Digital Filtering Exercises

The **due date** for this homework is **Sun 14 Apr 2013 8:00 PM EDT**.

Question 1

An interesting digital filter is described by the difference equation

$$y(n) = ay(n-1) + ax(n) - x(n-1),\ a = \frac{1}{\sqrt{2}}$$

Let's see why it is "interesting."

Find the unit-sample response of this filter. In other words, what is the output $y(n)$ when the input $x(n)$ equals $\delta(n)$? When we have a unit-sample input, the output is frequently called $h(n)$.

Provide **numeric** answers for the first four values of $h(n)$: $h(0)$, $h(1)$, $h(2)$, and $h(3)$ using $a = \frac{1}{\sqrt{2}}$. Separate each value by a space.

Question 2

What is this filter's transfer function? Express your answer using the filter parameter a rather than is numeric value.

$$H\left(e^{j2\pi f}\right) = ?$$

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Question 3

The “interesting” aspect arises when we consider the transfer function's magnitude and phase. What is the magnitude $\left|H\left(e^{j2\pi f}\right)\right|$?

You should answer by starting with an expression that involves the filter's parameter a.

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Question 4

What is the phase of this transfer function?

Your answer should be an expression that involves the filter's parameter a. Use the function `atan2` anywhere the arc-tangent function is needed. For example, the phase of $a+jb$ would be typed as `atan2(b,a)` (note the order of `atan2`'s arguments).

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Question 5

What is this filter's output to $\sin\left(\frac{\pi}{4}n\right)$ when $a=\frac{1}{\sqrt{2}}$?

Type your answer as a expression for the output signal.

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Question 6

A discrete-time, linear, shift-invariant system has an output $y(n)$ for $n = 0, 1, 2, 3, \ldots$ equal to $1, -1, 0, 0, \ldots$.

Assuming the difference equation is of the form

$y(n) = a_1 y(n-1) + a_2 y(n-2) + b_0 x(n) + b_1 x(n-1)$, what are the filter coefficients that correspond to this input-output signal pair?

Your answer should be numeric values for the coefficients, typed in the order a_1 a_2 b_0 b_1 and separated by spaces.

Question 7

What is this filter's transfer function?

Your answer should be the complex-valued transfer function as a function of frequency f.

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Question 8

What description best fits this filter's action on its inputs?

- Highpass

- Lowpass
- Something else.
- Bandpass
- Allpass

Question 9

Echoes

Echoes occur not only in canyons and deep valleys, but also in auditoriums and telephone systems. In one case in which the signal has been sampled, the input signal $x(n)$ emerges from an echo system along with scaled and delayed copies of itself: $y(n) = x(n) + a_1 x(n - n_1) + a_2 x(n - n_2)$.

To simulate this echo system the FEE students want to write the most efficient (quickest) program that implements this input/output relationship. Suppose the duration of the input $x(n)$ is 1000 and that $a_1 = \frac{1}{2}$, $n_1 = 10$, $a_2 = \frac{1}{5}$, and $n_2 = 25$. In the Discussion Forum, half the students votes just to program the difference equation and the other half votes to program a frequency-domain approach that exploits the speed of the FFT. Which approach is the most efficient?

- Time domain.
- Frequency domain.
- Makes no difference.

Question 10

What is the transfer function of the digital filter that removes echoes? In other words, when $y(n)$ is the input, we want the original signal $x(n)$ to be the output.

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☐ In accordance with the Honor Code, I certify that my answers here are my own work.

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