PyFiberAmp theory basics

Joona Rissanen

March 30, 2018

1 Giles model

The original Giles model [1] describes an active fiber as a two-level system using four spectroscopic parameters:

1. Absorption spectrum α (m^{-1} vs. wavelength)

2. Gain spectrum g^* (m^{-1})

3. Saturation parameter ζ ($m^{-1}s-1$)

4. Linear loss l (m^{-1})

The absorption and gain spectra are defined as

$$\alpha(\lambda) = \sigma_a(\lambda)\Gamma(\lambda)n_t$$
$$g^*(\lambda) = \sigma_e(\lambda)\Gamma(\lambda)n_t$$

where $\sigma_a(\lambda)$ and $\sigma_e(\lambda)$ are the absorption and emission cross sections, respectively, $\Gamma(\lambda)$ is the overlap integral between the optical beam and the doped core, and n_t is the number density of the dopant ions. The absorption spectrum refers to the small signal absorption with all the dopant ions in the lower manifold. The gain spectrum refers to the small signal gain with all the ions in the upper manifold.

The saturation parameter is defined as

$$\zeta = \pi b_{eff}^2 n_t/\tau$$

where b_{eff} is the effective doped radius (equals to core radius in a uniformly doped fiber), and τ is the upper state lifetime of the ions. The loss l can typically be assumed to be constant within the limited bandwidth of simulated wavelengths.

The optical power of the kth beam P_k and the fraction of ions in the excited state $\frac{\bar{n}_2}{\bar{n}_t}$ (the overline denotes spatial averaging) are then described by the following system of equations

$$\frac{\bar{n}_2}{\bar{n}_t} = \frac{\sum_k \dfrac{P_k(z)\alpha_k}{h\nu_k\zeta}}{1 + \sum_k \dfrac{P_k(z)(\alpha_k + g_k^*)}{h\nu_k\zeta}}$$

$$\frac{dP_k}{dz} = u_k(\alpha_k + g_k^*)\frac{\bar{n}_2}{\bar{n}_t}P_k(z) + u_k g_k^* \frac{\bar{n}_2}{\bar{n}_t}mh\nu_k\Delta\nu_k - u_k(\alpha_k + l_k)P_k$$

where $h\nu_k$ is the photon energy at frequency ν_k, u_k is the propagation direction of the beam (forward=+1 and backward=-1), $m = 2$ is the number of modes in a single-mode fiber (two polarization modes) and $\Delta\nu_k$ is the frequency bandwidth of the beam (can be assumed zero for CW signals and pumps and non-zero for amplified spontaneous emission and Raman).

The Giles model is a 1D boundary value problem. In the simplest case, the boundary values are the input powers of the beams. If only forward-propagating beams are considered, the problem is reduced to an easily integrable initial value problem, but typically both backward-propagating ASE and pump beams must be modeled, so some input powers are know only at the end of the fiber and some at the beginning. PyFiberAmp uses the *solve_bvp* solver from SciPy. The solver typically converges rapidly provided that the initial guess is chosen well.

The Giles model suffers from several limitations, the most important of which are listed below:

- As a two-level model, it cannot describe complex transitions such as excited state absorption.

- As a steady-state model, it cannot describe pulse propagation at low repetition rates, when the gain varies from pulse to pulse.

- As a spatially averaged model, it cannot describe radially varying dopant distributions or power-dependent overlap between the optical beams and the dopant.

2 Raman model

Stimulated and spontaneous Raman scattering is simulated with the following rate equations (adapted from [2]):

$$\frac{dP_R^\pm}{dz} = \mp(g_R I_s P_R^\pm + g_R I_s h\nu_R \Delta\nu_R)$$

where the first term on the right hand side describes stimulated Raman scattering and the second term describes spontaneous Raman scattering, g_R is the Raman gain coefficient (about $1 \times 10^{-13} m/W$), $I_s = \frac{P_s}{A_{eff}}$ is the peak intensity of the signal, P_s is the peak intensity and A_{eff} is the nonlinear effective area. The subsripts s and R refer to the signal and Raman channels, respectively. The derivative of backward-propagating Raman beam is negative.

The signal is depleted by the amount of power gained by the Raman beams multiplied by the ratio of photon frequencies (energies) of the signal and Raman beams because the number of photons is conserved but the Raman photons have smaller energies than the signal photons.

$$\frac{dP_s}{dz} = -\frac{\nu_s}{\nu_R}\left(\frac{dP_R^+}{dz} - \frac{dP_R^-}{dz}\right)$$

In practice, terms on the right hand side of these equations are added to the Giles model rate equations to simulated Raman generation in active fibers, where the Raman signal interacts with the dopant ions as well.

References

[1] C.R. Giles and E. Desurvire, "Modeling erbium-doped fiber amplifiers," in Journal of Lightwave Technology, vol. 9, no. 2, pp. 271-283, Feb 1991. doi: 10.1109/50.65886
[2] R.G. Smith, "Optical Power Handling Capacity of Low Loss Optical Fibers as Determined by Stimulated Raman and Brillouin Scattering," Appl. Opt. 11, 2489-2494 (1972)