Homework 15

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Use implicit differentiation to find $\frac{dy}{dx}$ from the equation $y^2 - y = \sin 2x$.

- $\frac{dy}{dx} = \frac{2y-1}{2\cos 2x}$
- $\frac{dy}{dx} = \frac{\sin 2x}{2y-1}$
- $\frac{dy}{dx} = \frac{2y-1}{\sin 2x}$
- $\frac{dy}{dx} = \frac{y^2-y}{2\cos 2x}$
- $\frac{dy}{dx} = \frac{2\cos 2x}{y^2-y}$
- $\frac{dy}{dx} = \frac{2\cos 2x}{2y-1}$

Question 2

Find the derivative $\frac{dy}{dx}$ if x and y are related through $xy = e^y$.

- $\frac{dy}{dx} = \frac{y}{x-e^y}$
- $\frac{dy}{dx} = \frac{x-e^y}{y}$
- $\frac{dy}{dx} = \frac{y}{e^y+x}$

- $\dfrac{dy}{dx} = \dfrac{e^y - x}{y}$
- $\dfrac{dy}{dx} = \dfrac{y}{e^y - x}$
- $\dfrac{dy}{dx} = \dfrac{e^y + x}{y}$

Question 3

Use implicit differentiation to find $\dfrac{dy}{dx}$ if $\sin x = e^{-y\cos x}$.

- $\dfrac{dy}{dx} = y\cos x - e^{y\cos x}\sin x$
- $\dfrac{dy}{dx} = y\tan x - e^{y\cos x}$
- $\dfrac{dy}{dx} = \dfrac{y - e^{y\cos x}\tan x}{\sin x}$
- $\dfrac{dy}{dx} = -y\sin x + e^{-y\cos x}\cos x$
- $\dfrac{dy}{dx} = e^{-y\cos x}(\cos x - y\sin x)$
- $\dfrac{dy}{dx} = \dfrac{y\sin x - e^{-y\cos x}}{\cos x}$

Question 4

Find the derivative $\dfrac{dy}{dx}$ from the equation $x\tan y - y^2\ln x = 4$.

- $\dfrac{dy}{dx} = \tan y - \dfrac{y^2}{\sec^2 y}$
- $\dfrac{dy}{dx} = \dfrac{x\tan y - y^2}{2xy\ln x - x^2\sec^2 y}$

- $\frac{dy}{dx} = \frac{x \tan y}{2xy \ln x}$
- $\frac{dy}{dx} = \frac{-y^2}{x^2 \sec^2 y}$
- $\frac{dy}{dx} = \frac{2xy \ln x - x^2 \sec^2 y}{x \tan y - y^2}$
- $\frac{dy}{dx} = \frac{y^2 - \tan y}{x^2 \sec^2 y - 2xy \ln x}$

Question 5

Model a hailstone as a round ball of radius R. As the hailstone falls from the sky, its radius increases at a constant rate C. At what rate does the volume V of the hailstone change?

- $\frac{dV}{dt} = 8\pi CR$
- $\frac{dV}{dt} = \frac{4}{3}\pi R^3$
- $\frac{dV}{dt} = 4\pi R^2$
- $\frac{dV}{dt} = \frac{4}{3}\pi CR^3$
- $\frac{dV}{dt} = 4\pi CR^2$
- $\frac{dV}{dt} = \frac{4}{3}\pi C^3$

Question 6

A large tank of oil is slowly leaking oil into a containment tank surrounding it. The oil tank is a vertical cylinder with a diameter of 10 meters. The containment tank has a square base with side length of 15 meters and tall vertical walls. The bottom of the oil tank and the bottom of the containment tank are concentric (the round base inside the square base). Denote by h_o the height of the oil inside of the oil tank, and by h_c the height of the oil in the containment tank. How are the rates of change of these two quantities related?

- $dh_c = -\dfrac{25\pi}{225}\, dh_o$
- $dh_c = -\dfrac{225}{25\pi}\, dh_o$
- $dh_c = -\dfrac{25\pi}{225 - 25\pi}\, dh_o$
- $dh_c = (225 - 25\pi) dh_o$
- $dh_c = (25\pi - 225) dh_o$
- $dh_c = -\dfrac{225 - 25\pi}{25\pi}\, dh_o$

Question 7

The volume of a box of height h and with a square base of side L is $V = L^2 h$. If increase L by 10%, and decrease h by 10%, by what percentage does the volume of the box change?

Hint: consider the relative rate of change of the volume of the box.

- It does not change.
- It grows 5%.
- It shrinks 5%.
- It grows 10%.
- It grows 15%.
- It shrinks 10%.

Question 8

The *stopping distance* D_{stop} is the distance traveled by a vehicle from the moment the driver becomes aware of an obstacle in the road until the car stops completely. This occurs in two phases.

The first one, the *reaction phase*, spans from the moment the driver sees the obstacle until he or she has completely depressed the brake pedal. This entails taking the decision to stop the vehicle, lifting the foot from the gas pedal and onto the brake pedal, and pressing the latter down its full distance to obtain maximum braking power. The amount of time necessary to do all this is called the *reaction time* t_{react}, and is independent of the speed at which the vehicle was traveling. Although this quantity varies from driver to driver, it is typically between $1.5\,\text{s}$ and $2.5\,\text{s}$. For the purposes of this problem, we will use an average value of $2\,\text{s}$. The distance traversed by the vehicle in this time is unsurprisingly called *reaction distance* D_{react} and is given by the formula

$$D_{\text{react}} = vt_{\text{react}}$$

where v is the initial speed of the vehicle.

In the *braking phase*, the vehicle decelerates and comes to a complete stop. The *braking distance* D_{brake} that the vehicle covers in this phase is proportional to the square of the initial speed of the vehicle:

$$D_{\text{brake}} = \alpha v^2$$

The constant of proportionality α depends on the vehicle type and condition, as well as on the road conditions. Consider a typical value of $10^{-2}\,\text{s}^2/\text{m}$.

If the initial speed of the vehicle is $108\,\text{km/h} = 30\,\text{m/s}$, what is the ratio between the relative rate of change of the stopping distance and the relative rate of change of the initial speed?

- $\dfrac{dD_{\text{stop}}/D_{\text{stop}}}{dv/v} = \dfrac{28}{26}$
- $\dfrac{dD_{\text{stop}}/D_{\text{stop}}}{dv/v} = \dfrac{26}{23}$

- $\dfrac{dD_{\text{stop}}/D_{\text{stop}}}{dv/v} = \dfrac{25}{23}$
- $\dfrac{dD_{\text{stop}}/D_{\text{stop}}}{dv/v} = 1$
- $\dfrac{dD_{\text{stop}}/D_{\text{stop}}}{dv/v} = \dfrac{27}{23}$
- $\dfrac{dD_{\text{stop}}/D_{\text{stop}}}{dv/v} = \dfrac{24}{23}$

- [ ] In accordance with the Honor Code, I certify that my answers here are my own work.

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