TD 2 : ARO

Exo

Exo 1 :

$$\begin{cases} \max & 5x_2 + 4x_1 \\ & x_2 \leq 3 \\ & x_1 \leq 4 \\ & x_2 + 2x_1 \leq 9 \\ & x_1, x_2 \geq 0 \end{cases}$$

(D_1)

$$\begin{aligned} x_3 &= 3 - x_2 \\ x_4 &= 4 - x_1 \\ x_5 &= 9 - 2x_1 - x_2 \\ z &= 4x_1 + 5x_2 \end{aligned}$$

$3 - x_2 \geq 0 \Rightarrow x_2 \leq 3$

$4 + 0x_2 \geq 0 \Rightarrow 0 \leq 4$

$9 - x_2 \geq 0 \Rightarrow x_2 \leq 9$

① - méthode du plus large coeff.

Variable entrant : x_2.

Variable sortant : x_3.

(D_2)

$$\begin{aligned} x_2 &= 3 - x_3 \\ x_4 &= 4 - x_1 \\ x_5 &= 9 - 2x_1 - (3 - x_3) = 6 - 2x_1 + x_3 \\ z &= 4x_1 + 5(3 - x_3) = 15 + 4x_1 - 5x_3 \end{aligned}$$

(D_2)

$$\begin{aligned} x_2 &= 3 - x_3 \\ x_4 &= 4 - x_1 \\ x_5 &= 6 - 2x_1 + x_3 \\ z &= 15 + 4x_2 - 5x_3 \end{aligned}$$

Solution

$x_1 = 0 = x_3$

$x_2 = 3$

$x_4 = 4$

$x_5 = 6$

$z = 15$

- La Variable entrant : x_1
- La Variable sortant : x_5.

$3 \geq 0$ | $3 \geq 0$
$4 - x_1 \geq 0$ | $x_1 \leq 4$
$6 - 2x_1 \geq 0$ | $x_1 \leq 3$

$$x_1 = 3 + \frac{x_3}{2} - \frac{x_5}{2}.$$
$$x_2 = 3 - x_3.$$

$$x_4 = 4 - \left(3 + \frac{1}{2}x_3 - \frac{x_5}{2}\right) = 1 - \frac{x_3}{2} + \frac{x_5}{2}.$$
$$z = 15 + 4\left(3 + \frac{x_3}{2} - \frac{x_5}{2}\right) - 5x_3 =$$
$$= 27 - 3x_3 - 2x_5$$

(D_3)

$$x_1 = 3 + \frac{1}{2}x_3 - \frac{1}{2}x_5$$
$$x_2 = 3 - x_3.$$
$$x_4 = 1 - \frac{1}{2}x_3 + \frac{1}{2}x_5.$$
$$z = 27 - 3x_3 - 2x_5.$$

Solution de Base : $x_3 = 0 = x_5$
$x_1 = 3 \quad x_2 = 3 \quad x_4 = 1.$

Optimale : $z = 27.$

② / méthode de la plus grande croissant

$x_2 \rightarrow$ Croissance de 15.
$x_1 \rightarrow$?

maintenant pour x_1.

(D_1)

$x_3 = 3 - x_2.$	$3 = 0.x_1 \geq 0.$	
$x_4 = 4 - x_1.$	$4 = x_1 \geq 0$	$x_1 \leq 4.$
$x_5 = 9 - 2x_1 - x_2.$	$9 - 2x_1 \geq 0$	$x_1 \leq 4,5.$
$z = 4x_1 + 5x_2.$	$\boxed{x_4 \text{ Variable sortante.}}$	

(D_2)

$$x_1 = 4 - x_4$$
$$x_3 = 3 - x_2$$
$$x_5 = 1 - x_2 + 2x_4$$

$$z = 16 + 5x_2 - 4x_4$$

$z = 16$

Variable entrant : x_2

Variable sortant : x_5

$\boxed{x_2 = 1 + 2x_5 - x_5}$

(D_3)

$$x_2 = 1 + 2x_4 - x_5 \quad \Big\{ 1 + 0x_4 \geq 0$$
$$x_1 = 4 - x_4 \quad \Big\{ 4 - x_4 \geq 0$$
$$x_3 = 2 - 2x_4 + x_5 \quad \Big\{ 2 - 2x_4 \geq x_5 + 1$$

$$z = 21 + 6x_4 - 5x_5$$

Var entrante : x_4

Var sortant : x_3

$\boxed{x_4 = 1 - \frac{1}{2}x_3 + \frac{1}{2}x_5}$

(D_4)

$$x_1 = 3 + \frac{1}{2}x_3 - \frac{1}{2}x_5$$
$$x_2 = 3 - x_3$$
$$x_4 = 1 - \frac{1}{2}x_3 + \frac{1}{2}x_5$$

$$z = 27 - 3x_3 - 2x_5$$

$x_1 = 3$, $x_2 = 3$

$x_4 = 1$ $x_3 = 0 = x_5$

$\boxed{z = 27 \quad \text{optimal}}$

$\boxed{3}$ - Regle de Bland revient a faire la methode 2 (car a partir $(D1)$ on a une seul variable entrante.

