Lower tail dependence as a risk metric

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

Financial markets sometimes experience large losses as a results of unforeseen events that have massive implications on the economy and the financial sector. These events are often characterized as "Black Swans": highly improbable extreme events, which are by nature unpredictable outliers. The question that naturally arises from these events in the context of financial assets are "How can risk be mitigated in case of these unforeseen events?".

Risk management is an essential part of portfolio construction and selection. There are various risk metrics used by portfolio managers to limit risk in accordance with a specified risk tolerance range. Common risk measures include alpha, beta, R-squared, standard deviation, and the Sharpe ratio; which compose the so called five principal risk measures. These give indications of the risk-return paradigm that portfolio managers use to optimize their return levels given a certain acceptable level of risk. Theses measures are common practice to use in normal market circumstances but fail to capture some aspects of risk management under extreme events.

A less frequently used risk metric is that of the lower tail dependence, χ. χ is a risk metric that could be used to describe the comovements between two variables in the far lower tails of their joint distribution. The derivation of χ comes from the branch of mathematics called Extreme Value Theory (EVT). An advantage of χ is that it can fairly accurately capture the dependence of for example, a stock and the market, at large market downturns that other risk metrics do not necessarily pick up on or consider. χ can be estimated without needing to specify the underlying joint distribution of the variables in consideration which simplifies assumptions.

1.2 AIM OF THIS PAPER

The aim of this paper is to develop a framework to measure the lower tail dependence of a stock or portfolio with the market and validate the use of χ as a valid risk metric. The paper follows similar methods developed by DiTraglia and Gerlach (2013) in their paper "Portfolio selection: An extreme value approach". Theses methods will then be applied to financial data from the South African stock market. χ as a risk metric will be evaluated by looking at the empirical results calculated and presented here. The analysis that will be done will be carried out at the stock and portfolio level.

A noteworthy event in financial markets in recent years has been that of the impact of Covid-19. During the this period the market in general experienced severe losses, and can be considered a Black Swan event. According to theory and intuition it is expected that stocks and portfolios with relatively low lower tail dependence will outperform stocks or portfolios with higher lower tail dependence in distressed market periods. This assertion will be empirically tested and χ will be overall assessed as a relevant risk metric and possible other applications will be looked at.

CHAPTER 2

UNDERLYING THEORY

2.1 INTRODUCTION

The lower tail dependence, χ, between pairs of random variables measures the correlation in the lower tail of the joint distribution of the variables. Random variables that display little to no dependence from normal correlations plots could frequently exhibit tail dependence in extreme deviations. For example, stocks returns often exhibit lower tail dependence with relevant markets even if the stock has a low market β.

It has been shown multiple times that stock returns display a log normal distribution, which is a fat tailed distribution, see for example Antoniou *et al.* (2004) and Rosenkrantz (2003). This means that extreme events (events far from the standard deviation) in stock returns have a larger probability than that of the normal distribution. Therefore if a normal distribution is used to model stock returns it can severely underestimate the risks associated. Many scholars and authors have brought to light the shortcomings of the normal distribution. Most notably Taleb (2005) has proposed in his book, "The black swan: Why don't we learn that we don't learn", that fat-tailed distributions such as the stable distributions determine asset returns often displayed in finance. Therefore if the distribution specified is incorrect, risks could be underestimated and lead to large losses. However, through results derived in EVT, we will find that the underlying distribution does not need necessarily need to be specified which simplifies matters a bit.

2.2 OVERVIEW

Let X be defined as the absolute stock return losses [1] (the negative of the normal returns), and let Y be defined as the associated absolute market index loss. The lower tail dependence between a stock and the market is the defined as:

$$\chi = \lim_{p \uparrow 1} P\left(X > F_X^{-1}(p) \mid Y > F_Y^{-1}(p)\right)$$

[1] This can easily be extended to portfolio losses.

where F_X^{-1} and F_Y^{-1} are the quantile functions of X and Y. A more intuitive way to look at χ is as the limiting condition Value at Risk (VAR). In other words χ is a risk metric that that conditions on large market losses, and captures the probability the stocks suffers losses beyond its p'th quantile, $F_X^{-1}(p)$ (DiTraglia and Gerlach, 2013).

If $\chi = 1$, the stock and the market are asymptotically perfectly dependant, and if $\chi = 0$, the stock and the market are asymptotically independent. Lower tail dependence has a few notable properties:

1. χ is not a moment. Therefore it exists regardless of the underlying distribution, even if the distribution has very fat tails. [2]
2. It is an asymmetric measure of dependence. Specifically it measures only dependence in large negatives losses.

DiTraglia and Gerlach (2013) states that another benefit of EVT, and sepcifically χ, is that it may prevent investors from overreacting to unexpected market movements. Overreaction to unforeseen events is a form of recency bias. For example, following the financial crisis investors tended to be overly sensitive to sudden price changes. EVT can aid as a tool for investors to avoid the associated economic costs in relation to overreaction of extreme events by providing a systematic technique for evaluating the lower tail of the joint distribution of an stock's returns with those of the market.

Estimating tail dependence can be approached via various methods. The broader class of methods distinguishing the approaches are parametric and non-parametric modeling. A parametric model could specify the marginal distributions and the dependence structure between the pair of variables. Maximum likelihood could then for example, be used to estimate the lower tail dependence. A drawback of this methods is that the underlying distributions of the variables are not known and if they are incorrectly specified, severe bias could be introduced into the model.

At the other end of the spectrum there is fully non-parametric models, which use the empirical cumulative distribution function to transform the variables X and Y into common marginal distribution. The drawback of this approach is that since the events that are in the far lower tail are

[2] Leptokurtic distributions often have an infinite variance.

the region of interest, few data points (if any) are available to estimate this region. Therefore even if the empirical cumulative distribution functions is a uniformly consistent estimator, it will always underestimate the thickness of the tails. This is a classic bias-variance trade-off. Fully parametric methods make much more efficient use of the information contained in the sample but are sensitive to the assumed distribution. Non-parametric methods, on the other hand, make few assumptions but yield wildly variable estimates as they try to study a region of the empirical distribution that contains practically no observations.

2.3 EXTREME VALUE THEORY APPLICATION

The main idea behind EVT is to model the probabilistic behaviour of unusually large or small observations. This paper derives the lower tail dependence through well known EVT theorems and results. Specifically the well known Fischer-Tippett theorem will be exploited and extended to use in a bivariate model between a given stock and the market that results in a convenient and flexible estimator for the lower tail dependence. The theorems and results are of an asymptotic nature and can be seen as approximately true for a large sample size given the right circumstances.

There are two main approach when an extreme value analysis is applied to a data set:

1. Block maxima: This approach involves dividing the data into different blocks and subsequently taking the maxima of each of the blocks as a new sequence of random variables that could be modelled through the results obatined by the Fischer-Tippett theorem.

2. Peaks over threshold (POT): This approach aims to model the conditional distribution of exceedances over a a specified threshold. The Pickands Balkema de Haan theorem gives that the limiting result is a generalized Pareto distribution (GPD).

