

Homework 48

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Consider the differential equation

$$\frac{dx}{dt} = \frac{1}{2x}$$

This is a separable O.D.E., so we know how to find all of its solutions: they are of the form

$$x(t) = \sqrt{t + C}$$

where C is a constant. Imposing the initial condition $x(1) = 1$ fixes $C = 0$.

Then we have $x(2) = \sqrt{2}$.

Use Euler's method with $h = 1/2$ to find an approximation to $\sqrt{2}$. Provide a numeric answer rounded to two decimal places.

Question 2

Recall from Lecture the fourth order Runge-Kutta method:

$$x_{n+1} = x_n + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_n, t_n)$$

$$k_2 = hf\left(x_n + \frac{1}{2} k_1, t_n + \frac{1}{2} h\right)$$

$$k_3 = hf\left(x_n + \frac{1}{2} k_2, t_n + \frac{1}{2} h\right)$$

$$k_4 = hf(x_n + k_3, t_n + h)$$

Apply it, with $h = 1$, to the initial value problem of the previous Question to find a (better) approximation to $\sqrt{2}$ (recall $\sqrt{2} \approx 1.41421$). Provide a numeric answer rounded to five decimal places.

☐ In accordance with the Honor Code, I certify that my answers here are my own work.