Homework 41

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

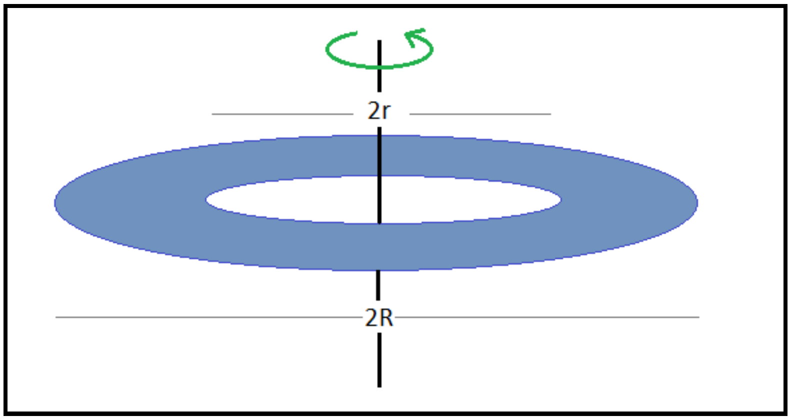
Three particles, each of mass m, are located at distances r_1, r_2 and r_3 respectively from a fixed axis of rotation A. We now place a fourth particle, also of mass m, at some distance r from the axis A. If the moment of inertia of all four particles is twice as big as the moment of inertia of the first three, what is r ?

Note: this question doesn't really use any calculus, but it will give you practice at remembering what moment of inertia *means*.

- $r = \sqrt[3]{2\left(r_1^3 + r_2^3 + r_3^3\right)}$
- $r = (r_1 + r_2 + r_3)\ln 2$
- $r = \sqrt{r_1^2 + r_2^2 + r_3^2}$
- $r = \sqrt[3]{r_1 r_2 r_3}$
- $r = \dfrac{2}{3}(r_1 + r_2 + r_3)$
- $r = \dfrac{r_1^2}{r_2} + \dfrac{r_2^2}{r_3} + \dfrac{r_3^2}{r_1}$

Question 2

In mathematics, an *annulus* is defined as the region between two circles with a common center. Assume you are given an annulus with outer radius R, inner radius r, and mass M distributed uniformly. What is its moment of inertia about the central axis shown in the picture below?

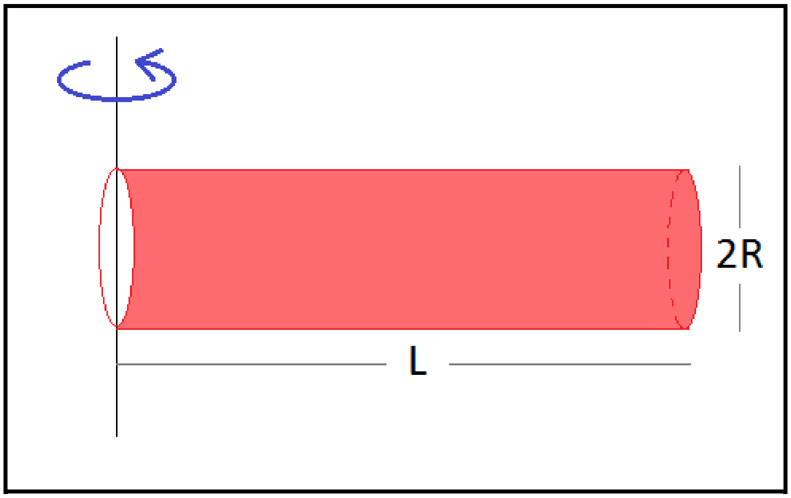


Hint: this problem becomes easier if you watch the bonus lecture first!

- $I_{\text{annulus}} = M(R-r)\sqrt{R^2 - r^2}$
- $I_{\text{annulus}} = \frac{1}{2}M(R^2 + r^2)$
- $I_{\text{annulus}} = \frac{1}{2}MR^2 - \frac{1}{4}Mr^2$
- $I_{\text{annulus}} = \frac{1}{2}M(R^2 - r^2)$
- $I_{\text{annulus}} = \frac{1}{4}M(R^2 - r^2)$
- $I_{\text{annulus}} = \frac{1}{4}M(R^2 + r^2)$

Question 3

A hollow cylindrical shell of length L and radius R is rotated about the an axis as shown in the picture.



