Homework 28

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

$$\int \sin^3 \frac{x}{2} \cos^3 \frac{x}{2}\ dx =$$

- $\frac{1}{3}\cos^6\frac{x}{2} - \frac{1}{2}\cos^4\frac{x}{2} + C$
- $-\frac{1}{3}\sin^6\frac{x}{2} + \frac{1}{2}\sin^4\frac{x}{2} + C$
- $-2\sin^5\frac{x}{2} + 2\sin^3\frac{x}{2} + C$
- $\frac{1}{6}\sin^6\frac{x}{2} - \frac{1}{4}\sin^4\frac{x}{2} + C$
- $\frac{1}{24}\cos^3 x - \frac{1}{8}\cos x + C$
- $\frac{1}{96}\big(\cos x - 9\cos 3x\big) + C$

Question 2

$$\int \frac{x^3\, dx}{\sqrt{9-x^2}} =$$

- $3(9-x^2)^{3/2} + C$
- $-\left(6 + \frac{x^2}{3}\right)\sqrt{9-x^2} + C$
- $\frac{1}{6}(9-x^2)^{1/2} + C$

- $-3\sqrt{9-x^2} - \frac{1}{81}\left(9-x^2\right)^{3/2} + C$
- $9\sqrt{9-x^2} - \frac{1}{3}\left(9-x^2\right)^{3/2} + C$
- $\left(\frac{1}{3} + 9x^2\right)\sqrt{9-x^2} + C$

Question 3

$$\int \sin^2 x \cos^2 x \, dx =$$

Hint: you may (or may not) need to use any of the following reduction formulae:

$$\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

$$\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

- $\frac{1}{8} x + \frac{1}{16} \sin 2x - \frac{1}{4} \sin x \cos^3 x + C$
- $\frac{1}{8} x - \frac{1}{16} \cos 2x + \frac{1}{4} \sin^3 x \cos x + C$
- $\frac{1}{8} x + \frac{1}{32} \cos 4x + C$
- $\frac{1}{8} x - \frac{1}{16} \sin 2x + \frac{1}{4} \sin^3 x \cos x + C$
- $\frac{1}{8} x + \frac{1}{16} \cos 2x - \frac{1}{4} \sin x \cos^3 x + C$
- $\frac{1}{8} x - \frac{1}{32} \sin 4x + C$

Question 4

$$\int 5\tan^5 x \sec^3 x\,dx =$$

- $\frac{5}{7}\sec^7 x - 2\sec^5 x + \frac{5}{3}\sec^3 x + C$
- $\frac{5}{7}\sec^{14} x - 2\sec^{10} x + \frac{5}{3}\sec^6 x + C$
- $\frac{5}{6}\tan^6 x + \frac{5}{8}\tan^8 x + C$
- $\frac{4}{5}\sec^5 x \tan^5 x - \frac{4}{3}\sec^3 x \tan^3 x + C$
- $\sec^5 x - \frac{10}{3}\sec^3 x + 5\sec x + C$
- $\frac{5}{7}\tan^7 x + \tan^5 x + C$

Question 5

$$\int 7\tan^4 x \sec^4 x\,dx =$$

- $\frac{1}{7}\tan^7 x + \frac{1}{5}\tan^5 x + C$
- $\tan^7 x + \frac{7}{5}\tan^5 x + C$
- $\frac{7}{5}\tan^5 x + \frac{7}{3}\tan^3 x + C$
- $\sec^7 x + \frac{7}{5}\sec^5 x + C$
- $\frac{1}{2}\sec^{14} x + \frac{7}{10}\sec^{10} x + C$
- $\tan^7 x - \frac{7}{5}\tan^5 x + C$

