MSc in Financial Markets with Information Systems

Financial Derivatives Coursework ECP019C

Double Barrier Replication

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Scenario

The scope of this paper will be the pricing of an at-the-money, non-resetting, zero rebate, double-barrier call option. Although pricing methodologies of these instruments may differ greatly between traders and modellers (pricing differences between front office and back office quants are an industry-wide acknowledged phenomenon), my intention is to approach the subject in an independent way from the market forces.

Basic theory and its application will be focused on, along with core concepts like improved convergence due to increased number of iterations in the Monte Carlo framework. Advanced practical topics like hedging difficulties around barrier levels and the subsequent liquidity holes will only be briefly touched upon.

(a) **Compare and contrast the pricing of this instrument using 10-branch binomial tree and a Monte Carlo simulation approach where the underlying track is generated on a daily bases.**

The binomial option pricing model was first introduced by Sharpe (1978) and detailed by Cox, Ross and Rubinstein (1979) (CRR). The primary practical use for the binomial model was, and still is, for pricing American-style options. The binomial model which will be analysed here is an at-the-money, non resetting, zero rebate, double barrier call option of non-dividend paying assets. The model assumes that the underling asset price follows a binomial process, that is at any time the asset price can only change to one of two possible values. Under this assumption the asset price has a binomial distribution. This can be valued in the same way as a regular call option except that, when we encounter a note above or below the barriers of 70 and 60 respectively, we set the value of the option equal to zero.
We price a 10 days maturity, at-the-money European double barrier call option with current asset price at 65. The binomial tree has ten steps and up and down proportional jumps of 1.0158 and 0.9844 respectively. The risk-free interest rate is assumed to be 5% per annum, $X = 65$, $T = 10/365 = 0.0274$, $S = 65$, $r = 0.05$, $N = 10$, $\sigma = 0.30$, $u = e^{\sigma\sqrt{\Delta t}} = 1.0158$,
$d = 1/u = 0.9844$

Firstly the constants; $\Delta t(dt)$, p, and $disc$ are precomputed;

$$\Delta t = \frac{T}{N} \times \frac{10}{365} = \frac{1}{10} \times \frac{10}{365} = 0.00274$$

$$p = \frac{e^{r \times \Delta t} - d}{u - d} = \frac{e^{0.05 \times 0.00274} - 0.9844}{1.0158 - 0.9844} = 0.5004$$

p*= 1 – p=1- 0.5004 = 0.4996

$$disc = e^{-r \times \Delta t} = e^{-0.05 \times 0.00274} = 0.9999$$

Then the asset prices are computed, for example the asset prices at nodes 1 and 2 respectively

$$S_{1,1} = S \times u^N = 65 \times 1.0158 = 66.0287$$
$$S_{1,0} = S \times d^N = 65 \times 0.9844 = 63.9872$$
$$S_{2,2} = S_{1,1} \times u^N = 66.0287 \times 1.0158^2 = 67.0737$$
$$S_{2,1} = S_{1,1} \times d^N = 66.0287 \times 0.9844^2 = 65$$
$$S_{2,0} = S_{1,0} \times d^N = 63.9872 \times 0.9844^2 = 62.9903$$

….

and at maturity

$$S_{10,10} = S_{9,9} \times u^N = 74.8668 \times 1.0158^{10} = 76.0517$$
$$S_{10,9} = S_{9,9} \times d^N = 74.8668 \times 0.9844^{10} = 73.7004$$
$$S_{10,8} = S_{9,8} \times d^N = 72.5522 \times 0.9844^{10} = 71.4218$$
$$S_{10,7} = S_{9,7} \times d^N = 70.3090 \times 0.9844^{10} = 69.2136$$
$$S_{10,6} = S_{9,6} \times d^N = 68.1353 \times 0.9844^{10} = 67.0737$$

….

$$S_{10,0} = S_{9,0} \times d^N = 56.4334 \times 0.9844^{10} = 55.5542$$

Next the option value at maturity are computed for nodes (10,8), (10,7), (10,6) respectively

