Homework 51

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT -0400**.

Question 1

Consider the sequence $a = \Delta b$, where $b = \left(\frac{1}{n}\right)$. Only one of the following statements is true. Which one?

- $\sum_{n=1}^{\infty} a_n$ converges, but $\sum_{n=1}^{\infty} b_n$ diverges.
- Both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge.
- Both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ diverge.
- $\sum_{n=1}^{\infty} a_n$ diverges, but $\sum_{n=1}^{\infty} b_n$ converges.

Question 2

Which of the following statements are true about the series $\sum_{n=1}^{\infty} ne^{-2n}$?

- [ ] $ne^{-2n} < e^{-n}$ for $n \geq 1$, so the comparison test says that the series converges.
- [ ] $\lim_{n\to+\infty} ne^{-n} = 0$, so the n-th term test says that the series diverges.
- [ ] $ne^{-2n} > n$ for $n \geq 1$, so the comparison test says that the series diverges.
- [ ] The integral $\int_{x=1}^{+\infty} xe^{-2x}\,dx$ converges, so the integral test says that the series converges.

- [ ] The integral $\int_{x=1}^{+\infty} xe^{-2x}\, dx$ diverges, so the integral test says that the series diverges.
- [ ] $\lim_{n\to+\infty} ne^{-2n} = 0$, so the n-th term test is inconclusive.

Question 3

Which of the following statements are true about the series $\sum_{n=1}^{\infty} \frac{1}{n^{0.999\ldots}}$?

- [ ] $\lim_{n\to+\infty} \frac{1}{n^{0.999\ldots}} \neq 0$, so the n-th term test says that the series diverges.
- [ ] $\frac{1}{n^{0.999\ldots}} < \frac{1}{n}$ for $n \geq 1$, so the comparison test says that the series converges.
- [ ] $\frac{1}{n^{0.999\ldots}} \geq \frac{1}{n}$ for $n \geq 1$, so the comparison test says that the series diverges.
- [ ] The integral $\int_{x=1}^{+\infty} \frac{dx}{x^{0.999\ldots}}$ diverges, so the integral test says that the series diverges.
- [ ] The integral $\int_{x=1}^{+\infty} \frac{dx}{x^{0.999\ldots}}$ converges, so the integral test says that the series converges.
- [ ] $\lim_{n\to+\infty} \frac{1}{n^{0.999\ldots}} = 0$, so the n-th term test says that the series converges.

Question 4

Which of the following statements are true about the series $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n}$?

- [ ] $\frac{(\ln n)^2}{n} < \frac{1}{n^2}$ for $n > 2$, so the comparison tests says that the series $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n}$ converges.
- [ ] $\frac{(\ln n)^2}{n} < \frac{1}{n}$ for $n = 1$, so the comparison tests says that the series $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n}$ converges.
- [ ] $\frac{(\ln n)^2}{n} > \frac{1}{n^2}$ for $n > 2$, so the comparison tests says that the series $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n}$ diverges.
- [ ] $\frac{(\ln n)^2}{n} > \frac{1}{n}$ for $n > 2$, so the comparison tests says that the series $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n}$ diverges.
- [ ] The integral $\int_{x=8}^{+\infty} \frac{(\ln x)^2}{x}\, dx$ diverges, so the integral test says that the series diverges.
- [ ] The integral $\int_{x=8}^{+\infty} \frac{(\ln x)^2}{x}\, dx$ converges, so the integral test says that the series converges.

Question 5

Which of the following statements are true about the series $\sum_{n=1}^{\infty} \frac{1}{2n+1}$?

- [ ] The integral $\int_{x=1}^{+\infty} \frac{dx}{2x+1}$ diverges, so the integral test says that the series diverges.
- [ ] The integral $\int_{x=1}^{+\infty} \frac{dx}{2x+1}$ converges, so the integral test says that the series converges.

- [ ] $\lim_{n\to\infty} \frac{1}{2n+1} = 0$, so the n-th term test is inconclusive.
- [ ] $\lim_{n\to+\infty} \frac{1/(2n+1)}{1/n} = \frac{1}{2}$, so the limit test says that the series converges.
- [ ] $\lim_{n\to+\infty} \frac{1/(2n+1)}{1/n} = \frac{1}{2}$, so the limit test says that the series diverges.
- [ ] $\lim_{n\to\infty} \frac{1}{2n+1} = 0$, so the n-th term test says that the series diverges.

Question 6

Which of the following statements are true about the series $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{(n+2)\sqrt{n}}$?

