Stewart Gough Platform

DESIGN AND VALIDATION OF LOW COST STEWART GOUGH PLATFORMS SUITABLE FOR INDUSTRIAL MICRO ASSEMBLY

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Institute: Utrecht University of applied Science / Knowledge Centre Science & Technology – Micro system technology

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Date: 22-07-2013

Location: Utrecht

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Preface

This Stewart Gough Platform project is an addition to the micro systems developments at the Science & Technology Knowledge Center of Utrecht University of Applied Science.

This report is written for my supervisors, as well for Mechanical Engineer student and Computer Engineer students. Which will continue this project to enlarge our Micro Systems and creating control software for these systems.

In this project I found difficulties creating a formula to express the movement of the end-effector necessary to validate the Stewart Gough Platform. With great help of Wouter Hijink I could create an understandable calculation model. I will thank Wouter Hijink for his kinematic knowledge and support.
As well I will thank Rik Lafeber for his knowledge en help during the whole project. And I will thank Erik Puik, Rob Sillen, Gertie Doppenberg, Janny Bakker and Bernard van Setten for all their help.

Kind Regards

Joost van Duijn
3th years Mechanical Engineer student

Summary
The design and validation of a Stewart Gough Platform driven by stepper motors is done for the Knowledge Center Science and Technology of Utrecht University of Applied Science. This to create a universal agile micro system with six degrees of freedom, which allows he platform to translate and rotated on each axis.
This micro system is an improvement of the redesigned Delta robot. Combining these micro systems in a grid improves the production with these universal micro assembly cells.
A design and validation of this low cost Stewart Gough Platform driven by brushless actuator is described. Two different designs are created according to product family engineering. To validate both designs a calculation model is made with simple matrix formulas. Of these two designs the parallel orientated motor axle design is chosen. By changing the length of the lower carbon arms to 350 mm the reach of the platform is improved compare to the other design and the handling of the Delta robot. To obtain this result the basis must as well be placed 50 mm higher on the equiplet.
Unfortunately this design cannot reach the required domain therefor further improvements on mainly the ball joints is preferred.

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Introduction

This report describes the design and validation of a Stewart-Gough platform. This project is done for the Knowledge Centre (KC) Science & Technology of Utrecht University of Applied Science (HU). This school project replace the general third years project of the Mechanical Engineering course.

The KC is a department on the HU, where a link is create between the professional companies and educational activities of the HU. The projects in the KC are supervised by a lecture, and performed by students and docents from the bachelor program.

HUniversal Production

The Stewart-Gough project is a development for the lectureship Micro System Technology / Embedded Systems (MST) which is focused on industrialization of Micro Systems. Which are highly accurate systems.
The production of Micro Systems is difficult. Therefore within the HU a novel production philosophy has been created, called HUniversal Production. This productions philosophy is modeled on the development of Agile Manufacturing (AM). Which increase the flexibility and response of the production requests.

Delta robot

For the lectureship MST already a development is made. In the project Delta Robot several Mechanical Engineer and Computer Engineer students develop a low cost Delta Robot (Figure 1). This Delta robot can be used as an agile micro system in an accurate micro assembly line.
This delta robot has 3 degrees of freedom (DOF) which result in a translation in the X, Y and Z direction of the effector.

Figure 1 Delta robot, a 3 DOF platform

Objective

To enlarge the product family of low cost agile micro systems, a low cost Stewart Gough Platform will be developed. This platform has even 6 DOF, which allows the effector to translate and rotate.
The combination of the Stewart Gough platforms and Delta robots will bring the production with universal micro systems to the next level, by placing a large number of these cells in a grid.
Creating an assembly line by using these cell even makes the assembly process less in depended of maintenance, by easily change a cell for maintenance without a breakdown time.

Problem definition

This project will develop a Stewart Gough Platform to meet the production philosophy of agile manufacturing. Which create the problem definition:

Design and validate a low cost six degree-of-freedom (DOF) Stewart-Gough Robot, suitable for industrial micro assembly and driven by brushless actuators.

The first chapter describes the Stewart Gough principle when after two different redesigns are detailed. In chapter 2 a calculation model will be described used to optimize both designs. Chapter 3 validate both designs and a final design will be chosen. Chapter 4 will describe the future outlook.
In the Appendices are the detailed calculations shown as well a list of all the parts. Also some calculation results are shown.

1. Redesigns

This chapter will describe the redesigns of the Stewart Gough Platform. First the principle of a Stewart Gough Platform will be described. Then the specifications for the redesigns. Finally two designs are created and the components will be specified.

1.1 Stewart Gough principle

A Stewart Gough Platform is an agile mechanical system with commonly hydraulic cylinders (Figure 2). By changing length of these cylinders the end-effector will move, the movement of the end-effector depends on the length of each of the six cylinders.
This results in a platforms with six degrees-of-freedom (DOF) which allows the platforms to translate and rotate in a certain domain. This principle of a Stewart Gough platform is widely used, for example in flight simulators, but surprisingly not for micro assembly.

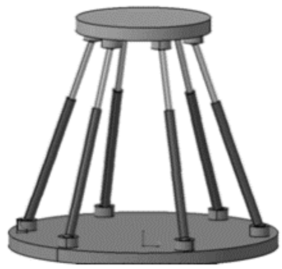

Figure 2 hydraulic Stewart Gough Platform

1.2 Specifications

The redesign of a Stewart Gough platform must comply the following specifications;

• Stroke in X and Y direction	minimal 150 mm
• Stroke in Z direction	minimal 50 mm
• Rotation around X, Y and Z-axis	minimal 15 degrees
• Suitable for different motors	Vexta Stepper motors Maxon brushless DC motors
• Industrial quality	safety protection
• Minimal cost price	€500

1.3 Product family engineering

The redesign of the Stewart Gough Platform is done according to product family engineering. This mains that the redesign mainly consist of reused components of the Delta Robot. This is chosen to minimize the number of different components for both platforms to create a product family. Product families have several benefits; higher productivity, faster time-to-market, lower labor needs and less storage needs. Which and makes it easier to storage spare parts for both micro systems, as well the batch size of each component can be bigger. This creates a less expensive product.

1.4 Equiplets

These micro systems, Stewart Gough platform and Delta robot will be mounted on the same universal equiplet (Figure 3). This equiplet gives the micro systems a rigid base.

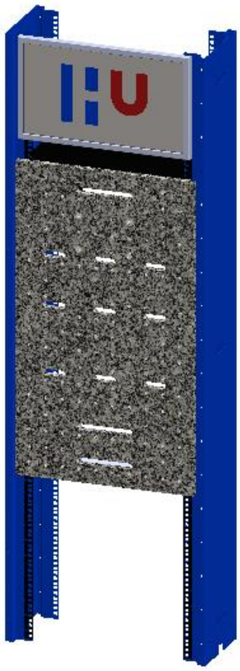
Figure 3 Equiplet

These equiplets place the micro systems at changeable working height around $915\,mm$ which makes it easier for the operator to check the systems.
All the hardware as well is fixated on the equiplet. Creates are replaceable equiplet, allows to change one equiplet easily for another for a maintenance check, without a long breakdown time.

1.5 Step motors

Compare to the redesigned Delta robot is for the redesign of the Stewart Gough Platform chosen for stepper motor. These motor replaces the hydraulic cylinders of the common Stewart Platforms.
A rotation of the stepper motor will result in a longer arm (cylinder) between the base and the end-effector (Figure 4). These stepper motors are highly accurate, and will be fixed on the basis. Result that the motors are not moving along with the effector. Which will improve the dynamic behavior, energy consumption and speed of the redesigned Stewart Gough Platform.

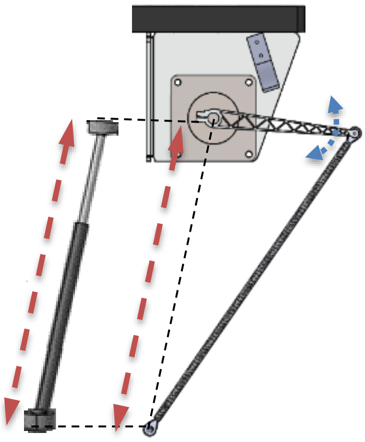

Figure 4 Cylinder(old) and Step Motor(new) solutions

1.6 Designs

Stewart Gough Platforms driven by step motor are rarely applied nowadays. In literature research only a few designs are found. Basically these can be divided in two different designs.
In the first design (Figure 5) the motors are orientated with their axle parallel with the inner circle (the circle where all the motors are placed on). Which result that the upper arms (blue) are positioned outwards. In the second design (Figure 6) are the motor are rotated with 90 degrees.

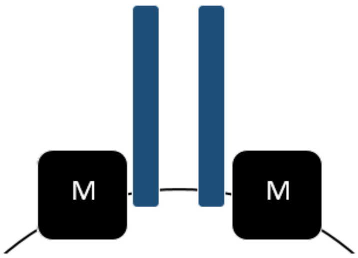

Figure 5 Parallel orientated motor axle

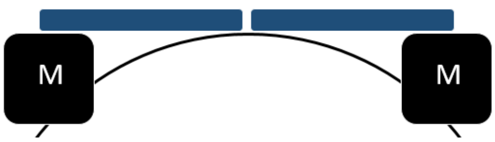

Figure 6 Perpendicular orientated motor axle

These different motor orientation will result in a different reach. Therefor is chosen to develop both designs and validate these.

1.7 Prototypes

There are created two different designs with reused components of the Delta Robot. One of the parallel orientated design (Figure 5) and one of the Perpendicular orientated design (Figure 6).

Both design have the same inner circle where the motors are placed on.
For the Perpendicular design there is as well another prototypes created, one where the motors are place beside each other and the upper arms would move outward. Because of the limited dimensions of the inner circle of $r = 100mm$ the upper arms would collide with each other in that design (Figure 7). This resulted in the design shown in Figure 9.

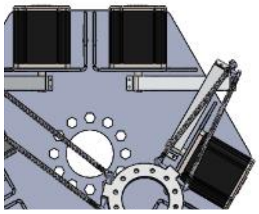

Figure 7 Upper arm collision

Figure 8 Prototype Parallel design

Figure 9 Prototype Perpendicular design

1.8 Design components

All the components are based on the Delta Robot components. Some components are the same and others are slightly different. But to maintain the benefits of a product family all of these components can be used for all the micro systems. Except the specific components for each of the platform, the basis and end-effector.
In appendix I an overview drawing is shown of the Stewart Gough Platform with all the different components appointed. And appendix II shows the part list with as well all the bolts, nuts and washers for all components.