These two approaches aim to answer the same question : "How does the far tail of the distribution of the data behave?". Cont (2001) has shown that volatility clustering is a common feature of stock returns and Coles *et al.* (2001) states that block maxima is a more robust approach for modeling clusters. For the purposes of this paper the block maxima methodology is therefore more appropriate. As such, only the necessary EVT of the block maxima as it pertains to the bivariate case will be discussed.

2.3.1 Univariate case

The starting point of the discussion will begin with that of the well known Fischer Tippet theorem, which a variation of is stated below:

Fischer Tippet Theorem: Let $X_1, X_2, \ldots, X_n$ be weakly dependent, identically distributed, random variables and define $M_{n,n} \equiv \max\{X_1, \ldots, X_n\}$, i.e the maximum of sample size n. If there exist sequences of constants $\{a_n > 0\}, \{b_n\}$ such that $(M_n - b_n)/a_n$ has a proper limit distribution, then that distribution is of the form following form:

$$G_\gamma(z) = \exp\left\{-\left[1 + \gamma\left(\frac{z-\mu}{\sigma}\right)\right]^{-1/\gamma}\right\}$$

where $1 + \gamma(z-\mu)/\sigma > 0$, $-\infty < \mu < \infty, \sigma > 0$ and $-\infty < \gamma < \infty$.

Note that this theorem is also sometimes called the Extremal Types Theorem. This equation gives the Generalized Extreme Value (GEV) distribution, a three parameter family with location parameter μ, scale parameter σ, and shape parameter γ, which describes the tail thickness of the underlying distribution. The parameter γ is called the extreme value index (EVI) and plays an important role in EVT. The limiting distrubution $G_\gamma(z)$ can be divided into the three standard extremal type distributions depending on the value of γ:

1. $\gamma > 0$. Positive values of γ indicate that limiting distribution belongs to the Fréchet-Pareto class, which indicates that the distribution has a very thick upper tail with polynomial decay.

2. $\gamma = 0$. When γ approaches zero, the GEV tends to the Gumbel Distribution and the upper tail exhibits exponential decay.

3. $\gamma < 0$. Negative values of γ indicate the reversed Weibull class and display a bounded upper tail.

The Fischer-Tippett theorem solves the extremal limit problem, which is to find all possible (non-degenerate) distributions that can appear as limiting distributions. The Fischer-Tippett theorem therefore allows to fully characterizes the limiting distribution of a univariate sample maxima. This can be thought of as the EVT analogue to the Central Limit theorem (CLT), which states that sample distribution asymptotically converges to a normal distribution. Therefore EVT allows a convenient way to use observed extremes to study distribution tails. Combining this theorem with certain assumptions about the joint distribution of the random variables, such as that they have the same marginal distribution, enables estimation of the lower tail dependence.

2.3.2 Bivariate case

For the purposes of this paper a brief overview of the bivariate anaylsis of extreme value distributions as it pertains to the data used will be given. A more in depth explanation of bivariate extreme value distributions can be found in Beirlant *et al.* (2004) and Coles *et al.* (2001). The limiting distributions of a bivariate model has an analogue results to that of the Fischer-Tippet theorem.

The derivation of the lower tail dependence makes the assumption that the asset's returns and the market index have common marginal distributions. Even though the Fischer-Tippett theorem is a limiting results, for a large enough sample the theorem approximately holds and therefore allows for a practical estimation procedure for the marginal distributions. Let $x_1, x_2, \ldots, x_N$ denote the daily returns for an individual stock and let $y_1, y_2, \ldots, y_N$ denote the daily returns for the chosen market index.

The method that will be followed in this paper is to split the sample observations of the stocks and market index into non-overlapping blocks of adequate size so that the maximum loss in each of the blocks is approximately GEV distributed. The maximum losses can then be transformed into common marginal distribution without the need to specify the underlying distribution of the individual stock returns and the market index.

The procedure involves dividing the sample returns into k equally spaced blocks of lengths r with $K = \lfloor N/r \rfloor$, i.e. the floor integer value. If there remains any additional observation, simply disregard them at the start of the series. Now let $\underline{x}_i$ denote the minimum return on stock x in block i and $\underline{y}_i$ denote he minimum return on the market index in the same block, where $i = 1, \ldots, K$. Although $\underline{x}_i$ and $\underline{y}_i$ are from the same block, they may or may not refer to the same day. The returns are compounded by a negative one to convert negative block minimum returns to positive block maximum losses.

If the series is stationary and meets a weak dependence criterion, maximum likelihood estimation could be utilized to generate parameter estimates from the GEV distributions. Although the normal standard errors are invalid, the estimation is based on a likelihood that implies independence will still yield consistent estimates. The log-likelihood is given as:

$$\begin{aligned}(\mu,\sigma,\gamma) = &- K\log\sigma - (1+1/\gamma)\sum_{n=i}^{K}\log\left[1+\gamma\left(\frac{z_i-\mu}{\sigma}\right)\right] \\ &- \sum_{n=i}^{K}\left[1+\gamma\left(\frac{z_i-\mu}{\sigma}\right)\right]^{-1/\gamma}\end{aligned}$$

defined wherever $1+\gamma\left(z_n-\mu\right)/\sigma > 0$ for all $i = 1,\ldots,K$.

The maximum likelihood estimates from the univariate models can be used to convert the block maximum losses for stock x and market index y to common marginals. The standard Fréchet distribution is a good choice, even if the distribution to which the transformation is applied has no effect on the estimated tail dependence. The standard Fréchet distribution is given by:

$$P(X \leq x) = \exp(-1/x)$$

for $x > 0$. This can be seen as a GEV distribution with parameters $\mu = \sigma = \gamma = 1$, and the transformation can then easily be derived. Suppose X is random variable and that $X \sim$ GEV. Then,

$$\tilde{X} \equiv f(X) = \left[1+\gamma\left(\frac{X-\mu}{\sigma}\right)\right]^{1/\gamma}$$

is standard Fréchet. This can be see by applying the inverse probability transformation:

$$\begin{aligned}P(\tilde{X}) &= P\left(X \leq f^{-1}(x)\right) = G\left(f^{-1}(x)\right) \\ &= \exp\left\{-\left[1+\frac{\gamma}{\sigma}\left(f^{-1}(x)-\mu\right)\right]^{-1/\gamma}\right\} \\ &= \exp\left\{-\left[1+\frac{\gamma}{\sigma}\left(\left[\left(\frac{\sigma}{\gamma}\right)(x^{\gamma}-1)+\mu\right]-\mu\right)\right]^{-1/\gamma}\right\} \\ &= \exp(\frac{-1}{x})\end{aligned}$$

Let $\{-\underline{\tilde{x}}_i\}_{i=1}^{M}$ and $\left\{-\underline{\tilde{y}}_j\right\}_{i=1}^{M}$ be the block maxima losses after the transformation to standard Fréchet has been applied. The tail dependence between stock x and market y can now be estimated using the transformed block maximum losses. The tail dependence parameter of the limiting distribution of transformed joint maxima $\left\{\left(-\underline{\tilde{x}}_i, -\underline{\tilde{y}}_i\right)\right\}_{i=1}^{M}$ fully characterizes the tail of the underlying distribution of returns and correspondes to the stock returns x and market index y, just as the shape parameter γ of the limiting distribution of univariate maxima fully characterizes the tail of the underlying distribution of returns. However, although univariate block maxima have a single limiting distribution, joint maxima do not. Instead, there is a class of limiting distributions defined by a rather complex integral equation.

