Feedback — Final Exam

Thank you. Your submission for this exam was received.

You submitted this exam on **Sun 14 Apr 2013 1:51 PM EDT -0400**. You got a score of **159.67** out of **170.00**. You can attempt again in 10 minutes.

Question 1

What is the centroid of the region bounded by the curves $y = x^2$ and $y = 5 - x^2$?

Hint: draw a picture of this region as your first step.

Your Answer		Score	Explanation
◉ $(\bar{x}, \bar{y}) = \left(0, \frac{5}{2}\right)$	✓	10.00	
○ $(\bar{x}, \bar{y}) = \left(0, \frac{\sqrt{3}}{2}\right)$			
○ $(\bar{x}, \bar{y}) = \left(0, \frac{5}{4}\right)$			
○ $(\bar{x}, \bar{y}) = \left(0, \frac{32}{5}\right)$			
○ $(\bar{x}, \bar{y}) = \left(0, \frac{3}{2}\right)$			
○ $(\bar{x}, \bar{y}) = (0, 2)$			
○ $(\bar{x}, \bar{y}) = (0, 0)$			
○ $(\bar{x}, \bar{y}) = \left(1, \frac{3}{2}\right)$			
Total		10.00 / 10.00	

Question 2

If $x(t)$ satisfies the differential equation

$$\frac{dx}{dt} = 2e^{t-x}$$

and $x(0) = 0$, then what is $x(1)$?

Your Answer		Score	Explanation
◉ $\ln(2e-1)$	✓	10.00	
○ $\ln(2e+1)$			
○ e^2-5			
○ $\ln(3e-2)$			
○ e^2-1			
○ $\ln(e+6)$			
○ e^2+5			
○ $\ln(3e+2)$			
Total		10.00 / 10.00	

Question 3

What is the area between the curves $y = -x$ and $y = \dfrac{2}{x(x+2)}$ for $1 \le x \le e$?

Your Answer		Score	Explanation
◉ $\dfrac{e^2+1}{2} + \ln\dfrac{3}{2+e}$	✓	10.00	
○ $\dfrac{e^2-1}{5} + \ln\dfrac{1}{3-e}$			
○ $\dfrac{e^2-1}{5}$			

$\frac{e^2}{3} + \ln \frac{1}{3+e}$	
$2e^2 + \ln \frac{7}{2+e}$	
$e^2 + \ln \frac{2}{4+e}$	
$\ln \frac{10}{3+2e}$	
$e^2 + \ln \frac{5}{4+e}$	
Total	10.00 / 10.00

Question 4

Compute the arc-length of the graph of $y = \frac{x^2}{4} - \frac{\ln x}{2}$ for $1 \leq x \leq e^2$.

Hint: your expression for the arclength element dL should admit a "miracle" factorization that eliminates a certain square root.

Your Answer		Score	Explanation
$\frac{e^4+3}{4}$	✓	10.00	
$\frac{e^6+11}{12}$			
$\frac{e-1}{2}$			
$\frac{e^2+1}{2}$			
$\frac{e^2-7}{4}$			
$\frac{e^4+7}{4}$			
$\frac{e^4-3}{2}$			

○ $\frac{e^4 - 11}{12}$			
Total		10.00 / 10.00	

Question 5

What is the volume of the solid generated by rotating about the y-axis the region defined by the inequalities:

1. $y \geq 2x^3$, and
2. $y \leq 2x$.

Your Answer		Score	Explanation
◉ $\frac{8\pi}{15}$	✓	10.00	
○ $\frac{32\pi}{105}$			
○ $\frac{\pi}{6}$			
○ $\frac{4\pi}{15}$			
○ $\frac{224\pi}{15}$			
○ $\frac{\pi}{3}$			
○ $\frac{2\pi}{5}$			
○ $\frac{16\pi}{21}$			
Total		10.00 / 10.00	

Question 6

You approximate $\ln \frac{3}{2}$ using the series:

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$$

If you use the first four terms as your approximation — that is, using terms up to and including the x^4 term — then what is the bound on your error E that comes from observing that the series above is alternating?

Your Answer		Score	Explanation
◉ $E < \frac{1}{160}$	✓	10.00	
○ $E < \frac{1}{324}$			
○ $E < \frac{1}{120}$			
○ $E < \frac{1}{24}$			
○ $E < \frac{1}{1024}$			
○ $E < \frac{1}{384}$			
○ $E < \frac{1}{720}$			
○ $E < \frac{1}{1215}$			
Total		10.00 / 10.00	

Question 7

Compute $\int e^{2x} \sin x \, dx$.

