Homework 46

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Use forward differencing to determine which of the following sequences (a_n) is **not** a polynomial of degree two? Choose all that apply.

- [ ] $(0, 2, 8, 18, 32, 50, 72, \ldots)$
- [ ] $(1, 6, 13, 22, 33, 46, 61, \ldots)$
- [ ] $(-2, 2, 6, 10, 14, 18, 22, \ldots)$
- [ ] $(-2, -4, 0, 16, 50, 108, 196, \ldots)$
- [ ] $(3, 8, 15, 24, 35, 48, 63, \ldots)$
- [ ] $(0, 2, 6, 14, 30, 62, 126, \ldots)$

Question 2

We have seen that the forward difference of the Fibonacci sequence (F_n) involves a shift. Which of the following is the *fourth* forward difference of the Fibonacci sequence, $\Delta^4 F$?

- $\Delta^4 F = (2, 1, 1, 0, 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots)$
- $\Delta^4 F = (-3, 2, -1, 1, 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots)$
- $\Delta^4 F = (3, 2, 1, 1, 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots)$
- $\Delta^4 F = (-2, 1, -1, 0, 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots)$
- $\Delta^4 F = (0, 0, 0, 0, 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots)$
- $\Delta^4 F = (-3, 2, 1, -1, 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots)$

Question 3

The discrete analogues of the second derivative include forward Δ^2 and backward ∇^2 versions that look ahead or behind respectively. There is a middle way — called the *second central difference*— and is defined as, equivalently, $\nabla\Delta$ or $\Delta\nabla$. Which of the following is the formula for the second central difference of a sequence $a = (a_n)$?

- $(\nabla\Delta a)_n = a_{n+1} - a_n + a_{n-1}$
- $(\nabla\Delta a)_n = a_{n+1} - 2a_n + a_{n-1}$
- $(\nabla\Delta a)_n = a_{n+1} + 2a_n - a_{n-1}$
- $(\nabla\Delta a)_n = a_{n+1} + a_n - 2a_{n-1}$
- $(\nabla\Delta a)_n = a_{n+1} + 2a_n + a_{n-1}$
- $(\nabla\Delta a)_n = a_{n+1} - a_{n-1}$

Question 4

Which (one) of the following sequences $a = (a_n)_{n=0}^{\infty}$ is an indefinite integral of $(n^3 - 4n^2)$ in the sense that $(\Delta a)_n = (n^3 - 4n^2)$ (for as much of the sequence as you are shown)?

- $a = (0, 0, 3, 8, 9, 0, -25, -72, \ldots)$
- $a = (0, 3, 8, 9, 0, -25, -72, \ldots)$
- $a = (0, -3, -8, -9, 0, 25, 72, \ldots)$
- $a = (1, 1, 4, 12, 21, 21, -4, -76, \ldots)$
- $a = (2, 2, -1, -4, -12, -21, 4, 76 \ldots)$
- $a = (-1, -1, -4, -12, -21, -21, 4, 76, \ldots)$

Question 5

We can rewrite any polynomial sequence in terms of falling powers. For example,

$$n^2 = n(n-1) + n = n^{\underline{2}} + n^{\underline{1}}$$

Which of the following equals n^3?

- $n^{\underline{3}} - n^{\underline{2}} + 3n^{\underline{1}}$
- $n^{\underline{3}} - n^{\underline{2}} + n^{\underline{1}}$
- $n^{\underline{3}} - 3n^{\underline{2}} + n^{\underline{1}}$
- $n^{\underline{3}} + 3n^{\underline{2}} + n^{\underline{1}}$
- $n^{\underline{3}} + n^{\underline{2}} - n^{\underline{1}}$
- $(n^{\underline{2}} - n^{\underline{1}})^2$

Question 6

We have seen that the difference of (2^n) is (2^n), but what is the (forward) difference of (3^n)?

- $(2 \cdot 3^n)$
- $(2 \cdot 3^{n-1})$
- $(3^{n-1} \cdot \ln 3)$
- $(2 \cdot 3^{n+1})$
- (3^n)
- (3^{n+1})

Question 7

There is a *product rule* for the forward and backward difference operators, but they may not be exactly what you think. Let u and v be sequences, and define the product sequence uv as $(uv)_n = u_n \cdot v_n$. By experimenting with different u and v sequences, try to guess which of the following is the correct product rule for the forward difference.

- $\Delta(uv) = u\Delta v + vE\Delta u$
- $\Delta(uv) = (E^{-1}u)\Delta v + (Ev)\Delta u$
- $\Delta(uv) = (Eu)\Delta v + v\Delta u$
- $\Delta(uv) = (E^{-1}u)\Delta v + v\Delta u$
- $\Delta(uv) = u\Delta v + v\Delta u$
- $\Delta(uv) = u\Delta v + (Ev)\Delta u$

In accordance with the Honor Code, I certify that my answers here are my own work.

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