Question 6

$$\int \frac{x^2\,dx}{\sqrt{1+x^2}} =$$

Hint: you may need to use the reduction formula

$$\int \sec^n x\,dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$$

Recall also that

$$\int \sec x\,dx = \ln\left|\sec x + \tan x\right|$$

- $\frac{1}{2}\,x\sqrt{1+x^2} - \frac{1}{2}\ln\left|x + \sqrt{1+x^2}\right| + C$
- $\frac{x}{2\sqrt{1+x^2}} - \frac{1}{2}\ln\left|x + \sqrt{1+x^2}\right| + C$
- $\frac{1}{2}\,x\sqrt{1+x^2} + \frac{1}{2}\ln\left|x + \sqrt{1+x^2}\right| + C$
- $\frac{x}{2\sqrt{1+x^2}} - \frac{1}{2}\ln\left|x - \sqrt{1+x^2}\right| + C$
- $\frac{1}{2}\ln\left|x - \sqrt{1+x^2}\right| + C$
- $-\frac{1}{2}\ln\left|x - \sqrt{1+x^2}\right| + C$

Question 7

For the brave: In this course, we have seen that every reasonable function can be expanded in a *Taylor series* about $x = a$:

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$$

The coefficients of this series are given in terms of the derivatives of the function $f(x)$ at $x = a$ as

$$c_n = \frac{f^{(n)}(a)}{n!}$$

Fourier Analysis is the study of how functions may be represented as a (possibly infinite) sum of sines and cosines. Here is a little taste of it.

Let $f(x)$ be a periodic function of period L —that is,a function that satisfies $f(x+L) = f(x)$. If $f(x)$ is reasonable (for a different notion of reasonableness), we can also write it as a *Fourier series*:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2\pi n x}{L} + b_n \sin\frac{2\pi n x}{L}$$

In order to find an expression for the coefficients a_0, a_n, b_n in terms of the function $f(x)$, notice that

$$\int_{x=-L/2}^{L/2} \sin\frac{2\pi n x}{L} \sin\frac{2\pi m x}{L}\, dx = \begin{cases} 0 & \text{if } n \neq m \\ L/2 & \text{if } n = m \end{cases}$$

$$\int_{x=-L/2}^{L/2} \cos\frac{2\pi n x}{L} \cos\frac{2\pi m x}{L}\, dx = \begin{cases} 0 & \text{if } n \neq m \\ L/2 & \text{if } n = m \end{cases}$$

$$\int_{x=-L/2}^{L/2} \sin\frac{2\pi n x}{L} \cos\frac{2\pi m x}{L}\, dx = 0$$

Which of the following is true? Select all that apply.

- [ ] $a_0 = \frac{2}{L}\int_{x=-L/2}^{L/2} f(x)\, dx$
- [ ] For $n > 0$, $a_n = \frac{2}{L}\int_{x=-L/2}^{L/2} f(x)\cos\frac{2\pi n x}{L}\, dx$
- [ ] For $n > 0$, $b_n = \frac{2}{L}\int_{x=-L/2}^{L/2} f(x)\sin\frac{2\pi n x}{L}\, dx$
- [ ] $a_0 = \frac{1}{L}\int_{x=-L/2}^{L/2} f(x)\, dx$
- [ ] For $n > 0$, $a_n = \frac{2}{L}\int_{x=-L/2}^{L/2} f(x)\sin\frac{2\pi n x}{L}\, dx$

- [ ] For $n > 0$, $b_n = \frac{2}{L}\int_{x=-L/2}^{L/2} f(x)\cos\frac{2\pi n x}{L}\,dx$

Question 8

Assuming $0 < x < \pi$, compute

$$\int \cot x\sqrt{1-\cos 2x}\,dx =$$

Hint: remember the double-angle formula

$$\cos 2x = \cos^2 x - \sin^2 x$$

- $\cos^2 x + C$
- $\sqrt{2}\cos x + C$
- $\frac{x}{2} + \frac{\cos 2x}{2} + C$
- $\sin x + C$
- $\cos x + C$
- $\sqrt{2}\sin x + C$

- [ ] In accordance with the Honor Code, I certify that my answers here are my own work.

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