Feedback — Chapter 5 Quiz: Discretization

Thank you. Your submission for this exam was received.

You submitted this exam on **Sun 7 Apr 2013 9:01 PM EDT -0400**. You got a score of **9.50** out of **10.00**.

Question 1

$$\lim_{n\to\infty}\left(\frac{2n+1}{2n-1}\right)^{2n-1} =$$

Your Answer		Score	Explanation
e			
$+\infty$			
1			
e^{-1}			
e^{-2}			
e^2	✓	1.00	
Total		1.00 / 1.00	

Question 2

Consider the sequence $b = \left(3n^2 - n\right)$. Which of the following sequences $a = (a_n)$ satisfies $\Delta a = b$?

Hint: rewrite the sequence a in terms of falling powers $n^{\underline{k}} = n(n-1)\cdots(n-k+1)$ and use $\Delta n^{\underline{k}} = kn^{\underline{k-1}}$.

Your Answer		Score	Explanation
$a = \left(n^{\underline{3}} + \frac{1}{2}n^{\underline{2}}\right)$			
$a = (n^{\underline{3}} + 2n^{\underline{2}} - n)$			
$a = \left(n^{\underline{3}} - \frac{7}{2}n^{\underline{2}} + \frac{5}{2}n\right)$			
$a = \left(n^{\underline{3}} + \frac{7}{2}n^{\underline{2}} - \frac{5}{2}n\right)$			
$a = \left(n^{\underline{3}} - \frac{1}{2}n^{\underline{2}}\right)$			
$a = (n^{\underline{3}} - 2n^{\underline{2}} + n)$	✓	1.00	
Total		1.00 / 1.00	

Question 3

Use **Euler's method** to compute the first three iterates, (x_1, x_2, x_3), of the differential equation

$$\frac{dx}{dt} = t - 2x$$

starting with $t_0 = 0$, $x_0 = x(t_0) = 3$ and using a constant step size $h = \Delta t = 0.1$. You do not need a calculator to do this (so long as you can do decimal arithmetic...), but you *are* allowed to use (a simple) calculator for this problem if you wish.

Your Answer	Score	Explanation
$(2.4, 1.88, 1.358)$		

◉ $(2.4, 1.93, 1.564)$	✓	1.00
○ $(2.4, 1.88, 1.529)$		
○ $(2.4, 1.93, 1.442)$		
○ $(2.4, 1.93, 1.391)$		
○ $(2.4, 1.88, 1.522)$		
○ $(2.4, 1.93, 1.612)$		
○ $(2.4, 1.88, 1.678)$		
Total		1.00 / 1.00

Question 4

Which of the following statements are true about the series $\sum_{n=2}^{\infty} \frac{3\pi}{n^2(\ln n)^2}$? Choose all that apply.

Your Answer		Score	Explanation
☐ $\frac{3\pi}{n^2(\ln n)^2} < \frac{3\pi}{n^4}$ for $n > 2$, so the comparison test says that the series converges.	✓	0.00	
☐ $\frac{3\pi}{n^2(\ln n)^2} > \frac{3\pi}{n}$ for $n > 2$, so the comparison test says that the series diverges.	✓	0.00	
☐ The integral $\int_{x=2}^{+\infty} \frac{3\pi}{x^2(\ln x)^2}\,dx$ diverges, so the integral test says that the series diverges.	✓	0.00	
☐ $\lim_{n\to\infty} \frac{3\pi}{n^2(\ln n)^2} = 0$, so the n-th term test says that the series converges.	✓	0.00	

$\dfrac{3\pi}{n^2(\ln n)^2} < \dfrac{3\pi}{n^2}$ for $n > 2$, so the comparison test says that the series converges.	✓	0.50
The integral $\displaystyle\int_{x=2}^{+\infty} \frac{3\pi}{x^2(\ln x)^2}\,dx$ converges, so the integral test says that the series converges.	✓	0.50
Total		1.00 / 1.00

Question 5

Determine the asymptotic behavior of the sequence $a = (a_n) = \left(n\,e^{1/n^2} - n\right)$ as $n \to +\infty$. Using this information, determine whether the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges or diverges.

