# Homework 50

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT -0400**.

## Question 1

Calculate the sum $S = \sum_{n=1}^{\infty} \frac{1}{3^n}$.

- $S = 1$
- $S = \frac{3}{2}$
- $S = \frac{1}{3}$
- $S = 3$
- $S = \frac{1}{2}$
- According to the $n$-th term test, the series diverges.

## Question 2

Calculate the sum $S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n}{n}$.

- $S = -\ln 3$
- $S = \ln 3$
- $S = \ln 2$
- $S = 1$
- $S = -\ln 2$
- According to the $n$-th term test, the series diverges.

# Question 3

Calculate the sum $S = \sum_{n=1}^{\infty} a_n$ of the sequence $a = \left( \frac{1}{n^2 + 3n + 2} \right)$.

**Hint:** remember from Question 1 of Homework 47 that $a = \Delta b$ for $b = \left( \frac{n}{n+1} \right)$. What are the partial sums of the series?

- $S = \frac{1}{2}$
- $S = \frac{1}{3}$
- $S = 0$
- According to the $n$-th term test, the series diverges.
- $S = 2$
- $S = 1$

# Question 4

All of the following statements are true, but only some of them are implied by the $n$-th term test. Which ones?

- The series $\sum_{n=0}^{\infty} \frac{n-1}{n+1}$ diverges.
- The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges.
- The series $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{3n+2}$ diverges.
- The series $\sum_{n=0}^{\infty} \frac{1}{n^2 + 3n + 2}$ converges.
- The series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

- [ ] The series $\sum_{n=0}^{\infty} e^{-2n^2+3}$ converges.

## Question 5

Below are given pairs of statements. Please mark those which are *contrapositives* (choose all that apply).

- [ ] "All fruit is delicious" : "Anything delicious is a fruit".
- [ ] "All cats speak French" : "Anyone who does not speak French is not a cat".
- [ ] "If I like you, then I will tell you I like you" : "If I tell you I like you, then I like you".
- [ ] "If $x = 2$ then $x^2 = 4$" : "If $x^2 = 4$ then $x = 2$".
- [ ] "If $x = 0$, then $\cos(x) = 1$" : "If $\cos(x) \neq 1$ then $x \neq 0$".
- [ ] "If I learn Calculus, I will be brilliant!" : "If I am not brilliant, then I did not learn Calculus".

## Question 6

Suppose you know that the partial sums $S_T$ of a series $a = (a_n)$ are given by:

$$s_T = \sum_{n=1}^{T} a_n = \frac{T-3}{T+1}$$

What is the sum $S = \sum_{n=1}^{\infty} a_n$?

- $S = \frac{1}{3}$
- $S = 2$
- The series diverges.
- $S = 3$
- $S = -3$
- $S = 1$

# Question 7

[Continued from the previous problem] Give an expression for $a_n$ in terms of $n$.

**Hint:** notice that $S_1 = a_1$.

- $a_1 = -1$ and $a_n = \dfrac{4}{(n+1)(n+2)}$ for $n > 1$.
- $a_1 = -1$ and $a_n = \dfrac{4}{n(n+1)}$ for $n > 1$.
- $a_1 = -2$ and $a_n = \dfrac{2-2n}{n(n+1)}$ for $n > 1$.
- $a_1 = -2$ and $a_n = \dfrac{4}{(n+1)(n+2)}$ for $n > 1$.
- $a_1 = -2$ and $a_n = \dfrac{4}{n(n+1)}$ for $n > 1$.
- $a_1 = -1$ and $a_n = \dfrac{2-2n}{n(n+1)}$ for $n > 1$.

☐ In accordance with the Honor Code, I certify that my answers here are my own work.

Submit Answers Save Answers