Your Answer		Score	Explanation
◉ $\frac{1}{5} e^{2x}(2\sin x - \cos x) + C$	✓	10.00	

- $-\frac{2}{5}e^{2x}(\cos x - 2\sin x) + C$
- $\frac{2}{5}e^{2x}(\sin x - 2\cos x) + C$
- $-\frac{1}{5}e^{2x}(\cos x - \sin x) + C$
- $\frac{2}{5}e^{2x}(\cos x - 2\sin x) + C$
- $\frac{2}{5}e^{2x}(2\cos x + \sin x) + C$
- $-\frac{1}{5}e^{2x}(2\cos x - \sin x) + C$
- $-\frac{2}{5}e^{x}(\cos x + \sin x) + C$

Total 10.00 / 10.00

Question 8

Which of the following is $\int_{x=0}^{2} \sqrt{4 - x^2}\, dx$?

Your Answer		Score	Explanation
π	✓	10.00	
$\frac{4}{3}$			
$\frac{\pi}{4}$			
$\frac{64}{15}$			
4π			
$\frac{\sqrt{2}}{3}$			
$\frac{\pi}{2}$			

○ $\frac{\sqrt{3}}{2}$		
Total	10.00 / 10.00	

Question 9

Which of the following functions is in $O(x^4)$ as $x \to +\infty$? Select all that apply.

Your Answer		Score	Explanation
☑ x^4	✓	1.25	
☐ $\frac{e^x}{x^5}$	✓	1.25	
☑ $\arctan x$	✓	1.25	
☑ $x(x+1)(x+2)$	✓	1.25	
☐ $\cosh x$	✓	1.25	
☑ $\frac{x^5}{1000 \ln x}$	✗	-1.25	
☑ $x^3 \ln(x^2)$	✓	1.25	
☑ $(x^2+1)^2\sqrt{3x}$	✗	-1.25	
Total		5.00 / 10.00	

Question 10

Evaluate the limit $\lim_{x \to 0} \frac{\ln\left(1+\sin^2(2x)\right)}{5x^2}$

Your Answer		Score	Explanation
◉ $\frac{4}{5}$	✓	10.00	

- $-\frac{4}{5}$
- $\frac{1}{5}$
- $\frac{2}{5}$
- $\frac{1}{10}$
- The limit does not exist.
- $-\frac{2}{5}$
- 0

Total 10.00 / 10.00

Question 11

Compute the expectation $\mathbb{E}$ of the probability distribution function

$$\rho(x) = \kappa\sqrt{x-1}, \qquad 1 \leq x \leq 2$$

where κ is the appropriate constant.

Your Answer		Score	Explanation
◉ $\mathbb{E} = \frac{8}{5}$	✓	10.00	
○ $\mathbb{E} = \frac{8}{3}$			
○ $\mathbb{E} = \frac{4}{3}$			
○ $\mathbb{E} = \frac{2}{5}$			
○ $\mathbb{E} = \frac{9}{8}$			
○ $\mathbb{E} = \frac{11}{6}$			

$\mathbb{E} = \frac{9}{5}$	
$\mathbb{E} = \frac{3}{2}$	
Total	10.00 / 10.00

Question 12

Which of the following sequences is the second forward difference, $\Delta^2 a$, of the sequence

$$a = (-1, 5, 3, 2, 5, 0, 0, 2, 1, 4, 8, 7, -1, 12, 0, 0, 0, 0, \ldots)$$

Your Answer		Score	Explanation
$\Delta^2 a = (-8, 1, 4, -8, 5, 2, -3, 4, 1, -5, -7, 21, -25, 12, 0, 0, \ldots)$	✓	10.00	
$\Delta^2 a = (6, -1, -5, 8, 0, 2, -1, 3, 3, -1, -8, 13, -25, 12, 0, 0, \ldots)$			
$\Delta^2 a = (-8, 3, -4, -1, 2, 4, -3, 1, 0, 9, -4, 2, -20, 12, 0, 0, \ldots)$			
$\Delta^2 a = (6, -2, -1, 3, -5, 0, 2, -1, 3, 4, -1, 8, 13, -12, 0, 0, \ldots)$			
$\Delta^2 a = (1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 134, 223, 0, 0, 0, \ldots)$			
$\Delta^2 a = (-6, -1, -4, 8, -3, 12, -2, -4, 0, 5, 9, 12, -25, 12, 0, 0, \ldots)$			
$\Delta^2 a = (-8, 0, 1, -2, 6, 4, 3, -4, 2, 4, -5, 7, -2, 12, 0, 0, \ldots)$			
$\Delta^2 a = (-8, 3, 4, 8, 1, 9, -13, 2, 4, 5, -7, 12, -21, -12, 0, 0, \ldots)$			
Total		10.00 / 10.00	

Question 13

Which of the following statements about series convergence are true? Select all that apply.