A parametric assumption must therefore be made before proceeding. The simplest conceivable limit, the bivariate logistic distribution, is assumed as the joint distribution for convenience sake. Coles *et al.* (2001) also gives other families that could be specified but are rather complex. Two random variables X and Y with standard Frećhet marginals follow a bivariate logistic distribution if their joint distribution is as follows:

$$G(x,y) = \exp\left\{-\left(x^{-1/\alpha} + y^{-1/\alpha}\right)^{\alpha}\right\}$$

for $x, y > 0$ and $\alpha \in [0,1]$. This is a one parameter distribution because the margins are already defined, and the single parameter α determines the strength of dependency. The two margins are asymptotically independent when $\alpha = 1$ and perfectly asymptotically dependent when $\alpha = 0$ The relationship between χ and α is defined as follows:

$$\chi = 2 - 2^{\alpha}$$

From this equation it can be seen that when $\alpha = 0$, $\chi = 1$ and when $\alpha = 1, \chi = 0$. The bivariate logistic model is fitted to the transformed block maximum losses

$$\left\{\left(-\underline{\tilde{x}}_i, -\underline{\tilde{y}}_j\right)\right\}_{i=1}^{M}$$

as the final phase of the estimate procedure by maximum likelihood. Using the invariance characteristic of maximum likelihood estimators, the resultant estimate $\hat{\alpha}$ may be transformed to $\hat{\chi} = 2 - 2^{\dot{\alpha}}$. The probability density function of the logistic family is given by cross partial derivative of G, that is

$$g(x, y) = \frac{\partial}{\partial x \partial y} G(x, y)$$

Thus,

$$g(x, y) = e^{-V} \left(V_x V_y - V_{xy} \right)$$

where

$$\begin{aligned}
&V = \left(x^{-1/\alpha} + y^{-1/\alpha} \right)^{\alpha} \\
&V_x = - \left(x^{-1/\alpha} + y^{-1/\alpha} \right)^{\alpha - 1} x^{-(\alpha+1)/\alpha} \\
&V_y = - \left(x^{-1/\alpha} + y^{-1/\alpha} \right)^{\alpha - 1} y^{-(\alpha+1)/\alpha} \\
&V_{xy} = \tfrac{\alpha - 1}{\alpha} \left(x^{-1/\alpha} + y^{-1/\alpha} \right)^{\alpha - 2} (xy)^{-(\alpha+1)/\alpha}
\end{aligned}$$

Temporal dependence can be neglected under weak conditions, and the likelihood for the entire sample can be written as the product of the likelihoods of each observation:

$$(\alpha) = \sum_{i=1}^{M} \log g \left(x_i, y_i; \alpha \right)$$

where g is defined as previously, and x, y have been transformed to standard Fréchet.

CHAPTER 3

DATA SET

3.1 BACKGROUND

Before the required analysis is done, a thorough examination of the data set will be layed out here. This will compromise of all the features and drawbacks of the data set constructed. The data set was artificially constructed in order to fit the purposes required as stated previously. All the data obtained was from Yahoo! Finance through their API in the program R. The data set constructed for the analysis consisted of 65 large-cap stocks that traded on the Johannesburg Stock Exchange (JSE).

3.2 SAMPLE PERIODS

The data set was divided into three different sample periods. The data was split into an in-sample period and two out-of-sample periods. These sample periods were chosen to test the proposed methodologies on the most recent data available to reflect the most recent market conditions. These periods are given here:

1. The in-sample period is from "2008/06/11 until June "2019/12/30".
2. The first out-of-sample period is from "2020/01/01" until "2021/11/30". This periods was chosen to test in general the use of the lower tail dependence.
3. The second out of sample period is the three month period of "2020/02/01" until "2020/05/01". This is chosen as such to reflect the effects of the Covid-19, and will henceforth be referred to as the Covid sample.

Figure 3.1 displays the market returns for the different sample periods. A very notable characteristic of the in-sample period is that it is the longest market bull run in the history of the stock market. Following the financial crisis of 2007/2008, stock have rallied to unprecedented levels. This has a few implications. Since market movements generally tended upwards over an extended period of time, with very few major downturns, the lower tail dependence might not pick up on the correlation in the far lower tails. A possible solution is to include the period of the financial crisis

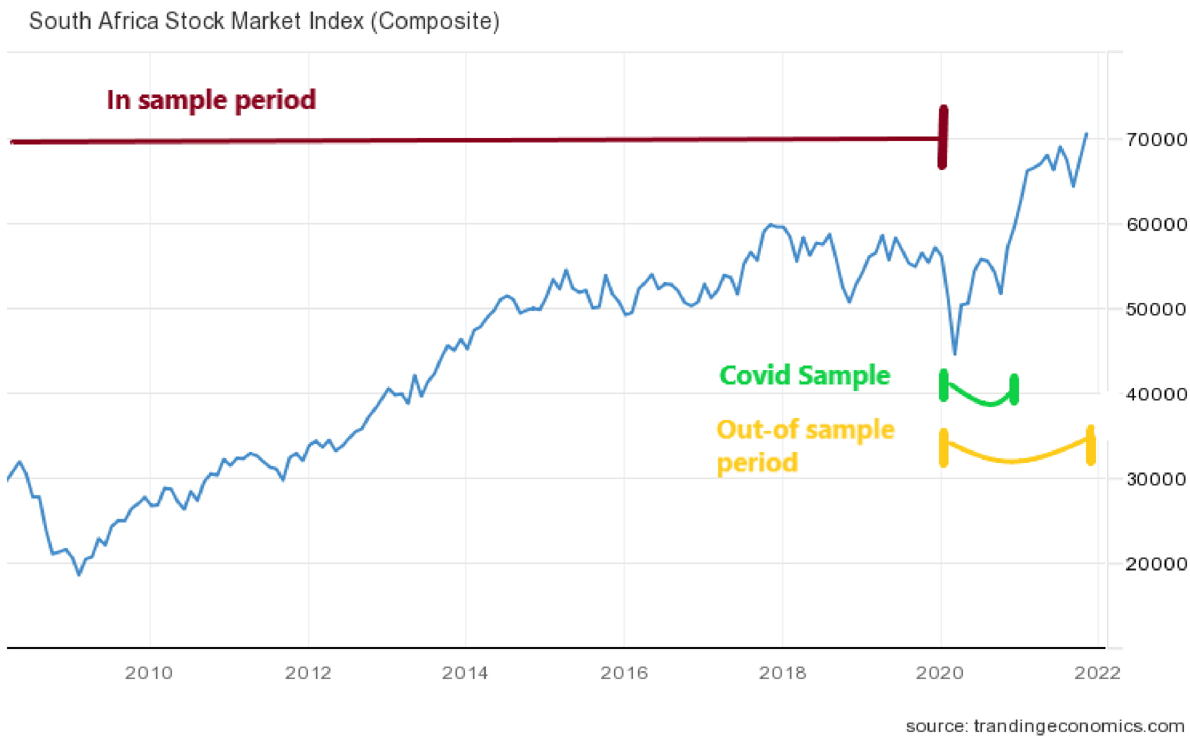

Figure 3.1: JSE Composite Market Index for differemt samples

in the in sample period. However for the analysis, not including the financial crises will also give a good indication of how the proposed methodologies can be used even when no major downturns are observed for the in-sample period.