1.8.1 Basis

The basis is made of a granite plate type Nero Impala of 18 mm thick. The granite plate will be honed on minimal one sides, which creates a flat surface preferred for precision alignment of the basis and the stepper motors.
The granite will be machined in a precision abrasive waterjet cutting machine. This machine can only cut in one direction. Already a specific mounting method has been developed for the redesigned delta robot. This method consist of lips on the motor and back plate holders and slots in the granite basis. The lips are slide into the slots and fixated by a M4*16mm thread forming screw.
The slots have different dimensions to prevent over constraining (Figure 10). The smallest slot alligns the holders and the other slots are to fixate the holder to the granite plate.

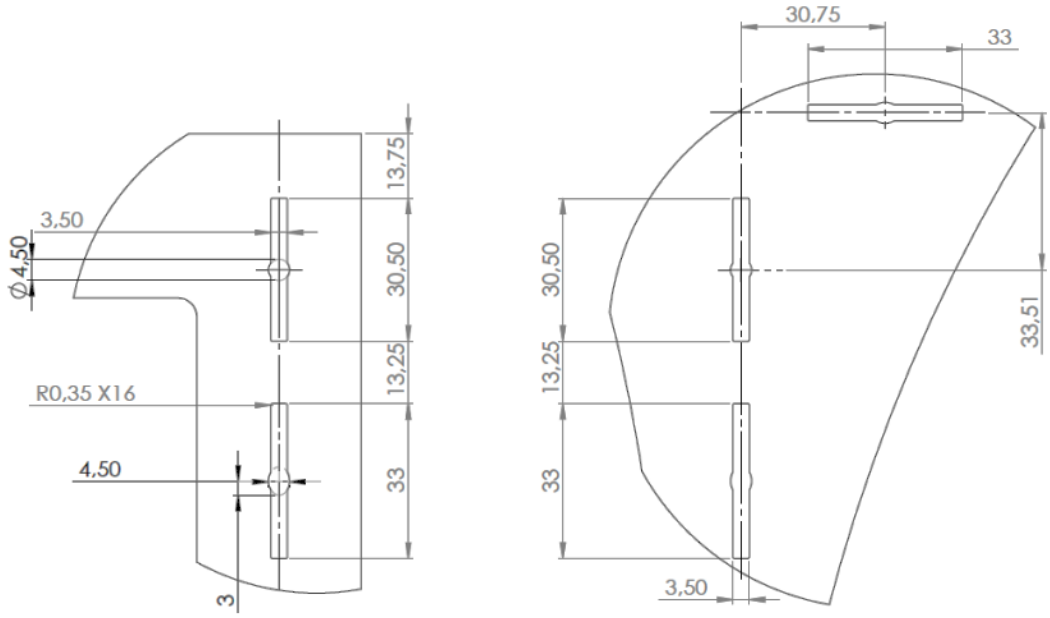

Figure 10 Slots in granite basis (left for back connector, right for motor holder)

The basis is different for the two designs. The basis for the Parallel design (Figure 11) is slightly smaller compare to the Perpendicular design. Al the slots for the lip connectors are the same, as well as the camera hole in the middle with hexagonal hole for the mounting. The distance from the center of the basis to the edge is 199 mm. Finally the distance to the equiplet will be 200 mm, results in a 1 mm gap between the equiplet and basis. This is done due to the roughness of the edges created by the waterjet machine, and to obtain a alignment by the back plate holders.

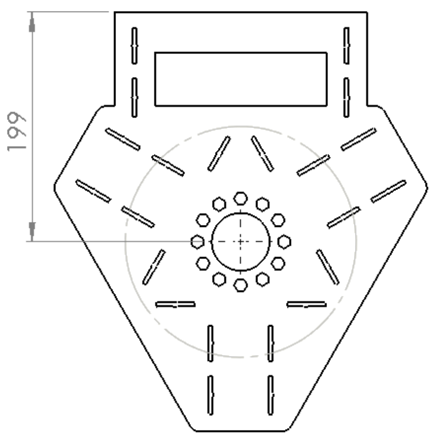

Figure 11 Basis parallel design

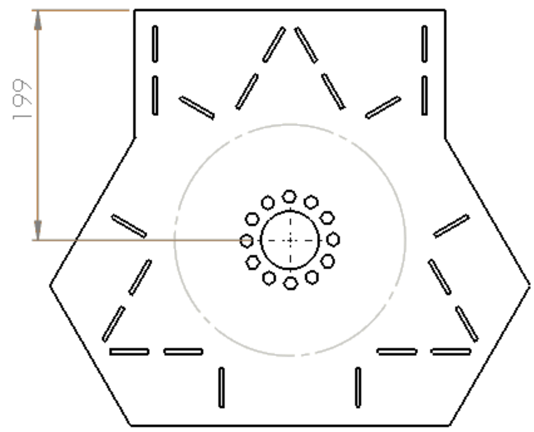

Figure 12 Basis perpendicular design

1.8.2 Motor holder

The stepper motors will be fixated on the motor holders, which are connected to the basis. The motor holders (Figure 14) are made out of a 3 mm thick aluminum 5754 plate. Chosen for its machinability and possibility to be bend with 90 degrees.

The Stewart Gough Platform must be suitable for two different stepper motors (Figure 13), a Vexta and Maxon motor.
Total there are four different motor holders. A right and left bended holder for both motors. The main difference between the two motors are the position and diameter of the mounting holes (M4 for the Vexta motors and M5 for the Maxon motors).
Beside those holes there are as well M2 holes machined for the sensor holders.

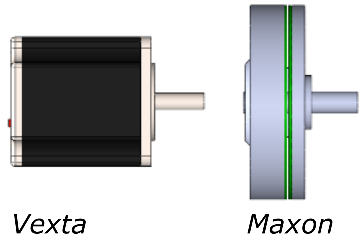

Figure 13 Stepper motors

1.8.3 Sensor holders

On the motor holders are sensor holders fixated (Figure 14). These holders contain a M8 hole for the sensor which are used for homing the machine. As well the sensor holder is a mechanical stop in case an error occurs.
The position of the sensor holder allows the upper arms to make an angle of 20 degrees.
These holders are slightly different compare to the holders of the Delta robot, however in combination with the motor holders they can be either applied in the Stewart Gough platforms and Delta robots.

1.8.4 Upper arms

The upper arms are attached to the stepper motors. These upper arms are machine out of a 15 mm thick aluminum 5083 plate. Due to thickness is chosen for another aluminum type. This to maintain a smooth waterjet cut but also a strong and stiff component.
For both motors are specific upper arms. The contour of these arms are similar only the holes for the motor axle are different.
For the redesign there is added an edge to secure the M2 nut, which makes it easier to fixate the upper arm with an M2x10 bolt around the motor axle.

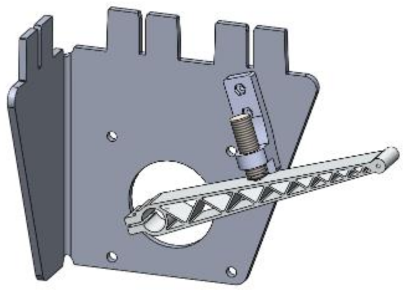
Figure 14 Assembly motor holder(Vexta), upper arm, sensor holder and sensor without bolts

For mounting the ball-joint the small waterjet hole will be threaded, for a wanted results these holes must be made <0,9 mm diameter in the waterjet machine, because of the tolerances and the thickness of the plate these hole easily become too wide.

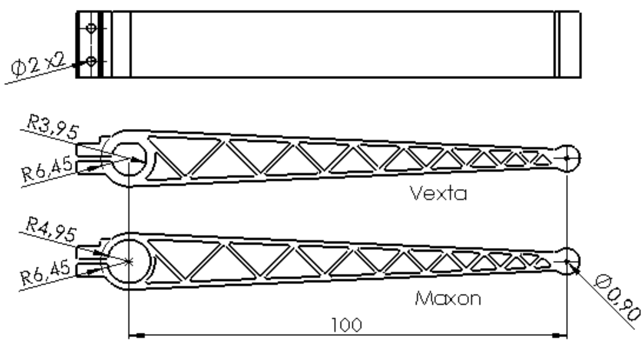

Figure 15 Different upper arms

1.8.5 Lower arms

The lower arms connecting the end-effector to the upper arms, these arms are made out of small carbon tubes. These tubes are bought and can be cut of at the wanted length.

1.8.6 Ball joints

The ball joints in the different Stewart Gough platforms are as well the same as the Delta robot. These joint are bought and be mounted on the upper arms and end-effector by an M2x20 Pozidriv countersunk screw. For the best result there are placed tapered parts between the upper arm and ball-joint. These are made with a 3D printer.
The connection of the joints to the lower arms is done by 3D printed plugs. To secure the connection it's glued on the inside.

1.8.7 End-effector

There are different end-effectors (Figure 16), for the Delta robot, Stewart Gough parallel design and perpendicular design. All these effectors are made by 3D printing.
The ball-joints with tapered parts can be mounted with the M2 bolts on the flat surfaces of the end-effectors (Figure 20).

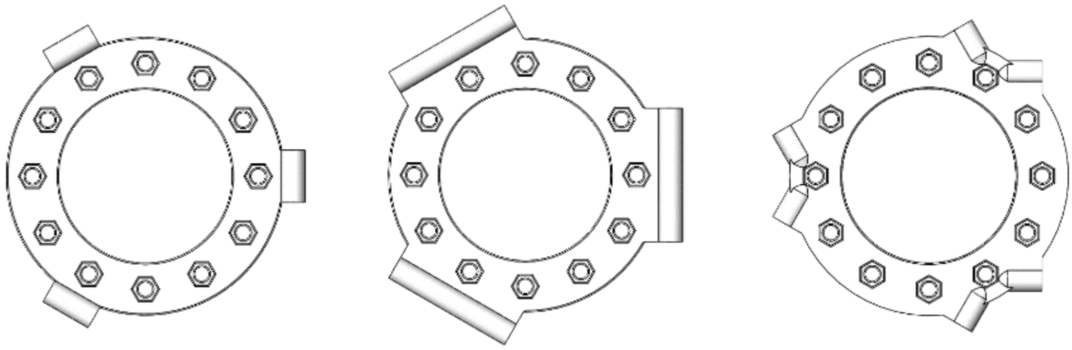

Figure 16 End-effectors (Left; Delta robot, Middle; Stewart Gough Platform parallel, Right; Stewart Gough Platform Perpendicular

1.8.8 Back plate holder

The back plate holder fixate and align the basis on the granite back plate of the equiplet (Figure 17). Similar to the motor holders these holders are made out of a 3 mm thick aluminum 5754 plate and are machined with the waterjet. After machining the holders lips will be bend with 90 degrees.
There are two varies of this holder, one with a right and the other left bended lips.

