# Homework 3

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

## Question 1

Compute the Taylor series about $x = 0$ of the polynomial $f(x) = x^4 + 4x^3 + x^2 + 3x + 6$. Be sure to fully simplify. What does this tell you about the Taylor series of a polynomial?

Hint: If you paid attention during the lecture, this will be a very simple problem!

- The Taylor series of $f(x)$ is $6 + 3x + x^2 + 4x^3 + x^4$: the Taylor series about $x = 0$ of a polynomial is the polynomial itself.
- A polynomial does not have a Taylor series.
- The Taylor series of $f(x)$ is 6: the Taylor series about $x = 0$ of a polynomial is just the lowest order term.
- The Taylor series of $f(x)$ is $3 + 2x + 12x^2 + 4x^3$: the Taylor series about $x = 0$ of a polynomial is its derivative.
- The Taylor series of $f(x)$ is $6x + \frac{3x^2}{2} + \frac{x^3}{3} + x^4 + \frac{x^5}{5} + C$: the Taylor series about $x = 0$ of a polynomial is its integral.
- The Taylor series of $f(x)$ is $x^4$: the Taylor series about $x = 0$ of a polynomial is just the highest order term.

## Question 2

Compute the first three terms of the Taylor series about $x = 0$ of $\sqrt{1 + x}$.

- $\sqrt{1+x} = 1 - \frac{1}{2}x + \frac{1}{8}x^2 + \cdots$
- $\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{4}x^2 + \cdots$

- $\sqrt{1+x} = 1 + 2x - 4x^2 + \cdots$
- $\sqrt{1+x} = 1 + x - \frac{1}{4}x^2 + \cdots$
- $\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots$
- $\sqrt{1+x} = 1 + 2x - 2x^2 + \cdots$

## Question 3

Find the first four non-zero terms of the Taylor series about $x = 0$ of the function $(x+2)^{-1}$.

- $(x+2)^{-1} = \frac{1}{2} + \frac{1}{4}x + \frac{1}{8}x^2 + \frac{1}{16}x^3 + \cdots$
- $(x+2)^{-1} = \frac{1}{2} + \frac{1}{4}x + \frac{1}{4}x^2 + \frac{3}{16}x^3 + \cdots$
- $(x+2)^{-1} = \frac{1}{2} - \frac{1}{4}x + \frac{1}{4}x^2 - \frac{3}{16}x^3 + \cdots$
- $(x+2)^{-1} = \frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{3}{16}x^3 + \cdots$
- $(x+2)^{-1} = \frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{1}{16}x^3 + \cdots$
- $(x+2)^{-1} = \frac{1}{2} + \frac{1}{4}x + \frac{1}{8}x^2 + \frac{3}{16}x^3 + \cdots$

## Question 4

Compute the coefficient of the $x^3$ term in the Taylor series about $x = 0$ of the function $e^{-2x}$.

- $-\frac{4}{3}$
- $-\frac{1}{3}$

- $-\dfrac{8}{3}$
- $2$
- $\dfrac{2}{3}$
- $\dfrac{4}{3}$
- $-\dfrac{2}{3}$

# Question 5

Which of the following is the Taylor series about $x = 0$ of $\dfrac{1}{1-x}$ ?

- $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$
- $\dfrac{1}{1-x} = 1 + x + 2x^2 + 3x^3 + \cdots$
- $\dfrac{1}{1-x} = 1 + x + \dfrac{1}{2}x^2 + \dfrac{1}{3}x^3 + \cdots$
- $\dfrac{1}{1-x} = 1 - x + x^2 - x^3 + \cdots$
- $\dfrac{1}{1-x} = 1 + x + \dfrac{1}{2!}x^2 + \dfrac{1}{3!}x^3 + \cdots$
- $\dfrac{1}{1-x} = x + x^2 + x^3 + \cdots$
- $\dfrac{1}{1-x} = 1 + 2x + 4x^2 + 8x^3 + \cdots$

# Question 6

What is the derivative of the Bessel function $J_0(x)$ at $x = 0$? Remember that $J_0(x)$ is defined through its Taylor series about $x = 0$:

$$J_0(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{2^{2k}(k!)^2}$$

- $-\frac{1}{2}$
- $\frac{1}{4}$
- $1$
- $0$
- $\frac{1}{2}$
- $-\frac{1}{4}$

# Question 7

The Taylor series about $x = 0$ of the arctangent function is

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1}$$

Given this, what is the 11th derivative of $\arctan x$ at $x = 0$?

Hint: think in terms of the definition of a Taylor series. The coefficient of the degree 11 term of arctan is $-1/11$: therefore...

- $-11$
- $-10$
- $-10!$
- $-23!$
- $-11!$
- $-23$

- [ ] In accordance with the Honor Code, I certify that my answers here are my own work.

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