Homework 38

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Consider a dam of height H and width W that has a perfectly vertical face facing the water, which reaches all the way up to the dam's height. If the water has weight density ρ, what is the total force the water exerts against the face of the dam?

- $H^2 W \rho$
- $\frac{1}{2} H W^2 \rho$
- $\frac{1}{4} H^2 W \rho$
- $\frac{1}{2} H W \rho$
- $\frac{1}{2} H^2 W \rho$
- $\frac{1}{4} H W^2 \rho$

Question 2

Consider two potential income streams, each valued based on an assumption of a constant return on investment at rate $r > 0$. The first, I_1, starts off slow, then peaks, and then decreases. The second, I_2, starts off high, then decreases. Both oscillate eventually with the same period. The specific formulae are:

$$I_1(t) = I_0 + A \sin \frac{\pi t}{P} \quad ; \quad I_2(t) = I_0 + A \cos \frac{\pi t}{P}$$

Here, $I_0 > 0$ is a constant (the baseline income), $A > 0$ is a constant (the amplitude of fluctuation) and $P > 0$ is a constant (the half-period). Assume $\pi > Pr$. Which income stream has the greater present value over the time interval $[0, P]$? Which has the greater present value over the time interval $[0, +\infty)$?

Hints: (1) Which constants are important? I_0 ? A? P? r? (2) You may want a reduction formula like that from Lecture 22. (3) If you get stuck in the algebra, try using WolframAlpha.

- On $[0, P]$, $PV_1 = PV_2$; but on $[0, +\infty)$, $PV_1 < PV_2$.
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Question 3

We have learned about *present value* of an income stream $I(t)$; one may also reverse the derivation to determine the *future value* of the income at a time $t=T$. The future value element of $I(t)$ is

$$dFV = e^{r(T-t)} I(t)dt,$$

assuming a continuous compounding at fixed interest rate r.

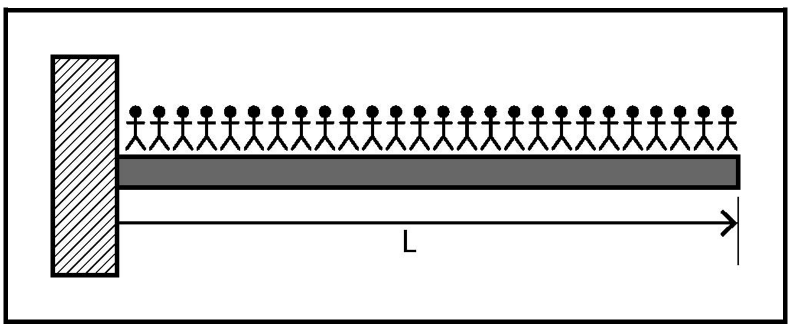
If you save for a child's college at a rate of $\$5{,}000/\mathrm{year}$ starting at the child's birth, how much money will be available when she is 20? Assume a fixed 5% return on investments.

- $FV = \$100{,}000$
- $FV = \$50{,}000e$
- $FV = \$50{,}000\sqrt{e}$

- $FV = \$500{,}000$
- $FV = \$100{,}000e$
- $FV = \$100{,}000(e-1)$

Question 4

Consider a cantilever beam of length L. Suppose that N people, each of mass m_0, stand on it equally spaced, so that their combined weight is supported uniformly along the beam. If $L = 20\,\mathrm{m}$, $m_0 = 75\,\mathrm{kg}$ and, at the point of attachment, the beam can withstand a maximum torque of $\tau_{\max} = 1.5 \cdot 10^6\ \mathrm{N \cdot m}$, what is the maximum number of people that can stand on it? Assume the acceleration of gravity to be $g = 10\,\mathrm{m/s^2}$.



- 200 people.
- 50 people.
- 100 people.
- 25 people.
- 400 people.
- 300 people.

Question 5

Suppose that a radiator is turned off at $t = 0$; after that, the amount of heat generated by the radiator is described by the *heat flow element*

$$dQ = Q_0 e^{-\lambda t} dt$$

where both Q_0 and λ are positive constants. What is the total amount of heat radiated from the moment it is turned off?

- $\lambda^2 Q_0$
- $\sqrt{\lambda} Q_0$
- λQ_0
- $Q_0 e^{-\lambda}$
- $\dfrac{Q_0}{\lambda}$
- $Q_0 e^{\lambda}$

Question 6

Air density ρ in the atmosphere varies according to altitude h, according to the formula

$$\rho(h) = \frac{MP_0}{RT_0}\left(1 - \frac{Lh}{T_0}\right)^{(gM/RL)-1}$$

Assume everything except the altitude h is a constant. Compute the mass of air in a cylinder of height H whose base has area A and rests at sea level — that is, at height $h = 0$.

Hint: compute the mass element for a disc of area A and infinitesimal thickness dh located at a height h by multiplying its volume element dV by the density $\rho(h)$ at height h.

For the curious: the constants in the formula above represent the following:

- the molar mass of dry air, $M = 2.896 \cdot 10^{-2}\ \mathrm{kg/mol}$,
- the ideal gas constant, $R = 8.314\,\mathrm{J/(mol\,K)}$,
- the sea level standard atmospheric pressure, $P_0 = 1.101 \cdot 10^5\ \mathrm{N/m^2}$,

- the sea level standard temperature, $T_0 = 288.2K$,
- the temperature lapse rate (that is, the rate of change of temperature with respect to altitude), $L = 6.5 \cdot 10^{-3}\,\mathrm{K/m}$, and
- the acceleration of gravity, $g = 9.806\,\mathrm{m/s^2}$.

With these values, you can now compute some accurate figures! For more information, consult the Wikipedia page on density of air.

- $\dfrac{AP_0}{g}\left(1 - \dfrac{LH}{T_0}\right)^{gM/RL}$
- $\dfrac{ARLP_0}{gM}\left[1 - \left(1 - \dfrac{LH}{T_0}\right)^{gM/RL}\right]$
- $\dfrac{ALP_0}{gT_0}\left(1 - \dfrac{LH}{T_0}\right)^{gM/RL}$
- $\dfrac{AP_0}{g}\left[1 - \left(1 - \dfrac{LH}{T_0}\right)^{gM/RL}\right]$
- $\dfrac{ARLP_0}{gT_0}\left[\left(1 - \dfrac{LH}{T_0}\right)^{gM/RL} - 1\right]$
- $\dfrac{ALP_0}{gT_0}\left[\left(1 - \dfrac{LH}{T_0}\right)^{gM/RL} - 1\right]$

☐ In accordance with the Honor Code, I certify that my answers here are my own work.

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