Homework 16

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Find the derivative of $f(x) = (\cos x)^x$.

- $f'(x) = -x(\cos x)^{x-1} \sin x$
- $f'(x) = (\ln \cos x - x \tan x)(\cos x)^x$
- $f'(x) = \ln \cos x - x \tan x$
- $f'(x) = (\ln \cos x + x \cot x)(\cos x)^x$
- $f'(x) = (\ln \cos x - x \tan x)(\cos x)^{x-1}$
- $f'(x) = -(\cos x)^{x-1} \sin x$

Question 2

Find the derivative of $f(x) = (\ln x)^x$.

- $f'(x) = (\ln x)^x \left(\dfrac{1}{e^x} + e^x \ln x \right)$
- $f'(x) = \dfrac{1}{e^x} + e^x \ln x$
- $f'(x) = \dfrac{1}{\ln x} + \ln(\ln x)$
- $f'(x) = (\ln x)^x \left(\dfrac{1}{\ln x} + \ln(\ln x) \right)$
- $f'(x) = (\ln x)^x \dfrac{\ln x}{x}$
- $f'(x) = (\ln x)^x \ln(\ln x)$

Question 3

Find the derivative of $f(x) = x^{\ln x}$.

- $f'(x) = 2x^{\ln x} \ln x$
- $f'(x) = 2 \ln x$
- $f'(x) = x^{\ln(x)-1} \ln x$
- $f'(x) = x^{\ln x} \ln x$
- $f'(x) = 2x^{\ln(x)-1} \ln x$
- $f'(x) = (\ln x + x)x^{\ln x}$

Question 4

We all know by heart the product and quotient rules for differentiation:

$$(fg)' = f'g + fg', \quad \text{and} \quad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

There are, of course, formulas for derivatives of functions that involve more than one product or quotient. In this problem, we will use logarithmic differentiation to find one such formula. From it you should be able to see a beautiful pattern that would allow you to write such formulas for any number of factors!

Consider the function

$$F = \frac{f_1 \, f_2}{g_1 g_2}$$

Apply to it the logarithm operator, followed by the differentiation operator, and isolate F'. Which of the following formulas do you get?

- $F' = \dfrac{f_1' \, f_2}{g_1 g_2} + \dfrac{f_1 \, f_2'}{g_1 g_2} + \dfrac{f_1 \, f_2}{g_1' g_2} + \dfrac{f_1 \, f_2}{g_1 g_2'}$

- $F' = \dfrac{f_1' f_2}{g_1 g_2} + \dfrac{f_1 f_2'}{g_1 g_2} - \dfrac{f_1 f_2}{g_1' g_2} - \dfrac{f_1 f_2}{g_1 g_2'}$
- $F' = \dfrac{f_1' f_2}{g_1 g_2} + \dfrac{f_1 f_2'}{g_1 g_2} + \dfrac{f_1 f_2 g_1'}{g_1^2 g_2} + \dfrac{f_1 f_2 g_2'}{g_1 g_2^2}$
- $F' = \dfrac{f_1' f_2 + f_1 f_2'}{g_1 g_2} - \dfrac{f_1 f_2}{g_1' g_2 + g_1 g_2'}$
- $F' = \dfrac{f_1' f_2}{g_1 g_2} + \dfrac{f_1 f_2'}{g_1 g_2} - \dfrac{f_1 f_2 g_1'}{g_1^2 g_2} - \dfrac{f_1 f_2 g_2'}{g_1 g_2^2}$
- $F' = \dfrac{f_1' f_2 + f_1 f_2'}{g_1 g_2} + \dfrac{f_1 f_2}{g_1' g_2 + g_1 g_2'}$

Question 5

$$\lim_{x \to 0^+} \left(\frac{2}{x} \right)^{\sin x} =$$

- The limit does not exist.
- $\sqrt{2}$
- 0
- 1
- 2
- $+\infty$

Question 6

$$\lim_{x \to +\infty} \left(\frac{x+2}{x+3} \right)^{2x} =$$

Hint: write the fraction $\dfrac{x+2}{x+3}$ as $1 + \text{something}$.

- $e^{2/3}$

- e^{-2}
- e^{2}
- 1
- $e^{4/3}$
- $e^{3/2}$

Question 7

$$\lim_{x\to 0}\left(1+\arctan\frac{x}{2}\right)^{2/x} =$$

- e
- 0
- $\sqrt{e}$
- 1
- $+\infty$
- e^{2}

Question 8

$\lim_{x\to 0^+}\,[\ln(1+x)]^x =$

- e^{2}
- 1
- The limit does not exist.
- e
- $\sqrt{e}$
- 0

Question 9

This question requires thinking as in the somewhat mysterious implicit methods of the bonus lecture. Let

$$\varphi = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}}}$$

What is the value of ϕ?

If you find this way of doing problems unsettling, don't worry; we will return to this example in Lecture 45 and do it properly.

- φ is infinite.
- $\varphi = \sqrt{1+\sqrt{5}}$
- $\varphi = \dfrac{1-\sqrt{5}}{2}$
- $\varphi = 1$
- $\varphi = \sqrt{5}$
- $\varphi = \dfrac{1+\sqrt{5}}{2}$

Question 10

This question requires thinking based on the somewhat mysterious implicit methods of the bonus lecture. Let

$$\alpha = 1 + \frac{2}{2+\frac{2}{2+\frac{2}{2+\cdots}}}$$

What is the value of α?

You may find this type of problem unsettling; don't worry, we will return to this example in Lecture 45 and do it properly.

- α is infinite.
- $\alpha = \dfrac{3}{16}$
- $\alpha = \sqrt{2}$
- $\alpha = \dfrac{1}{\sqrt{2}-1}$
- $\alpha = \sqrt{3}$
- $\alpha = \dfrac{1+\sqrt{5}}{2}$

In accordance with the Honor Code, I certify that my answers here are my own work.

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