# TD 2 : ARO

## Exo 1 :

$$\begin{cases} \max \; 5x_2 + 4x_1 \\ x_2 \leq 3 \\ x_1 \leq 4 \\ x_2 + 2x_1 \leq 9 \\ x_1, x_2 \geq 0 \end{cases}$$

$(D_1)$

$$\begin{aligned} x_3 &= 3 - x_2 \\ x_4 &= 4 - x_1 \\ x_5 &= 9 - 2x_1 - x_2 \\ z &= 4x_1 + 5x_2 \end{aligned}$$

$3 - x_2 \geq 0 \Rightarrow x_2 \leq 3$

$4 + 0x_2 \geq 0 \Rightarrow 0 \leq 4$

$9 - x_2 \geq 0 \Rightarrow x_2 \leq 9$

② - méthode du plus large coeff.

Variable entrant : $x_2$.

Variable sortant : $x_3$.

$(D_2)$ $\quad x_2 = 3 - x_3$

$x_4 = 4 - x_1$

$x_5 = 9 - 2x_1 - (3 - x_3) = 6 - 2x_1 + x_3$

$z = 4x_1 + 5(3 - x_3) = 15 + 4x_1 - 5x_3$

$(D_2)$

$$\begin{aligned} x_2 &= 3 - x_3 \\ x_4 &= 4 - x_1 \\ x_5 &= 6 - 2x_1 + x_3 \\ z &= 15 + 4x_2 - 5x_3 \end{aligned}$$

Solution

$x_1 = 0 = x_3$

$x_2 = 3$

$x_4 = 4$

$x_5 = 6$.

$z = 15$.

- La Variable entrant : $x_1$
- La Variable sortant : $x_5$.

$3 \geq 0$ | $3 \geq 0$
$4 - x_1 \geq 0$ | $x_1 \leq 4$
$6 - 2x_1 \geq 0$ | $x_1 \leq 3$

$x_1 = 3 + \frac{x_3}{2} - \frac{x_5}{2}$.

$x_2 = 3 - x_3$.

$x_4 = 4 - \left(3 + \frac{1}{2}x_3 - \frac{x_5}{2}\right) = 1 - \frac{x_3}{2} + \frac{x_5}{2}$.

$z = 15 + 4\left(3 + \frac{x_3}{2} - \frac{x_5}{2}\right) - 5x_3 =$
$= 27 - 3x_3 - 2x_5$

$(D_3)$

$x_1 = 3 + \frac{1}{2}x_3 - \frac{1}{2}x_5$
$x_2 = 3 - x_3$.
$x_4 = 1 - \frac{1}{2}x_3 + \frac{1}{2}x_5$.

$z = 27 - 3x_3 - 2x_5$.

Solution de Base : $x_3 = 0 = x_5$

$x_1 = 3 \quad x_2 = 3 \quad x_4 = 1$.

Optimale : $z = 27$.

② / méthode de la plus grande croissant

$x_2 \rightarrow$ Croissance de 15.
$x_1 \rightarrow$ ?

maintenant pour $x_1$.

$(D_1)$ $x_3 = 3 - x_2$. $\quad 3 = 0.x_1 \geq 0$.

$x_4 = 4 - x_1$. $\quad 4 = x_1 \geq 0 \quad x_1 \leq 4$.

$x_5 = 9 - 2x_1 - x_2$. $\quad 9 - 2x_1 \geq 0 \quad x_1 \leq 4,5$.

$z = 4x_1 + 5x_2$. $\quad$ $x_4$ Variable sortante.

$(D_2)$ $\begin{cases} x_1 = 4 - x_4 \\ x_3 = 3 - x_2 \\ x_5 = 1 - x_2 + 2x_4 \end{cases}$

$z = 16 + 5x_2 - 4x_4$

$z = 16$

Variable entrant : $x_2$.

Variable sortant : $x_5$.

