Homework 6

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Which of the following are Taylor series about $x = 1$?

- [ ] $25\ln(x-1) + (x-1)^2 + (x-1)^4 + \text{H.O.T.}$
- [ ] $\frac{1}{2} + 3(x-1) + \frac{4}{45}(x-1)^2 + \frac{1}{90}(x-1)^3$
- [ ] $1 + x^2 + \frac{3}{16}x^3 + \frac{1}{90}x^4 + \text{H.O.T.}$
- [ ] $1 + (x-1) + (x-1)^2 + (x-1)^3 + \text{H.O.T.}$
- [ ] $\sum_{k=0}^{\infty} \frac{\pi^{2k}}{(2k+1)!}(x-1)^{k-1}$
- [ ] $\sum_{k=0}^{\infty} \frac{2^k}{k!}(x-1)^k$

Question 2

Which of the following is the Taylor series expansion about $x = \pi$ of $\cos 2x$?

- $\cos 2x = \sum_{k=0}^{\infty}(-1)^k 2^{2k}\frac{(x-\pi)^{2k}}{(2k)!}$
- $\cos 2x = \sum_{k=0}^{\infty}(-1)^k 2^{k}\frac{(x-\pi)^{2k+1}}{(2k+1)!}$
- $\cos 2x = \sum_{k=0}^{\infty}(-1)^k 2^{k}\frac{(x-\pi)^{2k}}{(2k)!}$

- $\cos 2x = \sum_{k=0}^{\infty} (-1)^k \dfrac{(x-\pi)^{2k+1}}{2^{2k+1}(2k+1)!}$
- $\cos 2x = \sum_{k=0}^{\infty} (-1)^k \dfrac{(x-\pi)^{2k}}{2^{2k}(2k)!}$
- $\cos 2x = \sum_{k=0}^{\infty} (-1)^k 2^{2k+1} \dfrac{(x-\pi)^{2k+1}}{(2k+1)!}$

Question 3

Which of the following is the Taylor series expansion about $x = 2$ of $\dfrac{1}{x^2}$?

- $\dfrac{1}{x^2} = \dfrac{1}{4} - \dfrac{1}{4}(x-2) + \dfrac{3}{8}(x-2)^2 + \text{H.O.T.}$
- $\dfrac{1}{x^2} = \dfrac{1}{4} - \dfrac{1}{4}(x-2) + \dfrac{3}{16}(x-2)^2 + \text{H.O.T.}$
- $\dfrac{1}{x^2} = \dfrac{1}{4} + \dfrac{1}{2}(x-2) + \dfrac{3}{64}(x-2)^2 + \text{H.O.T.}$
- $\dfrac{1}{x^2} = \dfrac{1}{4} + \dfrac{1}{4}(x-2) + \dfrac{3}{8}(x-2)^2 + \text{H.O.T.}$
- $\dfrac{1}{x^2} = \dfrac{1}{4} + \dfrac{1}{4}(x-2) + \dfrac{3}{16}(x-2)^2 + \text{H.O.T.}$
- $\dfrac{1}{x^2} = \dfrac{1}{4} - \dfrac{1}{2}(x-2) + \dfrac{3}{64}(x-2)^2 + \text{H.O.T.}$

Question 4

Which of the following is the Taylor series expansion about $x = 1$ of $\arctan x$?

- $\arctan x = \dfrac{\pi}{4} + \dfrac{1}{2}(x-1) + \dfrac{1}{8}(x-1)^2 + \text{H.O.T.}$
- $\arctan x = \dfrac{\pi}{4} + \dfrac{1}{2}(x-1) + \dfrac{1}{4}(x-1)^2 + \text{H.O.T.}$

- $\arctan x = \frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \mathrm{H.O.T.}$
- $\arctan x = \frac{\pi}{4} + \frac{1}{\sqrt{2}}(x-1) - \frac{1}{4}(x-1)^2 + \mathrm{H.O.T.}$
- $\arctan x = \frac{\pi}{4} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{4}(x-1)^2 + \mathrm{H.O.T.}$
- $\arctan x = \frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \mathrm{H.O.T.}$

Question 5

We know that $\frac{1}{x}$ does not have a Taylor series expansion about $x = 0$, since the function blows up at that point. But we can find a Taylor series about the point $x = 1$. The obvious strategy is to calculate, using induction, all the derivatives of $\frac{1}{x}$ at $x = 1$. A more interesting approach (and one that will be useful in cases in which computing derivatives would be too burdensome) is to use what we know about Taylor series about the origin: write $x = 1 + h$ and expand $\frac{1}{x}$ in a polynomial series on h. Remember to substitute h in terms of x at the end. What is the resulting series and for which values of x does it converge to the function?