Figure 17 Stewart Gough Platform on transparent equiplet

1.8.9 Glass plate / Glass plate holder

The glass plate is mounted on the equiplet by two glass plate holders (Figure 17). These holders are made of a 3 mm thick aluminum 5754 plate. Both the glass plate as holder are machined by the waterjet cutting machine.
There are varies of this holder, one with a right and the other left bended lips, to create a holder for both sides.

1.8.10 Camera holder

The last component is the camera holder (Figure 18) which fixate the camera with a 200 mm offset from the equiplet. The same offset as the camera hole on the basis.
As well this holder is made of a 3 mm aluminum 5754 plate, and is machine by a waterjet machine. When after the scribed

bend lines will be bended by 80 degrees. This is chosen to create a simple but stiff construction for the camera. And as well this will protect the camera from small impacts and vibrations.
This holder will be mounted by four M6*16 mm bolts to the back plate of the equiplet and the camera is mounted by four M2 bolts.

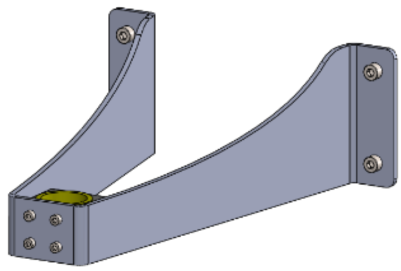

Figure 18 Camera holder with camera mounted

2. Optimization

In the previous chapter are two designs are created and specified. To optimize and validate these designs two calculation models are made. This chapter will describe the approach for these calculation models.
The Excel calculation calculates the angle of the motors for a specific position and orientation of the end-effector, which depends on several parameters.
Beside the optimization and validation these calculation can as well be important for the final control software. Therefore is chosen to make a model based on simple matric summations.
The calculations for both designs are divided in several steps; first the positions of the ball-joints in the end-effector (C_i) are calculated (Figure 19). Secondly the ball-joint positions of basis (B_i) on the end of the upper arm (r_i) are calculated.
The distance between those C_i and B_i points is the length of the lower arm. By rotating the motor and consequently the position of B_i the length can be equaled to the wanted value.
Finally the brute force method is used to define the motor angles of each position and orientation of the tool center point (TCP).
All the detailed formulas can be found in appendix III.

Situation sketch parallel orientated motor axis:

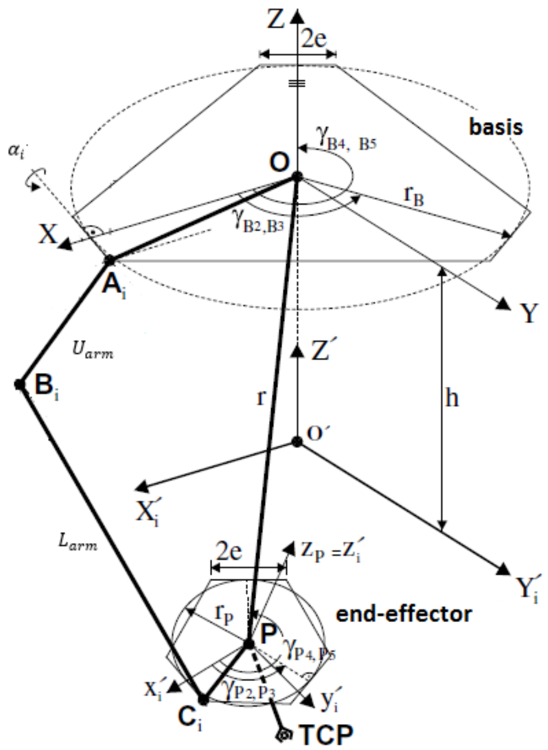

Figure 19 Kinematic parameters Stewart Gough Platform

2.1 End-effector

The position of C_i is similar for both designs. In this paragraph this calculation will be described.
All the point are defined by specific coordinates from the origin (O) (Figure 19).

The first step is to place the TCP without any translation or rotation on the origin. Figure 20, 19 and 20 shows the translations on the effector, from the TCP to the first ball-joint for both designs.

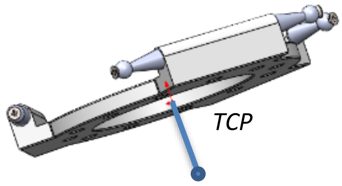

Figure 20 Tool center point

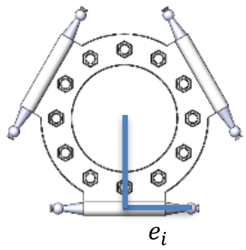

Figure 21 End effector Parallel design

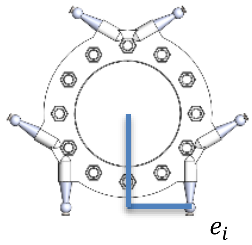

Figure 22 End-effector Perpendicular design

This results in the following matrix;

$$\begin{bmatrix} X_{S01} \\ Y_{S01} \\ Z_{S01} \end{bmatrix} = \begin{bmatrix} r_p \\ e_i \\ -TCP \end{bmatrix}$$

$r_p \ = radius\ effector$
$e_i \ = distance\ between\ ball\ joint\ and\ X\ axis$
$TCP \ = Tool\ Center\ Point$

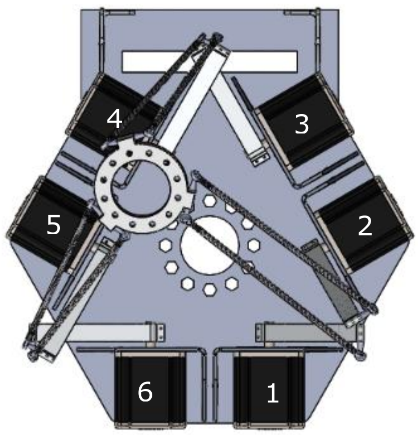

Figure 23 Specify stepper motor

2.1.1 Specify stepper motor

In the Stewart Gough Platform are six stepper motors positioned (Figure 23). The calculation of each of these motors is different. This step defines the motors by rotating the previous formula ($[X_{S01} \quad Y_{S01} \quad Z_{S01}]^T$) around the Z-axis. This will be done by multiply the matrix with the rotation operator around the Z-axis (A_z).

$$\begin{bmatrix} X_{S02} \\ Y_{S02} \\ Z_{S02} \end{bmatrix} = A_z \cdot \begin{bmatrix} X_{S01} \\ Y_{S01} \\ Z_{S01} \end{bmatrix}$$

$$= \begin{bmatrix} cos\ \gamma_{Pi} & -\sin\gamma_{Pi} & 0 \\ sin\ \gamma_{Pi} & \cos\gamma_{Pi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_p \\ e_i \\ -TCP \end{bmatrix}$$

$\gamma_{Pi} \ = angle\ between\ X\ axis\ and\ shifted\ X'\ axis$

Each of the motors have their specific rotation around the Z-axis. As well changes the lengths e_i for each specific motors.

Table 1 Parameters effector

$\boldsymbol{Situation\ (i)}$	$\boldsymbol{\gamma_{Pi}\ [deg]*}$	$\boldsymbol{e_i\ [mm]}$
1	0	e_i
2	120	-e_i
3	120	e_i
4	240	-e_i
5	240	e_i
6	0	-e_i

** The angles should be convert to radians before insert in the formula.*

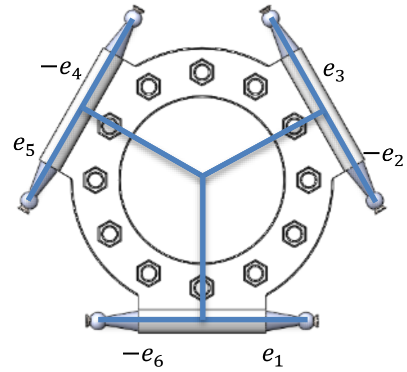

Figure 24 Dimensions effector

2.1.2 Rotation of the end-effector

The end-effector can rotate around the 3 axis. In this step the rotation will be added. This is done by the rotation operator A, which will rotate the points around the X-, Y- and Z-axis.

$$\begin{bmatrix} X_{S03} \\ Y_{S03} \\ Z_{S03} \end{bmatrix} = A \cdot \begin{bmatrix} X_{S02} \\ Y_{S02} \\ Z_{S02} \end{bmatrix}$$

$$= \begin{bmatrix} cos\psi & -sin\psi & 0 \\ sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix} \cdot \begin{bmatrix} cos\varphi & 0 & sin\varphi \\ 0 & 1 & 0 \\ -sin\varphi & 0 & cos\varphi \end{bmatrix} \cdot \begin{bmatrix} X_{S02} \\ Y_{S02} \\ Z_{S02} \end{bmatrix}$$

$$\psi = rotation\ Z - axis$$
$$\theta = rotation\ X - axis$$
$$\varphi = rotation\ Y - axis$$

2.1.3 Translate the end-effector

Until this step the effector has only been rotated around the origin, but the effector can also translate in each direction. To translate the effector it should be multiplied with a translation matrix T. Which will result in the exact positions (C_i) of the ball-joints of the end-effector from the origin.

$$\vec{C_i} = \begin{bmatrix} X_{C_i} \\ Y_{C_i} \\ Z_{C_i} \\ 1 \end{bmatrix} = T \cdot \begin{bmatrix} X_{S03} \\ Y_{S03} \\ Z_{S03} \\ 1 \end{bmatrix}$$

$$\vec{C_i} = \begin{bmatrix} X_{C_i} \\ Y_{C_i} \\ Z_{C_i} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & X_T \\ 0 & 1 & 0 & Y_T \\ 0 & 0 & 1 & Z_T \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_{S03} \\ Y_{S03} \\ Z_{S03} \\ 1 \end{bmatrix}$$

In appendix III are these calculation detailed. Which resulted in the following formula;

$$\vec{C_i} = \begin{bmatrix} \left((cos\psi\ cos\varphi - sin\psi\ sin\theta\ sin\varphi) \cdot \vartheta - sin\psi \cdot\ cos\theta \cdot \rho + (cos\psi \cdot sin\varphi + sin\psi\ sin\theta\ cos\varphi) \cdot -TCP\right) + X_T \\ \left((sin\psi \cdot cos\varphi + cos\psi \cdot sin\theta \cdot sin\varphi) \cdot \vartheta + cos\psi \cdot cos\theta \cdot \rho + (sin\psi \cdot sin\varphi - cos\psi\ sin\theta\ cos\varphi) \cdot -TCP\right) + Y_T \\ (-\ cos\theta\ sin\varphi \cdot \vartheta + sin\theta \cdot \rho + cos\theta \cdot cos\varphi \cdot -TCP) + Z_T \end{bmatrix}$$

$$\vartheta = \ cos\,\gamma_{Pi} \cdot r_p - \sin\gamma_{Pi} \cdot e_p$$
$$\rho = sin\,\gamma_{Pi} \cdot r_p + \cos\gamma_{Pi} \cdot e_p$$

