# Homework 42

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

## Question 1

The result of flipping a single coin is either heads, H, or tails ,T, each one of them with probability $1/2$ —such a coin is said to be *fair*. If you flip the same coin a second time, there are four possible combinations of the results of both tosses —HH, HT, TH and TT—, each one of them equally probable. Think of what happens when you do it yet once more: what is the probability of obtaining two heads and one tail, in whatever order?

- $\frac{1}{8}$
- $\frac{3}{8}$
- $\frac{5}{8}$
- $\frac{1}{2}$
- $\frac{1}{4}$
- $\frac{7}{8}$

## Question 2

A bus line runs every 30 minutes. If you arrive at a stop randomly, what is the probability that you will have to wait more than 10 minutes for the next bus?

**Hint:** this probability is a "volume" fraction over some domain. What is the

domain, and what is its dimension?

- $1$
- $\frac{1}{2}$
- $\frac{1}{4}$
- $\frac{3}{4}$
- $\frac{1}{3}$
- $\frac{2}{3}$

## Question 3

What is the probability that a randomly chosen point of a square of side length $L$ is more than a distance $r$ away from every corner? Suppose $r < L/2$.

- $\pi\left(\frac{r}{L}\right)^2 - 1$
- $1 - \frac{\pi r^2}{L}$
- $1 - \pi\left(\frac{r}{L}\right)^2$
- $L^2 - \frac{\pi r^2}{4}$
- $\frac{\pi r^2}{L^2}$
- $L^2 - \pi r^2$

## Question 4

Two people decide to meet at a café, some time between 10am and 10:30am. They agree that they will wait for the other person at most 15 minutes, and never past 10:30am. If both of them arrive randomly between the agreed-upon hours, what is the probability that they will indeed meet?

**Hint:** Let $x$ and $y$ be the arrival times of the two persons; then $0 \leq x, y \leq 30$ (minutes after 10am). Consider the subset of arrival times that give a successful meeting. What geometric figure does this set form?

- $\frac{3}{4}$
- $\frac{1}{4}$
- $\frac{1}{3}$
- $\frac{1}{2}$
- $1$
- $\frac{1}{6}$

# Question 5

What is the probability that a randomly chosen point on the surface of the Earth of the surface has a latitude between $-30^{\circ}$ and $30^{\circ}$?

**Hint:** We computed a surface area element for a sphere of radius $R$ in Lecture 36. Recall also from Question 6 of Homework 39 that latitude is related to the $x$-coordinate in this model by $x = R \sin \frac{\pi\phi}{180}$.

- $\frac{1}{4}$
- $\frac{1}{2}$

- $\frac{1}{6}$
- $\frac{\sqrt{2}}{2}$
- $\frac{\sqrt{3}}{2}$
- $\frac{1}{3}$

## Question 6

Consider an $n$-dimensional "hypercube" $C$ of all side lengths equal to $L$. Its $n$-dimensional volume is, clearly, $L^n$. Now consider a point chosen at random within this "cube". What is the probability that the chosen point is "within 1 percent of the boundary"? Your answer will definitely depend on $n$.

**Hint:** the set of points *not* within 1 percent of the boundary is a concentric hypercube whose dimensions are a bit smaller than $L$. What side-length cube does this yield? To get the probability of being in the "crust", what (hyper)volume fraction should you compute? What happens as $n \to +\infty$? So many interesting questions! You may wish to refer to Lecture/Homework 34.

- $0.98^n$
- $1-(0.99)^n$
- $1$
- $1-(0.98)^n$
- $0.99^n$
- $1-L^n$

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