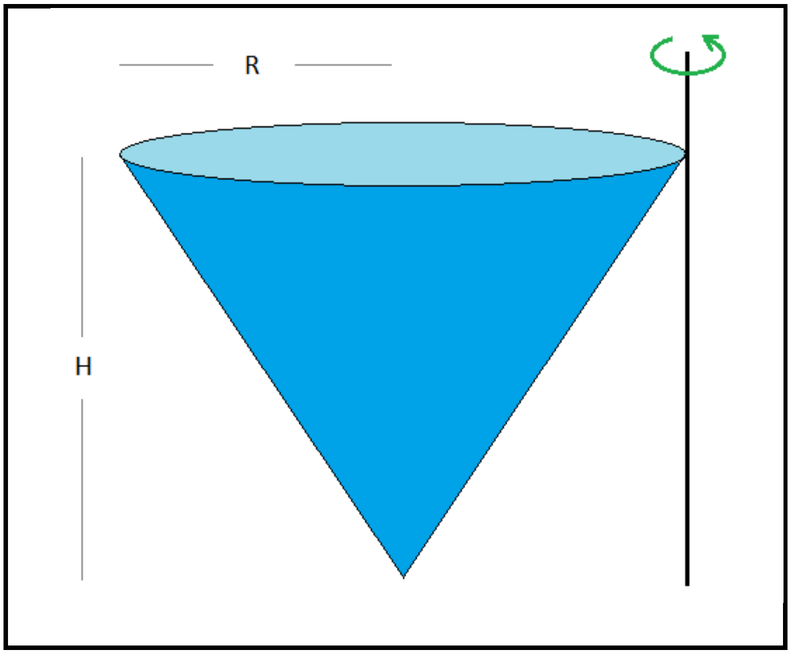
You may assume that this cylindrical shell does not have "caps" at either the left or the right edge, and that its mass M is distributed uniformly along the surface.You may also assume that R is so small that the piece of this cylinder at any distance r from the axis of rotation is a circle. What is its moment of inertia?

HInt: start by computing the area A and then the density $\rho = M/A$. Then, setting r to be a radial coordinate (distance-to-axis), the moment-of-inertia element is $dI = \rho r^2 dA$. For dA, use the approximation implied by the "R is small" assumption.

- $\frac{2}{5}M(L^2 + \pi R^2)$
- $\frac{1}{3}ML^2$
- $\frac{1}{4}MR^2$
- $\frac{1}{4}ML^2$
- $\frac{2}{3}ML^2$
- $\frac{2\pi}{3}MLR$

Question 4

A solid cone of height H and radius R is rotated about an axis as shown in the picture below.



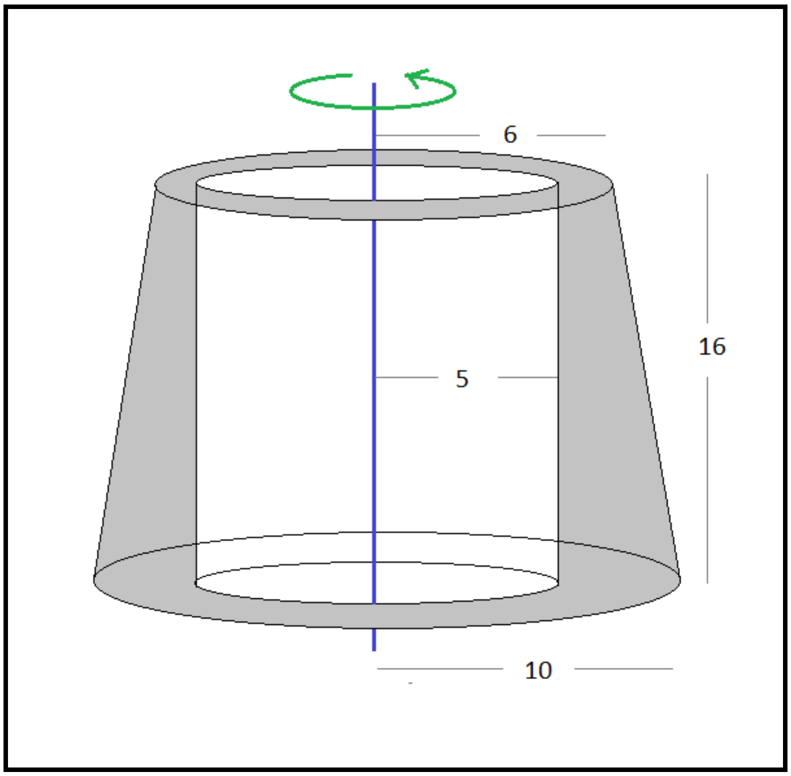
Assuming the mass M of this cone is evenly distributed, what is its moment of inertia I_{cone} ?

Hint: first compute the moment of inertia of this solid cone about the axis that is parallel to the given one and runs through the center of the cone. Then use the Parallel Axis Theorem. It might help to know that the volume of this cone is $\frac{1}{3}\pi R^2 H$.

- $I_{\text{cone}} = \frac{2}{5} MR^2$
- $I_{\text{cone}} = \frac{3}{10} MR^2$
- $I_{\text{cone}} = \frac{3}{5} MR^2$
- $I_{\text{cone}} = \frac{13}{10} MR^2$
- $I_{\text{cone}} = \frac{17}{10} MR^2$
- $I_{\text{cone}} = \frac{1}{10} MR^2$

Question 5

Here is the $16\,\mathrm{m}$ high dragon's cage from Question 6 of Homework 37: the inner cylinder of radius $5\,\mathrm{m}$ is hollow, and the solid walls slope from an outer radius of $6\,\mathrm{m}$ at the top to an outer radius of $10\,\mathrm{m}$ near the base. There are no "caps" at the top or at the bottom. The mass $M\,\mathrm{kg}$ is distributed throughout the walls with uniform density $\rho\,\mathrm{kg}/\mathrm{m}^3$. Let I be the moment of inertia of the cage about the axis shown in the picture. What is the ratio I/M?