Then first out-of-sample period selected is for a medium term of 22 months, beginning from of 2020 until the end of October 2021. This period displayed extreme volatility; from major market downturns to excessive increases in stock prices. This is mostly due to the effects of the Covid-19 virus, where the largest fall in the history of the JSE occurred, followed by a subsequent bull run in stock prices. An second out-of-sample periods was therefore chosen to specifically look at the initial impact of Covid-19. The reason is that the low χ stocks are expected to have lower losses during this periods and the effect could therefore be more isolated.

3.3 STOCK AND PORTFOLIO DATA

3.3.1 Stocks

The stocks selected were subject to the requirements that they traded on the date for the in-sample period to the end of the out-of-sample period. This means that certain stocks were excluded that could display important contributions to the analysis, such as stocks for companies that went bankrupt or only recently emerged. Company spins offs and mergers further also complicated the construction of the stock data and were therefore excluded. The daily adjusted prices of the stocks were used to calculate the daily returns. This means effects such as stock splits were removed to represent the true movement of the underlying stock price. The daily returns of the stocks therefore represent the sample that was used in all calculations.

Stocks were selected in decreasing order from their market capitalisation at the end of the out-of-sample period. This introduces a type of survivorship bias since these stocks tended to outperform the market and therefore have a higher market capitalisation. 65 large-cap stocks were selected and make up the sample stocks that will be used throughout that analysis.

The market index that was used to estimate the lower tail dependence to that of the stocks was the JSE Top 40 index. This also introduces some bias in the estimate as not all the stocks selected form a part of the JSE Top 40 index. The reason that this index was chosen is because that it is the only index with values that stretches all the was back to start of the in-sample period on Yahoo! Finance. The JSE All Share index or the JSE Large-Cap index would be better indices in the context of the chosen stocks but the JSE Top 40 index could serve as proxy for the correlation. Around 0.1% of the values were missing for the index and were subsequently removed and replaced with zero. The return for the market index (JSE Top 40) for the in-sample, out-of-sample, Covid sample, was respectively 4% 4,9% and -7,1%; where the returns for the in-sample and out-of-sample periods are annualized and the return for Covid sample is the return over the period.

3.3.2 Portfolios

Random portfolios were generated from the 65 sample stocks. The portfolios all consist out of 5 stocks that have equal weighting. In further analysis different weights could be generated as well as increasing the number of the stocks to use in the portfolio construction. The reason that 5 stocks were chosen is that it included enough stocks to represent a diversified portfolio but not too many that the low χ stocks will not really have an influence. 100 portfolios were generated and this was increased to 10 000 portfolios later one. The results were similar but for simplicity sake the analysis will be done on 100 portfolios to explain the results better. The returns for each of the portfolios was calculated and is the sample data that will be used in the portfolio analysis. A possible extension of this procedure is to investigate certain stocks and find balance of the stock weightings so that the portfolios either has a low or high χ value.

CHAPTER 4

METHODOLOGY

4.1 METHOD

The basic methodology for estimating the lower dependence in this paper is summarrixed below:

1. Multiply the returns by negative one and partition the daily returns of the individual stocks and the market index selected into blocks of 22 daily returns. Find the maximum return for each block (greatest loss) and use as the sample maximum for the estimation.
2. Estimate the parameters of the GEV distribution, μ, σ, and γ by using maximum likelihood on each of the individual stocks and the market index.
3. Transform the sample block maxima of the individual stocks and the market index into a standard Fréchet series by using the estimated parameters from the GEV distrbition.
4. Specify the logistic family as the relationship of the bivariate sequences of the stocks with market index. Using maixmum likelihood, estimate α and convert it to χ.
5. Rank χ in ascending order to find the stocks with lowest and highest χ.

The same procedure is done for the portfolios, where the daily returns of the stocks is simply replaced by the daily returns of the portfolio.

In the analysis that follows, the lower tail dependence is estimated between individual portfolios and the market index using the three-step maximum likelihood procedure previously described with blocks of 22 trading days. The reason that 22 tradings were chosen is that DiTraglia and Gerlach (2013) found this to be the optimal block size. These are also approximately the length of a month and the maximum could therefore represent monthly minimum losses . Although one-step estimation is theoretically more efficient, it is much more difficult to find good starting values for seven parameters at once than for blocks of three, three and one parameter. Under relatively weak dependence assumptions the procedure should produce good estimates of lower tail dependence. However, standard errors are not reported, as correcting them for temporal dependence would require much stronger assumptions.

CHAPTER 5

RESULTS

5.1 BACKGROUND

Lower tail dependence is a reasonable risk metric if, all else equal, investors or portfolio managers want to hold portfolios that relatively outperform other portfolios when financial markets experience significant losses. For this assumption to hold investors should weight a positive probability to economic disaster in order for tail dependency to be a significant risk metric for portfolio selection. Economic disasters are states of the world where the majority of assets have experienced a significant drop in value. From historical stock and derivatives markets this assumptions seems to hold well. For example DiTraglia and Gerlach (2013) cites an article by Barro (2006) that estimated that the probability of a economic disaster occurring in any given year is between 1.5–2% from data from the 20'th century. This estimates seem even low if extrapolated to the 21st century where disaster such as the dot.com bubble, 2007/2008 financial crises and the Covid-19 pandemic have already occurred.

For χ to have any chance of being useful in portfolio selection, it must satisfy two minimal conditions. First, it must capture different information from more traditional risk measures. Second, it must be relatively stable over time. DiTraglia and Gerlach (2013) have shown that both these condition are met for χ. The real question, however, is whether it can be used to construct portfolios that will weather severe market downturns. To answer this question, the following out-of-sample is exercise is carried out.

5.2 INDIVIDUAL STOCK RESULTS

In Appendix A the full results are displayed for all the stocks, however to simplify matters only the bottom and top 10 stocks will be displayed in this section. The tickers of the stocks are displayed in all tables for convenience sake and where applicable the stock will be described fully. Table 5.1 displays the results for the ten stocks with the lowest χ. Firstly note that the EVI (shape parameters) is positive for all stocks which indicates that they belong to the Fréchet-Pareto class, except for Clicks(CLS), which has a very small negative EVI. Most of the EVI's are relatively small and close to zero. If returns are log normally distributed we would expect the EVI's of the stocks to be in the Gumbel class [1]. These results indicate that stocks in general have thicker lower tails with polynomial decay than that of the log normal distribution. The χ is between 0.18 and 0.2 for the ten lowest χ stocks, indicating very low correlation in the very far lower tails with that of the market index.

