Homework 5

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Determine the full Taylor series expansion about $x = 0$ of the function $f(x) = \frac{1}{2-x} + \frac{1}{2-3x}$. Where does this series converge?

- $f(x) = \sum_{k=0}^{\infty} (-1)^k \frac{1+3^k}{2^{k+1}} x^k$. The series converges for $|x| < \frac{2}{3}$.
- $f(x) = \sum_{k=0}^{\infty} \frac{1+3^k}{2^{k+1}} x^k$. The series converges for $|x| < \frac{3}{2}$.
- $f(x) = \sum_{k=0}^{\infty} \frac{1+3^k}{2^{k+1}} x^k$. The series converges for $|x| < 1$.
- $f(x) = \sum_{k=0}^{\infty} (-1)^k \frac{1+3^k}{2^{k+1}} x^k$. The series converges for $|x| < \frac{3}{2}$.
- $f(x) = \sum_{k=0}^{\infty} (-1)^k \frac{1+3^k}{2^{k+1}} x^k$. The series converges for $|x| < 1$.
- $f(x) = \sum_{k=0}^{\infty} \frac{1+3^k}{2^{k+1}} x^k$. The series converges for $|x| < \frac{2}{3}$.

Question 2

In the lecture we saw that the sum of the infinite series $1 + x + x^2 + \cdots$ equals $1/(1-x)$ as long as $|x| < 1$. In this problem, we will derive a formula for summing the first n terms of the series. That is, we want to calculate

$$s_n = 1 + x + x^2 + \cdots + x^n$$

The strategy is exactly that of the algebraic proof given in lecture for the sum of the full geometric series: compute the difference $s_n - xs_n$ and the isolate s_n. What formula do you get?

- $s_n = \dfrac{1+nx}{1-x}$
- $s_n = \dfrac{1+x^n}{1-x}$
- $s_n = \dfrac{1-x^n}{1-x}$
- $s_n = \dfrac{1-x^{n+1}}{1-x}$
- $s_n = \dfrac{1-nx}{1-x}$
- $s_n = \dfrac{1+x^{n+1}}{1-x}$

Question 3

Which of the following is the Taylor series of $\ln \dfrac{1}{1-x}$ about $x = 0$ up to and including the terms of order three?

- $\ln \dfrac{1}{1-x} = x + \dfrac{1}{2}x^2 + \dfrac{1}{6}x^3 + \mathrm{H.O.T.}$
- $\ln \dfrac{1}{1-x} = 1 + x + \dfrac{1}{2}x^2 + \dfrac{1}{3}x^3 + \mathrm{H.O.T.}$
- $\ln \dfrac{1}{1-x} = x - \dfrac{1}{2}x^2 + \dfrac{1}{3}x^3 + \mathrm{H.O.T.}$
- $\ln \dfrac{1}{1-x} = 1 + x - x^2 + \dfrac{1}{3}x^3 + \mathrm{H.O.T.}$
- $\ln \dfrac{1}{1-x} = 1 + x + \dfrac{3}{2}x^2 + x^3 + \mathrm{H.O.T.}$
- $\ln \dfrac{1}{1-x} = x + \dfrac{1}{2}x^2 + \dfrac{1}{3}x^3 + \mathrm{H.O.T.}$

Question 4

Compute the Taylor series expansion about $x = 0$ of the function $f(x) = \ln \dfrac{1+2x}{1-2x}$. For what values of x does the series converge?

Hint: use the properties of the logarithm function to separate the quotient inside into two pieces.

- $f(x) = \sum_{k=1}^{\infty} \dfrac{2^{2k}}{k} x^{2k}$ for $|x| < \dfrac{1}{2}$.
- $f(x) = \sum_{k=1}^{\infty} \dfrac{2^{2k+2}}{2k+1} x^{2k+1}$ for $|x| < 1$.
- $f(x) = \sum_{k=1}^{\infty} \dfrac{2^{2k}}{k} x^{2k}$ for $|x| < 1$.
- $f(x) = \sum_{k=1}^{\infty} \dfrac{2^{2k}}{2k-1} x^{2k-1}$ for $|x| < 1$.
- $f(x) = \sum_{k=1}^{\infty} \dfrac{2^{2k+2}}{2k+1} x^{2k+1}$ for $|x| < \dfrac{1}{2}$.
- $f(x) = \sum_{k=1}^{\infty} \dfrac{2^{2k}}{2k-1} x^{2k-1}$ for $|x| < \dfrac{1}{2}$.

