THE UNIVERSITY OF THE WEST INDIES

Semester I ☐ Semester II ■ Supplemental/Summer School ☐

Examinations of December ☐ April/May ■ July ☐ 2021

Originating Campus:	**Cave Hill**	☐	**Mona**	■	**St. Augustine**	☐	**Open**	☐
Mode:	**On Campus**	■			**By Distance**	☐		

Course Code and Title: **INFO2100 – Mathematics & Statistics for IT**

Date:	**May 6, 2021**	Time:	**2 pm**
Duration:	**2 Hours**	Paper No:	**1**

Materials required:

Answer booklet:	**Normal**	■	**Special**	☐	**Not required**	☐		
Calculator: (where applicable)	**Programmable**	☐	**Non-Programmable**	■	**Not required**	☐		
Multiple Choice answer sheets:	**numerical**	☐	**alphabetical**	☐	**1-20**	☐	**1-100**	☐

Auxiliary/Other material(s): None

Instructions to Candidates: This paper has 9 page(s) and 8 questions

Candidates are reminded that the examiners shall take into account the proper use of the English Language in determining the mark for each response.

Answer three (3) questions in Section 1 and three (3) questions in Section 2.

The maximum number of marks you may earn for the entire paper is **60**. The number in **[]** by each question indicates the number of marks allotted to the question. Justify all your answers; full credit will be given only for properly supported answers, partial credit will be given where applicable.

Statistical tables are provided for you on pages 6–9 of this exam. Use them where appropriate.

Since this paper is a take-home exam, you may reference your notes, and your textbook, but you may not seek nor receive help on any questions from any unauthorised person. You are also not allowed to use search engines to look-up information. You should note that the exam has been written so that you are not expected to have to look up anything. Therefore, please be mindful of the time you have for the exam, and avoid losing too much time chasing references.

Please write legibly and keep your answers concise. Good skill!

Section 1

Question 1 *Basic probability* **[10]**

a. Based on a recent clinical trial for a COVID-19 vaccine, carried out before a surge in cases, the probability that a study participant contracted COVID-19 was 0.005. Study participants were randomly assigned to groups to receive either a placebo or the actual vaccine. The probability that a study participant received a placebo was 0.5, while the probability that a placebo recipient contracted COVID-19 was 0.009.

(i) What is the probability of being a placebo recipient given that the participant has contracted COVID-19? **[3]**

(ii) What is the probability that a vaccine recipient contracted COVID-19? **[2]**

(iii) What is the probability that a vaccine recipient does *not* contract COVID-19? **[1]**

(iv) What is the probability that a placebo recipient does *not* contract COVID-19? **[1]**

Anosmia (a temporary loss of the sense of smell) is listed as a possible symptom of COVID-19. Suppose that: the probability of having anosmia given that the study participant has COVID-19 is 0.7, while the probability of having anosmia is 0.03 among study participants concerned about having COVID-19.

(v) What is the probability of having anosmia and COVID-19, given that you have received the vaccine? **[3]**

Question 2 *Discrete random variables* **[10]**

The number of 4 GiB memory banks, B, required in a personal computer depends on how many application programs, A, the owner wants to run simultaneously. The number of memory banks B and number of application programs A are described by:

$$B = \begin{cases} 2 & \text{banks for 1 program} \\ 2 & \text{banks for 2 programs} \\ 3 & \text{banks for 3 programs} \\ 4 & \text{banks for 4 programs} \end{cases} \qquad \Pr(A = a) = \begin{cases} 0.1 & a = 1 \\ 0.2 & a = 2 \\ 0.3 & a = 3 \\ 0.4 & a = 4 \\ 0 & \text{otherwise} \end{cases}$$

a. What is the probability of running 3 or more programs? **[1]**

b. What is the expected number of programs $\mu_A = \mathbb{E}[A]$? **[2]**

c. What is the variance of the number of programs $\mathrm{Var}(A)$? **[3]**

d. Express B as a function $B = g(A)$ of the number of application programs A **[2]**

e. What is the expected number of memory banks $\mathbb{E}[B] = \mathbb{E}[g(A)]$? **[2]**

