Homework 36

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Think of the sphere of radius 1 as obtained by revolving the curve $y = \sqrt{1-x^2}$ about the x-axis. For any $-1 \le a < b \le 1$, calculate the surface area of the slice between $x = a$ and $x = b$.

- $2\pi(b+a)$
- $2\pi(b-a)$
- $2\pi(b^2-a^2)$
- $4\pi\sqrt{\frac{b+a}{2}}$
- $4\pi\sqrt{\frac{b-a}{2}}$
- $2\pi(b^2+a^2)$

Question 2

A typical dish antenna is built as a surface of revolution obtained by revolving a parabola about an axis of symmetry. One of the main benefits of this design is that the resulting antenna exhibits very high gains in the direction towards which it points, making it well-suited for applications in which a strong directionality is needed —such as TV reception and radar.

We can model such a *parabolic antenna* as the surface of revolution obtained by revolving the function

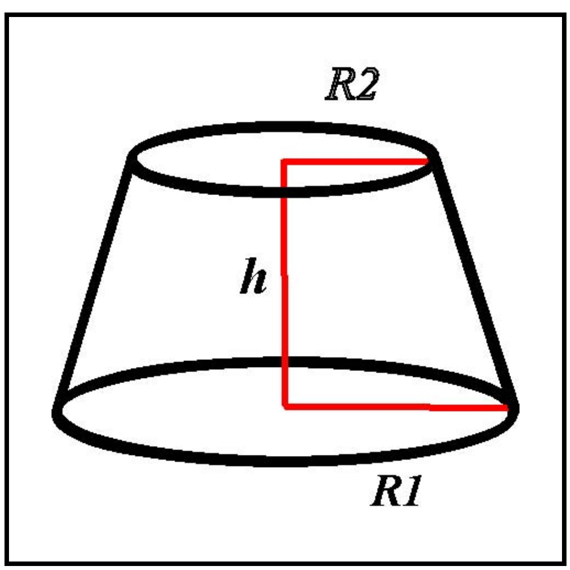
$$y = \sqrt{\frac{K}{4}}x^2, \qquad 0 \le x \le R$$

about the y-axis. Here R is the radius of the antenna, and K —the curvature at the tip— controls how flat it is. Compute the surface area of this antenna in terms of the parameters R and K.

- $\dfrac{2\sqrt{2}}{3} R^{1/2} K^{-3/4}$
- $\dfrac{2\pi}{3K} \left[\left(1+KR^2\right)^{3/2} - 1\right]$
- $\dfrac{\pi}{K} \left[(1+2RK)^{1/2} - 1\right]$
- $\dfrac{2\pi}{3K} \left[(1+2RK)^{3/2} - 1\right]$
- $\dfrac{\pi}{K} \left[\left(1+KR^2\right)^{1/2} - 1\right]$
- $\dfrac{4\sqrt{2}}{3} R^{3/2} K^{-1/4}$

Question 3

Consider the truncated circular cone in the figure (without the bottom and top circles).



It can be modeled as the surface of revolution obtained by revolving the line

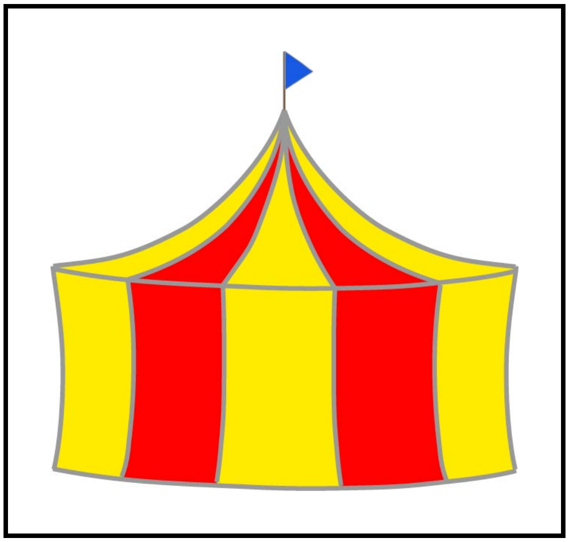
$$y = R_1 + (R_2 - R_1)\,\frac{x}{h}\,, \qquad 0 \le x \le h$$

about the x-axis. Which of the following expressions describes its surface area in terms of the parameters h, R_1 and R_2 ?

- $\frac{\pi}{2}\,(R_1 + R_2)\sqrt{h^2 + (R_2 - R_1)^2}$
- $\frac{\pi(R_1 + R_2)}{2\sqrt{h^2 + (R_2 - R_1)^2}}$
- $\frac{\pi}{2}\,(R_1 + R_2)\left(h^2 + (R_2 - R_1)^2\right)^{3/2}$
- $\pi(R_1 + R_2)\left(h^2 + (R_2 - R_1)^2\right)^{3/2}$
- $\frac{\pi(R_1 + R_2)}{\sqrt{h^2 + (R_2 - R_1)^2}}$
- $\pi(R_1 + R_2)\sqrt{h^2 + (R_2 - R_1)^2}$

Question 4

Consider a circular tent whose roof is made of fabric hanging from the rim of the walls of the tent and supported at a central pole.



If you look at the curve that the fabric roof forms along any radial cross-section, you will discover a catenary —that is, a hyperbolic cosine. Modeling the roof as the surface of revolution obtained by revolving the curve

$$y = R\cosh\left(1 - \frac{x}{R}\right), \qquad 0 \leq x \leq R$$

around the y-axis, which of the following integrals computes its surface area?

- $2\pi R \displaystyle\int_{u=0}^{1} u\cosh(1-u)\,du$
- $2\pi R^2 \displaystyle\int_{u=0}^{1} u\cosh(1-u)\,du$
- $2\pi R \displaystyle\int_{u=0}^{1} u\sinh(1-u)\,du$
- $2\pi R^2 \displaystyle\int_{u=0}^{1} u\sinh(1-u)\,du$
- $2\pi \displaystyle\int_{u=0}^{1} u\cosh(1-u)\,du$
- $2\pi \displaystyle\int_{u=0}^{1} u\sinh(1-u)\,du$

Question 5

A *diabolo* is a juggling prop that can be modeled as the surface of revolution obtained by revolving the parametrized curve

$$x = t^3, \quad y = t^2, \qquad -2/3 \leq t \leq 2/3$$

about the x-axis. What is its surface area?

Hint: you might remember this curve as the *cuspidal cubic* of Homework 35, so you know what its arc length element dL is. Remember that the surface area element is given by $2\pi r dL$, where r is the radius of an infinitesimal ring. You may also use that

$$\int_{u=0}^{1} u^3 \sqrt{1+u^2}\, du = \frac{2}{15}\left(1+\sqrt{2}\right)$$

[Image courtesy of Wikimedia Commons]

- $\frac{16\pi}{15}\left(\frac{2}{3}\right)^4 \left(1+\sqrt{2}\right)$
- $\frac{8\pi}{15}\left(\frac{2}{3}\right)^4 \left(1+\sqrt{2}\right)$

- $\frac{8\pi}{15}\left(\frac{2}{3}\right)^3(1+\sqrt{2})$
- $\frac{16\pi}{15}\left(\frac{2}{3}\right)^2(1+\sqrt{2})$
- $\frac{8\pi}{15}\left(\frac{2}{3}\right)^2(1+\sqrt{2})$
- $\frac{16\pi}{15}\left(\frac{2}{3}\right)^3(1+\sqrt{2})$

- [ ] In accordance with the Honor Code, I certify that my answers here are my own work.

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