Homework 14

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Find all the local maxima and minima of the function xe^{-x^2}.

- The function has a local maximum at $x = \frac{\sqrt{2}}{2}$, and a local minimum at $x = -\frac{\sqrt{2}}{2}$.
- The function has local maxima at $x = \frac{\sqrt{2}}{2}$ and $x = -\frac{\sqrt{2}}{2}$, but no local minima.
- The function has local minima at $x = \frac{\sqrt{2}}{2}$ and $x = -\frac{\sqrt{2}}{2}$, and a local maximum at $x = 0$.
- The function has a local maximum at $x = -\frac{\sqrt{2}}{2}$, and a local minimum at $x = \frac{\sqrt{2}}{2}$.
- The function has local minima at $x = \frac{\sqrt{2}}{2}$ and $x = -\frac{\sqrt{2}}{2}$, but no local maxima.
- The function has local maxima at $x = \frac{\sqrt{2}}{2}$ and $x = -\frac{\sqrt{2}}{2}$, and a local minimum at $x = 0$.

Question 2

Which of the following statements is true about the function $f(x) = e^{\sin(x^4)} \cos(x^2)$?

- Its Taylor series expansion about $x = 0$ is $1 + \frac{x^4}{2} + O(x^5)$. Hence it has a local maximum at $x = 0$.
- Its Taylor series expansion about $x = 0$ is $1 - \frac{x^2}{2} + O(x^5)$. Hence it has a local minimum at $x = 0$.
- Its Taylor series expansion about $x = 0$ is $1 + \frac{x^4}{2} + O(x^5)$. Hence it has a local minimum at $x = 0$.
- Its Taylor series expansion about $x = 0$ is $1 + \frac{x^3}{2} + O(x^4)$. Hence $x = 0$ is a critical point of $f(x)$ that is neither a local maximum nor a local minimum.
- Its Taylor series expansion about $x = 0$ is $1 - \frac{x^2}{2} + O(x^5)$. Hence it has a local maximum at $x = 0$.
- Its Taylor series expansion about $x = 0$ is $1 - \frac{x}{3} + O(x^2)$. Hence $x = 0$ is not a critical point of $f(x)$.

Question 3

Use a Taylor series about $x = 0$ to determine whether the function $f(x) = \sin^3(x^3)$ has a local maximum or local minimum at the origin.

- $x = 0$ is a local minimum of f.
- $x = 0$ is a critical point of f, but it is neither a local maximum nor a local minimum.
- $x = 0$ is not a critical point of f.
- $x = 0$ is a local maximum of f.

Question 4

Find the location of the global maximum and minimum of $f(x) = x^3 - 6x^2 + 1$ on the interval $[-1, 7]$.

- The global maximum is attained at $x = 0$ and the global minimum at $x = -1$.
- The global maximum is attained at $x = 0$, but there is no global minimum.
- The global maximum is attained at $x = 0$ and the global minimum at $x = 4$.
- The global maximum is attained at $x = 7$ and the global minimum at $x = -1$.
- The global maximum is attained at $x = 7$, but there is no global minimum.
- The global maximum is attained at $x = 7$ and the global minimum at $x = 4$.

Question 5

Which of the following statements are true for the function $f(x) = x^3 + \frac{48}{x^2}$?

- [ ] $x = 2$ is the global maximum of f in $[-3, 3]$
- [ ] $x = -1$ is the global maximum of f in $[-3, -1]$
- [ ] $x = -2$ is the global maximum of f in $[-3, -1]$
- [ ] $x = 1$ is the global maximum of f in $[1, 3]$
- [ ] $x = 1$ is the global minimum of f in $[1, 3]$
- [ ] $x = 2$ is the global minimum of f in $[1, 3]$

Question 6

In this problem we will study a very basic traffic model. Consider a stretch of highway in which cars are traveling at an average speed v. The *traffic density* u is the total amount of cars on our stretch of road divided by its length. These two quantities are related: the less cars on the road, the faster drivers are able to go. On the other hand, if traffic becomes heavy, drivers will naturally decrease their speed. The so-called *parabolic model* assumes that this relationship is dictated by the equation

$$u = u_{\max}\left(1 - \frac{v}{v_{\max}}\right)$$

where $u_{\max}$ represents the capacity of the road, and $v_{\max}$ the speed limit on it.

The amount of cars passing through our road is called the *traffic flux* or *throughput*, and is given by the product of the traffic density and the average speed:

$$F = uv$$

Using the parabolic model, find out at what average speed v_* the flux through our road is maximized.

- $v_* = \dfrac{v_{\max}}{4}$
- $v_* = \dfrac{3v_{\max}}{4}$
- $v_* = v_{\max}$
- $v_* = 0$
- $v_* = \dfrac{v_{\max}}{2}$

Question 7

A manufacturing company wants to know how many workers it should hire. If it employs too many people, the machines in the factory will be overutilized and the workers will have to wait until they are free, thus reducing the number of units each one will produce in a day's work. On the other hand, too few workers would leave the machines idle for long periods of time.

A rough model for the relationship between the number n of workers and their productivity p is given by the equation

$$p = p_{\max}\left(1 - \frac{n}{n_{\max}}\right)$$

where $p_{\max} = 10$ is the maximum number of units a worker can produce in a day and $n_{\max} = 100$ is the maximum number of workers the factory can accomodate.

The amount of units U manufactured in the whole factory in one day is equal to the product of the number of workers and the number of units each one produces:

$$U = np$$

How many workers should the company hire in order to maximize its productivity?

- 10 workers
- 50 workers
- 67 workers
- 75 workers
- 25 workers
- 90 workers

Question 8

A technology company has just invented a new gadget. In order to maximize the profit derived from its sale, the company must make a critical decision: at what price should it be sold? A market study suggests that the number N of units sold would approximately follow the equation

$$N = N_{\max} e^{-P/\lambda}$$

where P is the sale price, $N_{\max} = 10,000,000$ is the number of units that would saturate the market, and $\lambda = \$50$.

If it costs $\$250$ to manufacture one of these gadgets, at what price P_* would be profit of the company be maximized?

- $P_* = \$275$

- $P_* = \$325$
- $P_* = \$300$
- $P_* = \$350$
- $P_* = \$375$
- $P_* = \$400$

Question 9

The manufacturing process of a certain chemical substance is exothermic, that is, it releases heat. The amount of heat released, Q, depends on the temperature T at which the process is carried out, and it is given by the equation

$$Q = \alpha (T - T_0)^{-2} e^{(T - T_0)/\lambda}$$

where $T_0 = 70^{\rm o} F$ is the room temperature of the manufacturing plant, and $\alpha = 3000\, J\, (^{\rm o} F)^2$ and $\lambda = 50^{\rm o} F$.

If the temperature T must be maintained above $100^{\rm o} F$, at what temperature T_* would be the heat loss be minimized?

- $T_* = 220^{\rm o} F$
- $T_* = 100^{\rm o} F$
- $T_* = 120^{\rm o} F$
- $T_* = 150^{\rm o} F$
- $T_* = 270^{\rm o} F$
- $T_* = 170^{\rm o} F$

In accordance with the Honor Code, I certify that my answers here are my own work.

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