Modeling Conservative Forces – HW1 Report

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***Abstract* — This report details various simulations and experiments conducted on MATLAB to highlight characteristics of conservative systems, leveraging the Newton-Raphson method. Implicit and Explicit methodologies are explored as well as the Euler-Bernoulli beam bending theorem.**

I. Assignment 1 – Section 4.2

Three rigid spheres of radii $R = 0.005, 0.025, 0.005m$, respectively, on an elastic beam are modeled to fall freely under gravity in a viscous fluid. The forces modeled are gravity, elastic forces from the beam, and viscous drag. The following MATLAB solver simulates this system under an Implicit and Explicit method. A timestep of $dt = 1E-2$ is used for the Implicit Newton-Raphson's method and a timestep of $dt = 1E-5$ is used for the Explicit method.

1. System Dynamics

The dynamics and shape of the system can be observed at various times between 0 and 10 seconds. This can be seen below in *Figure I.1.1*.

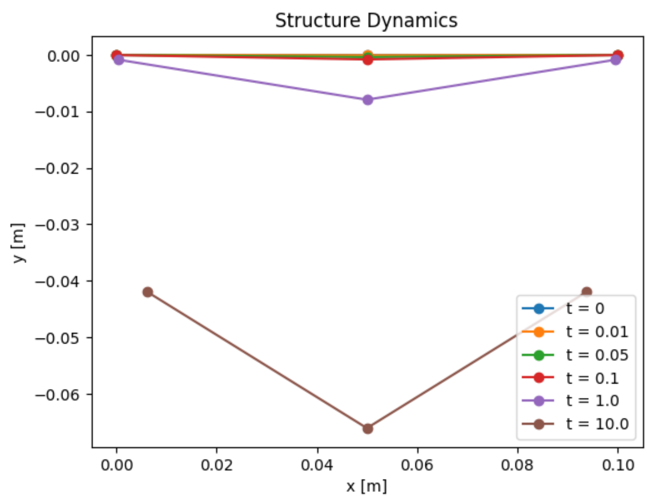

Figure I.1.1: Shape of structure at various times

The position and velocity of the middle node can also be seen changing over time as seen below in *Figure I.1.2* and *Figure I.1.3*.

The figures depict an accurate representation of real-world physics. The position graph sees a slight nonlinear relationship until the velocity eventually reaches a steady state.

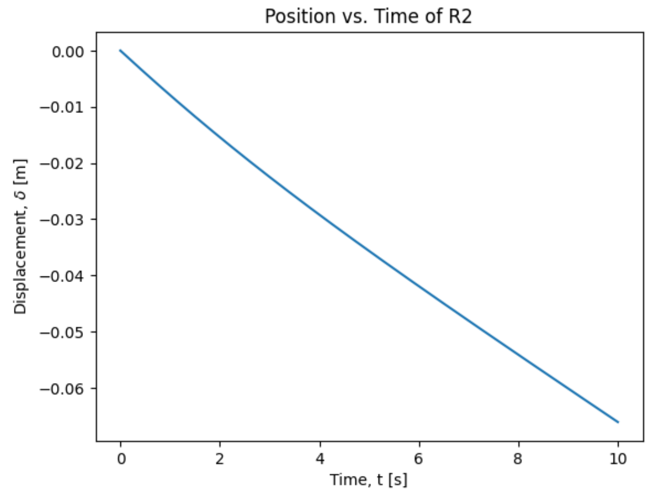

Figure I.1.2: Position of the middle node over time

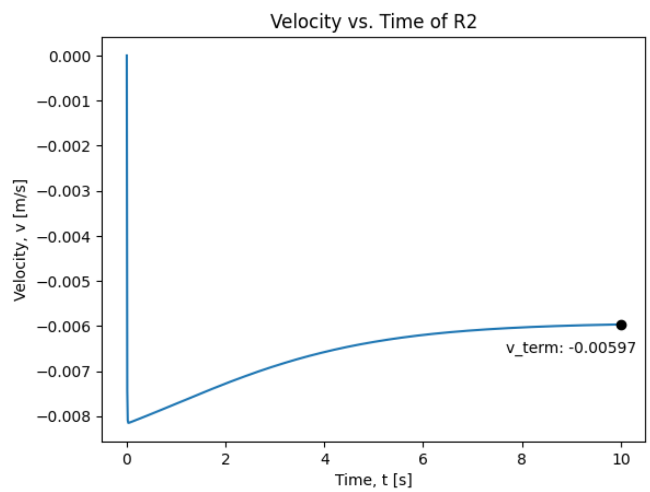

Figure I.1.3: Velocity of the middle node over time

2. Terminal Velocity

As seen above in *Figure I.1.3*, the velocity eventually reaches a steady state value where the viscous drag forces equilibrate with the force of gravity. This terminal velocity is around *-0.006 m/s*.