Question 3 *Continuous random variables* **[10]**

A bank of satellite modems has been configured to send telemetry data from a research site in Antarctica to a university's main campus. Observations show that "calls" with this system are dropped at random intervals. Analysis shows that the PDF that more than t minutes elapse between call drops is given by:

$$f_T(t) = \begin{cases} \frac{1}{50}e^{-t/50} & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

a. What is the probability that between 2 and 4 minutes elapse before a call is dropped? **[3]**

b. What is the expected number of minutes between call drops? **[3]**

c. What is the variance of the number of minutes between call drops? **[4]**

Question 4 *Central Limit Theorem* **[10]**

The waiting time W for accessing one record from a database is a random variable uniformly distributed between 0 and 5 milliseconds. The read time, R, for moving a block of information from the disk to main memory is 1 millisecond. The random variable X milliseconds is the total access time (waiting time + read time) to get one block of information from the disk. Before performing a certain task, the computer must read 8 different blocks of information from the disk. (Access times for different blocks are independent of one another). The total access time for all the information is a random variable A milliseconds.

a. What is $\mathbb{E}[X]$, the expected value of the access time? **[1]**

b. What is the variance, $\text{Var}(X)$, the variance of the access time? **[2]**

c. What is $\mathbb{E}[A]$, the expected value of the total access time? **[1]**

d. What is σ_A, the standard deviation of the total access time? **[2]**

e. Use the Central Limit Theorem to estimate $\Pr(A > 32)$, the probability that the total access time exceeds 32 ms. **[4]**

Section 2

Question 5 *One-sample hypothesis tests* **[10]**

a. The authors of an examination scheduling system wish to compare the (clock) running time of a new scheduling algorithm to the one currently implemented. They apply the new algorithm to 50 randomly generated course schedules; it averages 10.5 seconds with a standard deviation of 0.50 second. The current algorithm averages 10.7 seconds on such schedules. Test, at the 1% level of significance the alternative hypothesis that the new algorithm has a lower average time than the current algorithm.

   (i) What is the critical value at a 1% level of significance? **[1]**

   (ii) What is the test statistic? **[2]**

   (iii) What is the p-value for this test? **[2]**

b. A calculator has a built-in algorithm for generating a random number according to the standard normal distribution. Twenty-five numbers thus generated have mean 0.1 and sample standard deviation 0.95. Test the null hypothesis that the mean of all numbers so generated is 0 versus the alternative that that it is different from 0, at the 5% level of significance. Assume that the numbers do follow a normal distribution.

   (i) Is this test one-sided or two-sided? **[1]**

   (ii) What is the critical value at a 5% level of significance? **[1]**

   (iii) What is the test statistic? **[2]**

   (iv) What is your conclusion regarding the null hypothesis? Why? **[1]**

Question 6 *Non-parametric tests* **[10]**

Given that most courses at UWI are primarily offered online this academic year, lecturers now have easy ways to to study relationships between engagement and performance. In one course with 60 students, the following relationship was identified between tutorial attendance and performance:

	Exam F/P	
Tutorials attended	Fail	Pass
Less than 60%	21	11
60% or more	6	22

Test the null hypothesis that tutorial attendance is independent of course performance versus the alternative that course performance is dependent on tutorial attendance at a 1% level of significance.

a. What is the critical value at a 1% level of significance? **[2]**

b. What is the χ^2 test statistic? Show all steps. **[5]**

c. What is your conclusion regarding the null hypothesis? Why do you make this conclusion? **[3]**

Question 7 *Confidence Intervals* **[10]**

a. Data is collected on the round trip time to a distant host on the Internet. We collect a data set of size 20 with sample mean $\bar{x} = 102$ ms and sample standard deviation $s = 77$ ms. Assume the data follows a normal random distribution.

  (i) Compute the 95% confidence interval on the mean. **[2]**

  (ii) Compute the 95% confidence interval on the variance. **[4]**

b. You have been monitoring the runtime of a daily query as part of your job. You would like to estimate the mean (clock) runtime of the query, $\mu = t$ sec. Suppose that the variance of the runtime is 4 sec^2. How many samples do you need to collect such that you can bound the sample mean runtime with a buffer of ± 2 sec with 95% confidence. **[4]**

Question 8 *Linear regression and correlation* **[10]**

A university has recorded mathematics (y) and verbal (English) (x) aptitude test scores for 162 incoming students. It is assumed that these scores are normally distributed. You are to build a model that shows how these scores are related. You may assume the following values:

$$
\begin{array}{rrr}
n = 162 & \sum x = 96,600 & \sum y = 99,160 \\
\sum xy = 60,205,200 & \sum x^2 = 59,196,800 & \sum y^2 = 62,246,200 \\
\bar{x} = 596 & \bar{y} = 612 & \\
s_{xy} = 6,686.08 & s_{xx} = 9,904.21 & s_{yy} = 9,630.35
\end{array}
$$

a. Calculate the regression model, $y = ax + b$ **[4]**

b. Interpret the values of a, b, and r. **[4]**

c. Give a forecast of the math score for a student whose verbal score is 650 **[2]**

