21AIE201 - Introduction to Robotics

Assignment 1 PART 1

Group 14 Batch B

Kalyana Sundaram	CB.EN.U4AIE21120
Kaushik Jonnada	CB.EN.U4AIE21122
Praneetha K	CB.EN.U4AIE21147
Sarvesh Shashikumar	CB.EN.U4AIE21163
Subikksha	CB.EN.U4AIE21167

1. Find out the global position of a body point at ${}^{B}P = [-5\ 5\ 5]^T$ after a rotation of 30° about the X – axis (global axis) followed by 45° about the Z– axis (global axis).

Given ${}^{B}P = [-5\ 5\ 5]^T$

$$= \begin{bmatrix} -5 \\ 5 \\ 5 \end{bmatrix}$$

It is given that it undergoes a rotation of 30° about x-axis so the required matrix for that is

$$R(X,30°) = \begin{bmatrix} \text{"1"} & \text{"0"} & \text{"0"} \\ \text{"0"} & \text{"cos 30"} & \text{"-sin 30"} \\ \text{"0"} & \text{"sin 30"} & \text{"cos 30"} \end{bmatrix} = \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 0.8660 & -0.5000 \\ 0 & 0.5000 & 0.8660 \end{bmatrix}$$

It's followed by rotation of 45° about the z-axis

$$R(Z, 45) = \begin{bmatrix} \text{"cos 45"} & \text{"-sin 45"} & \text{"0"} \\ \text{"sin 45"} & \text{"cos 45"} & \text{"0"} \\ \text{"0"} & \text{"0"} & \text{"1"} \end{bmatrix} = \begin{bmatrix} 0.7071 & -0.7071 & 0 \\ 0.7071 & 0.7071 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix}$$

The matrices are pre multiplied because the rotations are being done with respect to the global axis(Fixed Angle Rotation)

$$R = \begin{bmatrix} 0.7071 & -0.7071 & 0 \\ 0.7071 & 0.7071 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix} \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 0.8660 & -0.5000 \\ 0 & 0.5000 & 0.8660 \end{bmatrix}$$

$$Ap = R.Bp$$

$$= \begin{bmatrix} 0.7071 & -0.7071 & 0 \\ 0.7071 & 0.7071 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix} \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 0.8660 & -0.5000 \\ 0 & 0.5000 & 0.8660 \end{bmatrix} \begin{bmatrix} -5 \\ 5 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} -4.8200 \\ -26.0000 \\ 6.8300 \end{bmatrix}$$

```python
# IMPORTING LIBRARIES
from spatialmath import *
from spatialmath.base import *
from spatialmath.base import symbolic as sym
import numpy as np
from matplotlib import pyplot as plt
```

QUESTION_1

```python
def prettyprint(A):
    # Pretty print a matrix
    print(np.array2string(A, precision=2, separator=', ', suppress_small=True))
```

```python
R_x = rotx(30,unit='deg')
print("Rx\n",R_x)
R_z = rotz(45, unit='deg')
print("Rz\n",R_z)
R = R_z@R_x
print("R\n",R)
print()
Bp =np.array([[-5],[5],[5]])
print("Bp\n",Bp)
Ap = R@Bp
print()
print("Ap\n",Ap)
```

```
Rx
 [[ 1.          0.          0.        ]
 [ 0.          0.8660254 -0.5       ]
 [ 0.          0.5        0.8660254]]
Rz
 [[ 0.70710678 -0.70710678  0.        ]
 [ 0.70710678  0.70710678  0.        ]
 [ 0.          0.          1.        ]]
R
 [[ 0.70710678 -0.61237244  0.35355339]
 [ 0.70710678  0.61237244 -0.35355339]
 [ 0.          0.5         0.8660254 ]]


Bp
 [[-5]
 [ 5]
 [ 5]]


Ap
 [[-4.82962913]
 [-2.24143868]
 [ 6.83012702]]
```

2. Find out the local position of a global point at ${}^{A}P = [3\ 5\ 3]^T$ after a rotation of 30° about the Y-axis (global axis) followed by 30° about the X-axis (global axis).