3. Turning Angle for Equal Radii

In *Figure I.1.1*, it is evident that the system adopts a V-shaped characteristic as the middle node is more massive than the first and last nodes. If all nodes are given the same radii, and thus they have the same mass, one can intuitively expect the V-shaped phenomenon to no longer hold true and for the structure to simply fall in the viscous fluid with no turning angle, always remaining parallel to the ground. This hypothesis holds as seen in *Figure I.3.1* below.

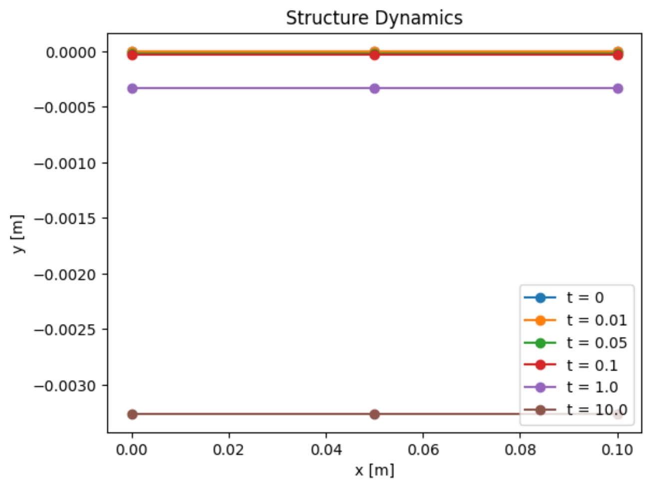

Figure I.3.1: Shape of structure at various times for nodes of equal radii

4. *Explicit Method Approach*

All previous figures are identical when produced through the Implicit or Explicit method. In *Figure I.4.1*, the final structure of the system can be seen at the last timestep under an Explicit Method.

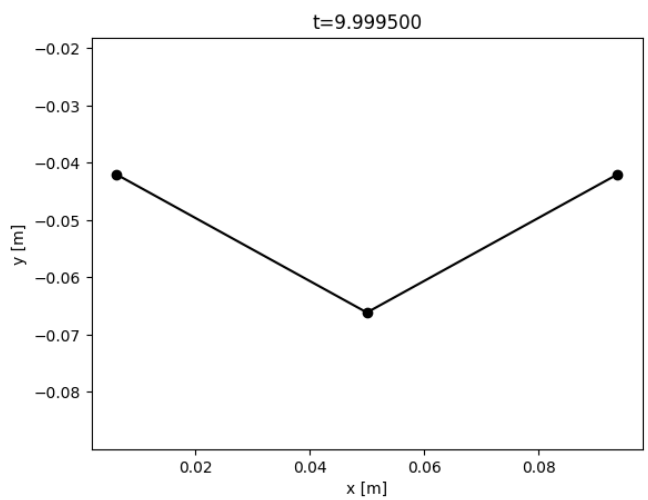

Figure I.4.1: Final shape of structure simulated with an Explicit Method

With an Implicit approach, a smaller timestep can be leveraged as Newton-Raphson's method can be used to approximate the position at a future time step. However, this approach is more mathematically complex to implement and simulate while offering negligibly different results from the Explicit approach.

The Explicit approach, however, requires a smaller timestep to account for the simplicity and ease of implementation it offers. A smaller timestep yields more accurate results than those of the Implicit approach but at the expense of a far slower computational time. While the Implicit approach took a few minutes to converge, the Explicit approach required around 6 hours to produce results.

The Implicit approach produced a shape structure plot almost identical to the one produced by the Explicit approach and thus has benefits that far outweigh its costs. Of course, this trade-off varies on a case-by-case basis.

II. Assignment 2 – Section 4.3

The previous simulation is tweaked to handle a more general case for N number of nodes. The middle sphere has a radius of $R = 0.025m$ and all other spheres have $R = 0.0005m$. For a simulation time of 50 seconds, 21 nodes are simulated using an Implicit method and a timestep of $1E-2$.

1. *Position and Velocity*

The position and velocity of the middle node can be seen changing over time as seen below in *Figure II.1.1* and *Figure II.1.2*. The terminal velocity of this node is found to be around *-0.00681m/s* as seen in *Figure II.1.2*.