2.2 Calculation ball-joint upper arms of parallel orientated design

In the previous steps the positions of the ball-joints of the effector are calculated. In the following steps the position of the ball-joint of the basis (B_i) for the first design will been calculated.
The first step for the formula is to define the length of the upper arm (U_{arm}) and the distance between the arm and ball-joint $(e_{B,i})$ (Figure 25).

$$\begin{bmatrix} X_{S04} \\ Y_{S04} \\ Z_{S04} \end{bmatrix} = \begin{bmatrix} U_{arm} \\ e_B \\ 0 \end{bmatrix}$$

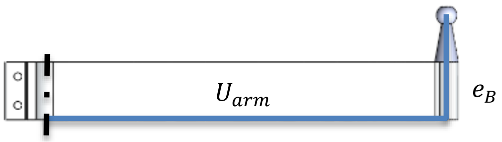

Figure 25 Dimensions Upper arm

2.2.1 Rotation upper-arm (motor angle)

The ball-joint position depends on the angle of the stepper motors. In this step the upper arm will be rotated around the Y-axis, which simulate the stepper motor angle (α_i).

$$\begin{bmatrix} X_{S05} \\ Y_{S05} \\ Z_{S05} \end{bmatrix} = A_y \cdot \begin{bmatrix} X_{S04} \\ Y_{S04} \\ Z_{S04} \end{bmatrix}$$

$$A_y = \begin{bmatrix} cos\ \alpha_i & 0 & sin\ \alpha_i \\ 0 & 1 & 0 \\ -sin\ \alpha_i & 0 & cos\ \alpha_i \end{bmatrix}$$

$\alpha_i\ =\ motor\ angle$ $\qquad$ $where: -20° \leq \alpha \leq 90°$ (Figure 26)

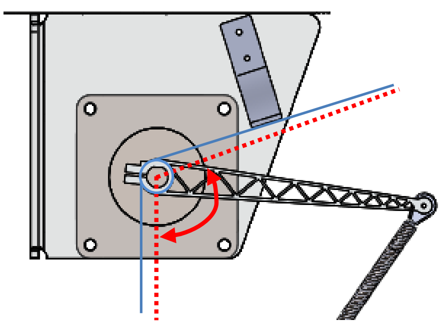

Figure 26 Reach of the stepper motor

2.2.2 Translation upper ball joint

The motor rotation of the upper arm is done around the Y-axis. But in the real situation the arm will be rotate around and axis with an offset from the origin. Therefor the previous formula will be multiplied with the translation operator (T).

$$\begin{bmatrix} X_{S05} \\ Y_{S05} \\ Z_{S05} \\ 1 \end{bmatrix} = T \cdot \begin{bmatrix} X_{S04} \\ Y_{S04} \\ Z_{S04} \\ 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & \Delta X \\ 0 & 1 & 0 & \Delta Y \\ 0 & 0 & 1 & \Delta Z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & r_b \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$r_b \ = \ radius\ basis$

2.2.3 Specify stepper motor for basis

Like the calculation of the end-effector in this step the different stepper motors will be specified by rotate the previous formula around the z-axis.
This is the last step to define the position of B_i;

$$\vec{B_i} = \begin{bmatrix} X_{B_i} \\ Y_{B_i} \\ Z_{B_i} \end{bmatrix} = A \cdot \begin{bmatrix} X_{S5} \\ Y_{S5} \\ Z_{S5} \end{bmatrix}$$

$$= \begin{bmatrix} cos\ \gamma_{Bi} & -\sin\gamma_{Bi} & 0 \\ sin\ \gamma_{Bi} & \cos\gamma_{Bi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_{S5} \\ Y_{S5} \\ Z_{S5} \end{bmatrix}$$

Table 2 Dimensions basis design I

$\boldsymbol{Situation\ (i)}$	$\boldsymbol{\gamma_{Bi}\ [deg]*}$	$\boldsymbol{e_B[mm]}$
1	0	22
2	120	-22
3	120	22
4	240	-22
5	240	22
6	0	-22

** The angles should be convert to radians before insert in the formula.*

In appendix IV are these calculation detailed. Which resulted in the following formula;

$$\vec{B_i} = \begin{bmatrix} cos\ \gamma_{Bi} \cdot (cos\ \alpha_i \cdot U_{arm} + r_b) - \sin\gamma_{Bi} \cdot e_B \\ sin\ \gamma_{Bi} \cdot (cos\ \alpha_i \cdot U_{arm} + r_b) + \cos\gamma_{Bi} \cdot e_B \\ -sin\ \alpha_i \cdot U_{arm} \end{bmatrix}$$

2.2.4 Lower arm length

Until now both ball-joints B_i and C_i are defined. The length between these points can be define by de following formula:

$$\overrightarrow{L_{arm}} = \overrightarrow{B_i} - \overrightarrow{C_i} = \begin{bmatrix} X_{B_i} \\ Y_{B_i} \\ Z_{B_i} \\ 1 \end{bmatrix} - \begin{bmatrix} X_{C_i} \\ Y_{C_i} \\ Z_{C_i} \\ 1 \end{bmatrix} = \begin{bmatrix} X_{B_i} - X_{C_i} \\ Y_{B_i} - Y_{C_i} \\ Z_{B_i} - Z_{C_i} \\ 1 \end{bmatrix} = \begin{bmatrix} X_L \\ Y_L \\ Z_L \\ 1 \end{bmatrix}$$

$$L_{arm} = \sqrt{{X_L}^2 + {Y_L}^2 + {Z_L}^2}$$

$$\Rightarrow {L_{arm}}^2 = \overrightarrow{X_L}^2 + \overrightarrow{Y_L}^2 + \overrightarrow{Z_L}^2 = \left(X_{B_i} - X_{C_i}\right)^2 + \left(Y_{B_i} - Y_{C_i}\right)^2 + \left(Z_{B_i} - Z_{C_i}\right)^2$$

Changing the angle of the step motors (α_i) will change the length of the arm. Then the length of the lower arms can be equaled with the desired length.
This can be done according to the Brute Force method, as done in the attached Excel document.

2.3 Limited angle parallel design

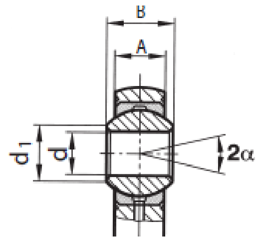

Figure 27 Limited angle ball-joint

The reach of the Stewart Platform is limited by the length of the upper arms, lower arms, dimensions of the basis and effector as calculated in the begin of the chapter but as well the limitations of the ball joints (Figure 27).
These limitations are not included in the formula. Therefor a separate formula is made to verify the feasibility of the platform, by calculating the angles of the joints. The ball joints used in the prototype can make an angle of 32 degrees ($2 \cdot \alpha = 2 \cdot 16° = 32°$).
The feasibility of this design can be calculated by two separate formulas described in this paragraph.

2.3.1 Verify angle I

The first verify angle is the angle between the ball-joints(B_i) and lower arms. By projecting the lower arm vector on the XZ-plane of the ball joint, the angle can be calculated between this projection (${L_{arm}}'^{XZ}$) and the original lower arm(L_{arm}).

$$\cos \sigma_1 = \frac{\overrightarrow{L_{arm}} \cdot \overrightarrow{{L_{arm}}'^{XZ}}}{\left|\overrightarrow{L_{arm}}\right| \cdot \left|\overrightarrow{{L_{arm}}'^{XZ}}\right|}$$

$$\Rightarrow \sigma_1 = \cos^{-1}\left(\frac{\begin{bmatrix} X_L \\ C \\ Z_L \end{bmatrix} \cdot \begin{bmatrix} X'_L \\ Y'_L \\ Z'_L \end{bmatrix}}{\sqrt{{X_L}^2 + {Y_L}^2 + {Z_L}^2} \cdot \sqrt{{X_L}^2 + {Y'_L}^2 + {Z'_L}^2}} \right)$$

$X'_L = X_L$
$Y'_L = 0$
$Z'_L = Z_L$

This situation only counts for stepper motor 1, for the other motors the vector $\overrightarrow{L_{arm}}$ should be rotated around the Z-axis to be compare to stepper motor 1.

$$\overrightarrow{L_{arm}}_R = \begin{bmatrix} cos\,\gamma_{Pi} & -\sin\gamma_{Pi} & 0 \\ sin\,\gamma_{Pi} & \cos\gamma_{Pi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} = \begin{bmatrix} \cos\gamma_{Pi} \cdot X_L - \sin\gamma_{Pi} \cdot Y_L \\ sin\,\gamma_{Pi} X_L + \cos\gamma_{Pi} \cdot Y_L \\ Z_L \end{bmatrix}$$

Table 3 Parameters verify angle I

$\boldsymbol{Situation\ (i)}$	$\boldsymbol{\gamma_{Bi}\ [deg]*}$
1	0
2	240
3	240
4	120
5	120
6	0

** The angles should be convert to radians before insert in the formula.*

2.3.2 Verify angle II

For the second verify angle the angle between $e_{b,i}$ and $e_{p,i}$ (e of the basis and effector/platform) is calculated. This because the first verify angle doesn't verify the angle cause by a rotation around the Z-axis.

$$\vec{\boldsymbol{e}}_{\boldsymbol{B}} = \begin{bmatrix} X_{B_i} - X_{Be_i} \\ Y_{B_i} - Y_{Be_i} \\ Z_{B_i} - Z_{Be_i} \end{bmatrix}$$

$$\vec{\boldsymbol{e}}_{\boldsymbol{P}} = \begin{bmatrix} X_{P_i} - X_{Pe_i} \\ Y_{P_i} - Y_{Pe_i} \\ Z_{P_i} - Z_{Pe_i} \end{bmatrix}$$

The vectors B_i en P_i are already calculated and vector Be_i and Pe_i are the same calculation only without a e_i translation($e_i = 0$).
The angle between the two vectors can be calculate with the following equation;

$$\cos\sigma_1 = \frac{\overrightarrow{e_B} \cdot \overrightarrow{e_P}}{|\vec{\boldsymbol{e}}_{\boldsymbol{B}}| \cdot |\overrightarrow{e_P}|}$$

$$\Rightarrow \sigma_1 = \cos^{-1}\left(\frac{\begin{bmatrix} X_{B_i} - X_{Be_i} \\ Y_{B_i} - Y_{Be_i} \\ Z_{B_i} - Z_{Be_i} \end{bmatrix} \cdot \begin{bmatrix} X_{P_i} - X_{Pe_i} \\ Y_{P_i} - Y_{Pe_i} \\ Z_{P_i} - Z_{Pe_i} \end{bmatrix}}{\sqrt{X_B{}^2 + Y_B{}^2 + Z_B{}^2} \cdot \sqrt{X_P{}^2 + Y_P{}^2 + Z_P{}^2}} \right)$$

Combining these formulas creates a valid model of the parallel orientated Stewart Gough Platform. These calculation are inserted in the Excel sheet.

