Homework 31

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Find the area enclosed by the curves $y = 1$, $x = 1$, and $y = \ln x$.

Hint: draw the three curves first and identify the region that they enclose. It should look like a right triangle but with a curved hypotenuse.

- 1
- $\ln 2$
- $e - 1$
- $e - 2$
- e
- $e - \frac{3}{2}$

Question 2

Find the area bound by the curves $y = \cos^2 x$ and $y = \frac{8x^2}{\pi^2}$.

Hint: the x coordinates of the intersection points of the two curves are some very familiar values...

- $\frac{1}{2} + \frac{\pi}{6}$
- $\frac{\pi}{2} - \frac{\pi^3}{12}$
- $\frac{\pi}{2} - \frac{\pi^3}{48}$

- $\pi - \dfrac{2\pi^3}{3}$
- $\dfrac{\pi}{4} - \dfrac{\pi^3}{96}$
- $1 - \dfrac{\pi}{6}$

Question 3

Compute the area bounded by $y = x(x-1)(x-2)$ and the x-axis.

- 2
- 0
- $\dfrac{1}{4}$
- $\dfrac{3}{4}$
- 1
- $\dfrac{1}{2}$

Question 4

Find the area of the bounded region enclosed by the x-axis, the lines $x = 1$ and $x = 2$ and the hyperbola $xy = 1$.

- 1
- $\ln 3$
- $\dfrac{1}{2}$
- $-\dfrac{1}{2}$
- 2

- $\ln 2$

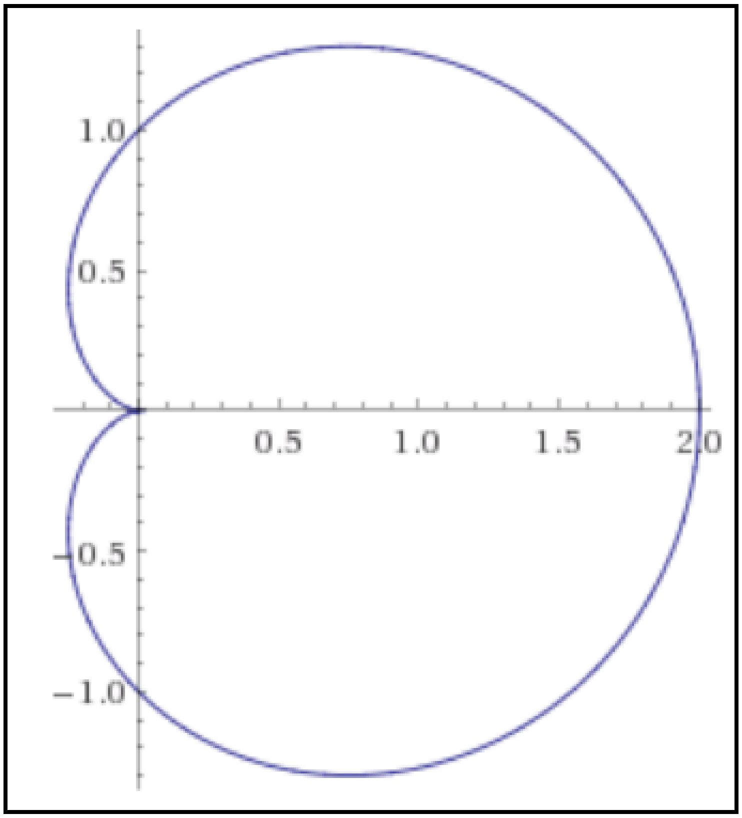
Question 5

Find the area of the sector of a circular disc of radius r (centered at the origin) given by $1 \leq \theta \leq 3$. (As usual, θ is in radians...)

- $2r^2$
- r^2
- $2r$
- $2\pi r^2$
- $\frac{2}{3}r^3$
- $\frac{\pi r^2}{2}$

Question 6

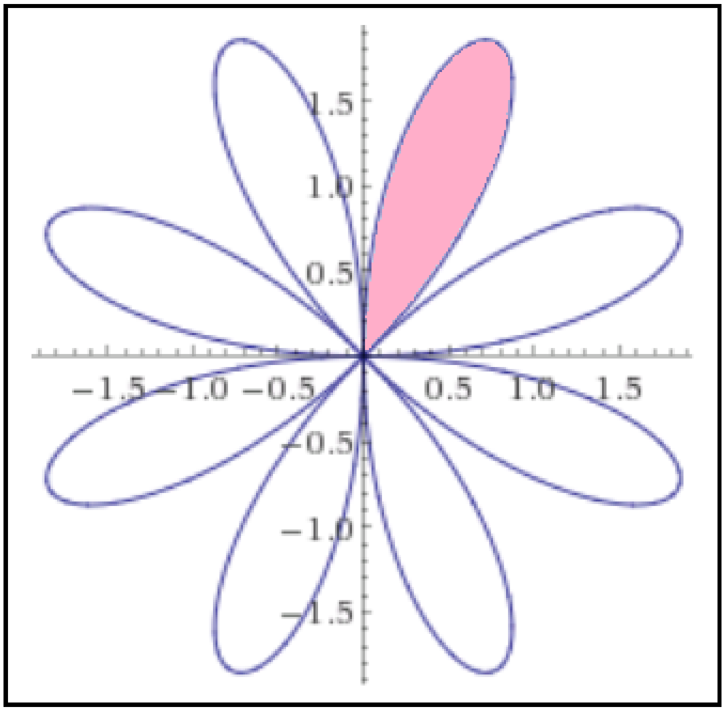
Compute the area enclosed by the *cardioid* in the figure below. This curve is described by the polar equation $r = 1 + \cos\theta$.