Statistical Tables

NORMAL CUMULATIVE DISTRIBUTION FUNCTION

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

STUDENT'S t PERCENTAGE POINTS

ν	60.0%	66.7%	75.0%	80.0%	87.5%	90.0%	95.0%	97.5%	99.0%	99.5%	99.9%
1	0.325	0.577	1.000	1.376	2.414	3.078	6.314	12.706	31.821	63.657	318.31
2	0.289	0.500	0.816	1.061	1.604	1.886	2.920	4.303	6.965	9.925	22.327
3	0.277	0.476	0.765	0.978	1.423	1.638	2.353	3.182	4.541	5.841	10.215
4	0.271	0.464	0.741	0.941	1.344	1.533	2.132	2.776	3.747	4.604	7.173
5	0.267	0.457	0.727	0.920	1.301	1.476	2.015	2.571	3.365	4.032	5.893
6	0.265	0.453	0.718	0.906	1.273	1.440	1.943	2.447	3.143	3.707	5.208
7	0.263	0.449	0.711	0.896	1.254	1.415	1.895	2.365	2.998	3.499	4.785
8	0.262	0.447	0.706	0.889	1.240	1.397	1.860	2.306	2.896	3.355	4.501
9	0.261	0.445	0.703	0.883	1.230	1.383	1.833	2.262	2.821	3.250	4.297
10	0.260	0.444	0.700	0.879	1.221	1.372	1.812	2.228	2.764	3.169	4.144
11	0.260	0.443	0.697	0.876	1.214	1.363	1.796	2.201	2.718	3.106	4.025
12	0.259	0.442	0.695	0.873	1.209	1.356	1.782	2.179	2.681	3.055	3.930
13	0.259	0.441	0.694	0.870	1.204	1.350	1.771	2.160	2.650	3.012	3.852
14	0.258	0.440	0.692	0.868	1.200	1.345	1.761	2.145	2.624	2.977	3.787
15	0.258	0.439	0.691	0.866	1.197	1.341	1.753	2.131	2.602	2.947	3.733
16	0.258	0.439	0.690	0.865	1.194	1.337	1.746	2.120	2.583	2.921	3.686
17	0.257	0.438	0.689	0.863	1.191	1.333	1.740	2.110	2.567	2.898	3.646
18	0.257	0.438	0.688	0.862	1.189	1.330	1.734	2.101	2.552	2.878	3.610
19	0.257	0.438	0.688	0.861	1.187	1.328	1.729	2.093	2.539	2.861	3.579
20	0.257	0.437	0.687	0.860	1.185	1.325	1.725	2.086	2.528	2.845	3.552
21	0.257	0.437	0.686	0.859	1.183	1.323	1.721	2.080	2.518	2.831	3.527
22	0.256	0.437	0.686	0.858	1.182	1.321	1.717	2.074	2.508	2.819	3.505
23	0.256	0.436	0.685	0.858	1.180	1.319	1.714	2.069	2.500	2.807	3.485
24	0.256	0.436	0.685	0.857	1.179	1.318	1.711	2.064	2.492	2.797	3.467
25	0.256	0.436	0.684	0.856	1.178	1.316	1.708	2.060	2.485	2.787	3.450
26	0.256	0.436	0.684	0.856	1.177	1.315	1.706	2.056	2.479	2.779	3.435
27	0.256	0.435	0.684	0.855	1.176	1.314	1.703	2.052	2.473	2.771	3.421
28	0.256	0.435	0.683	0.855	1.175	1.313	1.701	2.048	2.467	2.763	3.408
29	0.256	0.435	0.683	0.854	1.174	1.311	1.699	2.045	2.462	2.756	3.396
30	0.256	0.435	0.683	0.854	1.173	1.310	1.697	2.042	2.457	2.750	3.385
35	0.255	0.434	0.682	0.852	1.170	1.306	1.690	2.030	2.438	2.724	3.340
40	0.255	0.434	0.681	0.851	1.167	1.303	1.684	2.021	2.423	2.704	3.307
45	0.255	0.434	0.680	0.850	1.165	1.301	1.679	2.014	2.412	2.690	3.281
50	0.255	0.433	0.679	0.849	1.164	1.299	1.676	2.009	2.403	2.678	3.261
55	0.255	0.433	0.679	0.848	1.163	1.297	1.673	2.004	2.396	2.668	3.245
60	0.254	0.433	0.679	0.848	1.162	1.296	1.671	2.000	2.390	2.660	3.232
∞	0.253	0.431	0.674	0.842	1.150	1.282	1.645	1.960	2.326	2.576	3.090