$C_{10,8} = \max(0, S_{10,8} - X) = \max(0,71.4218 - 65) = 0$ as the option is knock-out at barrier of 70
$C_{10,7} = \max(0, S_{10,7} - X) = \max(0,69.2136 - 65) = 4.2136$
$C_{10,6} = \max(0, S_{10,6} - X) = \max(0,67.0737 - 65) = 2.0737$

Finally We perform discounted expectations back through the tree. For node (9,6)

$C_{9,6} = disc \times (p \times C_{10,7} + p^* \times C_{10,6}) = 0.9999 \times (0.5004 \times 4.2136 + 0.4996 \times 2.0737 = 3.1442$
….
And the final result for the option value is

$C_{0,0} = disc \times (p \times C_{1,1} + p^* \times C_{1,0}) = 0.9999 \times (0.5004 \times 0.8692 + 0.4996 \times 0.5595 = 0.71445$

Cox, Ross & Rubinstein Binomial Tree Method to price ATM European Double Barrier Call Option

Stock price (S)		65.00		*dt*	0.0027
Exercise price (X)		65.00		*erdt*	1.0001
Int rate-cont (r)		5.00%		*ermqdt*	1.0001
				u	1.0158
Dividend yield - cont (q)		0.00%		*d*	0.9844
				p	0.5004
upper barrier		70		*p**	0.4996
lower barrier		60		*disc*	0.9999
Time now (0, years)		0.000000			
Time maturity (T, years)		0.027397			
Option life (t, years)		0.027397			
Volatility (s)		30.00%			
dt		0.002740			
Steps in tree (n)		10			

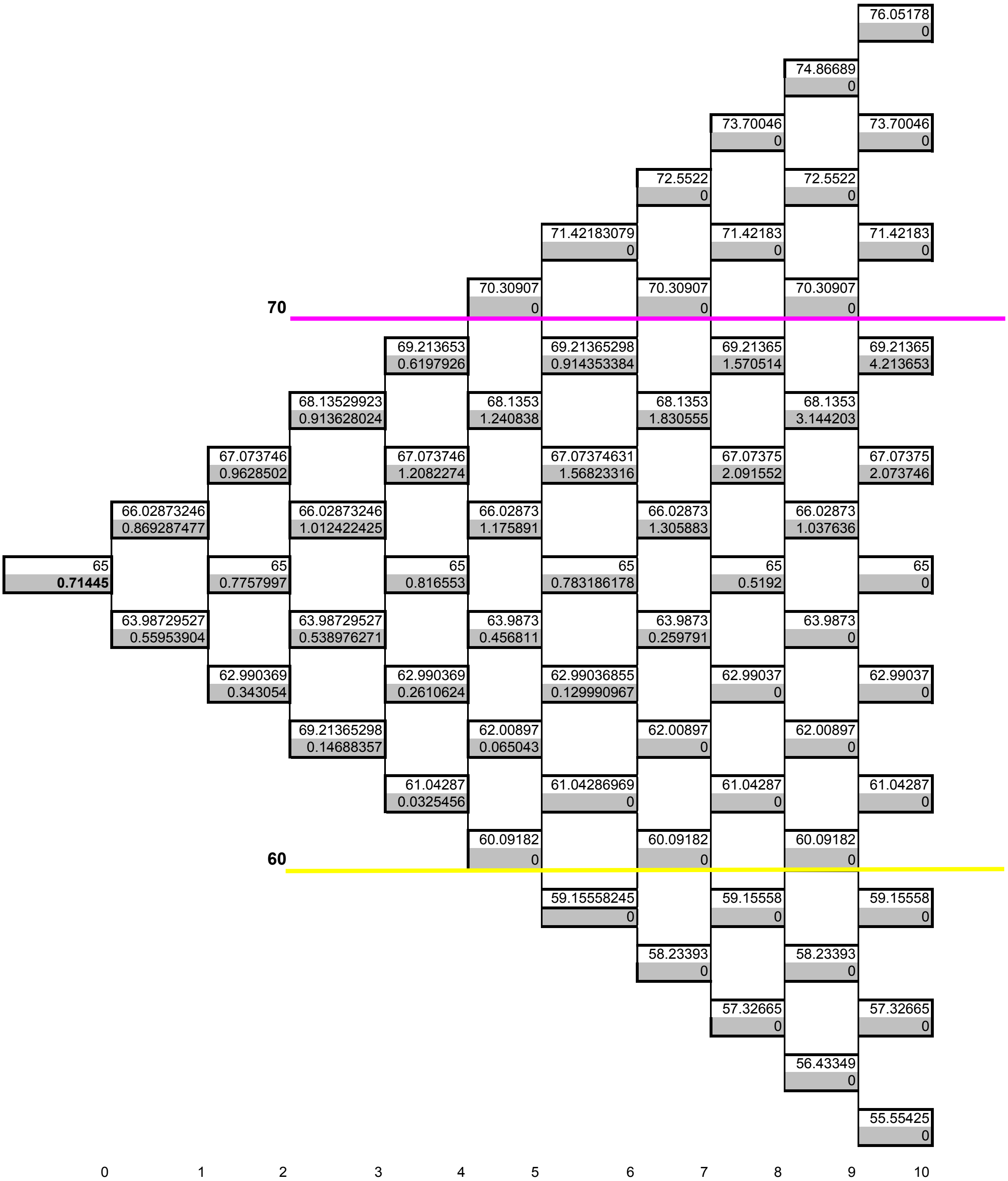
Stock price evalution for 10 step for 10 days

