Homework 54

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT -0400**.

Question 1

For which values of x does the power series $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ converge?

- $-1 \leq x < 1$
- $-1 < x \leq 1$
- Only at $x = 0$
- $-1 < x < 1$
- $-1 \leq x \leq 1$
- $-\infty < x < +\infty$

Question 2

For which values of x does the power series $\sum_{n=1}^{\infty} n^2 x^n$ converge?

- $-1 < x \leq 1$
- $-1 < x < 1$
- $-1 \leq x < 1$
- $-\infty < x < +\infty$
- $-1 \leq x \leq 1$
- Only at $x = 0$

Question 3

For which values of x does the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n \cdot n} (x-1)^n$ converge?

- $-4 < x < 2$
- $-2 < x \leq 4$
- $-3 < x \leq 3$
- $-4 < x \leq 2$
- $-3 < x < 3$
- $-2 < x < 4$

Question 4

What is the radius of convergence, R, of the power series $\sum_{n=1}^{\infty} \frac{n!(x+2)^n}{n^n}$?

- $R = e^{-1}$
- $R = 0$
- $R = 2$
- $R = e$
- $R = +\infty$
- $R = 1$

Question 5

For which values of x does the power series $\sum_{n=1}^{\infty} n^n (x-2)^n$ converge?

- $-\infty < x < +\infty$
- Only at $x = 2$
- $2 - e < x < 2 + e$
- $2 - e \leq x < 2 + e$

- $1 \le x < 3$
- $1 < x < 3$

Question 6

For which values of x does the power series $\sum_{n=1}^{\infty} \left(\frac{n-1}{n}\right)^{n^2} x^n$ converge?

Hint: instead of the ratio test, apply the root test.

- Only at $x = 0$
- $-e \le x < e$
- $-e \le x \le e$
- $-e < x < e$
- $-e < x \le e$
- $-\infty < x < +\infty$

Question 7

For which values of x does the series $\sum_{n=1}^{\infty} \frac{1}{n^x}$ converge?

Hint: this is *not* a power series, but it is important! It is known as the *Riemann zeta function* and is usually denoted by $\zeta(x)$. It is crucial to number theory and is the subject of the *Riemann hypothesis*, one of the grandest open questions in Mathematics.

- Only at $x = 1$
- $x < 1$
- $x \ge 1$
- $-\infty < x < +\infty$

- $x > 1$
- $x \leq 1$

Question 8

For which values of x does the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!x^n}$ converge?

Hint: this is also *not* a power series (series on negative powers of x are called *asymptotic series*), but you can apply the same reasoning as in the Lecture to find out where it converges.

- $-\infty < x < +\infty$
- $-1 < x < 1$
- $x \neq 0$
- $0 < x < e$
- $0 < x < 1$
- $0 < x < +\infty$

Question 9

Consider two power series,

$$A(x) = \sum_{n=0}^{\infty} a_n x^n, \qquad B(x) = \sum_{n=0}^{\infty} b_n x^n$$

and let

$$C(x) = \sum_{n=0}^{\infty} c_n x^n = A(x)B(x)$$

It is not difficult to find an expression for the sequence $c = (c_n)$ in terms of the sequences $a = (a_n)$ and $b = (b_n)$. For example, think about c_3. It is the coefficient of the x^3 term in $C(x)$. But which terms in the product $A(x)B(x)$ yield cubic terms? We either have a constant term coming from $A(x)$ and a cubic term from $B(x)$, or a linear term from $A(x)$ and a quadratic one from $B(x)$, or... This yields

$$c_3 = a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0$$

The same argument generalizes easily to give

$$c_n = \sum_{j=0}^{n} a_j b_{n-j} = a_0 b_n + a_1 b_{n-1} + a_2 b_{n-2} + \cdots + a_{n-2} b_2 + a_{n-1} b_1 + a_n b_0$$

which you might recognize from Homework 47 as the *discrete convolution* $a * b$.

Armed with this knowledge, suppose $a_n = b_n = 1$, so that

$$A(x) = B(x) = \sum_{n=0}^{\infty} x^n$$

What are in this case the coefficients c_n ?

Note: you should have recognized the power series $\sum_{n=0}^{\infty} x^n$ as the Taylor series about $x = 0$ of the function $\frac{1}{1-x}$. The power series $C(x)$ is then nothing but the Taylor series about $x = 0$ of the function $\frac{1}{(1-x)^2}$.

- $c_n = n$
- $c_n = \binom{n}{2}$
- $c_n = 2^n$
- $c_n = 1$
- $c_n = n + 1$
- $c_n = 2$

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