The five lowest χ stocks are Blue Label Telecoms , DRD Gold, Shoprite Holdings Limited, JSE Limited and Clicks. Intuitively one would think that stocks that are dependant on the economy would have higher χ values, therefore Shoprite and Clicks are a relative surprise. The low ranking of these stocks could however be attributed to a number of reasons such as that they are not correlated with the JSE Top 40 index or display some other characteristics that are resilient to market downturns. The sectors of Blue Label Telecoms, DRD Gold and the JSE are respectively technology, natural resources and the financial sector. These stocks make more intuitively sense to have a low χ value since they are expected to have little correlation with the market. An interesting feature of these stocks are that they are all value stocks. This could indicate that value stocks are more resistant to market downturns than growth stocks but further analysis should be first carried out on this topic before drawing any conclusions. The average monthly minimum returns seem to be in the range from 2.7% 6.7% to.

	GEV parameter estimates				*Percenatge returns*		
Stocks	**Location**	**Scale**	**Shape**	χ	**In-sample**	**Out-of-sample**	**Covid-19 Sample**
BLU	0,029	0,014	0,312	0,18	-7%	50%	-20%
DRD	0,049	0,019	0,095	0,18	5%	47%	96%
SHP	0,026	0,010	0,123	0,18	14%	27%	-9%
JSE	0,023	0,010	0,117	0,18	8%	-2%	-6%
CLS	0,027	0,009	-0,074	0,19	32%	7%	-6%
IVT	0,027	0,015	0,069	0,19	3%	33%	-64%
MUR	0,035	0,014	0,178	0,19	-13%	15%	-49%
WBO	0,026	0,012	0,047	0,19	4%	-12%	-26%
SNH	0,028	0,017	0,481	0,19	-21%	78%	3%
HAR	0,045	0,018	0,101	0,20	-4%	5%	40%
JBL	0,067	0,025	0,092	0,20	-20%	133%	-5%

Table 5.1: 10 Low χ stocks

[1]This is done through the Von Mises theorem

Table 5.2 displays the results for the ten stocks with the highest χ. The EVI is again positive for all stocks which indicates that they belong to the Fréchet-Pareto class, and the conclusion drawn from this is similar to the previous discussion. The five highest χ stocks are Investec, BHP Group, African Rainbows Minerals, Remgro, Bidvest Group. These stocks are a mixture of different industries. The χ values range from 0,41 to 0,57. The in-sample percentage returns are all positive except for ARI which has a return of %1. The average monthly minimum return range is from 2.2% to 3.5% and seem to be a lot more stable than for the the low χ stocks.

	GEV parameter estimates				*Percenatge returns*		
Stocks	**Location**	**Scale**	**Shape**		**In-sample**	**Out-of-sample**	**Covid-19 Sample**
CFR	0,025	0,010	0,236	0,41	9%	35%	-5%
EXX	0,034	0,016	0,134	0,42	6%	31%	-4%
FSR	0,026	0,010	0,182	0,45	22%	0%	-27%
AGL	0,033	0,014	0,233	0,45	1%	25%	-13%
SBK	0,025	0,010	0,135	0,46	12%	-6%	-32%
MNP	0,025	0,012	0,197	0,47	20%	10%	9%
BVT	0,023	0,009	0,158	0,49	30%	-1%	-26%
REM	0,022	0,008	0,033	0,51	14%	-2%	-23%
ARI	0,035	0,016	0,197	0,51	-1%	27%	-12%
BHP	0,028	0,011	0,194	0,55	5%	17%	-3%
INP	0,023	0,011	0,234	0,57	14%	26%	-19%

Table 5.2: High χ stocks

Table 5.3 displays the results for percentage returns for the different groups of *chi* values for in the in-sample, out-of-sample and Covid-19 sample tests. The returns displayed for the in and out of sample test are annualized returns while for the Covid-19 period it is the return over the period. In a efficient market prices should reflect all relevant information. This includes measures such as the lower tail dependence for stocks. It is therefore theoretically expected that low *chi* stocks should have lower average returns than high *chi* stocks when there is no significant market downturns to compensate for the protection in times of market losses.

Since the in-sample test period consist of a long market bull run the results should theoretically indicate this. However when looking at the returns for this period the average return for the low χ stocks is 11%; for the high χ stocks it is 12%, and for the middle χ it is 8%. This indicates that low

χ stocks tends to perform relatively well even in times when market is generally trending upwards. It should also be noted that for the bottom half χ stocks the return is 5% while the top half is 11%. This indicates that stocks should be selected that either have either low or high χ values in order to capture the effects of market upswings or adequate protection against significant losses.

For the out-of-sample test period the average returns is 26 % for the low χ stocks, 14% for the high χ stocks and 19% for the middle χ stocks. This period was dominated by the effects of the Covid-19 pandemic that resulted in a extreme market downturns and subsequent steep rises in stock prices. The low χ stocks substantially outperformed the high χ stocks by 12% in this period, indicating thaT χ is a very good risk measure. This is even more apparent when comparing the average returns for these stocks from the Covid-19 sample. The low χ stocks still produce a positive return of 11 % while the high χ stocks produce a negative returns of -17 %. It should be noted that positive return is mostly due to DRD Gold producing a phenomenal return of 96% in that period. This is expected since in times of great market uncertainty investors tend to buy gold as an hedge.

	Percentage returns		
	In-sample	**Out-of-sample**	**Covid-19 Sample**
Lowest 5	11%	26%	11%
Highest 5	12%	14%	-17%
Middle 55	8%	19%	-16%
Bottom half	5%	24%	-13%
Top half	11%	14%	-14%

Table 5.3: Stock results

5.3 PORTFOLIO RESULTS

This section display and interprets results of the portfolios generated. Table 5.4 displays the results of the portfolios with the 10 lowest χ values. The table also displays the tickers of the stocks randomly selected in the portfolio. Table 5.5 displays the results of the portfolios with the 10 highest χ values. The full table of results for the portfolios can be found in Appendix B. An interesting results here is that the lowest χ portfolio has a χ value even lower then the the lowest stocks. Therefore through careful portfolio construction a very low χ portfolio could be constructed.