Given, Ap = $[3\ 5\ 3]^T$

So, Ap = $\begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix}$

It is given that it undergoes a rotation about 30° of Y-axis. So, the matrix is

$$R(Y,30) = \begin{bmatrix} \text{"cos30"} & \text{"0"} & \text{"sin30"} \\ \text{"0"} & \text{"1"} & \text{"0"} \\ \text{"-sin30"} & \text{"0"} & \text{"cos30"} \end{bmatrix} = \begin{bmatrix} 0.8660 & 0 & 0.5000 \\ 0 & 1.0000 & 0 \\ -0.5000 & 0 & 0.8660 \end{bmatrix}$$

And it is followed by 30° about X-axis

$$R(X,30) = \begin{bmatrix} \text{"1"} & \text{"0"} & \text{"0"} \\ \text{"0"} & \text{"cos 30"} & \text{"-sin 30"} \\ \text{"0"} & \text{"sin30"} & \text{"cos30"} \end{bmatrix} = \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 0.8660 & -0.5000 \\ 0 & 0.5000 & -0.8660 \end{bmatrix}$$

It's rotated with respect to the global axis so the matrices are multiplied as it is.

Fixed angular rotation(FAR) = $\begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 0.8660 & -0.5000 \\ 0 & 0.5000 & -0.8660 \end{bmatrix} \begin{bmatrix} 0.8660 & 0 & 0.5000 \\ 0 & 1.0000 & 0 \\ -0.5000 & 0 & 0.8660 \end{bmatrix}$

$Bp = Ap.R^T$

$$= \begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix} \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 0.8660 & -0.5000 \\ 0 & 0.5000 & -0.8660 \end{bmatrix} \begin{bmatrix} 0.8660 & 0 & 0.5000 \\ 0 & 1.0000 & 0 \\ -0.5000 & 0 & 0.8660 \end{bmatrix}$$

$$= \begin{bmatrix} 2.5400 \\ 5.8300 \\ 1.5800 \end{bmatrix}$$

```python
R_y = SO3.Ry(30,unit='deg')
R_x = SO3.Rx(30, unit='deg')

R=R_x @ R_y #FAR
print(R)
Rt = np.transpose(R)
print(Rt)
```

```
  0.866      0          0.5
  0.25       0.866     -0.433
 -0.433      0.5        0.75

[[ 0.8660254  0.25       -0.4330127]
 [ 0.         0.8660254  0.5       ]
 [ 0.5       -0.4330127  0.75      ]]
```

```python
Ap = np.array([[3],[5],[3]])
Bp = Rt @ Ap
print(Bp)
```

```
[[2.54903811]
 [5.83012702]
 [1.58493649]]
```

3. Find out the global position of a body point at ${}^{B}P = [-10\ 10\ 10]^T$ after a rotation of 45° about the X – axis (global axis) followed by 60° about the Z– axis (local axis).

$$Bp = [-10\ 10\ 10]^T = \begin{bmatrix} -10 \\ 10 \\ 10 \end{bmatrix}$$

It's rotated about the x-axis according to the question, so the required matrix is

$$R(X, 45) = \begin{bmatrix} \text{"1"} & \text{"0"} & \text{"0"} \\ \text{"0"} & \text{"cos 45"} & \text{"-sin 45"} \\ \text{"0"} & \text{"sin 45"} & \text{"cos 45"} \end{bmatrix} = \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 0.7071 & -0.7071 \\ 0 & 0.7071 & 0.7071 \end{bmatrix}$$

And it is followed by 60° about the z-axis on the local coordinate. So, the required matrix is R(Z, 60)

$$= \begin{bmatrix} \text{"cos 60"} & \text{"-sin 60"} & \text{"0"} \\ \text{"sin 60"} & \text{"cos 60"} & \text{"0"} \\ \text{"0"} & \text{"0"} & \text{"1"} \end{bmatrix} = \begin{bmatrix} 0.5000 & -0.8660 & 0 \\ 0.8660 & 0.5000 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix}$$

