Homework 29

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Let m and n be positive integers. Consider the function $f(x) = x^n e^{mx}$. Use WolframAlpha or your favorite computer algebra system to *guess* the coefficient of the $x^{n-1}e^{mx}$ term in the antiderivative $\int f(x)\,dx$.

Hint: first try many small values of m and n. Can you figure out what happens when you fix $m = 1$ and change n? And when you fix $n = 1$ while m varies?

- $-\dfrac{1}{m^2}$
- $-\dfrac{m+n}{2}$
- $\dfrac{-m}{n}$
- $-\dfrac{n}{m^2}$
- $-m^2 n$
- $-n$

Question 2

The function

$$f(x) = \frac{1}{\sqrt{2\pi}}\, e^{-x^2/2}$$

does not have an antiderivative that can be expressed in terms of elementary functions. In particular, none of the methods that we have studied in this course helps in computing such an antiderivative. Use WolframAlpha or your favorite computer algebra system to estimate the area under this curve between $x = -1$ and $x = 1$.

For the curious: The function $f(x)$ is the *Gaussian* function (with mean 0 and standard deviation 1), also called the *bell curve*. The answer you will compute has a statistical interpretation: if a random variable (say, the outcome of an experiment with some uncertainty) is a real number that is *normally distributed* with mean μ and standard deviation σ, your answer gives the probability that a single outcome gives you a value that is within σ of μ (that is, less than one standard deviation away from the mean). If you are confused, don't worry: we will cover this topic in more detail in Lectures 43 and 44.

- 0.831548
- 0.682689
- 1
- 1.71125
- 0.595879
- 0.736098

Question 3

The integral

$$\int_{t=0}^{2\pi} \sqrt{1 + 5\cos^2 t}\, dt$$

computes the arc length L of a certain ellipse. Integrals such as this one are called *elliptic integrals* and are rather difficult to calculate —the integrand does not have an antiderivative in terms of elementary functions. Use WolframAlpha or your favorite computer algebra system to find the approximate value of L.

- $10\sin^3 t$

- 10
- $\frac{3\pi}{7}$
- $\frac{\pi^2}{6}$
- 11.3208
- 5.6604

Question 4

For any nice function $f(t)$ defined on $t > 0$, the integral

$$L\{f(t)\}(s) = \int_{t=0}^{+\infty} f(t)e^{-st}\,dt$$

is called the *Laplace transform* of f and is a function of s with domain $s > 0$.

Use WolframAlpha or your favorite computer algebra system to compute the Laplace transforms of some simple functions —such as $t^2, e^t, \sin t$, etc.— and compare them to the Laplace transforms of the same simple functions multiplied by t. For instance, find and compare

$$L\{e^t\}(s) = \int_{t=0}^{+\infty} e^t e^{-st}\,dt, \qquad \text{and} \qquad L\{te^t\}(s) = \int_{t=0}^{+\infty} te^t e^{-st}\,dt.$$

Based on your experiments, what is the relationship between the Laplace transform $L\{f(t)\}(s)$ of a function $f(t)$ and the Laplace transform $L\{tf(t)\}(s)$ of $tf(t)$?

- $L\{tf(t)\}(s) = -\frac{d}{ds}\,L\{f(t)\}(s)$
- $L\{tf(t)\}(s) = -\frac{1}{s-1}\,L\{f(t)\}(s)$
- $L\{f(t)\}(s) = -\frac{d}{ds}\,L\{tf(t)\}(s)$

- $L\{tf(t)\}(s) = \frac{d}{ds} L\{f(t)\}(s)$
- $L\{f(t)\}(s) = -\frac{1}{s-1} L\{tf(t)\}(s)$
- $L\{tf(t)\}(s) = \frac{1}{s-1} L\{f(t)\}(s)$

☐ In accordance with the Honor Code, I certify that my answers here are my own work.

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