- π
- 3π
- $\frac{3\pi}{2}$
- $\frac{\pi}{2}$
- $\frac{5\pi}{2}$
- 2π

Question 7

Find the area enclosed by a single petal of the *polar rose* which given by the equation $r = 2\sin(4\theta)$.



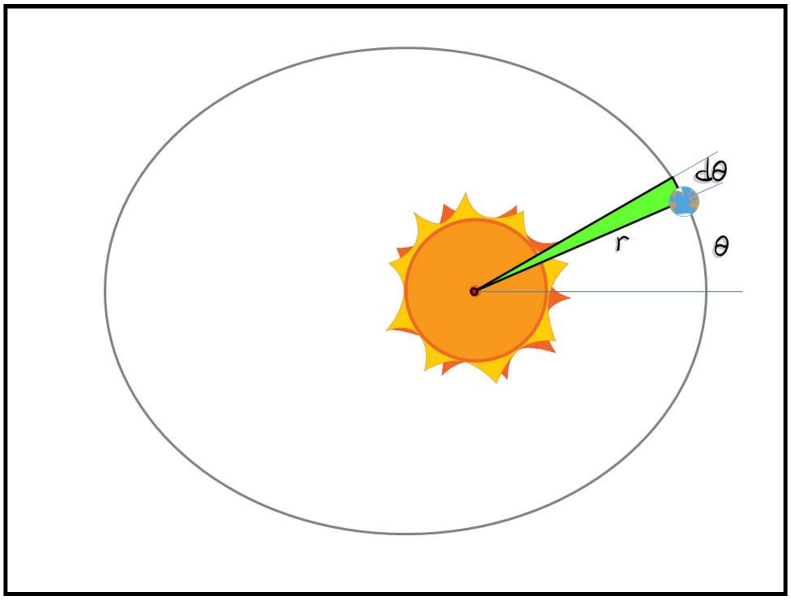
- $\frac{\pi}{2}$
- $\frac{\pi-2}{8}$
- $\frac{\pi}{8}$
- π
- $\frac{\pi+1}{4}$
- $\frac{\pi}{4}$

Question 8

Kepler's First Law states that the orbit of every planet is an ellipse with the Sun at one of its two foci. If we think of the Sun as being situated at the origin, we can describe the orbit with the equation

$$r = \frac{p}{1+\varepsilon\cos\theta}$$

The point at which the planet is closest to the Sun —so-called *perihelion*— corresponds to $\theta = 0$, while the planet is furthest away from the Sun at $\theta = \pi$ — the *aphelion*. Knowing the distance between the Sun and the planet at these two points would allow you to fix the values of the constants p and ε. Notice that $\varepsilon = 0$ describes a perfect circle, so that the *eccentricity* ε measures how far the orbit is from being a circle.



Kepler's Second Law states that the line joining a planet and the Sun sweeps out equal areas during equal intervals of time. Another way of expressing this fact is by saying that the *areal velocity* $v_A = \dfrac{dA}{dt}$ of that line is constant in time.

Express the area element dA in terms of the angle element $d\theta$ and use Kepler's Second Law to deduce the differential equation governing the time evolution of θ.

- $\dfrac{d\theta}{dt} = \dfrac{v_A}{p^2}\left(1 + \varepsilon\cos\theta\right)^2$
- $\dfrac{d\theta}{dt} = \dfrac{v_A}{p}\left(1 + \varepsilon\cos\theta\right)$
- $\dfrac{d\theta}{dt} = \dfrac{2v_A}{p}\left(1 + \varepsilon\cos\theta\right)$
- $\dfrac{d\theta}{dt} = \dfrac{2v_A p^2}{\left(1 + \varepsilon\cos\theta\right)^2}$

- $\dfrac{d\theta}{dt} = \dfrac{2v_A}{p^2}\left(1+\varepsilon\cos\theta\right)^2$
- $\dfrac{d\theta}{dt} = \dfrac{2v_A p}{1+\varepsilon\cos\theta}$

☐ In accordance with the Honor Code, I certify that my answers here are my own work.

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