							Percentage Reurns		
Stocks in portfolio					χ	**In-sample**	**Out-of sample**	**Covid-19**
SUI	MTA	DRD	ABSP	MIX	0,084	10%	17%	-5%
AGL	CSB	MIX	MTA	VOD	0,162	13%	18%	-15%
TRU	VOD	HAR	CSB	WHL	0,187	13%	12%	-6%
CML	GND	ANG	MTM	ZED	0,217	16%	10%	2%
CLH	TBS	ACL	FBR	ACT	0,237	10%	36%	-30%
WHL	ART	CAT	SNH	JBL	0,239	6%	63%	-16%
MIX	BTI	CLI	KIO	SNT	0,239	15%	2%	-6%
CAT	FBR	MTM	SNH	ANG	0,242	9%	19%	-4%
ABSP	BAW	KIO	MSM	CPI	0,246	28%	13%	-25%
CSB	MSM	SHP	PPC	IMP	0,249	9%	28%	-30%

Table 5.4: Low χ portfolios

Table 5.6 shows the results obtained for the different sample periods. The average return for the low χ outperform the high χ portfolios across all the sample periods. The in-sample returns for the low χ portfolios are 13 % and for the high χ stocks are 9 %. Holding low χ portfolios should in theory produce inferior returns when experiencing a bull run in an efficient market to compensate for the downside protection that they offer. This results indicates that this is not necessarily true in practice and that and low χ offer value even when the market does not experience distressed periods. A possible reason for this is that these portfolios minimize the losses (even if they are small) over the period and still make use of the market movements when when returns are not in the far tails. The bottom half of the table also outperforms the top half for the in-sample period.

The out-of-sample test returns for the low χ portfolios are 22 % and for the high χ portfolios are 14%. This once again demonstrates the ability of low χ portfolios to absorb economic disaster better then high χ portfolios while still taking advantage of positive market movements. The bottom half of the table averages returns of 20 % while he top half averages returns of 11 % indicating a similar results. For Covid-19 sample test the low χ portfolios mean returns is -14 % while for the high χ portfolios it is -22 %. This indicates that the returns were similar post Covid-19 but that the low χ portfolios offered protection in this period and a results produced superior returns over the period.

Because most portfolio optimization approaches in use don't account for lower tail dependence, the

						Percentage Reurns		
Stocks in portfolio					χ	**In-sample**	**Out-of sample**	**Covid-19**
AFE	DSY	MTA	ˆJ200	IMP	0,485	9%	15%	-20%
KIO	CCO	AFE	BAT	INP	0,495	8%	4%	-21%
TBS	EXX	WBO	CCO	DTC	0,498	6%	4%	-17%
BLU	REM	MUR	ART	MRF	0,505	1%	32%	-29%
PPC	SPG	FSR	TBS	BTI	0,506	12%	12%	-24%
SHP	INP	SLM	LEW	AFT	0,517	15%	22%	-22%
INP	SPG	GFI	ABG	SLM	0,542	11%	12%	-12%
WBO	CCO	MRF	SPG	ˆJ200	0,547	0%	4%	-25%
MNP	AFT	BVT	IPL	ˆJ200	0,560	20%	11%	-15%
HCI	PPC	EXX	MRF	REM	0,607	5%	21%	-36%

Table 5.5: High χ portfolios

	Percentage returns		
	In-sample	**Out-of-sample**	**Covid-19**
Lowest 10 %	13%	22%	-14%
Highest 10%	9%	14%	-22%
Middle 80%	14%	19%	-15%
Bottom half	15%	20%	-13%
Top half	11%	11%	18%

Table 5.6: Portfolio results

conclusions presented here shouldn't come as a surprise. The benefits of reducing significant losses will not be fully valued until a large number of investors start employing EVT strategies. The underlying question is why, despite its obvious importance and the availability of methodologies to assess it, tail dependency has not received greater attention. Investors will inevitably exclude certain information due to the broad array of financial market data and portfolio optimization tools available. It's possible that rationally inattentive investors would prefer well-established methodologies like mean–variance portfolio optimization over notions like tail dependency that have only lately been introduced.

5.4 COMPARISON TO OTHER RISK METRICS

For χ to be a relevant risk metric it should captures additional information that is not captured by other risk metrics. A simple way to do this is to calculate the the relevant risk metrics and find the correlation with χ. The risk metric that will be considered here is the variance of the stocks. The correlation coefficient between the variance of the stock and the lower tail dependence is -0,23. This value is low enough that there is no significant correlation between and therefore χ captures information that is not displayed the variance. The correlation is also negative which shows that high χ stock have smaller variation although this relationship is relatively weak. .

Skewness and kurtosis are also measures that could be looked at to determine the distribution of returns. Specifically excess kurtosis could be used to determine how fat tailed the distribution is. The correlation between χ and skewness and kurtosis is repsectively -0.13 and 0.1, but these correlations were found not to be significant at a 5 % confidence level. Therefore we can conclude that no significant relationship exist between these risk measures and that χ captures information that skewness and kurtosis do not. DiTraglia and Gerlach (2013) looks at this concept more in depth by covering other risk metric such as CAPM Beta and Semi Variance and over extende periods of time.

5.5 TRADING STRATEGIES

There are a number of possible trading strategies that could be used given the information presented in this paper. The first and most obvious strategy is to go long on low χ stocks or portfolios and short high χ stocks or portfolios. This should produce superior returns for an adequate level of risk. Another possible strategy is to sell long dated out of the money put options on low χ stocks and buy long dated out of the money put options on high χ stocks. This should produce a positive return since these option prices do not take into account χ. The idea behind this strategy is that the option sold should expire worthless most of the time while the options bought will be more often in the money. If the option sold is in the money, the option bought will be further in the money producing still a positive return.

An interesting strategy would be to set up a portfolio that will gain massively in unforeseen events. This can be regarded as a anti-fragile portfolio. This idea behind behind is this strategy is “We don’t know what ill go wrong, but *something* will go wrong”. This could entail the opposite view from that of usual investments: most of the time small losses are incurred until an eventual large pay-off. Whether or not an investor could sustain the psychological impacts of losing money every day in hopes of a event that can not be unforeseen is another debate. Various other strategies could be used based on the same principles.

CHAPTER 6

CONCLUSION

6.1 CONCLUSION

It is theoretically shown that lower tail dependence χ, a measure of the probability that a portfolio will suffer large losses given that the market does, contains important information for risk-averse investors. χ as a risk measure also substantially deviates from the normal risk measures and through the analysis done it it shown that portfolio managers overlook χ as useful investment tool. The most noteworthy results is that low χ stocks and portfolios outperformed high χ stocks and portfolios for theh in-sample, out-of-sample and the Covid-19 sample. The application of χ in portfolio selection is two-fold: Firstly it offers downside protection in volatile market conditions, and secondly, it produces greater returns by protecting for small downside market movements while taking advantage of positive market movements.

6.2 FURTHER APPLICATIONS/RESEARCH

There a number of ways in which the underlying theory and results displayed here could be modified or adjusted to reflect more accurate results. The number of trading days used could be experimented with to find the optimal trading days. Different types of stock could be experimented with, for example looking at momentum, growth and value stocks and see if χ can provide any additional information. Also further robustness test should be applied to check the validity of the model as well as the assumption made throughout.

REFERENCES

Antoniou, I., Ivanov, V.V., Ivanov, V.V. and Zrelov, P. (2004). On the log-normal distribution of stock market data. *Physica A: Statistical Mechanics and its Applications*, vol. 331, no. 3-4, pp. 617–638.

