Problem Set IV

Warning: The hard deadline has passed. You can attempt it, but **you will not get credit for it**. You are welcome to try it as a learning exercise.

In this problem set, you will be given a total of ten attempts. We will accept late submission until the fifth day after the due date, and late submission will receive half credit. Explanations and answers to the problem set will be available after the due date. Since the homework problems will become gradually more challenging as the course proceeds, we highly recommend you to start the habit of printing out the problems and working on them with paper and pencil. Also, please be sure to read the problem statements carefully and double check your expressions before you submit.

A pdf version of this problem set is available for you to print. Note: all mathematical expressions have to be exact, even when involving constants. Such an expression is required when a function and/or a variable is required in the answer. For example, if the answer is $\sqrt{3}x$, you must type `sqrt(3)*x`, not `1.732*x` for the answer to be graded as being correct.

Question 1

Find the Fourier series coefficients c_0, c_1, c_2, c_3 for $\sin t$ *without* explicitly calculating integrals. Use Euler's formula, Fourier series properties and any appropriate mathematical "tricks."

Express your answer numerically, typing the real and imaginary parts for each answer separated by spaces. So, an answer of $c_0 = 0$, $c_1 = 1 + j$, $c_2 = -j$, $c_3 = 0$ would be typed as 0 0 1 1 0 -1 0 0.

Question 2

Find the Fourier series coefficients c_0, c_1, c_2, c_3 for $\sin^2 t$ *without* explicitly calculating integrals. Use Euler's formula, Fourier series properties and any appropriate mathematical "tricks."

Express your answer numerically, typing the real and imaginary parts for each answer separated by spaces. So, an answer of $c_0 = 0$, $c_1 = 1 + j$, $c_2 = -j$, $c_3 = 0$ would be typed as 0 0 1 1 0 -1 0 0.

Question 3

Find the Fourier series coefficients c_0, c_1, c_2, c_3 for $\cos t + 2\cos 2t$ *without* explicitly calculating integrals. Use Euler's formula, Fourier series properties and any appropriate mathematical "tricks."

Express your answer numerically, typing the real and imaginary parts for each answer separated by spaces. So, an answer of $c_0 = 0$, $c_1 = 1 + j$, $c_2 = -j$, $c_3 = 0$ would be typed as 0 0 1 1 0 -1 0 0.

Question 4

Find the Fourier series coefficients c_0, c_1, c_2, c_3 for $\cos(t)\cos(2t)$ *without* explicitly calculating integrals. Use Euler's formula, Fourier series properties and any appropriate mathematical "tricks."

Express your answer numerically, typing the real and imaginary parts for each answer separated by spaces. So, an answer of $c_0 = 0$, $c_1 = 1 + j$, $c_2 = -j$,

$c_3 = 0$ would be typed as 0 0 1 1 0 -1 0 0.

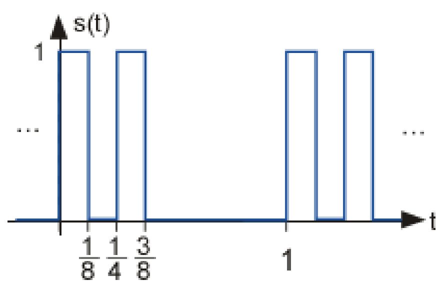
Question 5

Find the Fourier series coefficients c_0, c_1, c_2, c_3 for $\cos(2\pi t + \pi/6) \cdot \cos 2\pi t$ *without* explicitly calculating integrals. Use Euler's formula, Fourier series properties and any appropriate mathematical "tricks."

Express your answer numerically, typing the real and imaginary parts for each answer separated by spaces. So, an answer of $c_0 = 0$, $c_1 = 1 + j$, $c_2 = -j$, $c_3 = 0$ would be typed as 0 0 1 1 0 -1 0 0. Please round all your answers to two decimal places. *Base your answer on a period of 1*.

Question 6

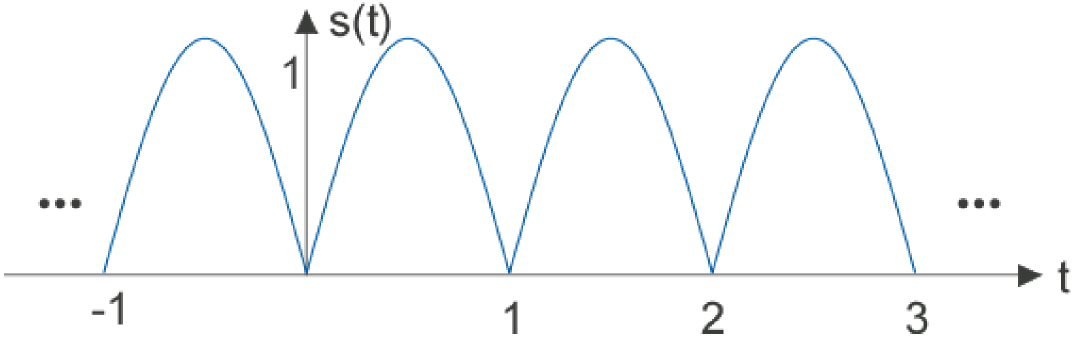
Find the Fourier series coefficients c_0 and c_1 for the depicted signal *without* explicitly calculating integrals. Use Euler's formula, Fourier series properties and any appropriate mathematical "tricks."