The matrices need not be pre multiplied here because we need to find the global position about the local axis, which gives Rotated Angular Rotation.

$$R = \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 0.7071 & -0.7071 \\ 0 & 0.7071 & 0.7071 \end{bmatrix} \begin{bmatrix} 0.5000 & -0.8660 & 0 \\ 0.8660 & 0.5000 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix}$$

$$Ap = R.\,Bp = \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 0.7071 & -0.7071 \\ 0 & 0.7071 & 0.7071 \end{bmatrix} \begin{bmatrix} 0.5000 & -0.8660 & 0 \\ 0.8660 & 0.5000 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix} \begin{bmatrix} -10 \\ 10 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} -13.6600 \\ -9.6500 \\ 4.4800 \end{bmatrix}$$

```python
R_x = rotx(45,unit='deg')
R_z = rotz(60, unit='deg')
R = R_x@R_z
print(R)

Bp =np.array([[-10],[10],[10]])
Ap = R@Bp
Ap
```

```
[[ 0.5        -0.8660254   0.        ]
 [ 0.61237244  0.35355339 -0.70710678]
 [ 0.61237244  0.35355339  0.70710678]]

array([[-13.66025404],
       [ -9.65925826],
       [  4.48287736]])
```

4. Find out the local position of a global point at ${}^{A}P = [4\ 4\ 3]^T$ after a rotation of 60° about the Z-axis (global axis) followed by 30° about the X-axis (local axis).

$$Ap = [4\ 4\ 3]^T = \begin{bmatrix} 4 \\ 4 \\ 3 \end{bmatrix}$$

It undergoes a rotation of 60° about Z-axis so the required matrix is

$$R(Z, 60) = \begin{bmatrix} \text{"cos 60"} & \text{"-sin 60"} & \text{"0"} \\ \text{"sin 60"} & \text{"cos 60"} & \text{"0"} \\ \text{"0"} & \text{"0"} & \text{"1"} \end{bmatrix} = \begin{bmatrix} 0.5000 & -0.8660 & 0 \\ 0.8660 & 0.5000 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix}$$

It is followed by rotation of 30° by X-axis on the local axis

$$R(X, 30) = \begin{bmatrix} \text{"1"} & \text{"0"} & \text{"0"} \\ \text{"0"} & \text{"cos 30"} & \text{"-sin 30"} \\ \text{"0"} & \text{"sin 30"} & \text{"cos 30"} \end{bmatrix} = \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 0.8660 & -0.5000 \\ 0 & 0.5000 & 0.8660 \end{bmatrix}$$

Since it's the local axes, the matrices need not be pre multiplied.

$$R = \begin{bmatrix} 0.5000 & -0.8660 & 0 \\ 0.8660 & 0.5000 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix} \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 0.8660 & -0.5000 \\ 0 & 0.5000 & 0.8660 \end{bmatrix}$$

$$Bp = R^{-1}Ap$$

$$= \begin{bmatrix} 5.4640 \\ 0.2320 \\ 3.3300 \end{bmatrix}$$

```python
R_z = SO3.Rz(60,unit='deg')
R_x = SO3.Rx(30, unit='deg')
R = R_z @ R_x
#print(R)
Rt = np.linalg.inv(R)
#print(Rt)
print()
Ap = np.array([[4],[4],[3]])
Bp =Rt@Ap
print(Bp)
```

```
[[5.46410162]
 [0.23205081]
 [3.33012702]]
```

5. A body frame B is rotated about 45° about global X-axis, then 60° about local Y -axis and finally by 30° about the local Z -axis. Then the origin of the body frame B is translated to ${}^{A}P_{B,org}$ = [-3 3 6]. Determine the corresponding homogeneous transformation matrix and inverse homogeneous transformation matrix.

- As given in the Question the rotations are Global axis and local axis so it is a Rotated Angle Rotation(RAR).

