Homework 43

The **due date** for this homework is **Tue 7 May 2013 12:00 AM EDT**.

Question 1

Which of the following **cannot** be a probability density function on the domain given? Select all that apply.

- [ ] $\rho(n) = \begin{cases} \dfrac{1}{5} & \text{if } n \text{ even} \\ 0 & \text{if } n \text{ odd} \end{cases}$ on $n = 0, 1, \ldots, 9$.
- [ ] $\rho(n) = \begin{cases} \dfrac{2}{5} & \text{if } n \text{ even} \\ -\dfrac{1}{5} & \text{if } n \text{ odd} \end{cases}$ on $n = 0, 1, \ldots, 9$.
- [ ] $\rho(n) = \dfrac{1}{n}$ on $n = 1, 2, \ldots$
- [ ] $\rho(n) = \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{otherwise} \end{cases}$ on $n = 1, 2, \ldots$
- [ ] $\rho(n) = \dfrac{1}{10}$ on $n = 0, 1, \ldots, 9$.
- [ ] $\rho(n) = \dfrac{1}{10}$ on $n = 0, 1, \ldots, 10$.

Question 2

Recall that a coin is said to be *fair* if the probability of it turning up heads (H) when tossed is the same one as that of it turning up tails (T): $1/2$ each. Suppose now that you have a non-fair coin, in which the probability of H is a fixed number p between 0 and 1; then, the probability of T is $(1-p)$. If you toss the same coin a second time, the four possible combinations of the results of both tosses —HH, HT, TH and TT— are not equally likely anymore: their probabilities are p^2,

$p(1-p)$, $(1-p)p$ and $(1-p)^2$, respectively. What is the probability of obtaining k heads and $n-k$ tails, in whatever order, after n repetitions of this experiment?

Hint: try writing down all the possible outcomes for small values of n, grouped by the number of heads, together with their probabilities. Each probability will be a power of p multiplied by a power of $(1-p)$ and some integer coefficient. Do you recognize these coefficients?

For the curious (a second hint, too): the discrete probability density function of tossing a single coin is called a *Bernouilli distribution*. That of of n repetitions of the latter experiment receives the name of *binomial distribution*.

- $p^{n-k}(1-p)^k$
- $\binom{n}{n-k}p^{n-k}(1-p)^{n-k}$
- $\binom{n}{k}p^{n-k}(1-p)^k$
- $p^k(1-p)^{n-k}$
- $\binom{n}{k}p^k(1-p)^{n-k}$
- $\binom{n}{k}p^k(1-p)^k$

Question 3

Which of the following **cannot** be a probability density function on the domain given? Select all that apply.

- $\rho(x) = \dfrac{1}{10}$ on $[0, 9]$
- $\rho(x) = \dfrac{2}{\pi}\dfrac{1}{1+x^2}$ on $[0, +\infty)$.

- [ ] $\rho(x) = \dfrac{1}{x^2}$ on $[1, +\infty)$.
- [ ] $\rho(x) = \dfrac{1}{10}$ on $[0, 10]$
- [ ] $\rho(x) = \dfrac{1}{2\pi} + \sin x$ on $[0, 2\pi]$
- [ ] $\rho(x) = \dfrac{2}{\pi}\dfrac{1}{1+x^2}$ on $\mathbb{R} = (-\infty, +\infty)$.

Question 4

For which value of λ is $\rho(x) = \lambda x^2 e^{-x}$ a probability density function on $[0, +\infty)$?

- $\lambda = \dfrac{1}{2}$
- $\rho(x)$ is not a probability density function for any value of λ
- $\lambda = 2$
- $\lambda = \dfrac{e}{2}$
- $\lambda = \dfrac{1}{e}$
- $\lambda = 1$

Question 5

Suppose $\rho(x)$ is a probability density function on $\mathbb{R} = (-\infty, +\infty)$. Its associated *cumulative distribution function* $F(y)$ is defined as

$$F(y) = \int_{x=-\infty}^{y} \rho(x)\, dx$$

that is, as the probability that x is at most y. Notice that, given this interpretation, it is easy to see that $\lim\limits_{y\to-\infty} F(y) = 0$ and $\lim\limits_{y\to+\infty} F(y) = 1$. But what is the rate of change of F with respect to y?

Hint: remember the Fundamental Theorem of Integral Calculus!

- $\rho(y) - \rho(0)$
- $\rho(y)$
- 1
- $\rho(y) - 1$
- $\rho(y) - \rho(x)$
- $\rho(y)\,dy$

Question 6

The probability that $a < x < b$ can also be expressed in terms of the cumulative distribution function F that we introduced in the last problem. How exactly?

- $P(\{a \leq x \leq b\}) = F(b) - F(a)$
- $P(\{a \leq x \leq b\}) = F(a) - F(b)$
- $P(\{a \leq x \leq b\}) = F(a) + F(b)$
- $P(\{a \leq x \leq b\}) = \dfrac{F(b) - F(a)}{2}$
- $P(\{a \leq x \leq b\}) = \dfrac{F(a) - F(b)}{2}$
- $P(\{a \leq x \leq b\}) = \dfrac{F(a) + F(a)}{2}$

Question 7

The amount of time between failures of a printer follows an exponential probability distribution —that is, right after being repaired, the probability that the printer will fail after a time at most T is given by

$$\int_{t=0}^{T} \alpha e^{-\alpha t}\, dt$$

for $\alpha = 0.01 \ln 2 \ \mathrm{h}^{-1}$ (notice that α has units of inverse time, in this case, inverse hours). What is the probability that the printer does not fail for $200\,\mathrm{h}$ after the last repair?

- e^{-2}
- $\frac{1}{4}$
- $\frac{1}{2}$
- $\frac{3}{4}$
- $e^{-1/2}$
- $1 - e^{-2}$

☐ In accordance with the Honor Code, I certify that my answers here are my own work.

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