Express your answer numerically, typing the real and imaginary parts for each answer separated by spaces. So, an answer of $c_0 = 0$, $c_1 = 1 + j$ would be typed as 0 0 1 1.

Question 7

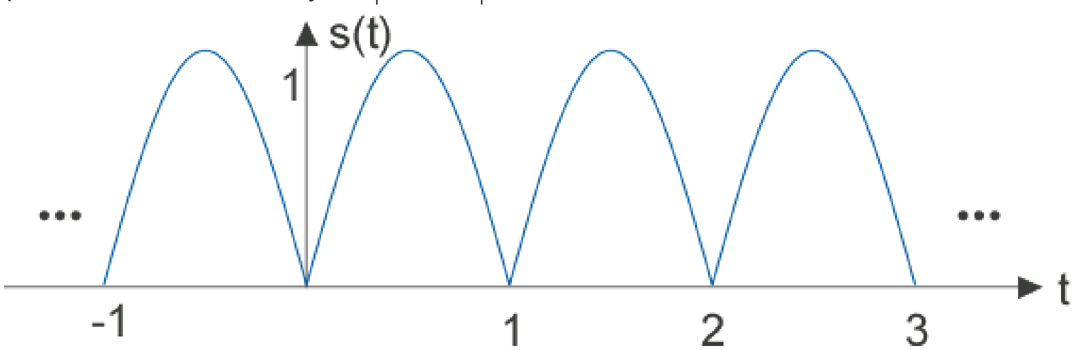
The depicted signal is known as a *full-wave rectiified sinusoid*, expressed mathematically as the absolute value of $\sin \pi t$: $|\sin \pi t|$.



What is this signal's period?

Question 8

Find the Fourier series coefficients c_k for this *full-wave rectiified sinusoid*, expressed mathematically as $|\sin \pi t|$.

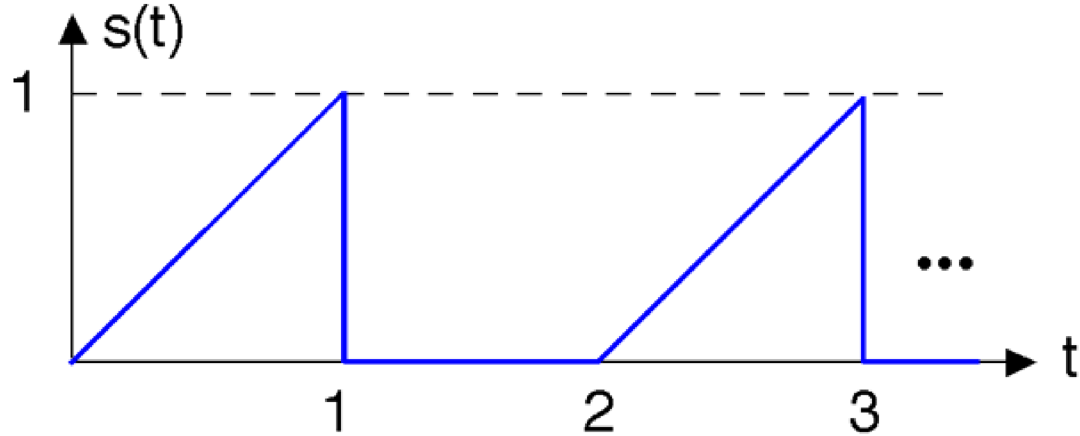


Express your answer as a mathematical expression; if $c_k = \frac{1}{k} - j\,\frac{1}{2k}$, you would enter 1/k-j/(2*k).

Preview

Question 9

Find the Fourier series coefficients c_k for this sawtooth-like waveform.