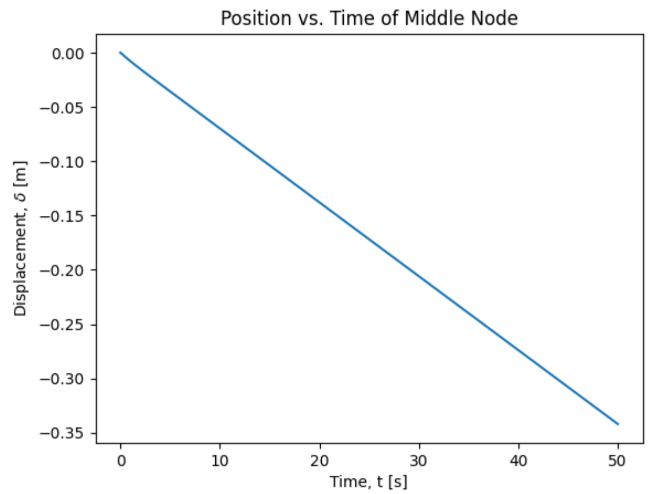

Figure II.1.1: Position of the middle node over time

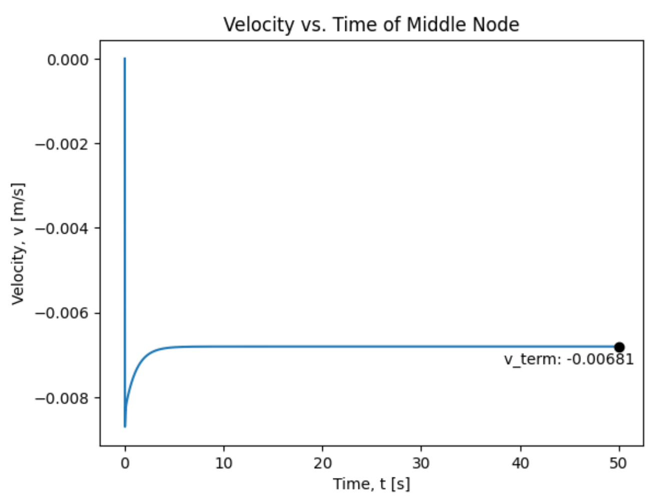

Figure II.1.2: Velocity of the middle node over time

2. *Final Shape Structure*

The final deformed shape of the beam meets expectations. The middle node has fallen farther than the others and a nonlinear relationship can be seen. This plot can be seen below in *Figure II.2.1*.

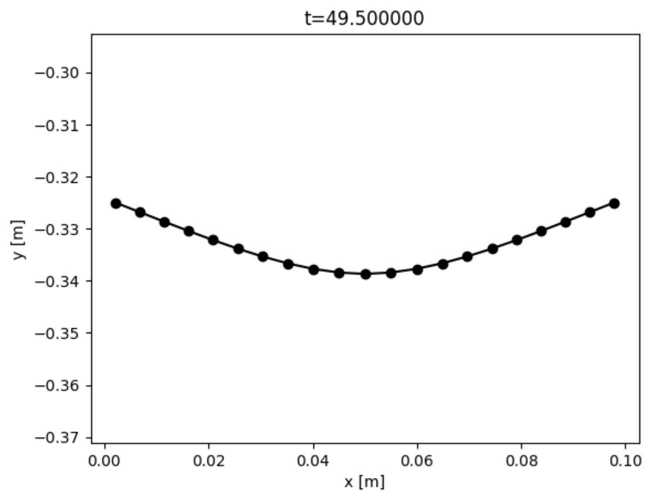

Figure II.2.1: Final deformed shape of the structure

3. *Spatial and Temporal Discretization*

For accurate simulation results, the model's parameters must be feasible and representative. The two most important characteristics, spatial and temporal discretization, can be changed through the node count and timestep, respectively. The influence that node count has on the terminal velocity of the system can be seen below in *Figure II.3.1*.

