Homework 2

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Using Euler's formula, compute the product $e^{ix} \cdot e^{iy}$. What is the real part (that is, the one without i)? Remember that $i^2 = -1$.

- $\cos x \cos y - \sin x \sin y$
- $\sin x \cos y + \cos x \sin y$
- $\cos x \cos y + \sin x \sin y$
- $\sin x \cos y - \cos x \sin y$

Question 2

Let n be an integer. Using Euler's formula we have

$$e^{inx} = \cos nx + i \sin nx$$

On the other hand, we also have

$$e^{inx} = \left(e^{ix}\right)^n = (\cos x + i \sin x)^n$$

Putting both of these expressions together, we obtain *de Moivre's formula*:

$$\cos nx + i \sin nx = (\cos x + i \sin x)^n$$

Use the latter to find an expression for $\sin 3x$ in terms of $\sin x$ and $\cos x$.

- $\sin 3x = 4\cos^3 x - 3\cos x$
- $\sin 3x = 3\sin x - 4\sin^3 x$
- $\sin 3x = 3\sin x \cos^2 x - \sin^3 x$

- $\sin 3x = \cos^3 x - 2\sin^2 x \cos x$

Question 3

Find all possible solutions to the equation $e^{ix} = i$.

- $x = \dfrac{(4n+1)\pi}{2}$ for all $n \in \mathbb{Z}$
- $x = \dfrac{(2n+1)\pi}{2}$ for all $n \in \mathbb{Z}$
- $x = n\pi$ for all $n \in \mathbb{Z}$
- $x = \dfrac{\pi}{2}$
- $x = \dfrac{n\pi}{2}$ for all $n \in \mathbb{Z}$
- $x = \dfrac{\pi}{4}$

Question 4

Calculate $\displaystyle\sum_{k=0}^{\infty}(-1)^k \frac{(\ln\ 4)^k}{k!}$.

- e^4
- -4
- e^{-4}
- $-\dfrac{1}{4}$
- $\dfrac{1}{4}$
- 4

Question 5

Calculate $\sum_{k=0}^{\infty}(-1)^k \frac{\pi^{2k}}{(2k)!}$.

- $-\pi$
- π
- 0
- e^{π}
- -1
- 1

Question 6

Write out the first four terms of the sum $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} 2^k}{2k-1}$.

- $\frac{2}{3} - \frac{4}{5} + \frac{8}{7} - \frac{16}{9} + \cdots$
- $2 + \frac{4}{3} - \frac{8}{5} + \frac{16}{7} + \cdots$
- $-2 + \frac{4}{3} - \frac{8}{5} + \frac{16}{7} + \cdots$
- $-1 + 2 - \frac{4}{3} + \frac{8}{5} + \cdots$
- $2 - \frac{4}{3} + \frac{8}{5} - \frac{16}{7} + \cdots$
- $-\frac{2}{3} + \frac{4}{5} - \frac{8}{7} + \frac{16}{9} + \cdots$

Question 7

Write out the first four terms of the sum $\sum_{k=0}^{\infty} \frac{(-1)^k \pi^{2k}}{k!(2k+1)}$.

- $-\frac{\pi^2}{3} + \frac{\pi^4}{10} - \frac{\pi^6}{42} + \frac{\pi^8}{216}$
- $1 - \frac{\pi^2}{3} + \frac{\pi^4}{10} - \frac{\pi^6}{42}$
- $\frac{\pi^2}{3} + \frac{\pi^4}{10} + \frac{\pi^6}{42} + \frac{\pi^8}{216}$
- $1 + \frac{\pi^2}{3} + \frac{\pi^4}{10} + \frac{\pi^6}{42}$
- $-\frac{\pi^2}{10} + \frac{\pi^4}{42} - \frac{\pi^6}{216} - \frac{\pi^8}{1320}$
- $1 - \frac{\pi^2}{10} + \frac{\pi^4}{42} - \frac{\pi^6}{216}$

Question 8

Which of the following expressions describes the sum $\frac{e}{2} - \frac{e^2}{4} + \frac{e^3}{6} - \frac{e^4}{8} + \cdots$?

- $\sum_{k=0}^{\infty} (-1)^{k+1} \frac{e^{k+1}}{2k+2}$
- $\sum_{k=1}^{\infty} (-1)^{k} \frac{e^{k}}{2k}$
- $\sum_{k=0}^{\infty} (-1)^{k+1} \frac{e^{k}}{2k}$
- $\sum_{k=1}^{\infty} (-1)^{k} \frac{e^{k+1}}{2k+2}$
- $\sum_{k=0}^{\infty} (-1)^{k} \frac{e^{k+1}}{2k+2}$

- $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\,\frac{e^k}{2k}$

Question 9

Which of the following expressions describes the sum

$-1+\dfrac{x}{2\cdot 1}-\dfrac{x^2}{3\cdot 2\cdot 1}+\dfrac{x^3}{4\cdot 3\cdot 2\cdot 1}+\cdots$?

- $\displaystyle\sum_{k=0}^{\infty}(-1)^{k+1}\,\frac{x^k}{(k+1)!}$
- $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\,\frac{x^k}{(k+1)!}$
- $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\,\frac{x^k}{(k+1)!}$
- $\displaystyle\sum_{k=1}^{\infty}(-1)^{k}\,\frac{x^{k-1}}{k!}$
- $\displaystyle\sum_{k=0}^{\infty}(-1)^{k}\,\frac{x^{k-1}}{k!}$
- $\displaystyle\sum_{k=0}^{\infty}(-1)^{k}\,\frac{x^k}{(k+1)!}$

Question 10

Engineers typically use powers of 10 and logarithms in base 10. In mathematics, we tend to prefer exponentials with base e and natural logarithms. We have seen in lecture one of the main reasons: the derivative of the exponential function e^x is itself. For applications, it is important that we know how to translate between logarithms in base e and those in base 10. In order to find such a formula, suppose

$$y=\ln x \quad \text{and} \quad z=\log_{10} x$$

Eliminate x between these two equations to find the relationship between y and z.

- $y = z \ln 10$
- $y = \dfrac{z}{\ln 10}$
- $z = y \log_{10} e$
- $z = \dfrac{y}{\log_{10} e}$

In accordance with the Honor Code, I certify that my answers here are my own work.

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