Homework 4

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Use a Taylor series to find a quadratic approximation for e^{2x^2} near $x = 0$.

- $e^{2x^2} \approx 1 + 2x^2$
- $e^{2x^2} \approx 2x^2$
- $e^{2x^2} \approx 1 - x - 2x^2$
- $e^{2x^2} \approx 1 + x + 2x^2$
- $e^{2x^2} \approx x + 2x^2$
- $e^{2x^2} \approx 1 - 2x^2$

Question 2

Determine the Taylor series of e^{u^2+u} up to terms of degree four.

- $e^{u^2+u} = 1 - u - \frac{1}{2}u^2 + \frac{5}{6}u^3 + \frac{25}{24}u^4 + \text{H.O.T.}$
- $e^{u^2+u} = 1 + u + \frac{3}{2}u^2 + \frac{7}{6}u^3 + \frac{5}{4}u^4 + \text{H.O.T.}$
- $e^{u^2+u} = 1 + u + \frac{3}{2}u^2 + \frac{7}{6}u^3 + \frac{25}{24}u^4 + \text{H.O.T.}$
- $e^{u^2+u} = 1 - u - \frac{1}{2}u^2 + \frac{2}{3}u^3 + \frac{25}{24}u^4 + \text{H.O.T.}$
- $e^{u^2+u} = 1 - u - \frac{1}{2}u^2 + \frac{2}{3}u^3 + \frac{5}{4}u^4 + \text{H.O.T.}$
- $e^{u^2+u} = 1 + u + \frac{3}{2}u^2 + \frac{4}{3}u^3 + \frac{5}{4}u^4 + \text{H.O.T.}$

Question 3

Compute the Taylor series expansion of $e^{1-\cos x}$ up to terms of degree four.

- $e^{1-\cos x} = 1 - \frac{x^2}{2} - \frac{x^4}{24} + \text{H.O.T.}$
- $e^{1-\cos x} = 1 + \frac{x^2}{2} + \frac{x^4}{8} + \text{H.O.T.}$
- $e^{1-\cos x} = 1 + \frac{x^2}{2} + \frac{x^4}{12} + \text{H.O.T.}$
- $e^{1-\cos x} = 1 + \frac{x^2}{2} - \frac{x^4}{24} + \text{H.O.T.}$
- $e^{1-\cos x} = 1 - \frac{x^2}{2} + \frac{x^4}{12} + \text{H.O.T.}$
- $e^{1-\cos x} = 1 - \frac{x^2}{2} + \frac{x^4}{8} + \text{H.O.T.}$

Question 4

Compute the first three nonzero terms of the Taylor series expansion of $\cos(\sin x)$.

- $\cos(\sin x) = 1 - \frac{x^2}{2} + \frac{x^4}{6} + \text{H.O.T.}$
- $\cos(\sin x) = 1 - \frac{x^2}{2} + \frac{5x^4}{6} + \text{H.O.T.}$
- $\cos(\sin x) = 1 - \frac{x^2}{2} + \frac{5x^4}{24} + \text{H.O.T.}$
- $\cos(\sin x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} + \text{H.O.T.}$
- $\cos(\sin x) = 1 - \frac{x^2}{2} + \frac{5x^4}{4} + \text{H.O.T.}$
- $\cos(\sin x) = 1 - \frac{x^2}{2} + \frac{x^4}{4} + \text{H.O.T.}$

Question 5

Compute the first three nonzero terms of the Taylor series of $\dfrac{\cos(2x)-1}{x^2}$.

- $\dfrac{\cos(2x)-1}{x^2} = -2 + \dfrac{2x^2}{3} - \dfrac{4x^4}{45} + \text{H.O.T.}$
- The function does not have a Taylor series about $x = 0$.
- $\dfrac{\cos(2x)-1}{x^2} = -\dfrac{1}{2} + \dfrac{x^2}{24} - \dfrac{x^4}{720} + \text{H.O.T.}$
- $\dfrac{\cos(2x)-1}{x^2} = -2 + \dfrac{2x^2}{3} - \dfrac{x^4}{45} + \text{H.O.T.}$
- $\dfrac{\cos(2x)-1}{x^2} = -2 + \dfrac{x^2}{6} - \dfrac{4x^4}{45} + \text{H.O.T.}$
- $\dfrac{\cos(2x)-1}{x^2} = -2 + \dfrac{x^2}{6} - \dfrac{x^4}{45} + \text{H.O.T.}$

Question 6

Determine the Taylor series expansion of $\cos x \sin 2x$ up to terms of degree five.

- $\cos x \sin 2x = 2x - \dfrac{4x^3}{3} + \dfrac{3x^5}{4} + \text{H.O.T.}$
- $\cos x \sin 2x = 2x - \dfrac{4x^3}{3} + \dfrac{61x^5}{60} + \text{H.O.T.}$
- $\cos x \sin 2x = 2x - \dfrac{4x^3}{3} + \dfrac{7x^5}{20} + \text{H.O.T.}$
- $\cos x \sin 2x = 2x - \dfrac{7x^3}{3} + \dfrac{3x^5}{4} + \text{H.O.T.}$
- $\cos x \sin 2x = 2x - \dfrac{7x^3}{3} + \dfrac{7x^5}{20} + \text{H.O.T.}$
- $\cos x \sin 2x = 2x - \dfrac{7x^3}{3} + \dfrac{61x^5}{60} + \text{H.O.T.}$

Question 7

Compute the Taylor series expansion of $x^{-1}e^x \sin x$ up to terms of degree four.