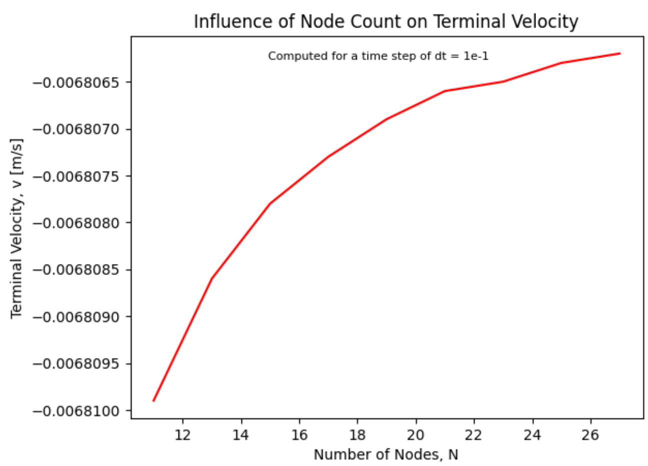

Figure II.3.1: As node count increases, terminal velocity begins to converge

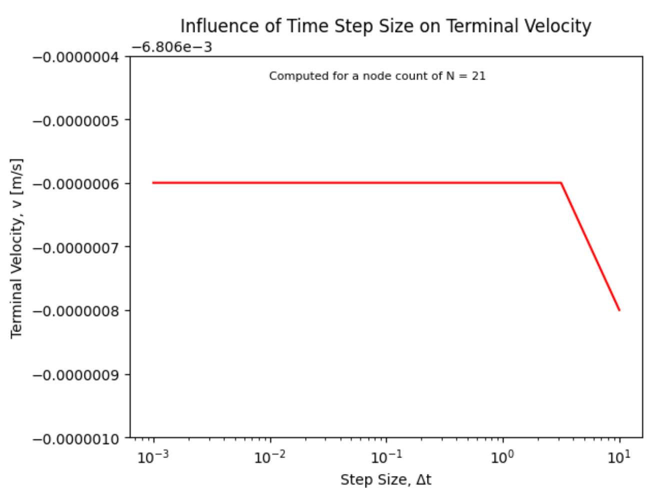

Figure II.3.1: Terminal velocity against timestep plotted logarithmically

While *Figure II.3.1* depicts a convergence on the terminal velocity as node count increases, *Figure II.3.2* depicts a divergence on the terminal velocity as timestep increases. Increasing the timestep reduces fidelity and thus accuracy. This is evident by the terminal velocity being almost uninfluenced by the timestep for steps less than 1 and only being affected for steps above 1.

Despite there being a slight divergence, the effect is incredibly small and only on the order of magnitude of 10^{-7}.

III. Assignment 3 – Section 4.4

The system modeled can be tweaked to resemble a simply supported beam with a circular-tube cross-section being represented as a mass-spring system with a mass at each node. A force of $P = 2000N$ is applied at the node corresponding to $d = 0.75m$ along the beam. After applying initial conditions and constraints, an Implicit solver can be used to simulate the beam as a function of time for $0 \leq t \leq 1$ and having 50 nodes.

1. *Maximum Vertical Deflection*

The simulation's results can be plotted to depict the structure of the system and showcase beam deflection due to the applied load. This can be seen below in *Figure III.1.1*.

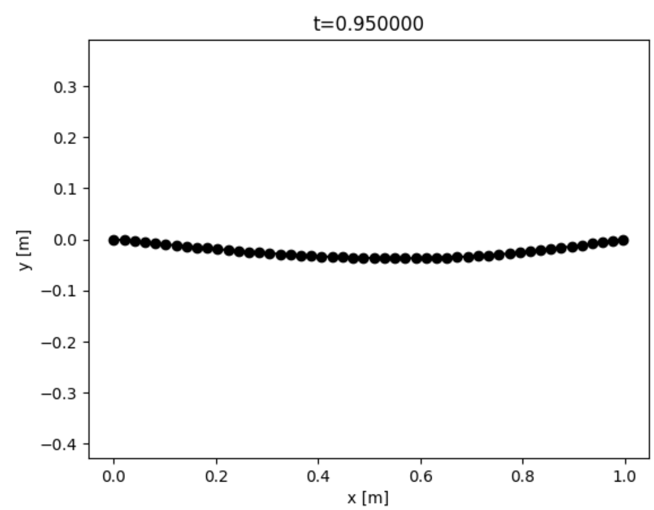

Figure III.1.1: Beam deflection simulation for a load of $P = 2000N$

The maximum vertical displacement of the beam can then be plotted as a function of time. The simulation's results can be compared with the Euler-Bernoulli Beam Theory approximation as well, as seen below in *Figure III.1.2*.

The plot highlights that the elastic forces in the beam induce a slight oscillation/overshoot in the beam when the force is applied. The system eventually converges to a steady state deflection value of $y_{ss_{sim}} = -0.03713m$.

