Homework 25

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Recall the definition of definite integrals through Riemann sums:

$$\int_{x=a}^{b} f(x)\,dx = \lim_{P:\Delta x \to 0} \sum_{i=1}^{n} f(x_i)(\Delta x)_i$$

Here P is a partition of the interval $[a, b]$ into n intervals P_i, each of width $(\Delta x)_i$. The point x_i is a sampling of P_i, that is, a point in the interval P_i. The limit is taken over all partitions as the width of the subdivisions gets smaller and smaller.

One particular choice of partition and sampling that can be used to numerically evaluate definite integrals is the following. With n fixed, divide the interval $[a, b]$ into n subintervals P_i of common length $(\Delta x)_i = (b - a)/n$. For the sampling, choose the right endpoint of each P_i; this gives you the formula

$$x_i = a + i\,\frac{b-a}{n}$$

With these choices, evaluate the Riemann sum for the integral

$$\int_{x=0}^{3} x^2\,dx$$

for an arbitrary number n of subdivisions.

You might need to use any of the following formulas:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

- $9 + \dfrac{27}{2n} + \dfrac{9}{2n^2}$
- 9

- $9 + \frac{9}{n} + \frac{9}{n^2}$
- $9 + \frac{9}{2n} + \frac{3}{2n^2}$
- $9 + \frac{2n}{27} + \frac{2n^2}{9}$
- $9 + \frac{2n}{9} + \frac{2n^2}{3}$

Question 2

The line $y = x$, the x-axis and the vertical line $x = 2$ bound a triangle of area 2. Thus,

$$I = \int_{x=0}^{2} x\,dx = 2$$

Evaluating the Riemann sum for n subdivisions for the above integral with the same choices of partition and sampling as in the previous problem yields an approximation $RS(n)$ for its value I. The error $E(n)$ we commit by using this approximation is defined to be the difference

$$E(n) = RS(n) - I$$

Some of the following statements are true, while others are false. Among the correct ones, which one is optimal (assuming that you want to know how bad the error can be)?

- $E(n)$ is in $O(n)$ as $n \to +\infty$
- $E(n)$ is in $O(1/n)$ as $n \to +\infty$
- $E(n)$ is in $O(1/n^3)$ as $n \to +\infty$
- $E(n)$ is in $O(1/n^2)$ as $n \to +\infty$
- $E(n)$ is in $O(n^2)$ as $n \to +\infty$
- $E(n)$ is in $O(1)$ as $n \to +\infty$

Question 3

In the next Lecture, we will learn that

$$\int_{x=1}^{2} \frac{dx}{x} = \ln 2 \simeq 0.693$$

With the same choices of partition and sampling as in the previous two problems, compute the Riemann sums for the above integral for $n = 1$, $n = 2$ and $n = 3$ subdivisions.

- $\frac{1}{2} = 0.5, \frac{7}{12} \simeq 0.583, \frac{37}{60} \simeq 0.617$
- $1, \frac{5}{6} \simeq 0.833, \frac{47}{60} \simeq 0.783$
- $1, \frac{3}{2} = 1.5, \frac{11}{6} \simeq 1.833$
- $\frac{1}{2} = 0.5, \frac{5}{6} \simeq 0.833, \frac{13}{12} \simeq 1.083$
- $1, \frac{1}{2} = 0.5, \frac{5}{6} \simeq 0.833$
- $\frac{1}{2} = 0.5, \frac{5}{6} \simeq 0.833, \frac{5}{12} \simeq 0.417$

Question 4

Evaluate $\int_{x=-\pi/4}^{\pi/4} \left(x^2 + \ln|\cos x|\right) \sin \frac{x}{2}\ dx$. Provide a numeric answer.

Question 5

Every function can be expressed as the sum of an even function —called its *even part*— and an odd function —its *odd part*. They are given by the formulas

$$f^{\text{even}}(x) = \frac{f(x) + f(-x)}{2}, \qquad f^{\text{odd}}(x) = \frac{f(x) - f(-x)}{2}$$

Notice that, indeed,

$$f^{\text{even}}(-x) = \frac{f(-x)+f(x)}{2} = f^{\text{even}}(x), \qquad f^{\text{odd}}(-x) = \frac{f(-x)-f(x)}{2} = -f^{\text{odd}}(x)$$

and

$$f^{\text{even}}(x) + f^{\text{odd}}(x) = \frac{f(x)+f(-x)}{2} + \frac{f(x)-f(-x)}{2} = f(x)$$

If $f(x) = e^x$, which of the following statements are true? Select all that apply.

- [ ] $f^{\text{odd}}(x) = \cosh x$
- [ ] $f^{\text{even}}(x) = \sinh x$
- [ ] $f^{\text{even}}(x) = 0$
- [ ] $f^{\text{odd}}(x) = \sinh x$
- [ ] $f^{\text{odd}}(x) = 0$
- [ ] $f^{\text{even}}(x) = \cosh x$

- [ ] In accordance with the Honor Code, I certify that my answers here are my own work.

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