- $x^{-1}e^x \sin x = 1 + x + \frac{7x^2}{6} + \frac{5x^3}{24} - \frac{x^4}{60} + \text{H.O.T.}$
- $x^{-1}e^x \sin x = 1 + x + \frac{7x^2}{6} + \frac{x^3}{3} + \frac{2x^4}{15} + \text{H.O.T.}$
- $x^{-1}e^x \sin x = 1 + x + \frac{x^2}{3} - \frac{x^4}{24} + \text{H.O.T.}$
- $x^{-1}e^x \sin x = 1 + x + \frac{7x^2}{6} + \frac{x^3}{24} + \frac{3x^4}{40} + \text{H.O.T.}$
- $x^{-1}e^x \sin x = 1 + x + \frac{x^2}{3} - \frac{x^4}{30} + \text{H.O.T.}$
- $x^{-1}e^x \sin x = 1 + x + \frac{x^2}{3} - \frac{3x^4}{40} + \text{H.O.T.}$

Question 8

Determine the first three nonzero terms of the Taylor expansion of $\frac{e^{2x}\sinh x}{2x}$.

- $\frac{e^{2x}\sinh x}{2x} = \frac{1}{2} + \frac{x}{2} + \frac{11x^2}{12} + \text{H.O.T.}$
- $\frac{e^{2x}\sinh x}{2x} = \frac{1}{2} + x + \frac{x^2}{12} + \text{H.O.T.}$
- $\frac{e^{2x}\sinh x}{2x} = \frac{1}{2} + x + \frac{13x^2}{12} + \text{H.O.T.}$
- $\frac{e^{2x}\sinh x}{2x} = \frac{1}{2} + x + \frac{11x^2}{12} + \text{H.O.T.}$
- $\frac{e^{2x}\sinh x}{2x} = \frac{1}{2} + \frac{x}{2} + \frac{13x^2}{12} + \text{H.O.T.}$
- $\frac{e^{2x}\sinh x}{2x} = \frac{1}{2} + \frac{x}{2} + \frac{x^2}{12} + \text{H.O.T.}$

Question 9

Suppose that a function $f(x)$ is *reasonable*, so that it has a Taylor series

$$f(x) = c_0 + c_1 x + c_2 x^2 + \mathrm{H.O.T.}$$

with $c_0 \neq 0$. Then the reciprocal function $g(x) = 1/f(x)$ is defined at $x = 0$ and is also reasonable. Let

$$g(x) = b_0 + b_1 x + b_2 x^2 + \mathrm{H.O.T.}$$

be its Taylor series. Because $f(x)g(x) = 1$, we have

$$\big(c_0 + c_1 x + c_2 x^2 + \mathrm{H.O.T.}\big)\big(b_0 + b_1 x + b_2 x^2 + \mathrm{H.O.T.}\big) = 1 + 0x + 0x^2 + \mathrm{H.O.T.}$$

Multiplying out the two series on the left hand side and combining like terms, we obtain

$$c_0 b_0 + \big(c_0 b_1 + c_1 b_0\big)x + \big(c_0 b_2 + c_1 b_1 + c_2 b_0\big)x^2 + \mathrm{H.O.T.} = 1 + 0x + 0x^2 + \mathrm{H.O.T.}$$

Equating the coefficients of each power of x on both sides of this expression, we arrive at the (infinite!) system of equations

$$\begin{aligned} c_0 b_0 &= 1 \\ c_0 b_1 + c_1 b_0 &= 0 \\ c_0 b_2 + c_1 b_1 + c_2 b_0 &= 0 \\ &\cdots \end{aligned}$$

relating the coefficients of the Taylor series of $f(x)$ to those of the Taylor series of $g(x)$. For example, the first equation yields $b_0 = 1/c_0$, while the second gives $b_1 = -c_1 b_0/c_0 = -c_1/c_0^2$.

Using the above reasoning for $f(x) = \cos x$, determine the Taylor series of $g(x) = \sec x$ up to terms of degree two.

- $\sec x = 1 - x^2 + \mathrm{H.O.T.}$
- $\sec x = 1 + x^2 + \mathrm{H.O.T.}$
- $\sec x = 1 + \dfrac{x^2}{2} + \mathrm{H.O.T.}$
- $\sec x = 1 - 2x^2 + \mathrm{H.O.T.}$
- $\sec x = 1 + 2x^2 + \mathrm{H.O.T.}$

- $\sec x = 1 - \dfrac{x^2}{2} + \text{H.O.T.}$

- [ ] In accordance with the Honor Code, I certify that my answers here are my own work.

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