- Given that

T = $[-3\ 3\ 6]^T$

- Formula:

${}^{A}P = R \times {}^{B}P$

R = $R_X \times R_Y \times R_Z$

- R = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 45 & -\sin 45 \\ 0 & \sin 45 & \cos 45 \end{bmatrix} \begin{bmatrix} \cos 60 & 0 & \sin 6\,0 \\ 0 & 1 & 0 \\ -\sin 60 & 0 & \cos 60 \end{bmatrix} \begin{bmatrix} \cos 30 & -\sin 30 & 0 \\ \sin 30 & \cos 30 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- R = $\begin{bmatrix} \frac{\sqrt{3}}{4} & -\frac{1}{4} & \frac{\sqrt{3}}{2} \\ \frac{5\sqrt{2}}{8} & \frac{\sqrt{2}\sqrt{3}}{8} & -\frac{\sqrt{2}}{4} \\ -\frac{\sqrt{2}}{8} & \frac{3\sqrt{2}\sqrt{3}}{8} & \frac{\sqrt{2}}{4} \end{bmatrix}$

```python
R_x = rotx(45,unit='deg')
R_y = roty(60,unit='deg')
R_z = rotz(30, unit='deg')
q = np.array([[-3],[3],[6]])
```

```python
# homogeneous transformation matrix..
R = R_x @ R_y @ R_z
#print(R)
t = np.c_[R, q]
T = np.r_[t, [np.array([0,0,0,1])]] # 2-D array
print(T)
```

```
[[ 0.4330127  -0.25        0.8660254  -3.        ]
 [ 0.88388348  0.30618622 -0.35355339  3.        ]
 [-0.1767767   0.91855865  0.35355339  6.        ]
 [ 0.          0.          0.          1.        ]]
```

```python
# inverse homogeneous transformation matrix..
RT = np.transpose(R)
a = -RT @ q
print(a)
T = np.c_[RT, -RT @ q]
T = np.r_[T, [np.array([0,0,0,1])]] # T inverse
print(T)
```

```
[[-0.29195215]
 [-7.17991057]
 [ 1.53741604]]
[[ 0.4330127   0.88388348 -0.1767767  -0.29195215]
 [-0.25        0.30618622  0.91855865 -7.17991057]
 [ 0.8660254  -0.35355339  0.35355339  1.53741604]
 [ 0.          0.          0.          1.        ]]
```

6. Frame B is rotated about 30 degrees in Y axis, 45 degrees about Z axis and 60 degrees again about Y axis. Assume all rotations are about global coordinate axis. The origin of frame B is then translated by a vector $q = [1\ 2\ -3]^T$. If P with reference to frame B is given as $[1\ 2\ 1]^T$, find out P with respect to global coordinate frame using homogeneous transformation matrix.

Let, global co-ordinate frame =A

Local co-ordinate frame(B) is rotated 30° in y-axis = $R_y = \theta = 30°$

Then it is again rotated 45 about z-axis , $R_z = \theta = 45°$

rotated 60 about y-axis, $R_y = \theta = 60°$

This question follows FAR(Fixed Angle Rotation),pre multiplication is required

$R = r(60°, y).R(45°, z).R(30°, Y)$

$$R = \begin{bmatrix} \frac{1}{2} & 0 & \sqrt{\frac{3}{2}} \\ 0 & 1 & 0 \\ -\sqrt{\frac{3}{2}} & 0 & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \sqrt{\frac{3}{2}} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \sqrt{\frac{3}{2}} \end{bmatrix}$$

θ=60° θ=45° θ= 30°

$$R = \begin{bmatrix} \sqrt{3} - \sqrt{6}/4 & -1/2\sqrt{2} & 3\sqrt{2} + 1/4\sqrt{2} \\ \sqrt{3}/2\sqrt{2} & 1/\sqrt{2} & 1/2\sqrt{2} \\ -3 - \sqrt{2}/4\sqrt{2} & \sqrt{3}/2\sqrt{2} & -\sqrt{3} + \sqrt{6}/4\sqrt{2} \end{bmatrix}$$

GIVEN ,

$${}^{A}p_{B},\ {}_{org} = [1\ \ 2\ \ -3]^{T} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} qx \\ qy \\ qz \end{bmatrix}$$

$$P^{B} = [1\ 2\ 1]T = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} xb \\ yb \\ zb \end{bmatrix}$$