It can also be seen that for a small load, the calculated Euler-Bernoulli approximation is fairly accurate and matches the results of the simulation to some decent extent.

$$y_{max_{sim}} = -0.03881 \approx y_{ma_{calc}} = -0.03804m$$

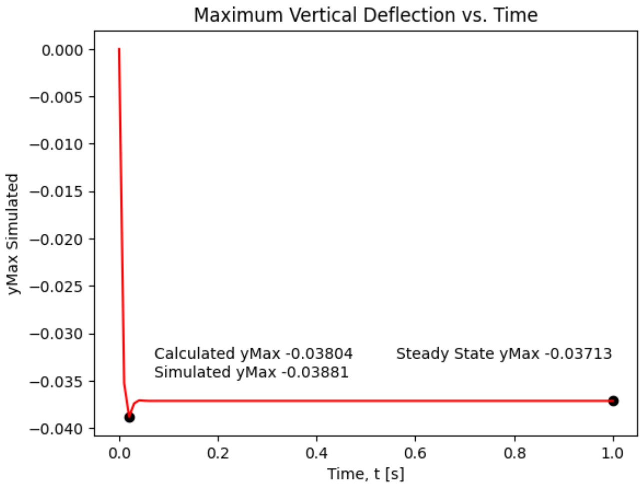

Figure III.1.2: Maximum beam deflection over time for $P = 2000N$

2. Euler-Bernoulli Accuracy

To test the accuracy of Euler-Bernoulli's equation, the system was simulated again but with a larger applied force of $P = 20000N$. The results of this simulation can be seen below in *Figure III.2.1*.

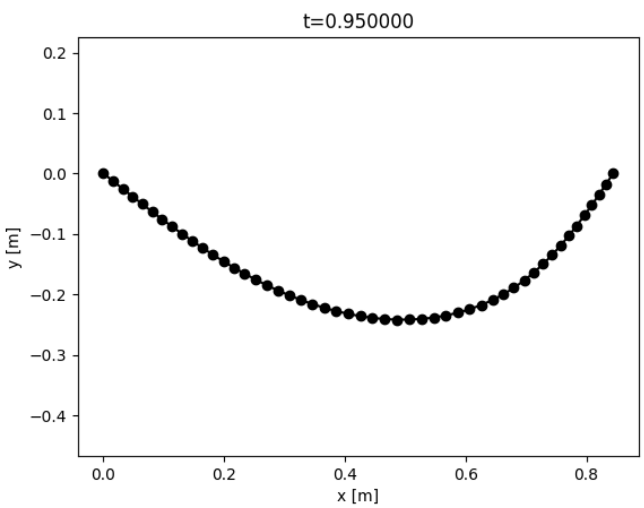

Figure III.2.1: Beam deflection simulation for a load of $P = 20000N$

Euler-Bernoulli's approximation holds true small loads but larger loads showcase a divergence from the simulation's results. For the larger load, the steady state value was found as $y_{SS_{sim}} = -0.24195m$.

It can also be seen that for the larger load, the calculated Euler-Bernoulli approximation is inaccurate. This phenomenon is evident in *Figure III.2.2*.

$$y_{max_{sim}} = -0.24790 \neq y_{max_{calc}} = -0.38045m$$

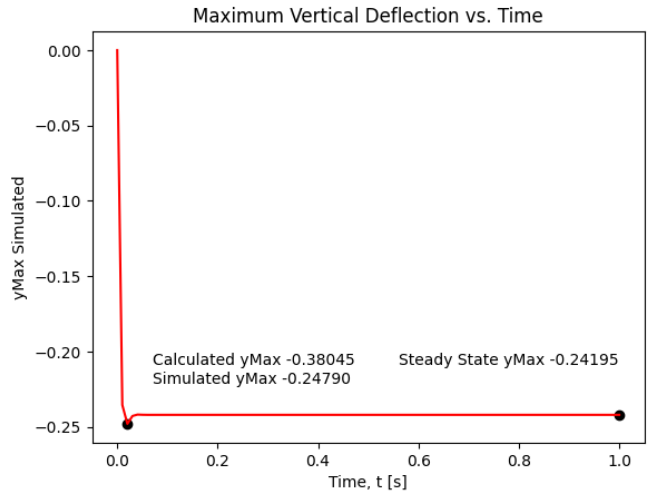

Figure III.2.2: Maximum beam deflection over time for $P = 20000N$

REFERENCES

[1] Professor M. Khalid Jawed, MAE 263F https://bruinlearn.ucla.edu/courses/193842/modules