Problem Set VIII

The **hard deadline** for this homework is **Sun 24 Mar 2013 12:59 AM EDT**.

In this problem set, you will be given a total of ten attempts. We will accept late submission until the fifth day after the due date, and late submission will receive half credit. Explanations and answers to the problem set will be available after the due date. Since the homework problems will become gradually more challenging as the course proceeds, we highly recommend you to start the habit of printing out the problems and working on them with paper and pencil. Also, please be sure to read the problem statements carefully and double check your expressions before you submit.

A pdf version of this problem set is available for you to print. Note: all mathematical expressions have to be exact, even when involving constants. Such an expression is required when a function and/or a variable is required in the answer. For example, if the answer is $\sqrt{3}x$, you must type `sqrt(3)*x`, not `1.732*x` for the answer to be graded as being correct.

Question 1

We can find the input-output relation for a discrete-time filter much more easily than for analog filters. The key idea is that a discrete-time signal can be written as a weighted linear combination of unit samples.

$$x(n) = \sum_i x(i)\delta(n-i),\ \delta(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\delta(n)$ is the unit-sample. In this expression, $x(i)$ is the amplitude of the i^{th} signal component and $x(n)$ denotes the signal being decomposed into a superposition of unit samples.

If $h(n)$ denotes the **unit-sample response**—the output of a discrete-time linear, shift-invariant filter to a unit-sample input—what is the output of the filter?

If $\delta(n) \rightarrow h(n), x(n) \rightarrow ?$

- $y(n) = \sum_i h(n)x(n-i)$
- $y(n) = \sum_i x(i)h(n-i)$
- $y(n) = \sum_i h(i)n(n-i)$
- $y(n) = \sum_i x(n)h(n-i)$

Question 2

Assume that we are using an FIR filter with the unit sample response having duration $q+1$. If the input has duration D, what is the duration of the filter's output to this signal?

Type your answer as an expression involving `q` and `D`.

Duration $=?$

Preview Help

Question 3

Assume that we are filtering a signal with an FIR boxcar averager: $h(n) = \frac{1}{q+1}$ for $n = \{0, \ldots, q\}$ and zero otherwise.

Let the input be a pulse of unit height and duration D. Find the filter's output when $D = \frac{q+1}{2}$, where q is an odd integer.

Type an expression for the output in terms of `n`, `q` and `D`.

For $n = 0, \ldots, D-1:\ y(n) = ?$

Preview

Preview Help

Question 4

Assume that we are filtering a signal with an FIR boxcar averager:
$h(n) = \frac{1}{q+1}$ for $n = \{0, \ldots, q\}$ and zero otherwise.
Let the input be a pulse of unit height and duration D. Find the filter's output when $D = \frac{q+1}{2}$, where q is an odd integer.
Type an expression for the output in terms of `n`, `q` and `D`.

For $n = D, \ldots, q: \ y(n) =?$

Preview Help

Question 5

Assume that we are filtering a signal with an FIR boxcar averager:
$h(n) = \frac{1}{q+1}$ for $n = \{0, \ldots, q\}$ and zero otherwise.
Let the input be a pulse of unit height and duration D. Find the filter's output when $D = \frac{q+1}{2}$, where q is an odd integer.
Type an expression for the output in terms of `n`, `q` and `D`.

For $n = q+1, \ldots, D+q+1: \ y(n) =?$

Preview Help

Question 6

A filter has an input-output relationship given by the difference equation

$$y(n) = \frac{1}{4}\,x(n) + \frac{1}{2}\,x(n-1) + \frac{1}{4}\,x(n-2).$$

What is the filter's transfer function?

$H(e^{j2\pi f}) = ?$

Preview Help

Question 7

For the same filter, which has an input-output relationship given by the difference equation,

$$y(n) = \frac{1}{4}\,x(n) + \frac{1}{2}\,x(n-1) + \frac{1}{4}\,x(n-2).$$

What is the filter's output when the input equals $\cos\left(\frac{\pi n}{2}\right)$?

$y(n) = ?$

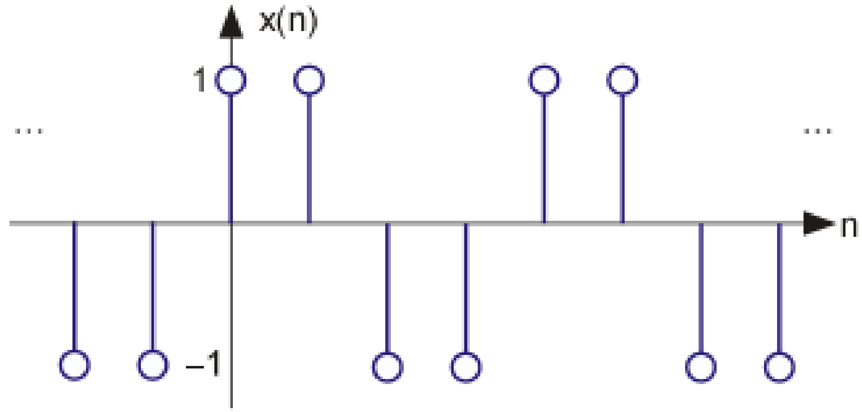
Preview Help

Question 8

A filter has an input-output relationship given by the difference equation

$$y(n) = \frac{1}{4}\,x(n) + \frac{1}{2}\,x(n-1) + \frac{1}{4}\,x(n-2).$$

What is the filter's output when the input is the depicted discrete-time square wave?



Your answer should be an expression involving well-known signals.

$y(n) =?$

Preview Help

Question 9

A discrete-time system is governed by the difference equation

$$y(n) = y(n-1) + \frac{x(n)+x(n-1)}{2}\,.$$

Find the transfer function for this system. Simplify your answer as much as possible.

$H(e^{j2\pi f}) =?$

Preview Help

Question 10

An electronics startup wants to develop a filter that would be used in analog applications, but that is implemented digitally. The filter is to operate on signals that have a 10 kHz bandwidth, and will serve as a lowpass filter. What minimum sampling rate (in kilohertz) must be used and how many bits must be used in the A/D converter for the acquired signal's signal-to-noise ratio to be at least 60 dB? (For the SNR calculation, assume the signal is a sinusoid.)

Enter your answer as a list. **Note:** the frequency has units of kHz for this problem. For example, if the necessary sampling rate was 1 kHz and the A/D converter needed 4 bits, you would type `1 4`

Question 11

Another consideration for the startup company's design is how to implement the filter. If the filter is a length-128 FIR filter (the duration of the filter's unit-sample response equals 128), should it be implemented in the time or frequency domain?

- Implement directly in the time domain
- Implement in the frequency domain using FFT

In accordance with the Honor Code, I certify that my answers here are my own work.

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