Homogeneous Transformation Matrix

$$P^A = \begin{bmatrix} r11 & r12 & r13 & qx \\ r21 & r22 & r23 & qy \\ r31 & r32 & r33 & qz \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} xb \\ yb \\ zb \\ 1 \end{bmatrix}$$

$$P^A = \begin{bmatrix} \sqrt{3} - \sqrt{6}/4\sqrt{2} & -1/2\sqrt{2} & 3\sqrt{2} + 1/4\sqrt{2} & 1 \\ \sqrt{3}/2\sqrt{2} & 1/\sqrt{2} & 1/2\sqrt{2} & 2 \\ -3 - \sqrt{2}/4\sqrt{2} & \sqrt{3}/2\sqrt{2} & -\sqrt{3} + \sqrt{6}/4\sqrt{2} & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$P^A = \begin{bmatrix} \sqrt{3} - \sqrt{6} - 3 + 7\sqrt{2}/4\sqrt{2} \\ 4\sqrt{2} + \sqrt{3} + 5/2\sqrt{2} \\ 3\sqrt{3} - 13\sqrt{2} + \sqrt{6} - 3/4\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} 1{\cdot}0928 \\ 4{\cdot}380 \\ -2{\cdot}428 \end{bmatrix}$$

So, we have found P with respect to global axis co-ordinate which is A.

QUESTION_6

```
R_Y = SO3.Ry(60,unit='deg')
R_y = SO3.Ry(30,unit='deg')
R_z = SO3.Rz(45, unit='deg')
q = np.array([[1],[2],[-3]])
R = R_Y @ R_z @ R_y
#print(R)
# homogeneous transformation matrix..
T = np.c_[R, q]
T = np.r_[T, [np.array([0,0,0,1])]]
print(T)

Bp =np.array([[1],[2],[1],[1]])
Ap = T@Bp
print(Ap[:3])
```

```
[[-0.12682648 -0.35355339  0.9267767   1.        ]
 [ 0.61237244  0.70710678  0.35355339  2.        ]
 [-0.78033009  0.61237244  0.12682648 -3.        ]
 [ 0.          0.          0.          1.        ]]
[[ 1.09284343]
 [ 4.38013939]
 [-2.42875873]]
```

7. Frame B is rotated by 90 degrees about X axis of A. Then it is again rotated by 45 degrees about Z axis of A. Translation vector q is given by $[2,3,1]^T$. If P with respect to frame A is $[2,2,3]^T$, find P with respect to local coordinate frame using homogeneous transformation matrix.

ANSWER :

Given ,

local co-ordinate frame =B

global co-ordinate frame =A

rotation of B along x-axis(global axis)=Px=90°

rotation of B along z-axis(global axis)=Pz=45°

This question follows FAR(fixed anlge rotation) , so pre multiplication is required.

$R = R(45^\circ, Z) \cdot R(90^\circ, x)$

$$R = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

(ɵ=45°) (ɵ=90°)

$$R = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix}$$

Given ,

$${}^{A}p_{B,org} = [2\ \ 3\ \ 1]^{T} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} qx \\ qy \\ qz \end{bmatrix}$$

$$P^{A} = [2\ 2\ 3]T = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$$P^A = \begin{bmatrix} r11 & r12 & r13 & qx \\ r21 & r22 & r23 & qy \\ r31 & r32 & r33 & qz \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} xb \\ yb \\ zb \\ 1 \end{bmatrix} \longrightarrow P^B$$

$$P^B = T^{-1} * P^A$$

$T^{-1} * P^A$, this is a inverse homogenous matrix ,inorder to get inverse homogenous we need to divide above r and q matrix into two .

$$PB_= \begin{bmatrix} r11 & r21 & r31 & 0 \\ r12 & r22 & r32 & 0 \\ r13 & r23 & r33 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -qx \\ 0 & 1 & 0 & -qy \\ 0 & 0 & 1 & -qz \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$$PB_= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$P^{B} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & -5/\sqrt{2} \\ 0 & 0 & 1 & -1 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$${}_{P}B = \begin{bmatrix} -1/\sqrt{2} \\ 2 \\ 1/\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} -0.707 \\ 2 \\ 0.707 \\ 1 \end{bmatrix}$$

So, we have found P with respect to local axis co-ordinate which is B.

