Problem Set VII

The **due date** for this homework is **Tue 12 Mar 2013 12:59 AM EDT**.

In this problem set, you will be given a total of ten attempts. We will accept late submission until the fifth day after the due date, and late submission will receive half credit. Explanations and answers to the problem set will be available after the due date. Since the homework problems will become gradually more challenging as the course proceeds, we highly recommend you to start the habit of printing out the problems and working on them with paper and pencil. Also, please be sure to read the problem statements carefully and double check your expressions before you submit.

A pdf version of this problem set is available for you to print. Note: all mathematical expressions have to be exact, even when involving constants. Such an expression is required when a function and/or a variable is required in the answer. For example, if the answer is $\sqrt{3}x$, you must type `sqrt(3)*x`, not `1.732*x` for the answer to be graded as being correct.

Question 1

Discrete-Time Fourier Transforms

Find the Fourier transform of the following sequence, where $s(n)$ is some sequence having Fourier transform $S(e^{j2\pi f})$.

$$x(n) = (-1)^n s(n)$$

$X(e^{j2\pi f}) = S(\_\_)$. Provide the expression that fills in the blank for this equation. If $e^{j2\pi f}$ is the answer, type `exp(j*2*pi*f)`.

Preview

Question 2

Questions 2-5 pertain to the DTFT of the following signal.

$$x(n) = s(n)\cos(2\pi f_0 n)$$

where $s(n)$ is some sequence having Fourier transform $S(e^{j2\pi f})$. The answer is of the form

$X(e^{j2\pi f}) = c_1 S(a_1) + c_2 S(a_2)$, where c_1, c_2 are constants and a_1, a_2 are functions of f. For example, if the first term were $S(e^{j2\pi f})$, $c_1 = 1$ and $a_1 = e^{j2\pi f}$. The first term corresponds to the *smallest* frequency component, the second term the largest.

What is c_1? To enter f_0, type `f0`.

Preview

Question 3

Questions 2-5 pertain to the DTFT of the following signal.

$$x(n) = s(n)\cos(2\pi f_0 n)$$

where $s(n)$ is some sequence having Fourier transform $S(e^{j2\pi f})$. The answer is of the form $X(e^{j2\pi f}) = c_1 S(a_1) + c_2 S(a_2)$, where c_1, c_2 are constants and a_1, a_2 are functions of f. For example, if the first term were $S(e^{j2\pi f})$, $c_1 = 1$ and $a_1 = e^{j2\pi f}$. The first term corresponds to the *smallest* frequency component, the second term the largest.

What is a_1? To enter f_0, type `f0`.

Preview

Question 4

Questions 2-5 pertain to the DTFT of the following signal.

$$x(n) = s(n)\cos(2\pi f_0 n)$$

where $s(n)$ is some sequence having Fourier transform $S(e^{j2\pi f})$. The answer is of the form $X(e^{j2\pi f}) = c_1 S(a_1) + c_2 S(a_2)$, where c_1, c_2 are constants and a_1, a_2 are functions of f. For example, if the first term were $S(e^{j2\pi f})$, $c_1 = 1$ and $a_1 = e^{j2\pi f}$. The first term corresponds to the *smallest* frequency component, the second term the largest.

What is c_2? To enter f_0, type `f0`.

Preview

Question 5

Questions 2-5 pertain to the DTFT of the following signal.

$$x(n) = s(n)\cos(2\pi f_0 n)$$

where $s(n)$ is some sequence having Fourier transform $S(e^{j2\pi f})$. The answer is of the form $X(e^{j2\pi f}) = c_1 S(a_1) + c_2 S(a_2)$, where c_1, c_2 are constants and a_1, a_2 are functions of f. For example, if the first term were $S(e^{j2\pi f})$, $c_1 = 1$ and $a_1 = e^{j2\pi f}$. The first term corresponds to the *smallest* frequency component, the second term the largest.

What is a_2? To enter f_0, type `f0`.

