Homework 40

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Find the coordinates $(\bar{x}, \bar{y})$ of the centroid of the region bounded by $y = \sin x$ and $y = \cos x$ for $0 \leq x \leq \frac{\pi}{4}$.

- $\bar{x} = \frac{\pi}{8}, \bar{y} = \sqrt{\frac{2-\sqrt{2}}{2}}$
- $\bar{x} = \frac{\pi\sqrt{2}}{\sqrt{2}-1}, \bar{y} = \frac{1}{\sqrt{2}-1}$
- $\bar{x} = \frac{\pi\sqrt{2}-4}{4(\sqrt{2}-1)}, \bar{y} = \frac{1}{4(\sqrt{2}-1)}$
- $\bar{x} = \frac{\sqrt{2}}{2}, \bar{y} = \frac{\sqrt{2}}{2}$
- $\bar{x} = \frac{1}{\sqrt{2}-1}, \bar{y} = \frac{1}{\sqrt{2}-1}$
- $\bar{x} = \pi\sqrt{2}, \bar{y} = 1$

Question 2

Find the the coordinates $(\bar{x}, \bar{y})$ of the centroid of the region defined by $|x + y| \leq 1$, $-1 \leq x \leq 1$, and $-1 \leq y \leq 1$.

Hint 1: draw a picture!

Hint 2: notice anything *interesting* about this region?

- $(\bar{x}, \bar{y}) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$
- $(\bar{x}, \bar{y}) = (1, 1)$
- $(\bar{x}, \bar{y}) = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right)$
- $(\bar{x}, \bar{y}) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$
- $(\bar{x}, \bar{y}) = (0, 0)$
- $(\bar{x}, \bar{y}) = \left(\frac{1}{2}, \frac{1}{2} \right)$

Question 3

Find the centroid $\bar{x}$ of an infinitely long solid *"horn"* (cf. Lecture 36) obtained by rotating the region $0 \leq y \leq \frac{1}{x^2}$ about the x-axis from $x = 1$ to $x = +\infty$.

Hint: begin by writing the volume element dV as a function of x. Compute the volume V and then...

- $\bar{x} = \frac{1}{3}$
- $\bar{x} = \frac{\pi}{2}$
- $\bar{x} = 1$
- $\bar{x} = \frac{2}{3}$
- $\bar{x} = \frac{3}{2}$
- $\bar{x} = +\infty$ (the integral diverges)

Question 4

Find the coordinates $(\bar{x}, \bar{y})$ of the centroid of the union of the following two discs:

$$D_1 : x^2 + y^2 \leq 4 \qquad \text{and} \qquad D_2 : (x-4)^2 + (y-2)^2 \leq 1$$

Hint: replace each disc with a vertex at its centroid. What "mass" should you assign to each vertex?

- $(\bar{x}, \bar{y}) = (4\pi, 2\pi)$
- $(\bar{x}, \bar{y}) = \left(\frac{4\pi}{5}, \frac{2\pi}{5}\right)$
- $(\bar{x}, \bar{y}) = \left(\frac{4}{\pi}, \frac{2}{\pi}\right)$
- $(\bar{x}, \bar{y}) = (0, 0)$
- $(\bar{x}, \bar{y}) = (2, 1)$
- $(\bar{x}, \bar{y}) = \left(\frac{4}{5}, \frac{2}{5}\right)$

Question 5

Find the x-coordinate of the center of mass of a thin plate of density (mass-per-unit-area) $\delta = \frac{1}{x^2}$ defined by the region $0 \leq y \leq \ln x$ for $1 \leq x \leq e$.

- $\frac{e-1}{2}$
- $\frac{e}{2e-4}$
- $\frac{e}{2-e}$
- $\frac{1}{1-\frac{2}{e}}$

- $\frac{e}{4e-8}$
- $\frac{e}{e-2}$

Question 6

Find the the coordinates $(\bar{x}, \bar{y})$ of the center of mass of the region between the x-axis, the y-axis, and the lines $x = 2$ and $y = x + 2$, with density (mass-per-unit-area) $\rho = 3x$.

Hint: remember, this is a center-of-mass, not a centroid, so you'll need to integrate with respect to $dM = \rho \cdot dA$.

- $(\bar{x}, \bar{y}) = \left(\frac{1}{3}, \frac{17}{3}\right)$
- $(\bar{x}, \bar{y}) = \left(\frac{7}{5}, \frac{17}{5}\right)$
- $(\bar{x}, \bar{y}) = (1, 2)$
- $(\bar{x}, \bar{y}) = \left(\frac{7}{5}, \frac{17}{10}\right)$
- $(\bar{x}, \bar{y}) = \left(10, \frac{17}{3}\right)$
- $(\bar{x}, \bar{y}) = \left(\frac{14}{3}, \frac{17}{3}\right)$

☐ In accordance with the Honor Code, I certify that my answers here are my own work.

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