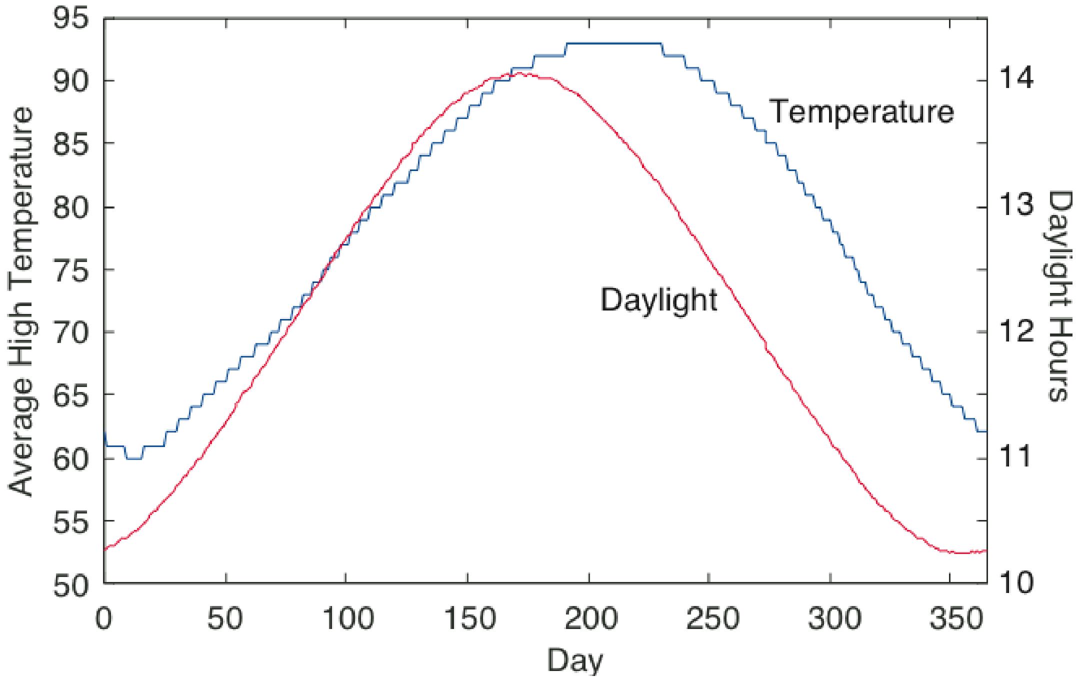


Express your answer as a mathematical expression; if $c_k = \frac{1}{k} - j\,\frac{1}{2k}$, you would enter 1/k-j/(2*k).

Preview

Question 10

The daily temperature is a consequence of several effects, one of them being the sun's heating. If this were the dominant effect, then daily temperatures would be proportional to the number of daylight hours. The plot shows that the average daily high temperature does **not** behave that way.



In the next four problems, we want to understand the temperature component of our environment using Fourier series and linear system theory. The file `temperature.mat` (`temperature.data` in ascii) contains these data (daylight hours in the first row, corresponding average daily highs in the second) for Houston, Texas. Once you download this data file, load it into Matlab or Octave using the `load` command. You will find two signals (vectors): `high` and `daylight`. Let the length of day serve as the sole input to a system having an output equal to the average daily temperature. Examining the plots of input and output, would you say that the system is linear or not?

- Yes
- No

Question 11

Using Matlab or Octave, find the first four Fourier series coefficients c_0, c_1, c_2, c_3 for the `high` dataset. Approximate the integral as a sum:

$$c_k \approx \frac{1}{366} \sum_{n=0}^{365} \mathtt{high}(n) e^{-j2\pi kn/366}$$

Express your answer numerically, typing the real and imaginary parts for each answer *to two decimal places* separated by spaces. So, an answer of $c_0 = 0$,

$c_1 = 1 + j$, $c_2 = -j\sqrt{3}$, $c_3 = 0$ would be typed as 0 0 1 1 0 -1.73 0 0. *note*: in Matlab you can simply use `j` to denote j.

Question 12

Using Matlab or Octave, find the first four Fourier series coefficients c_0, c_1, c_2, c_3 for the `daylight` dataset. Approximate the integral as a sum:

$$c_k \approx \frac{1}{366} \sum_{n=0}^{365} \mathtt{daylight}(n) e^{-j2\pi kn/366}$$

Express your answer numerically, typing the real and imaginary parts *to two decimal places* for each answer separated by spaces. So, an answer of $c_0 = 0$, $c_1 = 1 + j$, $c_2 = -j\sqrt{3}$, $c_3 = 0$ would be typed as 0 0 1 1 0 -1 0 0.

Question 13

What is the harmonic distortion for the two datasets? Enter your numeric answers as "daylight_harmonic_distortion high_harmonic_distortion" (note the space between the two values). Provide as many decimal places that are needed to reach the first non-zero digit. For example, a distortion of 0.0012 would be entered as `0.001`.

Question 14

Because the harmonic distortion is small, let's concentrate only on the first harmonic. What is the phase shift between input (`daylight`) and output (`high`) signals? This phase shift is the key quantity that will allow us to model the earth's temperature environment as a linear system. Express your answer in radians.

Question 15

It is an old axiom that the simplest model is usually the best. We are assuming that the model is a linear filter of some kind. *Because of the phase shift*, which models the response of the earth to the sun's heating process?

- First-order (RC) highpass filter
- Simple gain (amplifier)
- First-order (RC) lowpass filter
- Something more complicated

In accordance with the Honor Code, I certify that my answers here are my own work.

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