Homework 34

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

This is a homework assignment for...an unusual Lecture. Consider it as optional, as this is not at the core of single-variable calculus. But if you enjoy thinking about the fourth dimension (and more), then enjoy!

Question 1

Consider a four-dimensional box (or "rectangular prism") with side lengths 1, $1/2$, $1/3$, and $1/4$. What is the 4-dimensional volume of this box?

- $\frac{1}{24}$
- $\frac{1}{12}$
- 1
- $\frac{1}{2}$
- $\frac{1}{6}$

Question 2

In the 4-d box of Question 1, what is the "diameter" —the farthest distance between two points in the box?

- $\frac{25}{12}$
- $\frac{\sqrt{205}}{12}$

- 2
- $\dfrac{1}{2\sqrt{6}}$
- $\dfrac{5}{2\sqrt{3}}$

Question 3

Consider an n-dimensional "hypercube" C of all side lengths equal to 1. Its n-dimensional volume is, clearly, 1. Now consider what happens when you shrink the hypercube's side lengths by 1 percent (concentrically, so that the shrunken cube has the same center as the original) and *remove* it from the original cube. By subtracting the n-dimensional volume of this slightly smaller hypercube, conclude how much volume remains in the 1-percent "shell".

- $n^{0.99}$
- 0.01
- $\dfrac{99-n}{100}$
- $n/100$
- $(0.99)^n$
- $1-(0.99)^n$

Question 4

In Question 3, what happens to the volume of the 1-percent shell as $n \to \infty$?

- The volume of the shell limits to 0.
- The volume of the shell limits to 1.
- The volume of the shell limits to $1/2$.

- The volume of the shell limits to $1/e$.
- The volume of the shell limits to 0.01.

Question 5

We showed in lecture that the n-dimensional volume of a *unit radius* ball in dimension n converges to zero as $n \to \infty$. But what about a really large ball? For a ball of radius $R = 10^{10}$ meters in dimension n, what is the limit as $n \to \infty$ of its volume? (in unit of meters-to-the-n^{th})

- 0
- ∞
- 10^{10}
- Surely, you can't be serious.

- [ ] In accordance with the Honor Code, I certify that my answers here are my own work.

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