2.4 Calculations ball-joints upper arms of perpendicular orientated design

For the second design a slightly different calculation model is made. In this model the stepper motors are tilted with 90 degrees and have a small offset.

2.4.1 Ball-joints upper arm

The first step is to describe the length of the upper arm (U_{arm}),.

$$\begin{bmatrix} X_{S11} \\ Y_{S11} \\ Z_{S11} \end{bmatrix} = \begin{bmatrix} 0 \\ -U_{arm} \\ 0 \end{bmatrix}$$

2.4.2 Rotation upper arm (motor angle)

In this step the motor angle will added. By rotation the upper arm around the X-axis.

$$\begin{bmatrix} X_{S12} \\ Y_{S12} \\ Z_{S12} \end{bmatrix} = A_x \cdot \begin{bmatrix} X_{S11} \\ Y_{S11} \\ Z_{S11} \end{bmatrix}$$

$$A_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\alpha & -sin\alpha \\ 0 & sin\alpha & cos\alpha \end{bmatrix}$$

$\alpha_i \ = \ Motor\ angle$

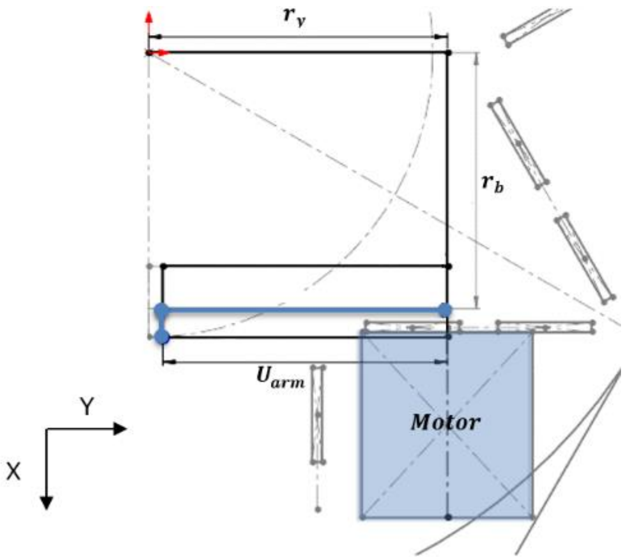

Figure 28 Detailed dimensions design II

2.4.3 Translation upper arm

Different than the first design are the position and orientation of the motors. In this design the motor will be translated with r_b and r_y (Figure 28). This result in the following formula;

$$\begin{bmatrix} X_{S13} \\ Y_{S13} \\ Z_{S13} \\ 1 \end{bmatrix} = T \cdot \begin{bmatrix} X_{S12} \\ Y_{S12} \\ Z_{S12} \\ 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & r_b \\ 0 & 1 & 0 & r_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$r_b \ = \ radius\ basis$

$r_y = distance\ between\ origin\ and\ axle\ (U_{arm} + free\ space)$

2.4.4 Specify stepper motor for basis

Similar to the calculation of the first design, each motor will be defined by rotation the previous formula around the Z-axis.

$$\begin{bmatrix} X_{S14} \\ Y_{S14} \\ Z_{S14} \end{bmatrix} = A_z \cdot \begin{bmatrix} X_{S13} \\ Y_{S13} \\ Z_{S13} \end{bmatrix}$$

$$A_z = \begin{bmatrix} cos\,\gamma_{Bi} & -\sin\gamma_{Bi} & 0 \\ sin\,\gamma_{Bi} & \cos\gamma_{Bi} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Table 4 Parameters design II

$Situation\ (i)$	$\gamma_{Bi}\ [deg]*$	$U_{arm}\ [mm]$	$r_y\ [mm]$	$Motor\ angle$
1	0	100	105	α_1
2	120	-100	-105	$-\alpha_2$
3	120	100	105	α_3
4	240	-100	-105	$-\alpha_4$
5	240	100	105	α_5
6	0	-100	-105	$-\alpha_6$

** The angles should be convert to radians before insert in the formula.*

In appendix V are these calculation detailed. Which resulted in the following formula;

$$\vec{B_i} = \begin{bmatrix} X_{B_i} \\ Y_{B_i} \\ Z_{B_i} \end{bmatrix} = \begin{bmatrix} cos\,\gamma_{Bi} \cdot r_b - \sin\gamma_{Bi} \cdot \left(cos\alpha \cdot -U_{arm} + r_y\right) \\ sin\,\gamma_{Bi} \cdot r_b + \cos\gamma_{Bi} \cdot \left(cos\alpha \cdot -U_{arm} + r_y\right) \\ sin\alpha \cdot -U_{arm} \end{bmatrix}$$

2.4.5 Lower arm length

With the position B_i and C_i is it possible with the following formula to calculate the length of the lower arm.

$$L_{arm} = \sqrt{{X_L}^2 + {Y_L}^2 + {Z_L}^2}$$

$$\Rightarrow {L_{arm}}^2 = {X_L}^2 + {Y_L}^2 + {Z_L}^2 = \left(X_{B_i} - X_{C_i}\right)^2 + \left(Y_{B_i} - Y_{C_i}\right)^2 + \left(Z_{B_i} - Z_{C_i}\right)^2$$

By changing the angle of the actuator the length of the arm will be changed to the desired length of the lower arms.

2.5 Limited angle perpendicular design

The limitation caused by the ball-joint is different in this concept. This because the ball-joint are as well tilled with 90 degrees (Figure 29). Creates a different angle compare to the first design

Figure 29 End-effector design II

2.5.1 Verify angle I

The first verify angle is the angle between the ball-joints B_i and lower arms. This is the angle between the lower arm vector and its protection on the YZ-plane of the ball joint. This angle can be calculated with the following formula

$$\cos\sigma_1 = \frac{\overrightarrow{L_{arm}} \cdot \overrightarrow{L_{arm}'^{YZ}}}{\left|\overrightarrow{L_{arm}}\right| \cdot \left|\overrightarrow{L_{arm}'^{YZ}}\right|}$$

$$\Rightarrow \sigma_1 = \cos^{-1}\left(\frac{\begin{bmatrix} X_L \\ C \\ Z_L \end{bmatrix} \cdot \begin{bmatrix} X'_L \\ Y'_L \\ Z'_L \end{bmatrix}}{\sqrt{X_L^2 + Y_L^2 + Z_L^2} \cdot \sqrt{X_L^2 + Y'^2_L + Z'^2_L}}\right)$$

$X'_L = 0$
$Y'_L = Y_L$
$Z'_L = Z_L$

This situation only counts for step motor 1, for the other motors the vector $\overrightarrow{L_{arm}}$ should be rotated around the Z-axis to be compare to situation 1.

$$\overrightarrow{L_{arm}}_R = \begin{bmatrix} cos\,\gamma_{Pi} & -\sin\gamma_{Pi} & 0 \\ sin\,\gamma_{Pi} & \cos\gamma_{Pi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} = \begin{bmatrix} \cos\gamma_{Pi} \cdot X_L - \sin\gamma_{Pi} \cdot Y_L \\ sin\,\gamma_{Pi} X_L + \cos\gamma_{Pi} \cdot Y_L \\ Z_L \end{bmatrix}$$

Table 5 Parameters verify angle I

$\boldsymbol{Situation\ (i)}$	$\boldsymbol{\gamma_{Bi}\ [deg]^*}$
1	0
2	240
3	240
4	120
5	120
6	0

** The angles should be convert to radians before insert in the formula.*

2.5.2 Verify angle II

The second verify angle is also different compare to design I. This angle calculates the angle between $\vec{\boldsymbol{e}}_{\boldsymbol{B}}$ and $\vec{\boldsymbol{e}}_{\boldsymbol{P}}$ (Figure 30).

$$\vec{\boldsymbol{e}}_{\boldsymbol{B}} = \begin{bmatrix} X_{B_i} - X_{B-1_i} \\ Y_{B_i} - Y_{B-1_i} \\ Z_{B_i} - Z_{B-1_i} \end{bmatrix}$$

$$\vec{\boldsymbol{e}}_{\boldsymbol{P}} = \begin{bmatrix} X_{P_i} - X_{P-1_i} \\ Y_{P_i} - Y_{P-1_i} \\ Z_{P_i} - Z_{P-1_i} \end{bmatrix}$$

Where;

$$\begin{bmatrix} X_{B-1_i} \\ Y_{B-1_i} \\ Z_{B-1_i} \end{bmatrix} = \begin{bmatrix} cos\,\gamma_{Bi} \cdot (r_b - 1) - \sin\gamma_{Bi} \cdot \left(cos\alpha \cdot -U_{arm} + r_y\right) \\ sin\,\gamma_{Bi} \cdot (r_b - 1) + \cos\gamma_{Bi} \cdot \left(cos\alpha \cdot -U_{arm} + r_y\right) \\ sin\alpha \cdot -U_{arm} \end{bmatrix}$$

$$\begin{bmatrix} X_{C-1_i} \\ Y_{C-1_i} \\ Z_{C-1_i} \end{bmatrix} =$$

$$\begin{bmatrix} \left((cos\psi\; cos\varphi - sin\psi\; sin\theta\; sin\varphi) \cdot \vartheta - sin\psi \cdot\; cos\theta \cdot \rho + (cos\psi \cdot sin\varphi + sin\psi\; sin\theta\; cos\varphi) \cdot -TCP\right) + X_T \\ \left((sin\psi \cdot cos\varphi + cos\psi \cdot sin\theta \cdot sin\varphi) \cdot \vartheta + cos\psi \cdot cos\theta \cdot \rho + (sin\psi \cdot sin\varphi - cos\psi\; sin\theta\; cos\varphi) \cdot -TCP\right) + Y_T \\ (-\,cos\theta\; sin\varphi \cdot \vartheta + sin\theta \cdot \rho + cos\theta \cdot cos\varphi \cdot -TCP) + Z_T \end{bmatrix}$$

$$\vartheta = \; cos\,\gamma_{Pi} \cdot \left(r_p - 1\right) - \sin\gamma_{Pi} \cdot e_p$$
$$\rho = sin\,\gamma_{Pi} \cdot \left(r_p - 1\right) + \cos\gamma_{Pi} \cdot e_p$$