Barro, R.J. (2006). Rare disasters and asset markets in the twentieth century. *The Quarterly Journal of Economics*, vol. 121, no. 3, pp. 823–866.

Beirlant, J., Goegebeur, Y., Segers, J. and Teugels, J.L. (2004). *Statistics of extremes: theory and applications*, vol. 558. John Wiley & Sons.

Coles, S., Bawa, J., Trenner, L. and Dorazio, P. (2001). *An introduction to statistical modeling of extreme values*, vol. 208. Springer.

Cont, R. (2001). Empirical properties of asset returns: stylized facts and statistical issues. *Quantitative finance*, vol. 1, no. 2, p. 223.

DiTraglia, F.J. and Gerlach, J.R. (2013). Portfolio selection: An extreme value approach. *Journal of Banking & Finance*, vol. 37, no. 2, pp. 305–323.

Rosenkrantz, W.A. (2003). Why stock prices have a lognormal distribution. *Department of Mathematics and Statistics, University of Massachusetts at Amhers.*

Taleb, N. (2005). The black swan: Why don't we learn that we don't learn. *NY: Random House.*

APPENDIX A

STOCKS

	GEV parameter estimates				*Percenatge returns*		
Stocks	**Location**	**Scale**	**Shape**		**In-sample**	**Out-of-sample**	**Covid-19 Sample**
BLU	0,029	0,014	0,312	0,18	-7%	50%	-20%
DRD	0,049	0,019	0,095	0,18	5%	47%	96%
SHP	0,026	0,010	0,123	0,18	14%	27%	-9%
JSE	0,023	0,010	0,117	0,18	8%	-2%	-6%
CLS	0,027	0,009	-0,074	0,19	32%	7%	-6%
IVT	0,027	0,015	0,069	0,19	3%	33%	-64%
MUR	0,035	0,014	0,178	0,19	-13%	15%	-49%
WBO	0,026	0,012	0,047	0,19	4%	-12%	-26%
SNH	0,028	0,017	0,481	0,19	-21%	78%	3%
HAR	0,045	0,018	0,101	0,20	-4%	5%	40%
JBL	0,067	0,025	0,092	0,20	-20%	133%	-5%
TFG	0,029	0,011	0,082	0,20	21%	-1%	-47%
WHL	0,028	0,011	0,014	0,20	20%	7%	-29%
ANG	0,040	0,016	0,093	0,21	2%	-4%	57%
SUI	0,025	0,012	0,096	0,21	-4%	-4%	-53%
RBX	0,034	0,014	0,062	0,22	0%	20%	-28%
SPP	0,021	0,008	0,067	0,23	17%	3%	-7%
ZED	0,028	0,009	0,087	0,23	8%	29%	-4%
HCI	0,023	0,010	0,213	0,23	5%	-13%	-74%
TRU	0,031	0,012	0,088	0,23	12%	12%	-28%
ART	0,034	0,018	0,115	0,24	-5%	58%	-20%
SNT	0,024	0,010	0,060	0,24	16%	-4%	0%
ACL	0,046	0,024	0,198	0,24	-36%	136%	-48%
AFT	0,033	0,014	0,250	0,25	19%	32%	-19%
MTA	0,025	0,014	0,354	0,26	9%	20%	-32%
KIO	0,039	0,018	0,094	0,26	11%	22%	10%
AFE	0,021	0,010	0,145	0,26	8%	12%	-25%
VOD	0,022	0,011	0,199	0,26	4%	15%	5%
DSY	0,022	0,011	0,135	0,27	17%	8%	-16%
ACT	0,044	0,022	0,286	0,27	10%	36%	11%
BTI	0,019	0,008	0,144	0,27	14%	-2%	8%
IPL	0,029	0,013	0,281	0,28	5%	9%	-31%
TBS	0,023	0,009	0,103	0,28	10%	-3%	-7%
BAW	0,031	0,012	0,122	0,29	6%	9%	-27%
CPI	0,023	0,011	0,139	0,29	45%	10%	-33%
CML	0,025	0,011	0,129	0,30	29%	22%	-13%

	GEV parameter estimates				*Percenatge returns*		
Stocks	**Location**	**Scale**	**Shape**		**In-sample**	**Out-of-sample**	**Covid-19 Sample**
MRF	0,043	0,017	0,191	0,30	-10%	31%	-31%
ABG	0,025	0,011	0,134	0,30	12%	1%	-29%
PSG	0,023	0,011	0,295	0,30	38%	24%	-26%
HDC	0,025	0,012	0,070	0,31	8%	21%	-31%
IMP	0,041	0,017	0,121	0,31	-5%	29%	-19%
GFI	0,039	0,016	0,265	0,31	3%	30%	48%
APN	0,026	0,011	0,262	0,31	13%	46%	2%
DTC	0,025	0,015	0,243	0,32	10%	16%	-27%
GND	0,031	0,016	0,070	0,32	-8%	2%	-23%
RLO	0,023	0,009	0,164	0,32	9%	-11%	-34%
AVI	0,022	0,009	0,236	0,32	23%	0%	4%
CCO	0,021	0,010	0,418	0,33	-8%	-17%	-19%
MTM	0,024	0,011	0,118	0,34	11%	-4%	-10%
SPG	0,025	0,012	0,396	0,36	1%	8%	-44%
PAN	0,037	0,019	0,301	0,36	12%	35%	32%
MTN	0,028	0,013	0,246	0,37	1%	36%	-35%
NPN	0,033	0,013	-0,148	0,39	31%	6%	16%
SLM	0,025	0,011	0,070	0,41	18%	-7%	-15%
CFR	0,025	0,010	0,236	0,41	9%	35%	-5%
EXX	0,034	0,016	0,134	0,42	6%	31%	-4%
FSR	0,026	0,010	0,182	0,45	22%	0%	-27%
AGL	0,033	0,014	0,233	0,45	1%	25%	-13%
SBK	0,025	0,010	0,135	0,46	12%	-6%	-32%
MNP	0,025	0,012	0,197	0,47	20%	10%	9%
BVT	0,023	0,009	0,158	0,49	30%	-1%	-26%
REM	0,022	0,008	0,033	0,51	14%	-2%	-23%
ARI	0,035	0,016	0,197	0,51	-1%	27%	-12%
BHP	0,028	0,011	0,194	0,55	5%	17%	-3%
INP	0,023	0,011	0,234	0,57	14%	26%	-19%

