Homework 56

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT -0400**.

Question 1

Using the Taylor series expansion of $\arctan x$ about $x = 0$ we get the following approximation for π:

$$\pi = 4\arctan 1 \approx 4\sum_{n=0}^{N} \frac{(-1)^n}{2n+1}$$

If we want to calculate the first 4 digits of π past the decimal point — that is, with an error E_N smaller than 0.00005— then how big must N be?

Note: we need an awful lot of terms! We saw a much better method —using Simpson's rule— in Homework 49.

- $N \geq 39{,}999$
- $N \geq 4{,}000$
- $N \geq 40{,}000$
- $N \geq 3{,}999$
- $N \geq 9{,}999$
- $N \geq 10{,}000$

Question 2

Apéry's constant is the value at $x = 3$ of the Riemann zeta function $\zeta(x) = \sum_{n=1}^{\infty} \frac{1}{n^x}$. If we approximate it using the N-th partial sum,

$$\zeta(3) \approx \sum_{n=1}^{N} \frac{1}{n^3}\ ,$$

how big must N be in order for the error E_N to be smaller than 0.00005 ?

- $N > 99$
- $N > 9$
- $N > 1{,}000$
- $N > 10$
- $N > 999$
- $N > 100$

Question 3

Consider the series $\sum_{n=1}^{\infty} ne^{-n}$. The integral test ensures that it converges. Now, if we want to approximate its value by using the N-th partial sum $\sum_{n=1}^{N} ne^{-n}$, how big must N be in order for the error E_N to be smaller than 0.00001 ?

- $e^{-N}(N+1) < 0.00001 \Rightarrow N \geq 15$
- $e^{-(N+1)}(N+1) < 0.00001 \Rightarrow N \geq 14$
- $e^{-N}N < 0.00001 \Rightarrow N \geq 15$
- $e^{-(N+1)}(N+2) < 0.00001 \Rightarrow N \geq 14$
- $e^{-N}(N+2) < 0.00001 \Rightarrow N \geq 15$
- $e^{-(N+1)}N < 0.00001 \Rightarrow N \geq 14$

Question 4

What is the error $E_N(x)$ that the "strong" version of Taylor's theorem gives for approximating $\sinh x$ by the partial sum

$$\sinh x \approx \sum_{n=0}^{N} \frac{1}{(2n+1)!}\, x^{2n+1}$$

- $E_N(x) = \dfrac{\cosh t}{(2N+3)!}\, t^{2N+3}$ for some $|t| < x$
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Question 5

According to Taylor's theorem, what is the minimum value of N for which the error E_N resulting from approximating

$$\cos 1 \approx \sum_{n=0}^{N} \frac{(-1)^n}{(2n)!}$$

is smaller than 10^{-6} ?

Hint: You know that the derivatives of $\cos$ are not too big, right? There is an obvious upper bound for any derivative of cosine... Also, just so you do not need to calculate lots of factorials, here are the first few ones to help:

$$\begin{array}{ll} 3! = 6 & 9! = 362{,}880 \\ 4! = 24 & 10! = 3{,}628{,}800 \\ 5! = 120 & 11! = 39{,}916{,}800 \\ 6! = 720 & 12! = 479{,}001{,}600 \\ 7! = 5{,}040 & 13! = 6{,}227{,}020{,}800 \\ 8! = 40{,}320 & 14! = 87{,}178{,}291{,}200 \end{array}$$

- $N \geq 1$
- $N \geq 4$
- $N \geq 6$
- $N \geq 5$
- $N \geq 2$
- $N \geq 3$

In accordance with the Honor Code, I certify that my answers here are my own work.

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