Homework 19

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Solve the differential equation $\frac{dx}{dt} = \frac{x}{t}$.

Note: observe that this equation can be rearranged as $\frac{dx}{x} = \frac{dt}{t}$, which says that the relatives rates of change of x and t are equal.

- $x(t) = Ce^t$
- $x(t) = \ln t + C$
- $x(t) = Ct$
- $x(t) = t + C$
- $x(t) = \ln(t + C)$
- $x(t) = e^t + C$

Question 2

Solve the differential equation $\frac{dx}{dt} = \frac{\sqrt{1-x^2}}{\sqrt{1-t^2}}$.

- $x(t) = \sin(\arcsin t + C)$
- $x(t) = t + C$
- $x(t) = \arcsin \sin(t + C)$
- $x(t) = \arcsin(\sin t + C)$
- $x(t) = Ct$
- $x(t) = \sin \arcsin(t + C)$

Question 3

Given that $x(0) = 0$ and $\frac{dx}{dt} = te^x$, compute $x(1)$.

- $x(1) = \sqrt{2}$
- $x(1) = 0$
- $x(1) = \ln \frac{1}{2}$
- $x(1) = \frac{1}{2}$
- $x(1) = \ln 2$
- $x(1) = 2$

Question 4

German physician Ernst Heinrich Weber (1795-1878) is considered one of the fathers of experimental psychology. In his study of perception, he noticed that the perceived difference between two almost-equal stimuli is proportional to the *percentual* difference between them. In terms of differentials, we can express Weber's law as

$$dp = k\frac{dS}{S}$$

where p is the perceived intensity of a stimulus and S its actual strength.

Observe that $\frac{dS}{S}$ is the *relative* rate of change of S.

In what way must the magnitude of a stimulus change in time for a human being to perceive a linear growth?

- Rationally.
- Proportional to the square root.
- None of these.

- Exponentially.
- Linearly.
- Logarithmically.

Question 5

Which of the following is the *integrating factor* used to solve the following linear differential equation?

$$t^2 \frac{dx}{dt} = 4t - t^5 x$$

- $I(t) = e^{t^6/6}$
- $I(t) = e^{1/2t^2}$
- $I(t) = e^{-t^6/6}$
- $I(t) = e^{-t^4/4}$
- $I(t) = e^{t^4/4}$
- $I(t) = e^{-1/2t^2}$

Question 6

Solve the differential equation $\frac{dx}{dt} - 5x = 3$.

- $x(t) = -\frac{3}{5} + Ce^{-5t}$
- $x(t) = -\frac{5}{3} + Ce^{5t}$
- $x(t) = \frac{5}{3} + Ce^{5t}$
- $x(t) = -\frac{3}{5} + Ce^{5t}$

- $x(t) = -\dfrac{5}{3} + Ce^{-5t}$
- $x(t) = \dfrac{3}{5} + Ce^{5t}$

Question 7

Solve the differential equation $\dfrac{dx}{dt} = \dfrac{x}{1+t} + 2$.

- $x(t) = 2t(1+t) + C(1+t)$
- $x(t) = 2C(1+t)\ln(1+t) + (1+t)$
- $x(t) = C(1+t) + \dfrac{1}{1+t}$
- $x(t) = 2(1+t)\ln(1+t) + C(1+t)$
- $x(t) = \dfrac{1+t}{2} + \dfrac{C}{1+t}$
- $x(t) = 1 + t + \dfrac{C}{1+t}$

Question 8

In the present economy, everyone (individuals and nations) has to understand *debt*. Suppose that, in order to buy a house, you obtain a mortgage. If the lender advertises an annual interest rate r, your debt D will increase exponentially according to the simple O.D.E.

$$\frac{dD}{dt} = rD.$$

If you pay your debt at a rate of P per year, the evolution of your debt will then (under assumptions of continual compounding and payment) obey the linear differential equation

$$\frac{dD}{dt} = rD - P$$

Using this model, answer the following question: if initial amount of the mortgage is for $400,000, the annual interest rate is 5%, and you pay at a rate of $40,000 every year, how many years will it take you to pay off the debt? Round your answer to the nearest integer.

Question 9

Some nonlinear differential equations can be reduced to linear ones by a clever change of variables. Bernouilli equations

$$\frac{dx}{dt} + p(t)x = q(t)x^{\alpha}, \qquad \alpha \in \mathbb{R}$$

constitute the most important case. Notice that for $\alpha = 0$ or $\alpha = 1$ the above equation is already linear. For other values of α, the substitution $u = x^{1-\alpha}$ yields a linear differential equation in the variable u.

Apply the above change of variables in the case

$$\frac{dx}{dt} + 2tx = x^3$$

to find the linear differential equations satisfied by u.

- $\frac{du}{dt} = 1 - 2t$
- $\frac{du}{dt} + 2tu = -2$
- $\frac{du}{dt} - 4tu = -2$
- $\frac{du}{dt} - 4tu = 1$

- $\frac{du}{dt} + 2tu = 1$
- $\frac{du}{dt} = -2 - 2t$

In accordance with the Honor Code, I certify that my answers here are my own work.

Submit Answers Save Answers