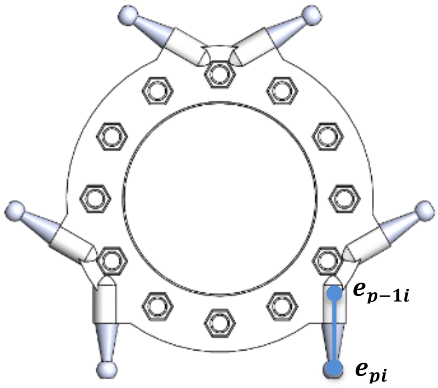

Figure 30 Verify angle II

The angle between $\vec{e}_B$ and $\vec{e}_P$ can be calculate with the following equation.

$$\sigma_1 = \cos^{-1}\left(\frac{\begin{bmatrix} X_{B_i} - X_{B-1_i} \\ Y_{B_i} - Y_{B-1_i} \\ Z_{B_i} - Z_{B-1_i} \end{bmatrix} \cdot \begin{bmatrix} X_{P_i} - X_{P-1_i} \\ Y_{P_i} - Y_{P-1_i} \\ Z_{P_i} - Z_{P-1_i} \end{bmatrix}}{\sqrt{{X_B}^2 + {Y_B}^2 + {Z_B}^2} \cdot \sqrt{{X_P}^2 + {Y_P}^2 + {Z_P}^2}} \right)$$

Combining these formulas creates a valid model of the perpendicular motor orientated Stewart Gough Platform design. These calculation are inserted in the second design Excel sheet.

2.6 Optimize reach

Both designs are defined in formulas, which can be used to define the maximal reach of the TCP. These calculations have several parameters and variables, by changing these an optimal design could be found with a large reach.
Unfortunately there are too many variables to use all of them. Therefor for optimization is chosen to take the lower arms as the main variable. This because the lower arms can easily be made of different lengths without losing the benefits of Product Family Engineering.

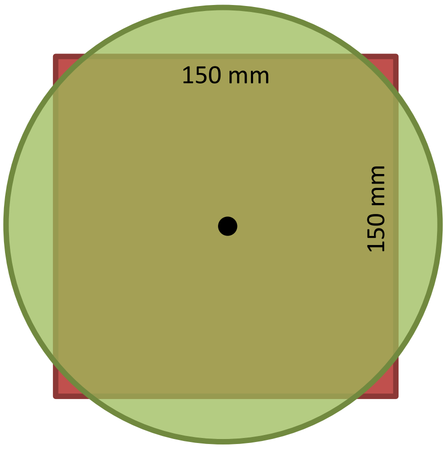

Figure 31 schematic overview of the reach

Mainly the reach of the platform is limited in the corners of the required domain (Figure 31, this is only a sample not the real domain). By only calculated the intersect point of the circle and the square the domain the design can be optimized. For these calculations the upper arm's length are changed in steps of 50 mm. First the used 250 mm arms of the Delta-robot are calculated, secondly 300 mm and finally 350 mm.
Because the longer arms as well cause a domain movement in Z direction, this dimension will also be variable and is changed with steps of 50 mm, according to the height between the mounting holes on the equiplet. Result in a maximum height and minimal height. In appendix VI are these results shown.

3. Validation

Two different designs are developed, one with parallel orientated motor and the other with 90 degrees rotated step motors the perpendicular orientated motor design. In the previous chapters the designs are defined and calculated.
This chapter will validate both design, which will better comply the requirements. Mainly the focus is on the cost and reach of the Stewart Gough Platforms.

3.1 Cost validation

Both designs are almost similar, they are made out of the same materials, and the way to produce and assembly is also almost the same. The main difference is the basis, for the parallel orientated design the basis is slightly smaller, which will result in less material cost. As well the total mass of the platform will be less which makes it easier to assemble and for the maintenance.

3.2 Reach validation

In the previous chapter some maximum translation and orientation reach are calculated for different parameters. These results (Appendix VI) shows that the parallel orientated motor design has a larger reach. Even the Perpendicular design could not reach the required domain of +75/-75 at all (Figure 32).

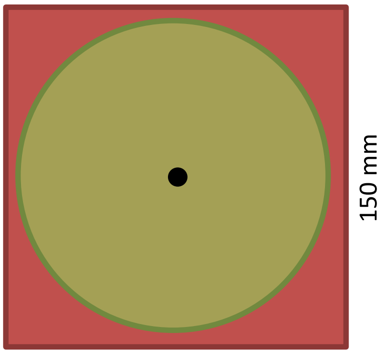

Figure 32 Schematic overview of the reach of Perpendicular design

The maximal reach of the parallel design can be obtain when create a platform with 350 mm long lower arms. And placing the basis at a distance of 373,17mm of the glass plate. Otherwise the reach cannot be obtained in the z-domain.
Unfortunately this design still doesn’t completely comply the requirements, but is the best design to create.

4. Further outlook

This chapter describes the further developments which could be done for the Stewart Gough Platform.

4.1 Reach improvements

The ball joints used in the Delta Robot and Stewart Platform are the main limitation of the total reach of both platforms. Changing this limitation would result in a better handling.
There are ball-joint available with slightly larger possible angles, with possibilities of an angle of 18 degrees.
As well there is a bearing type MBY-CR[1] which have a maximum angle of 29 degrees (Figure 33). Unfortunately for these bearing a redesign of the upper arms, lower arms, end-effector and the connection is necessary.

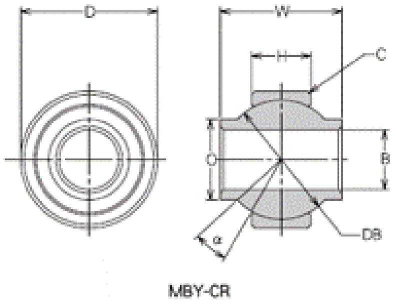

Figure 33 Spherical bearing

4.2 Industrial design

The current Stewart Gough design is not ready to be used in the industry. A safety cover is preferred in many industries, to protect the moving object, as well to protect all the electrical component from for instance dust.

[1] http://www.eminebea.com/en/product/bearing/spherical/mby-crmby-vcr.shtml

Results

In this project two different designs of Stewart Gough platforms are developed. One with parallel orientated motor axles and the other with perpendicular orientated motor axles. Both designs are made with reused components of the redesigned Delta robot of the Knowledge Centre Science & Technology of Utrecht University of Applied Science. This to create low cost micro system and use the benefits of a product family.

There are made two calculation excel models for both design to express the position and orientation of the end-effector in the angle of each of the six stepper motors. These simple matrix formulas are used to validate and optimize both design.

To maintain the product family benefits there is chosen to only adjust the upper arms to obtain a larger reach. This is done with steps of 50 (250, 300, 350 mm). Because this causes a shift in the height domain the height value was also set as a variable value. The height would be change with steps of 50 mm according to the equiplet mounting holes.

The calculations results in an optimized design with lower arms of 350 mm and a height lifted basis with 50 mm of the parallel orientated motor axles design.
The perpendicular design wasn't able at all to apply the specifications. But unfortunately the chosen parallel orientated design didn't completely apply the specifications as well. This parallel platform has a much larger reach but further development mainly of the ball joints is prefer to completely apply the specifications.

Figure 34 Parallel orientated motor axle final design

Reverences

Articles/Documents

#	Author	Title	Year	Publisher
1	Erik Puik, Leo van Moergestel	Agile Multi-Parallel Micro Manufacturing Using a Grid of Equip lets	-	Utrecht University of Applied Science
2	Bernard van Setten	Redesign of the Delta Robot	2012	Utrecht University of applied science
3	J. Hesselbach, C. Bier, I. Pietsch, N. Plitea, S. Büttgenbach, A. Wogersien, J. Güttler	Passive-joint sensors for parallel robots	2004	Elsevier

Websites

#	Link	Datum
1	www.intmath.com/vectors/7-vectors-in-3d-space.php	June 2013
2	www.eminebea.com	June 2013
3	www.mcb.nl	May 2013
4	www.rvspaleis.nl	June 2013
5	http://en.wikipedia.org/wiki/Rotation_matrix	May 2013
6	http://en.wikipedia.org/wiki/Stewart_platform	May 2013
7	http://en.wikipedia.org/wiki/Product_family_engineering	May 2013