CHI-SQUARED PERCENTAGE POINTS

ν	0.1%	0.5%	1.0%	2.5%	5.0%	10.0%	12.5%	20.0%	25.0%	33.3%	50.0%
1	0.000	0.000	0.000	0.001	0.004	0.016	0.025	0.064	0.102	0.186	0.455
2	0.002	0.010	0.020	0.051	0.103	0.211	0.267	0.446	0.575	0.811	1.386
3	0.024	0.072	0.115	0.216	0.352	0.584	0.692	1.005	1.213	1.568	2.366
4	0.091	0.207	0.297	0.484	0.711	1.064	1.219	1.649	1.923	2.378	3.357
5	0.210	0.412	0.554	0.831	1.145	1.610	1.808	2.343	2.675	3.216	4.351
6	0.381	0.676	0.872	1.237	1.635	2.204	2.441	3.070	3.455	4.074	5.348
7	0.598	0.989	1.239	1.690	2.167	2.833	3.106	3.822	4.255	4.945	6.346
8	0.857	1.344	1.646	2.180	2.733	3.490	3.797	4.594	5.071	5.826	7.344
9	1.152	1.735	2.088	2.700	3.325	4.168	4.507	5.380	5.899	6.716	8.343
10	1.479	2.156	2.558	3.247	3.940	4.865	5.234	6.179	6.737	7.612	9.342
11	1.834	2.603	3.053	3.816	4.575	5.578	5.975	6.989	7.584	8.514	10.341
12	2.214	3.074	3.571	4.404	5.226	6.304	6.729	7.807	8.438	9.420	11.340
13	2.617	3.565	4.107	5.009	5.892	7.042	7.493	8.634	9.299	10.331	12.340
14	3.041	4.075	4.660	5.629	6.571	7.790	8.266	9.467	10.165	11.245	13.339
15	3.483	4.601	5.229	6.262	7.261	8.547	9.048	10.307	11.037	12.163	14.339
16	3.942	5.142	5.812	6.908	7.962	9.312	9.837	11.152	11.912	13.083	15.338
17	4.416	5.697	6.408	7.564	8.672	10.085	10.633	12.002	12.792	14.006	16.338
18	4.905	6.265	7.015	8.231	9.390	10.865	11.435	12.857	13.675	14.931	17.338
19	5.407	6.844	7.633	8.907	10.117	11.651	12.242	13.716	14.562	15.859	18.338
20	5.921	7.434	8.260	9.591	10.851	12.443	13.055	14.578	15.452	16.788	19.337
21	6.447	8.034	8.897	10.283	11.591	13.240	13.873	15.445	16.344	17.720	20.337
22	6.983	8.643	9.542	10.982	12.338	14.041	14.695	16.314	17.240	18.653	21.337
23	7.529	9.260	10.196	11.689	13.091	14.848	15.521	17.187	18.137	19.587	22.337
24	8.085	9.886	10.856	12.401	13.848	15.659	16.351	18.062	19.037	20.523	23.337
25	8.649	10.520	11.524	13.120	14.611	16.473	17.184	18.940	19.939	21.461	24.337
26	9.222	11.160	12.198	13.844	15.379	17.292	18.021	19.820	20.843	22.399	25.336
27	9.803	11.808	12.879	14.573	16.151	18.114	18.861	20.703	21.749	23.339	26.336
28	10.391	12.461	13.565	15.308	16.928	18.939	19.704	21.588	22.657	24.280	27.336
29	10.986	13.121	14.256	16.047	17.708	19.768	20.550	22.475	23.567	25.222	28.336
30	11.588	13.787	14.953	16.791	18.493	20.599	21.399	23.364	24.478	26.165	29.336
35	14.688	17.192	18.509	20.569	22.465	24.797	25.678	27.836	29.054	30.894	34.336
40	17.916	20.707	22.164	24.433	26.509	29.051	30.008	32.345	33.660	35.643	39.335
45	21.251	24.311	25.901	28.366	30.612	33.350	34.379	36.884	38.291	40.407	44.335
50	24.674	27.991	29.707	32.357	34.764	37.689	38.785	41.449	42.942	45.184	49.335
55	28.173	31.735	33.570	36.398	38.958	42.060	43.220	46.036	47.610	49.972	54.335
60	31.738	35.534	37.485	40.482	43.188	46.459	47.680	50.641	52.294	54.770	59.335

