Problem Set VI

The **due date** for this homework is **Tue 5 Mar 2013 12:59 AM EST**.

In this problem set, you will be given a total of ten attempts. We will accept late submission until the fifth day after the due date, and late submission will receive half credit. Explanations and answers to the problem set will be available after the due date. Since the homework problems will become gradually more challenging as the course proceeds, we highly recommend you to start the habit of printing out the problems and working on them with paper and pencil. Also, please be sure to read the problem statements carefully and double check your expressions before you submit.

A pdf version of this problem set is available for you to print. Note: all mathematical expressions have to be exact, even when involving constants. Such an expression is required when a function and/or a variable is required in the answer. For example, if the answer is $\sqrt{3}x$, you must type `sqrt(3)*x`, not `1.732*x` for the answer to be graded as being correct.

Question 1

The signal $s(t)$ is bandlimited to 4 kHz. We want to sample it, but it has been subjected to various signal processing manipulations.

What minimum sampling frequency, F_s, can be used to sample the result of passing $s(t)$ through an RC *highpass* filter with $R = 10\ \mathrm{k\Omega}$ and $C = 8$ nF? If no frequencies work, enter 0.

$F_s = ?$ Hz. **NOTE:** Answer in Hertz, not kHz.

Question 2

What frequency can be used to sample the **derivative** of $s(t)$ mentioned in the previous problem? If no frequencies work, enter 0.

$F_s = ?$ Hz. **NOTE:** Answer in Hertz, not kHz.

Question 3

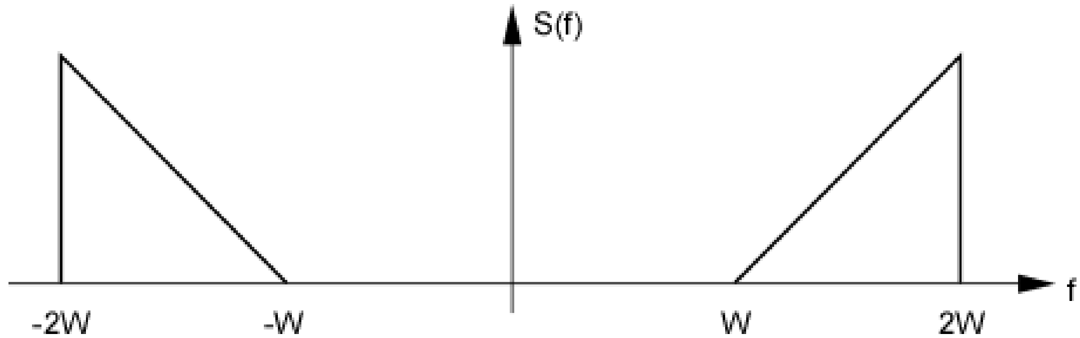
The signal $s(t)$ has been modulated by an 8 kHz sinusoid having an unknown phase: the resulting signal is $s(t)\sin(2\pi f_0 t + \phi)$ with $f_0 = 8$ kHz and $\phi = ?$ Can the modulated signal be sampled so that the **original** signal can be recovered from the modulated signal regardless of the phase value ϕ? Enter the smallest sampling rate that allows for full recovery of the original signal. If the original signal can not be recovered enter `0`.

$F_s = ?$ Hz **NOTE:** Answer in Hertz, not kHz.

Question 4

Bandpass Sampling

The signal $s(t)$ has the indicated spectrum.



What is the minimum sampling rate for this signal suggested by the Sampling Theorem? Express your answer as an expression in terms of W (type `W` in your answer).

$F_s = ?$ Hz

Preview

Question 5

Because of the particular structure of this bandpass spectrum, you might wonder whether a lower sampling rate could be used. This is indeed the case, first find the lower sampling rate that can be used to reconstruct $s(t)$ from its samples. Express your answer in terms of `W`.

$F_s = ?$ Hz

Preview

Question 6

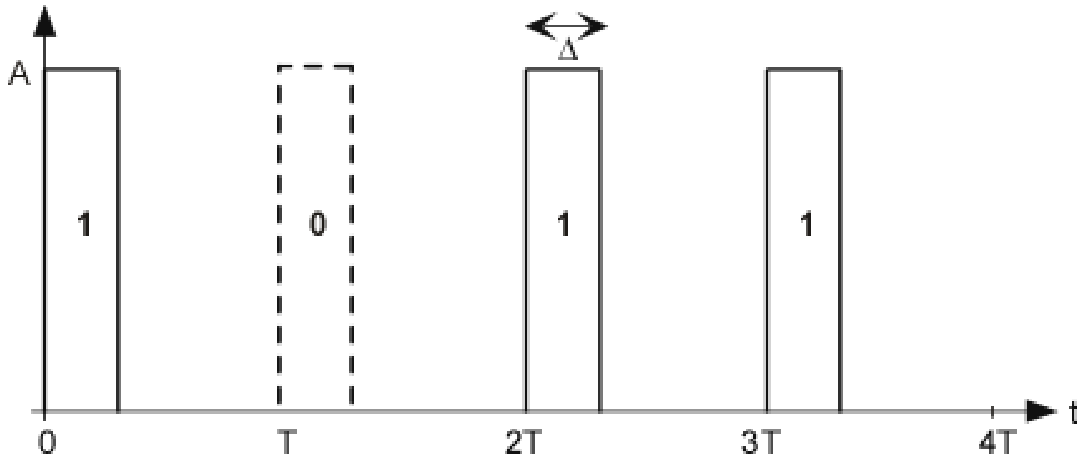
After using the lower sampling frequency for the above problem, it is necessary to filter the sampled signal. What filter is required to complete the system?

- Lowpass to W
- Bandpass
- Lowpass to $2W$
- Highpass

Question 7

Simple D/A Converter

Commercial digital-to-analog converters don't work this way, but a simple circuit illustrates how they work. Let's assume we have a B-bit converter. Thus we want to convert numbers having a B-bit representation into a voltage proportional to that number. The first step taken by our simple converter is to represent the number by a sequence of B pulses occurring at multiples of a time interval T. The presence of a pulse indicates a "1" in the corresponding bit position, and pulse absence means a "0" occurred. For a 4-bit converter, the number 13 has the binary representation 1101 ($13_{10} = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$) and would be represented by the depicted pulse sequence.



Note that the pulse sequence is "backwards" from the binary representation. We'll see why that is in the next three questions.

This signal serves as the input to a first-order RC lowpass filter. We want to design the filter and the parameters Δ and T so that the output voltage at time $4T$ (for a 4-bit converter) is proportional to the (decimal) number. This combination of pulse creation and filtering constitutes our simple D/A converter. The requirements are:

- The voltage at time $t = 4T$ should diminish by a factor of 2 the further the pulse occurs from this time. In other words, the voltage due to a pulse at $3T$ should be twice that of a pulse produced at $2T$, which in turn is twice that of a pulse at T.
- The 4-bit D/A converter must support a 10 kHz sampling rate.

What is the response to a pulse when $0 \leq t \leq \Delta$? Express your answer in terms of $A, R, C, t,$ and Δ.

(If necessary, enter Δ by typing `Delta`.)

Output = ?, $0 \leq t \leq \Delta$

Preview

Question 8

What is the response to a single pulse when $t > \Delta$? Express your answer in terms of $A, R, C, t,$ and Δ. (If necessary, enter Δ by typing `Delta`.)

Output = ?, $t > \Delta$

Preview

Question 9

What **numerical** value of the product RC satisfies the design constraints?

$RC = ?$ seconds

☐ In accordance with the Honor Code, I certify that my answers here are my own work.

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