- [ ] $\frac{\sqrt[3]{n}}{(n+2)\sqrt{n}} < \frac{1}{n^{7/6}}$ for $n \geq 1$, so the comparison test says that the series converges.
- [ ] $\lim_{n\to+\infty} \frac{\sqrt[3]{n}}{(n+2)\sqrt{n}} = 0$, so the n-th term test says that the series converges.
- [ ] $\lim_{n\to+\infty} \frac{\sqrt[3]{n}}{(n+2)\sqrt{n}} \Big/ \frac{1}{n^{7/6}} = 1$, so the limit test says that the series converges.
- [ ] $\frac{\sqrt[3]{n}}{(n+2)\sqrt{n}} > \frac{1}{n}$ for $n \geq 1$, so the comparison test says that the series diverges.
- [ ] The integral $\int_{x=1}^{+\infty} \frac{\sqrt[3]{x}}{(x+2)\sqrt{x}}\, dx$ converges, so the integral test says that the series converges.
- [ ] $\lim_{n\to+\infty} \frac{\sqrt[3]{n}}{(n+2)\sqrt{n}} \Big/ \frac{1}{n^{2}} = 0$, so the limit test says that the series converges.

Question 7

Determine the asymptotic behavior of the sequence $a = \left(\frac{n(n+1)}{n^2+4}\right)$ as $n \to +\infty$. Using this information, determine whether the series $\sum_{n=1}^{\infty} a_n$ converges or diverges.

- $a_n = 1 + O\left(\frac{1}{n}\right)$ as $n \to +\infty$, so the series diverges.
- $a_n = 1 + O\left(\frac{1}{n^2}\right)$ as $n \to +\infty$, so the series diverges.
- $a_n = O\left(\frac{1}{n^2}\right)$ as $n \to +\infty$, so the series diverges.
- $a_n = O\left(\frac{1}{n^2}\right)$ as $n \to +\infty$, so the series converges.
- $a_n = 1 + O\left(\frac{1}{n}\right)$ as $n \to +\infty$, so the series converges.
- $a_n = O\left(\frac{1}{n}\right)$ as $n \to +\infty$, so the series diverges.

Question 8

Determine the asymptotic behavior of the sequence $a = \left(n^2 \tan \frac{1}{n^3}\right)$ as $n \to +\infty$. Using this information, determine whether the series $\sum_{n=1}^{\infty} a_n$ converges or diverges.

- $a_n = \frac{1}{n} + O\left(\frac{1}{n^3}\right)$ as $n \to +\infty$, so the series converges.
- $a_n = O\left(\frac{1}{n^2}\right)$ as $n \to +\infty$, so the series diverges.

- $a_n = \frac{1}{n} + O\left(\frac{1}{n^7}\right)$ as $n \to +\infty$, so the series diverges.
- $a_n = O\left(n^2\right)$ as $n \to +\infty$, so the series diverges.
- $a_n = O\left(\frac{1}{n^3}\right)$ as $n \to +\infty$, so the series converges.
- $a_n = \frac{1}{n} + O\left(\frac{1}{n^7}\right)$ as $n \to +\infty$, so the series converges.

Question 9

Determine the asymptotic behavior of the sequence $a = \left(\sin \frac{1}{n^2}\right)$ as $n \to +\infty$. Using this information, determine whether the series $\sum_{n=1}^{\infty} a_n$ converges or diverges.

- $a_n = \frac{1}{n} + O\left(\frac{1}{n^3}\right)$ as $n \to +\infty$, so the series converges.
- $a_n = O\left(\frac{1}{n}\right)$ as $n \to +\infty$, so the series converges.
- $a_n = O(1)$ as $n \to +\infty$, so the series diverges.
- $a_n = \frac{1}{n^2} + O\left(\frac{1}{n^6}\right)$ as $n \to +\infty$, so the series converges.
- $a_n = \frac{1}{n} + O\left(\frac{1}{n^3}\right)$ as $n \to +\infty$, so the series diverges.
- $a_n = \frac{1}{n^2} + O\left(\frac{1}{n^6}\right)$ as $n \to +\infty$, so the series diverges.

Question 10

Determine the asymptotic behavior of the sequence $a = \left(\ln \frac{n^2+3}{n^2+1}\right)$ as $n \to +\infty$. Using this information, determine whether the series $\sum_{n=1}^{\infty} a_n$ converges or diverges.

- $a_n = \frac{1}{n} + O\left(\frac{1}{n^2}\right)$ as $n \to +\infty$, so the series converges.
- $a_n = \frac{1}{n} + O\left(\frac{1}{n^2}\right)$ as $n \to +\infty$, so the series diverges.
- $a_n = \frac{2}{n^2} + O\left(\frac{1}{n^4}\right)$ as $n \to +\infty$, so the series converges.
- $a_n = 1 + O\left(\frac{1}{n^2}\right)$ as $n \to +\infty$, so the series converges.
- $a_n = 1 + O\left(\frac{1}{n^2}\right)$ as $n \to +\infty$, so the series diverges.
- $a_n = \frac{2}{n^2} + O\left(\frac{1}{n^4}\right)$ as $n \to +\infty$, so the series diverges.

☐ In accordance with the Honor Code, I certify that my answers here are my own work.

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