Preview

Question 6

$s(n)$ is some sequence having Fourier transform $S(e^{j2\pi f})$. Find the Fourier transform of the following sequence derived from $s(n)$.

$$x(n) = \begin{cases} s\left(\frac{n}{2}\right) & \text{if } n \text{ (even)} \\ 0 & \text{if } n \text{ (odd)} \end{cases}$$

$X(e^{j2\pi f}) = S(\_\_)$. Provide the expression that fills in the blank for this equation.

Preview

Question 7

Questions 7-8 pertain to the DTFT of the following signal derived from $s(n)$, a signal having Fourier transform $S(e^{j2\pi f})$.

$$x(n) = ns(n)$$

The answer has the form $X(e^{j2\pi f}) = ?S(?)$.
The answer has the form $X(e^{j2\pi f}) = \_\_S(?)$. Fill in the blank.

NOTE: You may need to enter a derivative operator (e.g. $\frac{d}{df}$). If so, type `d/df`. So, if the answer is $X(e^{j2\pi f}) = 2\,\frac{d}{df}\,S(e^{j2\pi f})$, enter `2*d/df`.

Preview

Question 8

Questions 7-8 pertain to the DTFT of the following signal derived from $s(n)$, a signal having Fourier transform $S(e^{j2\pi f})$.

$$x(n) = ns(n)$$

The answer has the form $X(e^{j2\pi f}) = ?S(?)$.

$X(e^{j2\pi f}) = ?S(\_\_)$. Fill in the blank in the equation. **NOTE:** to enter a derivative operator (e.g. $\frac{d}{df}$), type `d/df`.

Preview

Question 9

Spectra of Finite-Duration Signals

Find the discrete-time Fourier transform of the following signal:

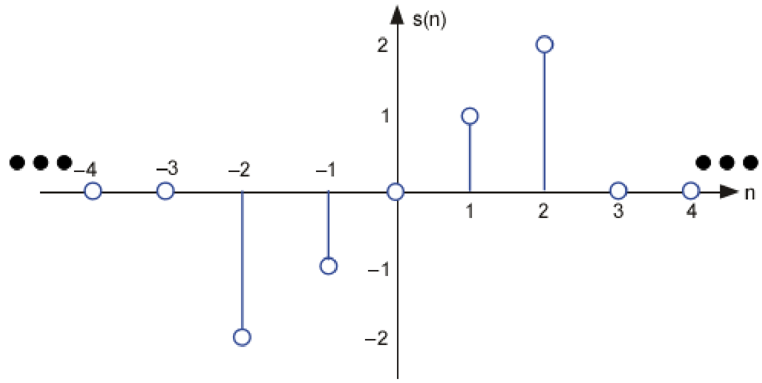
$$s(n) = \begin{cases} \cos^2\left(\frac{\pi}{4}\,n\right) & \text{if } n = \{-1, 0, 1\} \\ 0 & \text{if otherwise} \end{cases}$$

$S(e^{j2\pi f}) = ?$ (If possible, simplify your answer using trigonometric identities for sinusiods.)

Preview

Question 10

Find the discrete-time Fourier transform (DTFT) of the following signal



$S(e^{j2\pi f})$ =?

If possible, simplify your answer using Euler's formula and trigonometric identities for sinusoids.

Preview

Question 11

Find the DTFT of the following signal.

$$s(n) = \begin{cases} \sin(\frac{\pi}{4}n) & \text{if } n = \{0, \ldots, 7\} \\ 0 & \text{if otherwise} \end{cases}$$

$S(e^{j2\pi f})$ =?

If possible, simplify your answer using Euler's formula and trigonometric identities for sinusoids.

Preview

Question 12

Find the length-8 **DFT** (discrete Fourier transform) of the previous signal:

$$s(n) = \begin{cases} \sin(\frac{\pi}{4}n) & \text{if } n = \{0, \dots, 7\} \\ 0 & \text{if otherwise} \end{cases}$$

$S(k) =?$

Preview

Question 13

DSP Tricks
Sammy is faced with computing lots of discrete Fourier transforms. He will, or course, use the FFT algorithm but he is behind schedule and needs to get his results as quickly as possible. He gets the idea of computing two transforms at one time by computing the transform of $s(n) = s_1(n) + js_2(n)$, where $s_1(n)$ and $s_2(n)$ are two real-valued signals for which he needs to compute the spectra. The issue is whether he can retrieve the individual DFTs $S_1(k)$ and $S_2(k)$ from the result $S(k)$ or not.
What will be the DFT $S(k)$ of this complex-valued signal in terms of $S_1(k)$ and $S_2(k)$, the DFTs of the original signals?