Exo 2 ex 1 :

$$\max \; x_2 + 3x_1$$

$$\begin{cases} x_2 + 2x_1 \leq 4 \\ -x_2 - x_1 \leq -3 \\ -x_2 + x_1 \leq -1 \\ x_1, x_2 \geq 0 \end{cases}$$

(PLA)

$$\begin{cases} \max \; w = -x_0 \\ x_2 + 2x_1 - x_0 \leq 4 \; (x_3) \\ -x_2 - x_1 - x_0 \leq -3 \; (x_4) \\ -x_2 + x_1 - x_0 \leq -1 \; (x_5) \\ x_0, x_1, x_2 \geq 0 \end{cases}$$

(DA1) :

$x_3 = 4 - x_2 - 2x_1 + x_0$

$x_4 = -3 + x_2 + x_1 + x_0$

$x_5 = -1 + x_2 - x_1 + x_0$

$w = \qquad\qquad -x_0$

Var entrant : x_0.

Var sortant : x_4 (le plus negative)

$$\boxed{x_0 = 3 - x_2 - x_1 + x_4}$$

(DA2)

$$\begin{aligned} x_0 &= 3 - x_2 - x_1 + x_4 \\ x_3 &= 7 - 2x_2 - 3x_1 + x_4 \\ x_5 &= 2 \qquad - 2x_1 + x_4 \\ \hline w &= -3 + x_2 + x_1 - x_4 \end{aligned} \quad \left| \begin{aligned} & 3 - x_1 \geq 0 \\ & 7 - 3x_1 \geq 0 \\ & 2 - 2x_1 \geq 0 \end{aligned} \right.$$

Var entrant : x_1.

Var sortant : x_5.

$$\boxed{x_1 = 1 + \frac{1}{2}x_4 - \frac{1}{2}x_5}$$

(DA3)

$$\begin{aligned} x_0 &= 2 - x_2 + \frac{1}{2}x_4 + \frac{1}{2}x_5 \\ x_1 &= 1 \qquad + \frac{1}{2}x_4 - \frac{1}{2}x_5 \\ x_3 &= 4 - 2x_2 - \frac{1}{2}x_4 + \frac{3}{2}x_5 \\ \hline w &= 2 + x_2 - \frac{1}{2}x_4 - \frac{1}{2}x_5 \end{aligned} \quad \left\{ \begin{aligned} & 2 - x_2 \geq 0 \\ & 1 + 0x_2 \geq 0 \\ & 4 - 2x_2 \geq 0 \end{aligned} \right.$$

Var entrant x_2.

Var sortant x_0.

(D A 4)

$$x_1 = 1 \quad + \frac{1}{2}x_4 - \frac{1}{2}x_5$$

$$x_2 = 2 - x_0 + \frac{1}{2}x_4 + \frac{1}{2}x_5$$

$$x_3 = \quad 2x_0 - \frac{3}{2}x_4 + \frac{1}{2}x_5$$

$$\boxed{w = -x_0}$$

OPT = 0

list (liste : ↑type de la list)

si list est vide alors

courant ↑type élément = list

TD 2 : ARO / Suite.

(DA4)

$$x_1 = 1 \qquad\qquad + \frac{1}{2}x_4 - \frac{1}{2}x_5$$
$$x_2 = 2 - x_0 + \frac{1}{2}x_4 + \frac{1}{2}x_5$$
$$x_3 = 2x_0 - \frac{3}{2}x_4 + \frac{1}{2}x_5$$

$$w = -x_0$$

$B = \{x_1, x_2, x_3\}$, $N = \{x_0, x_4, x_5\}$

$w_0 = 0 \cdot x_0 = \quad x_1 = x_0 \qquad x_1 = 1, x_2 = 2, x_3 = 0.$

premier dictionnaire :

(D1)
$$x_1 = 1 + \frac{1}{2}x_4 - \frac{1}{2}x_5$$
$$x_2 = 2 + \frac{1}{2}x_4 + \frac{1}{2}x_5$$
$$x_3 = \quad - \frac{1}{2}x_4 + \frac{1}{2}x_5$$
$$z = 5 + 2x_4 - x_5$$

$1 + \frac{1}{2}x_4 \cdot x_4 \geq 0$

$2 + \frac{1}{2}x_4 \geq 0 \quad x_4 \geq 0$

$-\frac{3}{2}x_4 \geq 0 \Rightarrow x_3 \leq 0 \Rightarrow$

$x_3 -$

$$z = x_0 + 3x_1 + \left(2 + \frac{1}{2}x_4 + \frac{1}{2}x_5\right) + 3\left(1 + \frac{1}{2}x_4 - \frac{1}{2}x_5\right) =$$

$$\boxed{= 5 + 2x_4 - x_5}$$

Deuxième dictionnaire :

$$x_4 = \frac{1}{3}x_5 - \frac{2}{3}x_3$$

$$x_1 = 1 - \frac{1}{3}x_5 - \frac{1}{3}x_3$$

$$x_2 = 2 + \frac{3}{2}x_5 - \frac{1}{3}x_3$$

$x_1 = 1 + \frac{1}{2}\left(\frac{1}{3}x_5 - \frac{2}{3}x_3\right) - \frac{1}{2}x_5$

$= 1 + \left(\frac{1}{6} - \frac{2}{6}\right)x_5 - \frac{1}{3}$

"Stop"

$$\boxed{\begin{array}{l} z_0 = 5. \\ x_1 = 1 . \quad x_2 = 2. \\ x_3 = 0 . \quad x_4 = 0. \\ x_5 = 0. \end{array}}$$

La valeur des variable.

max

$x_2 + 3x_1$

$x_2 + 2x_1 \leq 4.$ $\quad 4 \leq 4$

$-x_2 - x_1 \leq -3.$ $\quad -3 \leq -3$

$-x_2 - x_1 \leq -1$ $\quad -2 + 1 \leq -1.$

$x_1, x_2 \geq 0$