Your Answer	Score	Explanation
$a_n = \dfrac{1}{n^2} + O\left(\dfrac{1}{n^3}\right)$ as $n \to +\infty$, so the series converges.		
$a_n = 1 + O\left(\dfrac{1}{n^2}\right)$ as $n \to +\infty$, so the series diverges.		
$a_n = \dfrac{1}{n^2} + O\left(\dfrac{1}{n^4}\right)$ as $n \to +\infty$, so the series converges.		
$a_n = \dfrac{1}{n} + O\left(\dfrac{1}{n^3}\right)$ as $n \to +\infty$, so the series converges.		
$a_n = n + O(1)$ as $n \to +\infty$, so the series diverges.		

$a_n = \frac{1}{n} + O\left(\frac{1}{n^3}\right)$ as $n \to +\infty$, so the series diverges.	✓	1.00
Total		1.00 / 1.00

Question 6

Which of the following statements are true? Select all that apply.

Your Answer		Score	Explanation
☐ $\sum_{n=1}^{\infty} \frac{3^n - 1000n^3}{3^n + 1}$ diverges.	✗	0.00	
☐ $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+3}$ diverges.	✓	0.00	
☐ $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{1+3^n}$ converges conditionally.	✓	0.00	
☐ $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+3}$ converges absolutely.	✓	0.00	
☑ $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+3}$ converges conditionally.	✓	0.33	
☐ $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{1+3^n}$ diverges.	✓	0.00	
☐ $\sum_{n=1}^{\infty} \frac{3^n - 1000n^3}{3^n + 1}$ converges conditionally.	✓	0.00	
☑ $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{1+3^n}$ converges absolutely.	✓	0.33	

☑ $\sum_{n=1}^{\infty} \frac{3^n - 1000n^3}{3^n + 1}$ converges absolutely.	✗	-0.17
Total		0.50 / 1.00

Question 7

For which values of x does the power series $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{4 \cdot 8 \cdot 12 \cdots (4n)} (x-1)^n$ converge?

Your Answer		Score	Explanation
○ $-4 < x < 4$			
○ $-1 < x \leq 3$			
○ $-1 < x < 3$			
○ $-4 < x \leq 4$			
○ $-3 < x \leq 5$			
◉ $-3 < x < 5$	✓	1.00	
Total		1.00 / 1.00	

Question 8

What is the radius of convergence R of the power series $\sum_{n=0}^{\infty} \left(\frac{3n+1}{2n+2}\right)^n x^n$?

Hint: the usual formula, derived from the ratio test, will not work so well... Can you think of a different convergence test to use to derive the radius of convergence?

Your Answer		Score	Explanation
$R = +\infty$			
$R = 2$			
$R = \frac{1}{2}$			
$R = 1$			
$R = \frac{3}{2}$			
$R = \frac{2}{3}$	✓	1.00	
Total		1.00 / 1.00	

Question 9

Consider the series

$$s = \sum_{n=1}^{\infty} (-1)^n \frac{16}{n^4}$$

Using the alternating series error estimate, what is the best (lowest) estimate for N such that the error E_N of the approximation

$$s = \sum_{n=1}^{N} (-1)^n \frac{16}{n^4} + E_N$$

satisfies $E_N < 10^{-8}$? You should not need a calculator for this problem, but you will need to be careful with your powers-of-10.

Your Answer	Score	Explanation
$N \geq 99$		
$N \geq 100$		

$N \geq 201$		
$N \geq 101$		
$N \geq 199$		
$N \geq 200$	✓	1.00
Total		1.00 / 1.00

Question 10

Suppose you try to approximate $e^{-1/2}$ using the first three terms in the Taylor series expansion of the exponential function:

$$e^x = 1 + x + \frac{x^2}{2} + E_2(x)$$

What is the the best bound for $\left|E_2\left(-\frac{1}{2}\right)\right|$ provided by Taylor's theorem?

Your Answer		Score	Explanation
$\left|E_2\left(-\frac{1}{2}\right)\right| \leq 0.00004$			
$\left|E_2\left(-\frac{1}{2}\right)\right| \leq \frac{e^{-1/2}}{48}$			
$\left|E_2\left(-\frac{1}{2}\right)\right| \leq \frac{1}{6}$			
$\left|E_2\left(-\frac{1}{2}\right)\right| \leq \frac{e^{-1/2}}{8}$			
$\left|E_2\left(-\frac{1}{2}\right)\right| \leq \frac{1}{8}$			
$\left|E_2\left(-\frac{1}{2}\right)\right| \leq \frac{1}{48}$	✓	1.00	
Total		1.00 / 1.00	