CHI-SQUARED PERCENTAGE POINTS

ν	60.0%	66.7%	75.0%	80.0%	87.5%	90.0%	95.0%	97.5%	99.0%	99.5%	99.9%
1	0.708	0.936	1.323	1.642	2.354	2.706	3.841	5.024	6.635	7.879	10.828
2	1.833	2.197	2.773	3.219	4.159	4.605	5.991	7.378	9.210	10.597	13.816
3	2.946	3.405	4.108	4.642	5.739	6.251	7.815	9.348	11.345	12.838	16.266
4	4.045	4.579	5.385	5.989	7.214	7.779	9.488	11.143	13.277	14.860	18.467
5	5.132	5.730	6.626	7.289	8.625	9.236	11.070	12.833	15.086	16.750	20.515
6	6.211	6.867	7.841	8.558	9.992	10.645	12.592	14.449	16.812	18.548	22.458
7	7.283	7.992	9.037	9.803	11.326	12.017	14.067	16.013	18.475	20.278	24.322
8	8.351	9.107	10.219	11.030	12.636	13.362	15.507	17.535	20.090	21.955	26.125
9	9.414	10.215	11.389	12.242	13.926	14.684	16.919	19.023	21.666	23.589	27.877
10	10.473	11.317	12.549	13.442	15.198	15.987	18.307	20.483	23.209	25.188	29.588
11	11.530	12.414	13.701	14.631	16.457	17.275	19.675	21.920	24.725	26.757	31.264
12	12.584	13.506	14.845	15.812	17.703	18.549	21.026	23.337	26.217	28.300	32.910
13	13.636	14.595	15.984	16.985	18.939	19.812	22.362	24.736	27.688	29.819	34.528
14	14.685	15.680	17.117	18.151	20.166	21.064	23.685	26.119	29.141	31.319	36.123
15	15.733	16.761	18.245	19.311	21.384	22.307	24.996	27.488	30.578	32.801	37.697
16	16.780	17.840	19.369	20.465	22.595	23.542	26.296	28.845	32.000	34.267	39.252
17	17.824	18.917	20.489	21.615	23.799	24.769	27.587	30.191	33.409	35.718	40.790
18	18.868	19.991	21.605	22.760	24.997	25.989	28.869	31.526	34.805	37.156	42.312
19	19.910	21.063	22.718	23.900	26.189	27.204	30.144	32.852	36.191	38.582	43.820
20	20.951	22.133	23.828	25.038	27.376	28.412	31.410	34.170	37.566	39.997	45.315
21	21.991	23.201	24.935	26.171	28.559	29.615	32.671	35.479	38.932	41.401	46.797
22	23.031	24.268	26.039	27.301	29.737	30.813	33.924	36.781	40.289	42.796	48.268
23	24.069	25.333	27.141	28.429	30.911	32.007	35.172	38.076	41.638	44.181	49.728
24	25.106	26.397	28.241	29.553	32.081	33.196	36.415	39.364	42.980	45.559	51.179
25	26.143	27.459	29.339	30.675	33.247	34.382	37.652	40.646	44.314	46.928	52.620
26	27.179	28.520	30.435	31.795	34.410	35.563	38.885	41.923	45.642	48.290	54.052
27	28.214	29.580	31.528	32.912	35.570	36.741	40.113	43.195	46.963	49.645	55.476
28	29.249	30.639	32.620	34.027	36.727	37.916	41.337	44.461	48.278	50.993	56.892
29	30.283	31.697	33.711	35.139	37.881	39.087	42.557	45.722	49.588	52.336	58.301
30	31.316	32.754	34.800	36.250	39.033	40.256	43.773	46.979	50.892	53.672	59.703
35	36.475	38.024	40.223	41.778	44.753	46.059	49.802	53.203	57.342	60.275	66.619
40	41.622	43.275	45.616	47.269	50.424	51.805	55.758	59.342	63.691	66.766	73.402
45	46.761	48.510	50.985	52.729	56.052	57.505	61.656	65.410	69.957	73.166	80.077
50	51.892	53.733	56.334	58.164	61.647	63.167	67.505	71.420	76.154	79.490	86.661
55	57.016	58.945	61.665	63.577	67.211	68.796	73.311	77.380	82.292	85.749	93.168
60	62.135	64.147	66.981	68.972	72.751	74.397	79.082	83.298	88.379	91.952	99.607