Appendices

I. Overview Stewart Gough Platform

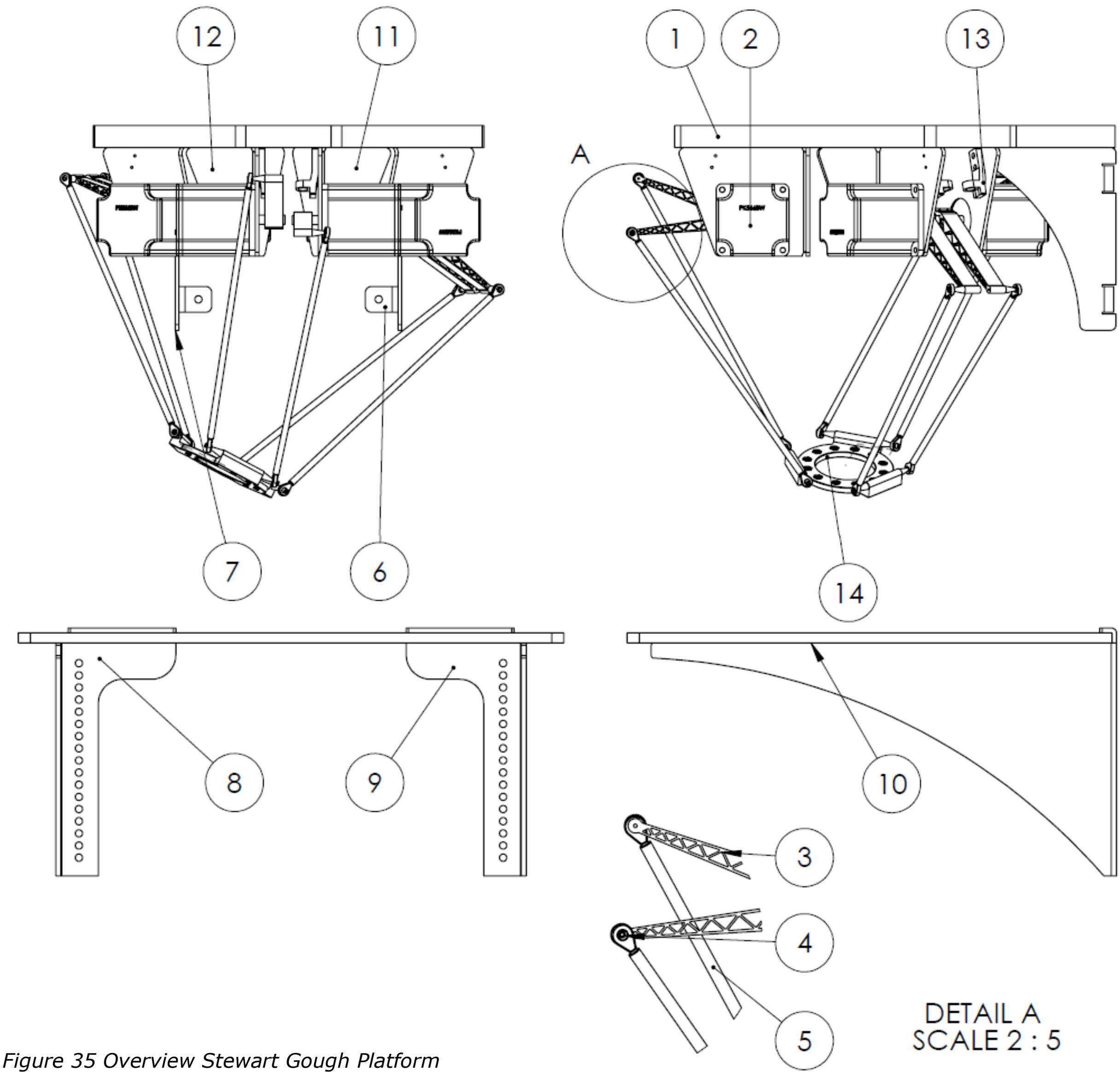

Figure 35 Overview Stewart Gough Platform

I.I Components

Item No.	Part description	Qty.
# 1	Basis	1
# 2	Step motor	6
# 3	Upper arm	6
# 4	Ball-joint	6
# 5	Lower arm	6
# 6	Back plate holder (Right)	1
# 7	Back plate holder (Left)	1

Item No.	Part description	Qty.
# 8	Glass plate holder (Right)	1
# 9	Glass plate holder (Left)	1
# 10	Glass plate	1
# 11	Motor holder (Right)	3
# 12	Motor holder (Left)	3
# 13	Precision switch	6
# 14	End-effector	1

II. Part list

No	Name	Information / types	Quantity				Part type
			Delta Robot		Stewart Gough Platform		
			Vexta	Maxon	Vexta	Maxon	
1.	Motor holder	Aluminum 5754					Create
		Type:					
		Vexta Right Bend	3		3		(Waterjet)
		Vexta Left Bend			3		
		Maxon Right Bend		3		3	
		Maxon left Bend				3	
2.	Stepper motor	Type:					Bought
		Vexta	3		6		
		CRK566PMbKD*		3		6	
		Maxon EC 90					
3.	Back plate holder	Aluminum 5754					Create
		Left bend	1	1	1	1	
		Right bend	1	1	1	1	(Waterjet)
4.	Upper arm	Aluminum 5083					Create
		Type:					
		Vexta	3		6		(Waterjet)
		Maxon		3		6	
5.	Sensor / precision switch	MY-COM H75/80	3	3	6	6	Bought
6.	Basis	Granite Nero Impala 18 mm thick	1	1	1	1	Create (Waterjet)
7.	Sensor holders	Sensor holders / mechanical stop	3	3	6	6	Create (3D printer)
8.	Camera holder	Aluminum 5754	1	1	1	1	Create (Waterjet)
9.	Glass plate holder	Aluminum 5754					Create
		Type:					
		Left bend	1	1	1	1	(Waterjet)
		Right bend	1	1	1	1	
10.	Glass plate		1	1	1	1	Create (Waterjet)
11.	M4*16mm thread forming screw	For lip-connections	13	13	22	22	Bought
12.	M4*16mm low head socket cap screw	Mounting Vexta motor. DIN7984	12	0	24	0	Bought

13.	M5*8mm socket cap screw	Mounting Maxon motor.	0	9	0	18	Bought
14.	M4 washer	DIN 125A	37	13	70	22	Bought
15.	M4 nuts	Only by use of PK566AW motor	12	0	24	0	Bought
16.	End-effector	End-effector/Platform Type: Delta robot Stewart Gough Platform	 1	 1	 1	 1	Create (3D printer)
17.	Glue	Bison Plastic Glue					Bought
18.	Plugs	Connect the ball-joint to the carbon tubes	12	12	12	12	Outsource (3D printer)
19.	Tapered parts	Creates a space between ball-joints and upper arm and end effector	12	12	12	12	Outsource (3D printer)
20.	Ball Joints	DIN 648	12	12	12	12	Bought
21.	Carbon Tubes		6	6	6	6	Create
22.	M2*20mm Pozidriv countersunk screw	Mounting the ball-joint on the Platform/upper arms DIN 965	6	6	12	12	Bought
23.	M2 Nut	For the clamping mechanism of the upper arms DIN 934	6	6	12	12	Bought
24.	M2 Washer	For the clamping mechanism of the upper arms DIN 125A	6	6	12	12	Bought
25.	M2*10mm Socket screw	For the clamping mechanism of the upper arms DIN 912	6	6	12	12	Bought
26.	Equiplet		1	1	1	1	Create/ bought
27.	M6*16mm bolt	Back plate connection	12	12	12	12	Bought
28.	M6 washer	Back plate connection	12	12	12	12	Bought
29.	Clean dampers	Attached to the Vexta motors	3	0	6	0	Bought

*Motor + Driver CRK566PMbKD (combination of PK566PMb +CRD514KD)

II.I Material list

Nr.	Description	Ordered dimension	Company
1.	Aluminum 5754	2000*1000*3 mm	MCB
2.	Aluminum 5083	XXXX*XXX*15 mm	algemenemethaalhandel.nl (via Gertie Doppenberg)
3.	Carbon Tubes	2500 mm	-

III. Effector calculations

III.I End-effector

$$\begin{bmatrix} X_{S01} \\ Y_{S01} \\ Z_{S01} \end{bmatrix} = \begin{bmatrix} r_p \\ e_i \\ -TCP \end{bmatrix}$$

III.II Specify motor

$$\begin{bmatrix} X_{S02} \\ Y_{S02} \\ Z_{S02} \end{bmatrix} = \begin{bmatrix} cos\,\gamma_{Pi} & -\sin\gamma_{Pi} & 0 \\ sin\,\gamma_{Pi} & \cos\gamma_{Pi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_p \\ e_i \\ -TCP \end{bmatrix} = \begin{bmatrix} cos\,\gamma_{Pi} \cdot r_p - \sin\gamma_{Pi} \cdot e_i \\ sin\,\gamma_{Pi} \cdot r_p + \cos\gamma_{Pi} \cdot e_i \\ -TCP \end{bmatrix}$$

III.III Rotation of the end-effector

$$\begin{bmatrix} X_{S03} \\ Y_{S03} \\ Z_{S03} \end{bmatrix} = \begin{bmatrix} cos\psi & -sin\psi & 0 \\ sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix} \cdot \begin{bmatrix} cos\varphi & 0 & sin\varphi \\ 0 & 1 & 0 \\ -sin\varphi & 0 & cos\varphi \end{bmatrix} \cdot \begin{bmatrix} cos\,\gamma_{Pi} \cdot r_p - \sin\gamma_{Pi} \cdot e_i \\ sin\,\gamma_{Pi} \cdot r_p + \cos\gamma_{Pi} \cdot e_i \\ -TCP \end{bmatrix}$$

$$= \begin{bmatrix} cos\psi & -sin\psi\, cos\theta & -sin\psi \cdot -sin\theta \\ sin\psi & cos\psi\, cos\theta & cos\psi \cdot -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix} \cdot \begin{bmatrix} cos\varphi & 0 & sin\varphi \\ 0 & 1 & 0 \\ -sin\varphi & 0 & cos\varphi \end{bmatrix} \cdot \begin{bmatrix} cos\,\gamma_{Pi} \cdot r_p - \sin\gamma_{Pi} \cdot e_p \\ sin\,\gamma_{Pi} \cdot r_p + \cos\gamma_{Pi} \cdot e_p \\ -TCP \end{bmatrix}$$

$$= \begin{bmatrix} cos\psi\, cos\varphi - sin\psi\, sin\theta\, sin\varphi & -sin\psi\, cos\theta & cos\psi \cdot sin\varphi + sin\psi\, sin\theta\, cos\varphi \\ sin\psi\, cos\varphi + cos\psi\, sin\theta\, sin\varphi & cos\psi\, cos\theta & sin\psi \cdot sin\varphi - cos\psi\, sin\theta\, cos\varphi \\ -\,cos\theta\, sin\varphi & sin\theta & cos\theta \cdot cos\varphi \end{bmatrix} \cdot \begin{bmatrix} cos\,\gamma_{Pi} \cdot r_p - sin\,\gamma_{Pi} \cdot e_p \\ sin\,\gamma_{Pi} \cdot r_p + cos\,\gamma_{Pi} \cdot e_p \\ -TCP \end{bmatrix}$$

$$= \begin{bmatrix} (cos\psi\, cos\varphi - sin\psi\, sin\theta\, sin\varphi) \cdot \left(cos\,\gamma_{Pi} \cdot r_p - \sin\gamma_{Pi} \cdot e_p\right) - sin\psi \cdot\; cos\theta \cdot \left(sin\,\gamma_{Pi} \cdot r_p + \cos\gamma_{Pi} \cdot e_p\right) + (cos\psi \cdot sin\varphi + sin\psi\, sin\theta\, cos\varphi) \cdot -TCP \\ (sin\psi \cdot cos\varphi + cos\psi \cdot sin\theta \cdot sin\varphi) \cdot \left(cos\,\gamma_{Pi} \cdot r_p - \sin\gamma_{Pi} \cdot e_p\right) + cos\psi \cdot cos\theta \cdot \left(sin\,\gamma_{Pi} \cdot r_p + \cos\gamma_{Pi} \cdot e_p\right) + (sin\psi \cdot sin\varphi - cos\psi\, sin\theta\, cos\varphi) \cdot -TCP \\ -\,cos\theta\, sin\varphi \cdot \left(cos\,\gamma_{Pi} \cdot r_p - \sin\gamma_{Pi} \cdot e_p\right) + sin\theta \cdot \left(sin\,\gamma_{Pi} \cdot r_p + \cos\gamma_{Pi} \cdot e_p\right) + cos\theta \cdot cos\varphi \cdot -TCP \end{bmatrix}$$

