Homework 39

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Find the average value of $f(x) = \dfrac{1}{\sqrt{4x-3}}$ from $x = 3$ to $x = 21$.

- $\dfrac{1}{18}$
- $\dfrac{1}{12}$
- 3
- $-\dfrac{2}{9}$
- $\dfrac{1}{6}$
- $\dfrac{3}{2}$

Question 2

Calculate the average of the function $f(x) = x^3\sqrt{1+x^2}$ over the interval $0 \le x \le \sqrt{3}$.

- $\dfrac{58}{15}$
- $\dfrac{2}{15}\left(1+\sqrt{2}\right)$
- $\dfrac{128}{15}$
- $\dfrac{2}{15\sqrt{3}}\left(1+\sqrt{2}\right)$

- $\dfrac{58}{15\sqrt{3}}$
- $\dfrac{128}{15\sqrt{3}}$

Question 3

A first approximation to the variation of temperature within the day at any given location on Earth is given by the function

$$T(t) = (T_{\max} - T_{\min}) \sin^2 \frac{\pi t}{24} + T_{\min}$$

where we measure t in hours. As a sanity check, notice that $T(t)$ is periodic of period 24 —that is, $T(t+24) = T(t)$. We have set up this model so that the minimum temperature is achieved at $t = 0$ —which we think of as midnight—, while the maximum is reached at $t = 12$ —noon.

According to our model, what is the average temperature over a full day?

Footnote: our model is too simplistic. For example, we would like to see a relatively sharp rise in temperature around sunrise. This could be achieved by using a Fourier series (which we passingly touched upon in Question 7 of Homework 28). For the purposes of this problem, let us be content with our simple approximation.

- $\dfrac{T_{\max} + 2T_{\min}}{3}$
- $\left(\dfrac{1}{2} - \dfrac{6}{\pi^2} \sin \dfrac{\pi^2}{12}\right) T_{\max} + \left(\dfrac{1}{2} + \dfrac{6}{\pi^2} \sin \dfrac{\pi^2}{12}\right) T_{\min}$
- $\dfrac{T_{\max} + T_{\min}}{2}$
- $24T_{\max} - 23T_{\min}$
- $\dfrac{T_{\max} + 3T_{\min}}{2}$

- $\frac{1}{2}T_{\max} - \frac{11}{24}T_{\min}$

Question 4

It is intuitively clear that the average value of x over a circle of radius 1 (given by the equation $x^2 + y^2 = 1$) is zero. But what is the average value of x^2 over this circle?

Hint: notice that this is an average over a curve, so you will need to integgrate with respect to the arc length element dL. In order to make your calculations easier, use the parametrization

$$x = \cos t, \quad y = \sin t, \qquad 0 \le t \le 2\pi$$

- $\frac{1}{2\pi}$
- $\frac{1}{2}$
- $\frac{\pi}{4}$
- $\frac{2}{\pi}$
- 0
- $\frac{1}{4}$

Question 5

Let us model a mountain as a circular cone of height h whose base has radius R. You can see it as the surface obtained by revolving the line

$$y = h\left(1 - \frac{x}{R}\right), \qquad 0 \le x \le R$$

about the y-axis. What is the average height of the points on the surface of the mountain?

Hint: This average is an integral with respect to area. You may wish to take as area element an infinitesimal annulus centered at the origin.

- $\frac{h}{2}$
- $\frac{h}{6}$
- $\frac{1}{2}\pi R^2 h$
- $\frac{1}{3}\pi R^2 h$
- $\frac{1}{6}\pi R^2 h$
- $\frac{h}{3}$

Question 6

The mean annual temperature of a point on the Earth's surface is essentially a function of its latitude, and can be modeled by the following expression:

$$T(\phi) = 48\cos^2\frac{\pi\phi}{180} - 23 \qquad (\text{in }^{\text{o}}\text{C})$$

The latitude ϕ is usually measured in degrees, with $\phi = 0^{\text{o}}$ corresponding to the equator (where the mean temperature is 25^{o}C), and $\phi = \pm 90^{\text{o}}$ to the poles (with a mean temperature of -23^{o}C).

In order to average this mean annual temperature over the whole surface of the Earth, start by taking our model of the sphere of radius R as the surface obtained by revolving the function

$$y = \sqrt{R^2 - x^2}, \qquad -R \le x \le R$$

about the x-axis. The x-coordinate is related to latitude by $x = R\sin\frac{\pi\phi}{180}$, so that the mean annual temperature can be expressed in terms of x as

$$T(x) = 25 - 48\left(\frac{x}{R}\right)^2$$

Find the average of this mean annual temperature over the whole surface of the Earth.

Hint: if you set you set up things correctly, you do not need to know an explicit value for R!

- 0^{o}C
- -1^{o}C
- 1^{o}C
- 9^{o}C
- 10^{o}C
- 5^{o}C

Question 7

What is the average of $(x-1)^2$ over the domain $1 \leq |x| \leq 3$. Be careful!

- 1
- $\frac{32}{3}$
- $\frac{4}{3}$
- $\frac{16}{3}$
- $\frac{8}{3}$
- 8

☐ In accordance with the Honor Code, I certify that my answers here are my own work.

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