Express your answer in terms of S_1 and S_2 (the (k) is implied), typed as `S1` and `S2`. For example, if the answer were $S(k) = S_1(k)/5 \cdot S_2(k)$ you would type `(S1/5) * S2` as the answer.

$S(k) =?$

Preview

Question 14

Sammy's friend, who is also learning about signal processing, says that retrieving the wanted DFTs is easy: "Just find the real and imaginary parts of $S(k)$." Show that this approach is too simplistic by expressing the real and imaginary parts of $S(k)$.
Express your answer in terms of real and imaginary components of $S_1(k)$ and $S_2(k)$.
NOTE: To write the real part of $S_1(k)$, which can also be expressed as Re$[S_1(k)]$, type `ReS_1`. To write the imaginary part of $S_1(k)$ type `ImS_1`. For example, the complex function $S_1(k) =$ Re$[S_1(k)] + j$Im$[S_1(k)]$ can be expressed by typing: `ReS_1 + j*ImS_1`.

What is the real part of $S(k)$?

Express your answer in terms of S_1 and S_2 (the (k) is implied), typed as `S1` and `S2`. To enter $\mathrm{Re}[S_1(k)]$ or $\mathrm{Im}[S_1(k)]$, you would type `re(S1)` or `im(S1)`.

Re$[S(k)] = ?$

Preview

Question 15

This is a continuation of the previous question.

What is the Imaginary component of $S(k)$?

Express your answer in terms of S_1 and S_2 (the (k) is implied), typed as `S1` and `S2`. To enter $\mathrm{Re}[S_1(k)]$ or $\mathrm{Im}[S_1(k)]$, you would type `re(S1)` or `im(S1)`.

Im$[S(k)] = ?$

Preview

Question 16

While Sammy's friend wasn't able to recover $S_1(k)$ and $S_2(k)$, his suggestion did give Sammy an idea. By using symmetry properties of the DFT Sammy was able to recover the Fourier transforms of both signals.

Sammy remembered that $\mathrm{Re}[S_1(k)]$ and $\mathrm{Re}[S_2(k)]$ are even (i.e $\mathrm{Re}[S_2(k)] = \mathrm{Re}[S_2(N-k)]$) and that $\mathrm{Im}[S_1(k)]$ and $\mathrm{Im}[S_2(k)]$ are odd (i.e. $\mathrm{Im}[S_2(k)] = -\mathrm{Im}[S_2(N-k)]$). Sammy then evaluated the even part of $S(k)$ in the following manner.

$$\begin{aligned}
\frac{\mathrm{Re}[S(k)] + \mathrm{Re}[S(N-k)]}{2} &= \frac{\mathrm{Re}[S_1(k)] - \mathrm{Im}[S_2(k)] + \mathrm{Re}[S_1(N-k)] - \mathrm{Im}[S_2(N-k)]}{2} \\
&= \frac{1}{2}\Big[\mathrm{Re}[S_1(k)] - \mathrm{Im}[S_2(k)] + \mathrm{Re}[S_1(k)] + \mathrm{Im}[S_2(k)]\Big] \\
&= \mathrm{Re}[S_1(k)]
\end{aligned}$$

Similarily,

$$
\begin{aligned}
1/2 \cdot \left[\mathrm{Im}[S(k)] - \ \mathrm{Im}[S(N-k)]\right] &= \ \mathrm{Im}[S_1(k)] \\
1/2 \cdot \left[\mathrm{Re}[S(k)] - \ \mathrm{Re}[S(N-k)]\right] &= - \ \mathrm{Im}[S_2(k)] \\
1/2 \cdot \left[\mathrm{Im}[S(k)] + \ \mathrm{Im}[S(N-k)]\right] &= \ \mathrm{Re}[S_2(k)]
\end{aligned}
$$

Sammy has sped up his FFT computation time by a factor of ?

☐ In accordance with the Honor Code, I certify that my answers here are my own work.

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