QUESTION_7

```python
R_x = SO3.Rx(90,unit='deg')
R_z = SO3.Rz(45, unit='deg')
q = np.array([[2],[3],[1]])
R = R_z @ R_x
print(R)

t = np.c_[R, q]
T = np.r_[t, [np.array([0,0,0,1])]]
print(T)
Ap = np.array([[2],[2],[3],[1]])
Bp = np.linalg.inv(T)@Ap
print(Bp[:3])
```

```
   0.7071    0         0.7071
   0.7071    0        -0.7071
   0         1         0

[[ 7.07106781e-01 -4.32978028e-17  7.07106781e-01  2.00000000e+00]
 [ 7.07106781e-01  4.32978028e-17 -7.07106781e-01  3.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  6.12323400e-17  1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00]]
[[-0.70710678]
 [ 2.        ]
 [ 0.70710678]]
```

8. Frame B is rotated by 60 degrees about Y axis of A. Then it is again rotated by 30 degrees about Z axis of B. Translation vector q is given by $[-3,3,-1]^T$. If P with respect to frame A is [2,2,1]T, find P with respect to local coordinate frame using homogeneous transformation matrix.

ANSWER :

Given ,

local co-ordinate frame = B

global co-ordinate frame = A

B is rotated by 60° about y-axis(global axis)=Ry = 60°

It is rotated by 30° about z-axis(local axis)=Rz=30°

This question follows RAR(Rotated Angle Rotation) , for this pre -multiplication is not required

$R = R(60^\circ, Y) . R(30^\circ, Z)$

$$R = \begin{bmatrix} \sqrt{3}/4 & -1/4 & \sqrt{3}/2 \\ 1/2 & \sqrt{3}/2 & 0 \\ -3/4 & \sqrt{3}/4 & 1/2 \end{bmatrix}$$

$$P^B = \begin{bmatrix} \sqrt{3}/4 & 1/2 & -3/4 & 0 \\ -1/4 & \sqrt{3}/2 & \sqrt{3}/4 & 0 \\ \sqrt{3}/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$P^B = \begin{bmatrix} \sqrt{3}/4 & 1/2 & -3/4 & 3\sqrt{3} - 9/4 \\ -1/4 & \sqrt{3}/2 & \sqrt{3}/4 & -(3 + 5\sqrt{3})/4 \\ \sqrt{3}/2 & 0 & 1/2 & 3\sqrt{3} + 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$P^B = \begin{bmatrix} 5\sqrt{3} - 8/4 \\ -5/4 \\ 5\sqrt{3} + 2/2 \end{bmatrix} = \begin{bmatrix} 0.165 \\ -1.25 \\ 5.33 \end{bmatrix}$$

so , we have P with respect to local co-ordinate frame B

```python
R_y = roty(60,unit='deg')
R_z = rotz(30, unit='deg')
q = np.array([[-3],[3],[-1]])
R = R_y @ R_z
print(R)
T = np.c_[R, q]
T = np.r_[T, [np.array([0,0,0,1])]]
print(T)
```

```
[[ 0.4330127 -0.25       0.8660254]
 [ 0.5        0.8660254  0.       ]
 [-0.75       0.4330127  0.5      ]]
[[ 0.4330127 -0.25       0.8660254 -3.        ]
 [ 0.5        0.8660254  0.         3.        ]
 [-0.75       0.4330127  0.5       -1.        ]
 [ 0.         0.         0.         1.        ]]
```

```python
Ap = np.array([[2],[2],[1],[1]])
Bp = np.linalg.inv(T)@Ap
print(Bp[:3])
```

```
[[ 0.16506351]
 [-1.25      ]
 [ 5.33012702]]
```

9. Find out the eigenvalues of the rotation matrix in 2 dimensions using characteristic equation assuming that the angle of rotation is theta. Substitute a value for theta as 45 degrees and find out the eigenvalues for the rotation matrix. Execute the solution in spatialmath package and compare the results. What do you infer from the eigenvalues of 2D rotation matrix?