Question 5

Use the binomial series to find the Taylor series about $x = 0$ of the function $f(x) = \left(9 - x^2\right)^{-1/2}$. Indicate for which values of x the series converges to the function.

- $f(x) = \sum_{k=0}^{\infty} (-1)^k \binom{-1/2}{k} \dfrac{x^{2k}}{3^{2k-1}}$ for $|x| < \dfrac{1}{3}$.
- $f(x) = \sum_{k=0}^{\infty} \binom{-1/2}{k} \dfrac{x^{2k}}{3^{2k}}$ for $|x| < 3$.

- $f(x) = \sum_{k=0}^{\infty} (-1)^k \binom{-1/2}{k} \frac{x^{2k}}{3^{2k-1}}$ for $|x| < 3$.
- $f(x) = \sum_{k=0}^{\infty} (-1)^k \binom{-1/2}{k} \frac{x^{2k}}{3^{2k+1}}$ for $|x| < \frac{1}{3}$.
- $f(x) = \sum_{k=0}^{\infty} (-1)^k \binom{-1/2}{k} \frac{x^{2k}}{3^{2k+1}}$ for $|x| < 3$.
- $f(x) = \sum_{k=0}^{\infty} \binom{-1/2}{k} \frac{x^{2k}}{3^{2k}}$ for $|x| < \frac{1}{3}$.

Question 6

Use the fact that

$$\arcsin x = \int \frac{dx}{\sqrt{1-x^2}}$$

and the binomial series to find the Taylor series about $x = 0$ of $\arcsin x$ up to terms of order five.

- $\arcsin x = x - \frac{x^3}{6} + \frac{3x^5}{20} + \text{H.O.T.}$
- $\arcsin x = 1 + x + \frac{x^3}{6} + \frac{3x^5}{40} + \text{H.O.T.}$
- $\arcsin x = x + \frac{x^3}{6} + \frac{3x^5}{40} + \text{H.O.T.}$
- $\arcsin x = 1 + x + \frac{x^3}{6} + \frac{3x^5}{20} + \text{H.O.T.}$
- $\arcsin x = x - \frac{x^3}{6} + \frac{3x^5}{40} + \text{H.O.T.}$
- $\arcsin x = x + \frac{x^3}{6} + \frac{3x^5}{20} + \text{H.O.T.}$

Question 7

Compute the Taylor series about $x = 0$ of the function $\arctan(e^x - 1)$ up to terms of degree three.

- $\arctan(e^x - 1) = x - \frac{x^2}{2} + \frac{x^3}{6} + \text{H.O.T.}$
- $\arctan(e^x - 1) = x - \frac{x^2}{2} - \frac{x^3}{3} + \text{H.O.T.}$
- $\arctan(e^x - 1) = x + \frac{x^2}{2} + \frac{x^3}{6} + \text{H.O.T.}$
- $\arctan(e^x - 1) = x + \frac{x^2}{2} - \frac{x^3}{6} + \text{H.O.T.}$
- $\arctan(e^x - 1) = x + \frac{x^2}{2} - \frac{x^3}{3} + \text{H.O.T.}$
- $\arctan(e^x - 1) = x - \frac{x^2}{2} - \frac{x^3}{6} + \text{H.O.T.}$

Question 8

We have derived Taylor series expansions about $x = 0$ for the sine and arctangent functions. The first one converges over the whole real line, but the second one does so only when its input is smaller than 1 in absolute value. If you try using these to find the Taylor series of

$$\arctan\left(\frac{1}{2}\sin x\right)$$

where would the resulting series converge to the function?

- $|x| < \frac{1}{2}$
- $\mathbb{R} = (-\infty, +\infty)$
- $|x| < 1$

- $|x| < 2$

- [ ] In accordance with the Honor Code, I certify that my answers here are my own work.

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