Your Answer		Score	Explanation
☐	$\sum_{n=3}^{\infty}(-1)^n \frac{\ln n}{n^{3/4}}$ converges conditionally.	✗	-1.00
☐	$\sum_{n=1}^{\infty}\frac{n\cdot 3^n}{(2n)!}$ converges conditionally by the ratio test.	✓	1.67
☑	$\sum_{n=1}^{\infty}\frac{n\cdot 3^n}{(2n)!}$ converges absolutely by the ratio test.	✓	1.00
☑	$\sum_{n=5}^{\infty}\frac{\ln n}{\ln(\ln n)}$ diverges by the n-th term test.	✓	1.00
☑	$\sum_{n=5}^{\infty}\frac{\ln n}{\ln(\ln n)}$ diverges by the comparison test with $\sum_{n=5}^{\infty}\frac{1}{n}$.	✓	1.00
☐	$\sum_{n=0}^{\infty}(-1)^n$ converges conditionally.	✓	1.67
☑	$\sum_{n=0}^{\infty}(-1)^n \frac{n}{n^2+1}$ converges conditionally.	✓	1.00
☑	$\sum_{n=3}^{\infty}\frac{\ln n}{n^{3/4}}$ diverges by the n-th term test.	✗	-1.67
Total			4.67 / 10.00

Question 14

Using your knowledge of Taylor series, evaluate the following infinite series:

$$\frac{\pi}{4}-\frac{\pi^3}{4^3\cdot 3!}+\frac{\pi^5}{4^5\cdot 5!}-\ldots+(-1)^n\frac{\pi^{2n+1}}{4^{2n+1}\cdot(2n+1)!}+\ldots$$

Your Answer	Score	Explanation
	✓ 10.00	

◉ $\sin\frac{\pi}{4}$	
○ $\cos\frac{\pi}{4}$	
○ $\frac{3}{1-\pi}$	
○ $\frac{1}{1-\pi/3}$	
○ The series does not converge.	
○ $\cosh\frac{\pi}{4}$	
○ $\sin\frac{\pi}{2}$	
○ $\tan\frac{\pi}{4}$	
Total	10.00 / 10.00

Question 15

If x and y are related by the equation

$$x^2 + 4x^3y + y^2 = 3x^2y + 5,$$

find a formula for $\frac{dy}{dx}$ in terms of x and y.

Your Answer		Score	Explanation
◉ $\frac{dy}{dx} = \frac{6xy - 2x - 12x^2y}{4x^3 + 2y - 3x^2}$	✓	10.00	
○ $\frac{dy}{dx} = \frac{2x - 12x^2y}{4x^3 - 4y + 3x^2}$			
○ $\frac{dy}{dx} = \frac{-6xy + 2x}{2y + 3x^2}$			
○ $\frac{dy}{dx} = \frac{-6xy + 2x - 12x^2y}{4x^3 - 2y + 3x^2}$			

- $\frac{dy}{dx} = -6xy + 2x - 12x^2y$
- $\frac{dy}{dx} = \frac{6xy - 2x}{4x^3 - 2y + 3x^2}$
- $\frac{dy}{dx} = \frac{6xy - 12x^2y}{4x^3 + 3x^2}$
- $\frac{dy}{dx} = \frac{-1}{4x^3 - 2y + 3x^2}$

Total 10.00 / 10.00

Question 16

Which of the following is the Taylor expansion about $x = 0$ of

$$f(x) = \int_{t=0}^{x} \sin(t^2)\, dt$$

up to and including terms of order x^{10} ?

Your Answer	Score	Explanation
◉ $f(x) = \frac{1}{3}x^3 - \frac{1}{42}x^7 + O(x^{11})$	✓ 10.00	
○ $f(x) = x^3 - \frac{1}{6}x^7 + O(x^{11})$		
○ $f(x) = \frac{1}{2}x^2 - \frac{1}{30}x^6 - \frac{1}{120}x^{10} + O(x^{11})$		
○ $f(x) = \frac{1}{3}x^3 - \frac{1}{42}x^7 + \frac{1}{120}x^{10} + O(x^{11})$		
○ $f(x) = x - \frac{2}{5}x^5 + \frac{2}{27}x^9 + O(x^{11})$		
○ $f(x) = x^2 - \frac{1}{2}x^6 + \frac{1}{6}x^{10} + O(x^{11})$		
○ $f(x) = x^2 - \frac{1}{6}x^6 + \frac{1}{120}x^{10} + O(x^{11})$		
○ $f(x) = \frac{1}{2}x^2 - \frac{1}{24}x^4 + \frac{1}{720}x^6 + O(x^{11})$		

Total	10.00 / 10.00

Question 17

Which of the following is the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{n+2}{4^n n^2} (2x+1)^n$?

Your Answer		Score	Explanation
◉ $-\frac{5}{2} \leq x < \frac{3}{2}$	✓	10.00	
○ $-2 \leq x < 2$			
○ $-\frac{5}{2} \leq x \leq \frac{3}{2}$			
○ $-\frac{5}{2} < x < \frac{3}{2}$			
○ $-2 < x < 2$			
○ $-2 < x \leq 2$			
○ $-\frac{5}{2} < x \leq \frac{3}{2}$			
○ $-2 \leq x \leq 2$			
Total		10.00 / 10.00	