Homework 17

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

$$\int (4x^3 + 3x^2 + 2x + 1)\, dx =$$

- $3x^2 + x + C$
- $\frac{1}{4}x^4 + \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + C$
- $4x^4 + 3x^3 + 2x^2 + x + C$
- $4x^3 + 3x^2 + x + C$
- $12x^2 + 6x + 1 + C$
- $x^4 + x^3 + x^2 + x + C$

Question 2

$$\frac{d}{dx} \int \ln \tan x \, dx =$$

- $\ln \tan x$
- $\ln \tan x + C$
- $\int \frac{\sec^2 x}{\tan x}\, dx$
- 0
- $\frac{\sec^2 x}{\tan x} + C$
- $\frac{\sec^2 x}{\tan x}$

Question 3

$$\int \left(\frac{d}{dx}\, e^{-x} \right) dx =$$

- $-e^{-x}$
- $e^{-x} + C$
- $e^{x} + C$
- $\dfrac{d}{dx} \displaystyle\int e^{-x}\, dx$
- $-e^{-x} + C$
- e^{-x}

Question 4

Find the general solution of the differential equation

$$\frac{dx}{dt} = t^2$$

- $x(t) = \dfrac{1}{3}\, t^3$
- $x(t) = \dfrac{1}{2}\, t^2 + C$
- $x(t) = t^2 + C$
- $x(t) = t^3$
- $x(t) = \dfrac{1}{3}\, t^3 + C$
- $x(t) = t^3 + C$

Question 5

Find the general solution of the differential equation

$$\frac{dx}{dt} = x^2$$

Hint: think in terms of differentials, as we did in the Lecture. This will allow you to bring everything that depends on x to the left hand side, and everything that depends on t to the right. Then integrate.

- $x(t) = -\frac{1}{t}$
- $x(t) = -\frac{1}{t^2}$
- $x(t) = \frac{1}{-t + C}$
- $x(t) = -\frac{1}{t} + C$
- $x(t) = \frac{1}{-t^2 + C}$
- $x(t) = -\frac{1}{t^2} + C$

Question 6

For the brave: There is a large class of differential equations —the so-called *linear* ones— for which we can find solutions using the Taylor series method discussed in the Lecture. One such differential equation is

$$t\,\frac{d^2x}{dt^2} + \frac{dx}{dt} + tx = 0 \qquad (*)$$

It is a particular case of the more general *Bessel differential equation*, and one solution of it is given by the Bessel function $J_0(t)$ that we saw in Chapter 1. Notice that $(*)$ involves not only the first derivative $\frac{dx}{dt}$ but also the second derivative $\frac{d^2x}{dt^2}$. For this reason, it is said to be a *second order* differential equation.

In this problem we will content ourselves with finding a relationship (specifically, a *recurrence relation*) on the coefficients of a Taylor series expansion about $t = 0$ of a solution to our equation. Hence consider the Taylor series

$$x(t) = \sum_{k=0}^{\infty} c_k t^k$$

Substituting this into $(*)$ will give you two conditions. The first one is $c_1 = 0$. What is the other one?

Note: this problem involves some nontrivial manipulation of indices in summation notation. Do not get discouraged if it feels more difficult than other problems: it is! If you can't get this one, don't be worried. This won't show up on the test ;-)

- $c_k = -\dfrac{c_{k-1}}{(k-1)^2}$
- $c_k = -\dfrac{c_{k-2}}{(k-2)^2}$
- $c_k = -\dfrac{c_{k-1}}{k^2}$
- $c_k = -\dfrac{c_{k-2}}{k^2}$
- $c_k = \dfrac{c_{k-1}}{k^2}$
- $c_k = \dfrac{c_{k-2}}{k^2}$

☐ In accordance with the Honor Code, I certify that my answers here are my own work.

Submit Answers Save Answers