- $\frac{1}{x} = \sum_{k=0}^{\infty} (-1)^k (x-1)^k$ for $|x| < 1$
- $\frac{1}{x} = \sum_{k=0}^{\infty} (x-1)^k$ for $|x| < 1$
- $\frac{1}{x} = \sum_{k=0}^{\infty} (-1)^k x^k$ for $0 < x < 2$
- $\frac{1}{x} = \sum_{k=0}^{\infty} x^k$ for $0 < x < 2$
- $\frac{1}{x} = \sum_{k=0}^{\infty} (x-1)^k$ for $0 < x < 2$

- $\frac{1}{x} = \sum_{k=0}^{\infty} (-1)^k (x-1)^k$ for $0 < x < 2$

Question 6

Which of the following is the Taylor series expansion about $x = 2$ of the function $f(x) = \frac{1}{1-x^2}$? For which values of x does the series converge to the function?

Hint: start by factoring the denominator, and then use the strategy in the previous problem, this time with $h = x - 2$.

- $f(x) = -\frac{1}{3} + \frac{4}{9}(x-2) - \frac{13}{27}(x-2)^2 + \mathrm{H.O.T.}$ for $|x| < 1$.
- $f(x) = 1 - \frac{4}{3}(x-2) + \frac{13}{9}(x-2)^2 + \mathrm{H.O.T.}$ for $|x| < 1$.
- $f(x) = -\frac{1}{3} + \frac{4}{9}(x-2) - \frac{13}{27}(x-2)^2 + \mathrm{H.O.T.}$ for $1 < x < 3$.
- $f(x) = 1 - \frac{4}{3}(x-2) + \frac{13}{9}(x-2)^2 + \mathrm{H.O.T.}$ for $1 < x < 3$.
- $f(x) = 1 + (x-2)^2 + (x-2)^4 + \mathrm{H.O.T.}$ for $1 < x < 3$.
- $f(x) = 1 + x^2 + x^4 + \mathrm{H.O.T.}$ for $|x| < 1$.

Question 7

Compute the Taylor series expansion about $x = -2$ of the function $f(x) = \frac{-1}{x^2+4x+3}$. For which values of x does the series converge to the function?

Hint: try completing the square in the denominator.

- $f(x) = \sum_{k=0}^{\infty} (-1)^k (x+2)^{2k}$ for $-3 < x < -1$.
- $f(x) = \sum_{k=0}^{\infty} \frac{1}{2^k} (x+2)^{2k}$ for $-3 < x < -1$.
- $f(x) = \sum_{k=0}^{\infty} (x+2)^{2k}$ for $|x| < 1$.
- $f(x) = \sum_{k=0}^{\infty} (-1)^k (x+2)^{2k}$ for $|x| < 1$.
- $f(x) = \sum_{k=0}^{\infty} (x+2)^{2k}$ for $-3 < x < -1$.
- $f(x) = \sum_{k=0}^{\infty} \frac{1}{2^k} (x+2)^{2k}$ for $|x| < 1$.

Question 8

Compute the Taylor series about $x = 2$ of $f(x) = \sqrt{x+2}$ up to terms of order two.

Hint: use the binomial series.

- $\sqrt{x+2} = 2 + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2 + \mathrm{H.O.T.}$
- $\sqrt{x+2} = 2 + \frac{1}{4}(x-2) - \frac{1}{32}(x-2)^2 + \mathrm{H.O.T.}$
- $\sqrt{x+2} = 2 + (x-2) - \frac{1}{16}(x-2)^2 + \mathrm{H.O.T.}$
- $\sqrt{x+2} = 2 + \frac{1}{4}(x-2) - \frac{1}{64}(x-2)^2 + \mathrm{H.O.T.}$
- $\sqrt{x+2} = 2 + \frac{1}{2}(x-2) - \frac{1}{64}(x-2)^2 + \mathrm{H.O.T.}$
- $\sqrt{x+2} = 2 + (x-2) - \frac{1}{4}(x-2)^2 + \mathrm{H.O.T.}$

☐ In accordance with the Honor Code, I certify that my answers here are my own work.

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