	0	1	2	3	4	5	6	7	8	9	10
10											76.05178
9										74.86689	73.70046
8									73.70046	72.5522	71.42183
7								72.5522	71.42183	70.30907	69.21365
6							71.42183079	70.30907	69.21365	68.1353	67.07375
5						70.30907	69.21365298	68.1353	67.07375	66.02873	65
4					69.213653	68.1353	67.07374631	66.02873	65	63.9873	62.99037
3				68.13529923	67.073746	66.02873	65	63.9873	62.99037	62.00897	61.04287
2			67.073746	66.02873246	65	63.9873	62.99036855	62.00897	61.04287	60.09182	59.15558
1		66.02873246	65	63.98729527	62.990369	62.00897	61.04286969	60.09182	59.15558	58.23393	57.32665
0	65.00	63.98729527	62.990369	62.00897402	61.04287	60.09182	59.15558245	58.23393	57.32665	56.43349	55.55425

Option Price discounted back for each step

	0	1	2	3	4	5	6	7	8	9	10
10											0
9										0	0
8									0	0	0
7								0	0	0	4.213653
6							0	0	1.570514	3.144203	2.073746
5						0	0.914353384	1.830555	2.091552	1.037636	0
4					0.6197926	1.240838	1.56823316	1.305883	0.5192	0	0
3				0.913628024	1.2082274	1.175891	0.783186178	0.259791	0	0	0
2			0.9628502	1.012422425	0.816553	0.456811	0.129990967	0	0	0	0
1		0.869287477	0.7757997	0.538976271	0.2610624	0.065043	0	0	0	0	0
0	**0.71445**	0.55953904	0.343054	0.14688357	0.0325456	0	0	0	0	0	0

Binomial tree used to value a stock option



Unfortunately, convergence is very slow when this approach is used. A large number of time steps are required to obtain a reasonably accurate result. The reason for this is that the barriers being assumed by the tree are different from the true barriers. For overcoming this problem we should use a trinomial tree rather than a binomial tree, which is beyond the scope of this paper.