$x_2 = 1 + 2x_5 - x_5$

$(D_3)$ $\begin{cases} x_2 = 1 + 2x_4 - x_5 \\ x_1 = 4 - x_4 \\ x_3 = 2 - 2x_4 + x_5 \end{cases}$ $\begin{cases} 1 + 0x_4 \geq 0 \\ 4 - x_4 \geq 0 \\ 2 - 2x_4 \geq x_5 - 1 \end{cases}$

$z = 21 + 6x_4 - 5x_5$

Var entrante : $x_4$.

Var sortant : $x_3$.

$x_4 = 1 - \frac{1}{2}x_3 + \frac{1}{2}x_5$

$(D_4)$ $\begin{cases} x_1 = 3 + \frac{1}{2}x_3 - \frac{1}{2}x_5 \\ x_2 = 3 - x_3 \\ x_4 = 1 - \frac{1}{2}x_3 + \frac{1}{2}x_5 \end{cases}$

$z = 27 - 3x_3 - 2x_5$

$x_1 = 3$, $x_2 = 3$.

$x_4 = 1$ $x_3 = 0 = x_5$.

$z = 27$ optimal.

3 - Règle de Bland revient à faire la méthode 2 (car à partir $(D_1)$ on a une seul variable entrante.

Exo 2 Ex 1 :

$\max x_2 + 3x_1$

$\begin{cases} x_2 + 2x_1 \leq 4 \\ -x_2 - x_1 \leq -3 \\ -x_2 + x_1 \leq -1 \\ x_1, x_2 \geq 0 \end{cases}$

(PLA)

$\max w = -x_0$

$\begin{cases} x_2 + 2x_1 - x_0 \leq 4 \ (x_3) \\ -x_2 - x_1 - x_0 \leq -3 \ (x_4) \\ -x_2 + x_1 - x_0 \leq -1 \ (x_5) \\ x_0, x_1, x_2 \geq 0 \end{cases}$

(DA1) :

$x_3 = 4 - x_2 - 2x_1 + x_0$.

$x_4 = -3 + x_2 + x_1 + x_0$.

$x_5 = -1 + x_2 - x_1 + x_0$.

$w = \quad - x_0$.

Var entrant : $x_0$.

Var sortant : $x_4$ (le plus negative)

$\boxed{x_0 = 3 - x_2 - x_1 + x_4}$.

(DA2)

$x_0 = 3 - x_2 - x_1 + x_4$. | $3 - x_1 \geq 0$.

$x_3 = 7 - 2x_2 - 3x_1 + x_4$. | $7 - 3x_1 \geq 0$

$x_5 = 2 \quad - 2x_1 + x_4$. | $2 - 2x_1 \geq 0$.

$w = -3 + x_2 + x_1 - x_4$

Var entrant : $x_1$.

Var sortant : $x_5$.

$\boxed{x_1 = 1 + \frac{1}{2} x_4 - \frac{1}{2} x_5}$

(DA3)

$x_0 = 2 - x_2 + \frac{1}{2} x_4 + \frac{1}{2} x_5$ | $2 - x_2 \geq 0$.

$x_1 = 1 \quad + \frac{1}{2} x_4 - \frac{1}{2} x_5$ | $1 + 0x_2 \geq 0$.

$x_3 = 4 - 2x_2 - \frac{1}{2} x_4 + \frac{3}{2} x_5$ | $4 - 2x_2 \geq 0$.

$w = 2 + x_2 - \frac{1}{2} x_4 - \frac{1}{2} x_5$

Var entrant $x_2$.

Var sortant $x_0$.

(DA4)

$$x_1 = 1 + \frac{1}{2}x_4 - \frac{1}{2}x_5$$
$$x_2 = 2 - x_0 + \frac{1}{2}x_4 + \frac{1}{2}x_5$$
$$x_3 = 2x_0 - \frac{3}{2}x_4 + \frac{1}{2}x_5$$

$$\boxed{w = -x_0}$$

OPT = 0

list (liste : type d'la lst)

si list est vide alors
courant type element = list