlight characteristics of discrete elastic rods, while expanding on concepts of twist and parallel transport theorems.**

## I. Assignment 1 – Section 7.3

An elastic rod of length $l = 20m$ and a defined curvature of $R_n = 2cm$ is modeled with 20 nodes and intrinsic elasticity. To simulate one end being clamped, the first two nodes and their twist angle are fixed in the simulation. Given the mechanical properties of the beam and the appropriate forces (gravity and elastic), the shape structure of the beam can be simulated.

### 1. *System Dynamics*

The dynamics and shape of the system can be observed at various times between 0 and 5 seconds. The beam's deformation under gravity at its initial position can be seen below in *Figure I.1.1*.

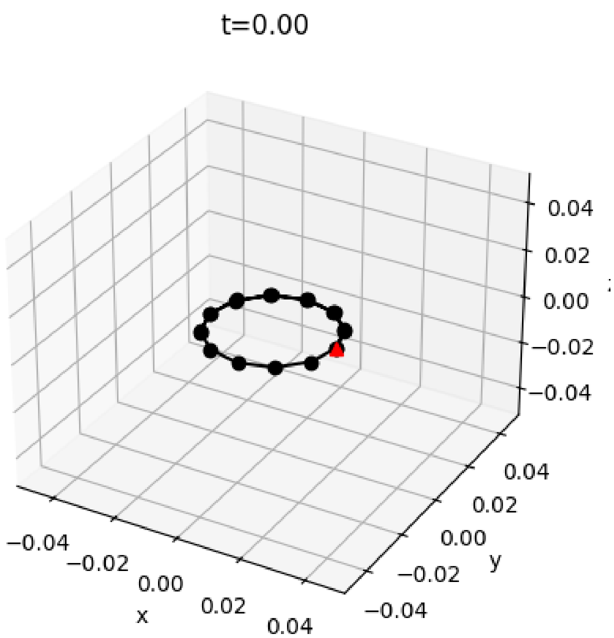


**Figure I.1.1**: Shape of structure at first time step

The final deformed position of the beam can be seen below in *Figure I.1.2*.

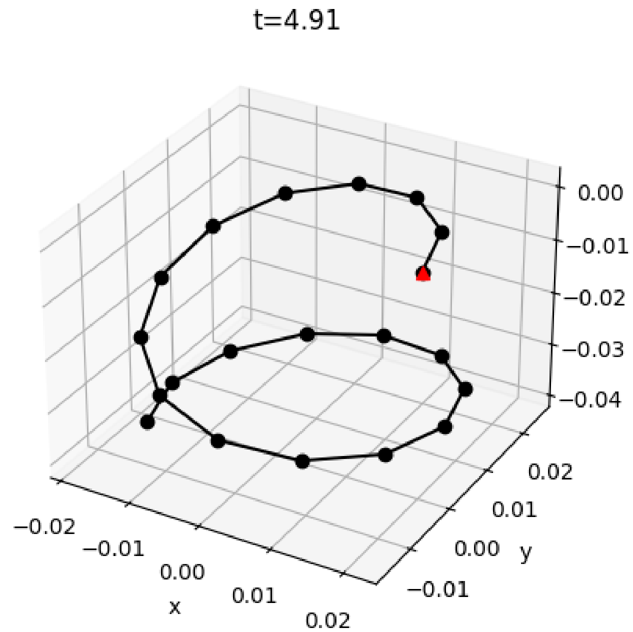


**Figure I.1.2**: Shape structure at the final time step

### 2. *Terminal Velocity*

The z-coordinate of the last node can also be plotted through time as the simulation runs. One can expect to see an oscillatory relationship as the node battles the elastic forces and gravitational forces until it reaches an equilibrium under steady state. This behavior can be seen below in *Figure I.2.1*.

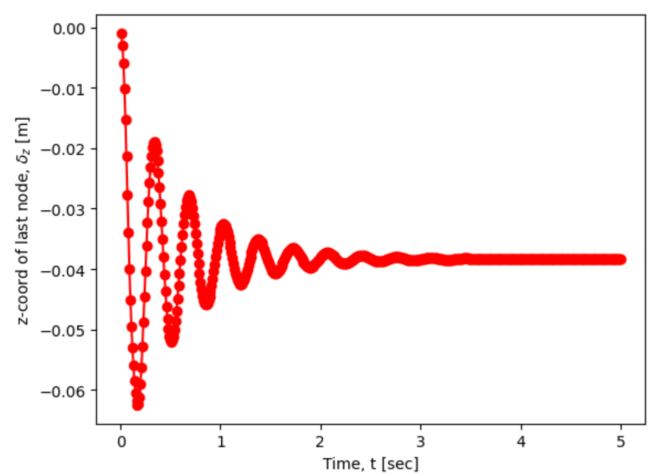


**Figure I.2.1**: z-coordinate of the last node over time

### 3. *Implementation of Algorithm*

The DER algorithm follows a series of steps. To begin, the rod must be discretized into nodes. This also includes the placement of material and reference frames in order to simulate twist and torsional energy.

To characterize the rod properly, the twisting, bending, and stretching energy terms are derived to penalize deviations from the rod's rest twist, stretched, and bent states. The twist is calculated based on the relative rotation of material frames between adjacent nodes.

Certain nodes are fixed due to boundary conditions. In this case, the first two nodes and their twist frame are fixed and excluded from the DOF vector that iterates within the numerical method.

The appropriate forces in effect are then computed. These include gravity and elastic forces (stretching, bending, twisting). The respective Jacobians are taken, and free components are extracted to solve for the changes in the DOF vector. The position and orientations are updated accordingly. Until the error is under a threshold, the numerical algorithm will continue to iterate by using the previous step's final values.

Once the method converges and the error is within the threshold, each node's position and orientation are returned and plotted for all time steps the method traverses through.

## References


[1] Professor M. Khalid Jawed, MAE 263F https://bruinlearn.ucla.edu/courses/193842/modules