III.IV Translate the end-effector

$$\vec{C_i} = \begin{bmatrix} X_{C_i} \\ Y_{C_i} \\ Z_{C_i} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & X_T \\ 0 & 1 & 0 & Y_T \\ 0 & 0 & 1 & Z_T \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} (cos\psi\ cos\varphi - sin\psi\ sin\theta\ sin\varphi) \cdot \left(\cos\gamma_{Pi} \cdot r_p - \sin\gamma_{Pi} \cdot e_p\right) - sin\psi \cdot\ cos\theta \cdot \left(\sin\gamma_{Pi} \cdot r_p + \cos\gamma_{Pi} \cdot e_p\right) + (cos\psi \cdot sin\varphi + sin\psi\ sin\theta\ cos\varphi) \cdot -TCP \\ (sin\psi \cdot cos\varphi + cos\psi \cdot sin\theta \cdot sin\varphi) \cdot \left(\cos\gamma_{Pi} \cdot r_p - \sin\gamma_{Pi} \cdot e_p\right) + cos\psi \cdot cos\theta \cdot \left(\sin\gamma_{Pi} \cdot r_p + \cos\gamma_{Pi} \cdot e_p\right) + (sin\psi \cdot sin\varphi - cos\psi\ sin\theta\ cos\varphi) \cdot -TCP \\ -cos\theta\ sin\varphi \cdot \left(\cos\gamma_{Pi} \cdot r_p - \sin\gamma_{Pi} \cdot e_p\right) + sin\theta \cdot \left(\sin\gamma_{Pi} \cdot r_p + \cos\gamma_{Pi} \cdot e_p\right) + cos\theta \cdot cos\varphi \cdot -TCP \end{bmatrix}$$

$$\begin{bmatrix} X_{C_i} \\ Y_{C_i} \\ Z_{C_i} \\ 1 \end{bmatrix} = \begin{bmatrix} \left((cos\psi\ cos\varphi - sin\psi\ sin\theta\ sin\varphi) \cdot \left(\cos\gamma_{Pi} \cdot r_p - \sin\gamma_{Pi} \cdot e_p\right) - sin\psi \cdot\ cos\theta \cdot \left(\sin\gamma_{Pi} \cdot r_p + \cos\gamma_{Pi} \cdot e_p\right) + (cos\psi \cdot sin\varphi + sin\psi\ sin\theta\ cos\varphi) \cdot -TCP\right) + X_T \\ \left((sin\psi \cdot cos\varphi + cos\psi \cdot sin\theta \cdot sin\varphi) \cdot \left(\cos\gamma_{Pi} \cdot r_p - \sin\gamma_{Pi} \cdot e_p\right) + cos\psi \cdot cos\theta \cdot \left(\sin\gamma_{Pi} \cdot r_p + \cos\gamma_{Pi} \cdot e_p\right) + (sin\psi \cdot sin\varphi - cos\psi\ sin\theta\ cos\varphi) \cdot -TCP\right) + Y_T \\ \left(-cos\theta\ sin\varphi \cdot \left(\cos\gamma_{Pi} \cdot r_p - \sin\gamma_{Pi} \cdot e_p\right) + sin\theta \cdot \left(\sin\gamma_{Pi} \cdot r_p + \cos\gamma_{Pi} \cdot e_p\right) + cos\theta \cdot cos\varphi \cdot -TCP\right) + Z_T \end{bmatrix}$$

IV. Calculations basis parallel design

IV.I Upper arm

$$\begin{bmatrix} X_{S04} \\ Y_{S04} \\ Z_{S04} \end{bmatrix} = \begin{bmatrix} U_{arm} \\ e_B \\ 0 \end{bmatrix}$$

IV.II Rotation upper-arm (motor angle)

$$\begin{bmatrix} X_{S05} \\ Y_{S05} \\ Z_{S05} \end{bmatrix} = \begin{bmatrix} cos\ \alpha_i & 0 & sin\ \alpha_i \\ 0 & 1 & 0 \\ -sin\ \alpha_i & 0 & cos\ \alpha_i \end{bmatrix} \cdot \begin{bmatrix} U_{arm} \\ e_B \\ 0 \end{bmatrix} = \begin{bmatrix} cos\ \alpha_i \cdot U_{arm} \\ e_B \\ -sin\ \alpha_i \cdot U_{arm} \end{bmatrix}$$

IV.III Translation upper ball joint

$$\begin{bmatrix} X_{S05} \\ Y_{S05} \\ Z_{S05} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & r_b \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} cos\ \alpha_i \cdot U_{arm} \\ e_B \\ -sin\ \alpha_i \cdot U_{arm} \\ 1 \end{bmatrix} = \begin{bmatrix} cos\ \alpha_i \cdot U_{arm} + r_b \\ e_B \\ -sin\ \alpha_i \cdot U_{arm} \\ 1 \end{bmatrix}$$

IV.IV Specify motor

$$\overrightarrow{B_{\imath\ des\imath gn\ I}} = \begin{bmatrix} cos\ \gamma_{Bi} & -\sin \gamma_{Bi} & 0 \\ sin\ \gamma_{Bi} & \cos \gamma_{Bi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} cos\ \alpha_i \cdot U_{arm} + r_b \\ e_B \\ -sin\ \alpha_i \cdot U_{arm} \end{bmatrix} = \begin{bmatrix} cos\ \gamma_{Bi} \cdot (cos\ \alpha_i \cdot U_{arm} + r_b) - \sin \gamma_{Bi} \cdot e_B \\ sin\ \gamma_{Bi} \cdot (cos\ \alpha_i \cdot U_{arm} + r_b) + \cos \gamma_{Bi} \cdot e_B \\ -sin\ \alpha_i \cdot U_{arm} \end{bmatrix}$$

V. Calculations basis perpendicular design

V.I Upper arms

$$\begin{bmatrix} X_{S11} \\ Y_{S11} \\ Z_{S11} \end{bmatrix} = \begin{bmatrix} 0 \\ -U_{arm} \\ 0 \end{bmatrix}$$

V.II Rotation upper arms (Motor angle)

$$\begin{bmatrix} X_{S12} \\ Y_{S12} \\ Z_{S12} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\alpha & -sin\alpha \\ 0 & sin\alpha & cos\alpha \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -U_{arm} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ cos\alpha \cdot -U_{arm} \\ sin\alpha \cdot -U_{arm} \end{bmatrix}$$

V.III Translation upper arm

$$\begin{bmatrix} X_{S13} \\ Y_{S13} \\ Z_{S13} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & r_b \\ 0 & 1 & 0 & r_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ cos\alpha \cdot -U_{arm} \\ sin\alpha \cdot -U_{arm} \\ 1 \end{bmatrix} = \begin{bmatrix} r_b \\ cos\alpha \cdot -U_{arm} + r_y \\ sin\alpha \cdot -U_{arm} \\ 1 \end{bmatrix}$$

V.IV Specify motor

$$\overrightarrow{B_{\imath\ des\imath gn\ II}} = \begin{bmatrix} X_{S14} \\ Y_{S14} \\ Z_{S14} \end{bmatrix} = \begin{bmatrix} cos\ \gamma_{Bi} & -\sin\gamma_{Bi} & 0 \\ sin\ \gamma_{Bi} & \cos\gamma_{Bi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_b \\ (cos\alpha \cdot -U_{arm}) + r_y \\ sin\alpha \cdot -U_{arm} \end{bmatrix} = \begin{bmatrix} cos\ \gamma_{Bi} \cdot r_b - \sin\gamma_{Bi} \cdot \left(cos\alpha \cdot -U_{arm} + r_y\right) \\ sin\ \gamma_{Bi} \cdot r_b + \cos\gamma_{Bi} \cdot \left(cos\alpha \cdot -U_{arm} + r_y\right) \\ sin\alpha \cdot -U_{arm} \end{bmatrix}$$

VI. Optimization

VI.I Parallel design

L_{arm}	X	Y	Z	θ	φ	ψ
250	43	0	-323,17	7,5	7,5	7,5
250	0	35	-323,17	7,5	7,5	7,5
300	75	45	-323,17	7,5	7,5	7,5
300	57	75	-323,17	7,5	7,5	7,5
300	-75	44	-323,17	7,5	7,5	7,5
300	45	-75	-323,17	7,5	7,5	7,5
300	-75	-24	-323,17	7,5	7,5	7,5
300	-45	-75	-323,17	7,5	7,5	7,5
300	75	-23	-323,17	7,5	7,5	7,5
350	-75	72	-323,17	7,5	7,5	7,5
350	60	-75	-323,17	7,5	7,5	7,5
350	75	72	-323,17	7,5	7,5	7,5
350	73	75	-323,17	7,5	7,5	7,5
350	-75	-51	-323,17	7,5	7,5	7,5
350	-61	-75	-323,17	7,5	7,5	7,5
350	75	-50	-323,17	7,5	7,5	7,5
350	-73	75	-323,17	7,5	7,5	7,5
	Reach of the platform at a positive or negative Z-domain (+50/-50)					
350	-73	75	**-373,17**	7,5	7,5	7,5
350	75	-50	**-373,17**	7,5	7,5	7,5
350	75	**0**	**-273,17**	7,5	7,5	7,5
350	**0**	75	**-273,17**	7,5	7,5	7,5

VI.II Perpendicular design

L_{arm}	X	Y	Z	θ	φ	ψ
250*	75	0	-323,17	7,5	7,5	7,5
250*	0	75	-323,17	7,5	7,5	7,5
300*	0	75	-323,17	7,5	7,5	7,5
300*	75	0	-323,17	7,5	7,5	7,5
350*	75	0	-323,17	7,5	7,5	7,5
350*	0	75	-323,17	7,5	7,5	7,5
	Changed Z-domain					
250*	0	75	-373,17	7,5	7,5	7,5
250*	75	0	-373,17	7,5	7,5	7,5
300*	75	0	-373,17	7,5	7,5	7,5

*300**	0	75	-373,17	7,5	7,5	7,5
*350**	0	75	-373,17	7,5	7,5	7,5
*350**	75	0	-373,17	7,5	7,5	7,5
	Changed Z-domain					
*250**	0	75	-423,17	7,5	7,5	7,5
*250**	75	0	-423,17	7,5	7,5	7,5
*300**	75	0	-423,17	7,5	7,5	7,5
*300**	0	75	-423,17	7,5	7,5	7,5
*350**	0	75	-423,17	7,5	7,5	7,5
*350**	75	0	-423,17	7,5	7,5	7,5

- These results are not possible with the given parameters.