APPENDIX B

PORTFOLIOS

Stocks in portfolio						Percentage Reurns In-sample	Out-of sample	Covid-19
SUI	MTA	DRD	ABSP	MIX	0,084	10%	17%	-5%
AGL	CSB	MIX	MTA	VOD	0,162	13%	18%	-15%
TRU	VOD	HAR	CSB	WHL	0,187	13%	12%	-6%
CML	GND	ANG	MTM	ZED	0,217	16%	10%	2%
CLH	TBS	ACL	FBR	ACT	0,237	10%	36%	-30%
WHL	ART	CAT	SNH	JBL	0,239	6%	63%	-16%
MIX	BTI	CLI	KIO	SNT	0,239	15%	2%	-6%
CAT	FBR	MTM	SNH	ANG	0,242	9%	19%	-4%
ABSP	BAW	KIO	MSM	CPI	0,246	28%	13%	-25%
CSB	MSM	SHP	PPC	IMP	0,249	9%	28%	-30%
TRU	DSY	IMP	DTC	MRP	0,252	17%	15%	-23%
DRD	MRP	BLU	HCI	MNP	0,253	17%	22%	-3%
IPL	SUI	SHP	MUR	CAT	0,258	4%	13%	-34%
CLH	HAR	CML	MTN	DRD	0,261	14%	17%	6%
MRP	TRU	WHL	ZED	PSG	0,262	26%	16%	-22%
CFR	MSM	ART	PAN	CLI	0,264	8%	27%	-17%
SHP	CLS	HDC	IVT	AFT	0,272	20%	24%	-26%
MTA	ANG	SPG	CPI	ABG	0,279	28%	7%	-16%
AGL	WHL	CAT	HAR	MTM	0,281	10%	10%	-8%
CLS	IPL	SNT	CLI	CCO	0,284	19%	-3%	-18%
CML	IMP	SHP	CCO	BAT	0,292	15%	8%	-22%
CLH	CPI	DRD	BLU	KIO	0,296	27%	21%	-2%
GND	CPI	SNT	DSY	AEG	0,296	28%	48%	-14%
CAT	CCO	TRU	SLM	CPI	0,301	28%	3%	-25%
PAN	IVT	RLO	MSM	PSG	0,302	22%	20%	-29%
CSB	PPC	JBL	TBS	CFR	0,306	8%	53%	-17%
DRD	FBR	AVI	GFI	ACL	0,308	13%	49%	11%
MIX	SLM	SUI	JSE	ABG	0,318	13%	-1%	-23%
WBO	SUI	NPN	CFR	MRP	0,321	21%	7%	-18%
SPP	WBO	ZED	SUI	MTM	0,322	9%	3%	-20%
CML	DTC	PPC	WHL	OCE	0,323	18%	20%	-23%
HCI	DRD	FBR	MSM	BVT	0,324	18%	10%	-20%
ˆJ200	WHL	WBO	AFE	CLS	0,326	20%	4%	-19%
EXX	GFI	ZED	CLS	CLI	0,326	18%	18%	0%
TRU	FBR	CML	HCI	ACT	0,328	18%	12%	-30%
SPG	ANG	ACT	DTC	DRD	0,333	6%	22%	19%
APN	JBL	GND	AVI	JSE	0,335	12%	44%	-6%

SHP	WBO	APN	SLM	AFE	0,338	12%	15%	-15%
PAN	GFI	SBK	IPL	MSM	0,339	7%	17%	-7%
DRD	SNT	ACT	KIO	ACL	0,341	9%	53%	14%
AFE	CML	ˆJ200	MNP	SPP	0,345	19%	10%	-9%
BAW	TRU	AVI	HDC	CML	0,346	19%	13%	-19%
AVI	SNH	VOD	BVT	FBR	0,347	20%	21%	-12%
SNH	SNT	JBL	AVI	WBO	0,348	12%	48%	-5%
ˆJ200	WHL	SHP	TBS	HAR	0,349	12%	8%	-3%
DRD	LEW	ABG	MRF	MTM	0,351	7%	21%	-4%
SNH	PPC	TFG	KIO	RLO	0,353	10%	31%	-23%
RLO	HCI	APN	DTC	ABG	0,353	10%	10%	-33%
MTN	CSB	FSR	PSG	ABG	0,353	24%	17%	-27%
ABG	SNT	PAN	TBS	CPI	0,363	28%	9%	-8%
MNP	CML	BAT	TRU	MTA	0,363	18%	7%	-23%
MTA	KIO	WHL	SNT	PPC	0,369	13%	19%	-20%
MTA	GFI	DTC	INP	CSB	0,369	12%	23%	-10%
CPI	NPN	CLH	BAW	IPL	0,369	30%	0%	-27%
RBX	ANG	WBO	IPL	ˆJ200	0,370	3%	4%	-7%
ABG	GND	HDC	PPC	SUI	0,372	3%	15%	-37%
AEG	DSY	SUI	ACT	CFR	0,373	8%	58%	-12%
ACT	CCO	ˆJ200	KIO	MTA	0,374	7%	14%	-7%
MUR	KIO	AFT	LEW	GFI	0,376	10%	25%	-11%
INP	EXX	AFE	FBR	ARI	0,379	11%	19%	-21%
WBO	JBL	DTC	ACL	IVT	0,383	2%	71%	-34%
SHP	NPN	SUI	SNH	HAR	0,389	16%	25%	-1%
AGL	ABSP	NPN	GND	IMP	0,391	16%	15%	-12%
ZED	JSE	LEW	ˆJ200	HAR	0,392	6%	13%	-4%
CCO	AVI	CLI	INP	HAR	0,392	13%	2%	-6%
APN	AEG	SLM	ANG	BHP	0,393	9%	55%	8%
RCL	SNT	SPG	BTI	AFE	0,397	10%	5%	-15%
MTN	REM	OCE	TBS	SUI	0,398	10%	7%	-23%
ACT	RLO	HDC	ARI	CFR	0,398	8%	23%	-14%
ZED	TBS	MRF	RLO	BLU	0,398	5%	21%	-19%
AVI	BAW	MUR	JBL	SNH	0,399	8%	54%	-15%
DSY	CAT	SBK	SPP	MTN	0,400	12%	12%	-24%
ABG	BAW	DTC	BLU	DSY	0,408	10%	18%	-24%
MRF	AGL	PAN	PSG	RLO	0,412	22%	22%	-15%
CPI	ZED	AGL	MNP	WHL	0,412	29%	17%	-14%
AGL	KIO	BAT	MUR	CCO	0,414	1%	4%	-25%
DSY	INP	WBO	RCL	JBL	0,415	9%	42%	-17%
SUI	BAW	JSE	GFI	AGL	0,417	3%	12%	-10%
ZED	SUI	SPG	HDC	BHP	0,422	5%	15%	-27%
DTC	CLI	FSR	BLU	NPN	0,423	20%	14%	-19%
KIO	SPG	SLM	CLI	WBO	0,428	11%	1%	-22%
SLM	AGL	NPN	ZED	SPP	0,430	20%	12%	-5%
BAW	SNT	GFI	MTN	ABSP	0,442	8%	17%	-7%
TBS	DTC	BHP	SNT	ABSP	0,443	10%	8%	-12%
ABSP	BTI	DSY	REM	AGL	0,468	12%	9%	-13%
VOD	DTC	RCL	MTN	SBK	0,474	6%	15%	-21%
BVT	MNP	ZED	MRF	MSM	0,482	18%	17%	-21%
AFE	DSY	MTA	ˆJ200	IMP	0,485	9%	15%	-20%
KIO	CCO	AFE	BAT	INP	0,495	8%	4%	-21%
TBS	EXX	WBO	CCO	DTC	0,498	6%	4%	-17%