Rotation matrix in 2 dimensions:

$$\begin{matrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{matrix}$$

θ=45 degrees

$$\begin{matrix} cos45 & -sin45 \\ sin45 & cos45 \end{matrix} \qquad \begin{matrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{matrix}$$

Characteristic equation is (R(θ) - $\lambda I)x = 0$

where I is the identity matrix.

$$R(\theta) - \lambda I = \begin{matrix} \frac{1}{\sqrt{2}} - \lambda & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{matrix}$$

Det(R(θ) - λI)=0 $\lambda^2 - \sqrt{2}\,\lambda + 1 = 0$

We get,

$$\lambda = \frac{\sqrt{2} + \sqrt{(\sqrt{2})^2 - 4(1)(1)}}{2(1)}, \frac{\sqrt{2} + \sqrt{(\sqrt{2})^2 - 4(1)(1)}}{2(1)} = 1+i/\sqrt{2}, 1-i/\sqrt{2}$$

$= 0.707+0.707i$, $0.707-0.707i$

- Therefore the eigen values of the given rotation matrix are 0.707+0.707i , 0.707-0.707i

QUESTION_9

```
sym.simplify(np.linalg.eigvals(SO2(45,unit='deg')))
```

```
array([0.70710678+0.70710678j, 0.70710678-0.70710678j])
```

10. Find out the eigenvalues of the rotation matrix about the X axis in 3 dimensions using characteristic equation assuming that the angle of rotation is theta. Substitute a value for theta as 30 degrees and find out the eigenvalues for the rotation matrix. Execute the solution in spatialmath package and compare the results. What do you infer from the non-imaginary eigenvalue of the rotation matrix?

Rotation Matrix in 3 dimensions

$$\begin{matrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{matrix}$$

θ=30 degrees

$$\begin{matrix} 1 & 0 & 0 \\ 0 & cos30^0 & -sin30^0 \\ 0 & sin30^0 & cos30^0 \end{matrix}$$

$$\begin{matrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\sqrt{3}}{2} & \mathbf{-1/2} \\ \mathbf{0} & \mathbf{-1/2} & \frac{\sqrt{3}}{2} \end{matrix}$$

Characteristic equation is (R(θ) - $\lambda I)x = 0$
where I is the identity matrix.

$$R(\theta) - \lambda I = \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2}-\lambda & -1/2 \\ 0 & 1/2 & \frac{\sqrt{3}}{2}-\lambda \end{bmatrix}$$

Det(R(θ) - λI)=0

$\Rightarrow (1-\lambda)[(3/4+\lambda^2-2(\frac{\sqrt{3}}{2})(\lambda))]=0$

$\Rightarrow \lambda_1=1$

$\Rightarrow (3/4+\lambda^2-2(\frac{\sqrt{3}}{2})(\lambda)) = 0 \Rightarrow \lambda^2 - \sqrt{3}\lambda + 1 = 0$

$$R(\theta) - \lambda I = \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2}-\lambda & -1/2 \\ 0 & 1/2 & \frac{\sqrt{3}}{2}-\lambda \end{bmatrix}$$

We get,

$$\lambda = \frac{\sqrt{3}+\sqrt{(\sqrt{3})^2-4(1)(1)}}{2(1)}, \frac{\sqrt{3}+\sqrt{(\sqrt{3})^2-4(1)(1)}}{2(1)}$$

$\Rightarrow \lambda 2 = \sqrt{3}$ +i /2,

$\Rightarrow \lambda 3 = \sqrt{3}$ - i /2

- Therefore the eigen values of the given rotation matrix are 1, $\sqrt{3}$ +i /2, $\sqrt{3}$ - i /2

QUESTION_10

```
sym.simplify(np.linalg.eigvals(rotx(30,unit='deg')))
```

```
array([0.8660254+0.5j, 0.8660254-0.5j, 1.        +0.j ])
```

Thank you!!!!