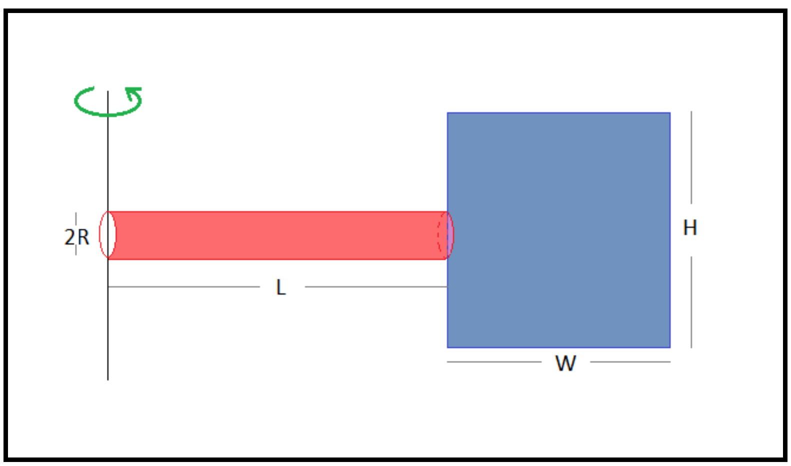
Hints: the volume of this cage is $\dfrac{1936\pi}{3}\ \mathrm{m}^3$. It will be helpful to break the region up into two pieces: one thick cylindrical shell when the distance from the axis is between $5\,\mathrm{m}$ and $6\,\mathrm{m}$, and the sloping region when the distance is between $6\,\mathrm{m}$ and $10\,\mathrm{m}$. The moment of inertia has units of mass times distance-squared, which in our case is $\mathrm{kg}\,\mathrm{m}^2$, so the ratio must have units m^2.

- $\dfrac{I}{M} = \dfrac{59793}{1210}\ \mathrm{m}^2$

- $\dfrac{I}{M} = \dfrac{19931}{1210}\ \mathrm{m}^2$
- $\dfrac{I}{M} = \dfrac{3177}{121}\ \mathrm{m}^2$
- $\dfrac{I}{M} = \dfrac{59793}{2420}\ \mathrm{m}^2$
- $\dfrac{I}{M} = \dfrac{19931\pi}{605}\ \mathrm{m}^2$
- $\dfrac{I}{M} = \dfrac{19931}{2420}\ \mathrm{m}^2$

Question 6

A fly-swatter consists of the hollow cylinder from Question 2 glued to a rectangle of width W and height H . What is the radius of gyration of this fly-swatter about the axis shown in the picture below?



You may assume that the mass of the fly swatter is evenly distributed across its surfaces. The mass of the "handle" is M_{cyl} and that of the rectangular "swatter" is M_{rect}.

- $R_g = \sqrt{\dfrac{\frac{1}{3}M_{\text{rect}}W^2 + \left(\frac{1}{3}M_{\text{cyl}} + M_{\text{rect}}\right)L^2 + M_{\text{rect}}LW}{M_{\text{rect}} + M_{\text{cyl}}}}$

- $R_g = \sqrt{\dfrac{\frac{1}{3}M_{\text{rect}}W^2 + \left(\frac{1}{3}M_{\text{cyl}} + M_{\text{rect}}\right)L^2}{M_{\text{rect}} + M_{\text{cyl}}}}$
- $R_g = \sqrt{\dfrac{\frac{1}{3}M_{\text{rect}}H^2 + \left(\frac{1}{3}M_{\text{cyl}} + M_{\text{rect}}\right)L^2}{M_{\text{rect}} + M_{\text{cyl}}}}$
- $R_g = \sqrt{\dfrac{\frac{4}{3}M_{\text{rect}}H^2 + \frac{1}{3}M_{\text{cyl}}L^2 + M_{\text{rect}}LH}{M_{\text{rect}} + M_{\text{cyl}}}}$
- $R_g = \sqrt{\dfrac{\frac{1}{3}M_{\text{rect}}H^2 + \left(\frac{1}{3}M_{\text{cyl}} + M_{\text{rect}}\right)L^2 + M_{\text{rect}}LW}{M_{\text{rect}} + M_{\text{cyl}}}}$
- $R_g = \sqrt{\dfrac{\frac{1}{3}M_{\text{rect}}L^2 + \left(\frac{1}{3}M_{\text{cyl}} + M_{\text{rect}}\right)W^2 + M_{\text{rect}}LW}{M_{\text{rect}} + M_{\text{cyl}}}}$

- [ ] In accordance with the Honor Code, I certify that my answers here are my own work.

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