

# Homework 9

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

## Question 1

$$\lim_{x \rightarrow +\infty} \frac{x^2 + x + 1}{x^4 - 3x^2 + 2} =$$

- ☐  $+\infty$
- ☐  $-\infty$
- ☐  $\frac{1}{2}$
- ☐  $0$
- ☐  $-\frac{1}{3}$
- ☐  $1$

## Question 2

$$\lim_{x \rightarrow +\infty} \frac{6x^2 - 3x + 1}{3x^2 + 4} =$$

- ☐  $\frac{1}{4}$
- ☐  $-\infty$
- ☐  $2$
- ☐  $\frac{1}{3}$
- ☐  $+\infty$
- ☐  $0$

### Question 3

$$\lim_{x \rightarrow +\infty} \frac{(x^2 - 3)(x^2 + 3)}{2x^4 - 2x^2 + 1} =$$

- ☐ 0
- ☐  $+\infty$
- ☐  $-9$
- ☐ The limit does not exist.
- ☐ 1
- ☐  $\frac{1}{2}$

### Question 4

$$\lim_{x \rightarrow +\infty} \frac{e^{2x}}{x^3 + 3x^2 + 4} =$$

- ☐  $e^2$
- ☐ 0
- ☐  $\frac{1}{3}$
- ☐  $+\infty$
- ☐  $\frac{1}{4}$
- ☐  $-\infty$

### Question 5

$$\lim_{x \rightarrow +\infty} \frac{\ln x}{\sqrt{x}} =$$

- ☐  $\ln 2$
- ☐  $\sqrt{2}$

- ☐ 1
- ☐  $-\infty$
- ☐ 0
- ☐  $+\infty$

## Question 6

$$\lim_{x \rightarrow +\infty} \frac{x^3 \ln x}{e^x} =$$

- ☐  $+\infty$
- ☐ The limit does not exist.
- ☐ 1
- ☐  $e^{-1}$
- ☐  $-\infty$
- ☐ 0

## Question 7

$$\lim_{x \rightarrow +\infty} \frac{e^{x^2}}{e^{3x}} =$$

- ☐ 0
- ☐  $+\infty$
- ☐  $e^{-1/3}$
- ☐  $e$
- ☐ The limit does not exist.
- ☐  $\frac{1}{3}$

## Question 8

$$\lim_{x \rightarrow +\infty} \frac{e^x (x-1)!}{x!} =$$

- ☐ 0
- ☐  $e$
- ☐  $+\infty$
- ☐ 1
- ☐  $e^x$
- ☐  $e$

## Question 9

$$\lim_{x \rightarrow +\infty} \frac{(x+1)!}{2^x + 1} =$$

- ☐  $-\infty$
- ☐  $+\infty$
- ☐ 1
- ☐  $\frac{1}{2}$
- ☐ 0
- ☐ 2

## Question 10

Evaluate the following limit, where  $n$  is a positive integer:  $\lim_{x \rightarrow +\infty} \frac{(3 \ln x)^n}{(2x)^n}$ .

- ☐  $\frac{3^n}{2^n}$

- ☐ 3
- ☐  $\frac{3}{2}$
- ☐ 0
- ☐ 2
- ☐  $+\infty$

## Question 11

Which of the following are in  $O(x^2)$  as  $x \rightarrow 0$ ? Select all that apply.

Hint: remember  $O(x^2)$  consists of those functions which go to zero at least as quickly as  $Cx^2$  for some constant  $C$ .

- ☐  $\sinh x$
- ☐  $\sin x^2$
- ☐  $5x$
- ☐  $\ln(1+x)$
- ☐  $5x^2 + 3x^4$
- ☐  $\sqrt{x + 3x^4}$

## Question 12

Which of the following are in  $O(x^2)$  as  $x \rightarrow +\infty$ ? Select all that apply.

Hint: recall  $O(x^2)$  consists of those functions that are  $\leq Cx^2$  for some constant  $C$  as  $x \rightarrow +\infty$ .

- ☐  $\ln(x^{10} + 1)$
- ☐  $\arctan x^2$
- ☐  $\sqrt{x^5 - 2x^3 + 1}$

- ☐  $5\sqrt{x^2 + x - 1}$
- ☐  $x^3 - 5x^2 - 11x + 4$
- ☐  $e^{\sqrt{x}}$

## Question 13

Simply the following asymptotic expression:

$$f(x) = \left(x - x^2 + O(x^3)\right) \cdot \left(1 + 2x + O(x^3)\right)$$

(here, the big-O means as  $x \rightarrow 0$ )

Hint: do not be intimidated by the notation; simply pretend that  $O(x^3)$  is a cubic monomial in  $x$  and use basic multiplication of polynomials.

- ☐  $f(x) = x + x^2 + O(x^4)$
- ☐  $f(x) = x + x^2 + O(x^3)$
- ☐  $f(x) = 1 + x + x^2 + O(x^3)$
- ☐  $f(x) = 1 + 3x - x^2 + O(x^3)$
- ☐  $f(x) = x + x^2 - 2x^3 + O(x^6)$
- ☐  $f(x) = x + x^2 - 2x^3 + O(x^3)$

## Question 14

Simplify the following asymptotic expression:

$$f(x) = \left(x^3 + 2x^2 + O(x)\right) \cdot \left(1 + \frac{1}{x} + O\left(\frac{1}{x^2}\right)\right)$$

(here, the big-O means as  $x \rightarrow +\infty$ )

Hint: do not be intimidated by the notation! Pretend that  $O(x)$  is of the form  $Cx$  for some  $C$  and likewise with  $O(1/x^2)$ . Multiply just like these are polynomials,

then simplify at the end.

- ☐  $f(x) = x^3 + 2x^2 + O(x)$
- ☐  $f(x) = x^3 + 3x^2 + 2x + O(x) + O(1) + O(\frac{1}{x})$
- ☐  $f(x) = x^3 + 3x^2 + 2x + O(x)$
- ☐  $f(x) = x^3 + 3x^2 + O(x)$
- ☐  $f(x) = x^3 + 3x^2 + 2x + O(\frac{1}{x})$

☐ In accordance with the Honor Code, I certify that my answers here are my own work.

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