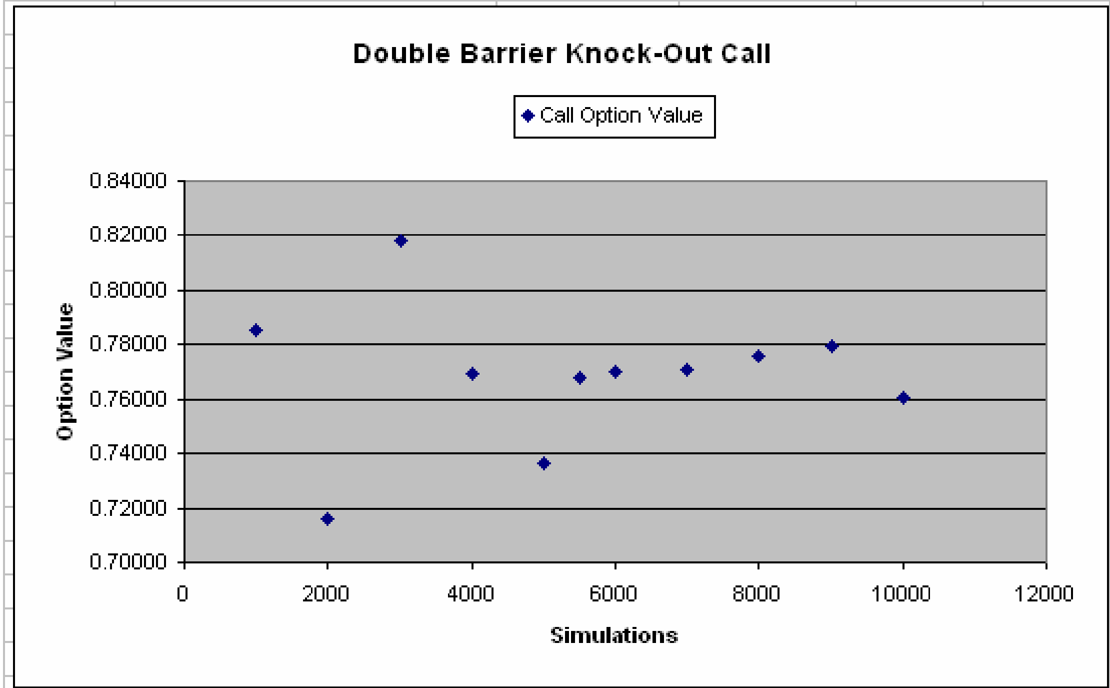
An important application of Monte Carlo simulation is in pricing complex or exotic path-dependent options. Simple analytical formulae exist for certain types of exotic options, these options being classified by the property that the path-dependent condition applies to continuous path. For example, a popular class of exotic option is the double barrier call option which has been used in this case below.

Monte Carlo Simulation Method to price ATM European Double Barrier Call Option

Share price (S)	65.00	
Exercise price (X)	65.00	
Int rate-cont (r)	5.00%	
Dividend yield (q)	0.00%	
Time now (0, years)	0.0000	
Time maturity (T, years)	0.0274	10 days
Option life (τ, years)	0.0274	
Volatility (σ)	30.00%	
Adjusted Standard Deviation	0.01570	
Daily Return	0.00014	
Hbarrier (H)	70.00	
Lbarrier (L)	60.00	
call or put (iopt)	1	
step (t-step)	10	every day
time step (dt)	0.00274	1 day in years
Exp(-r τ)	0.9986	

Run MC

Number of Simulations	*Option Value*	
1000	0.78531	
2000	0.71640	
3000	0.81815	
4000	0.76920	
5000	0.73617	
6000	0.77029	
7000	0.77088	
8000	0.77568	
9000	0.77928	
10000	0.76025	
5500	**0.76816**	Average Value



These are standard European options except where the option ceases to exist if the underlying asset price is checked continuously for the crossing of the barriers, then simple formula has been used for the price of these options. In contrast, with double barrier options the underlying asset price is checked (fixed) at most once a day and often much less frequently. This significantly affects the price of the option since the price is much less likely to be observed crossing the barriers if the fixings occur infrequently, and it also complicates the pricing formulae. Monte Carlo simulation is a good method for pricing these options.

We consider pricing a ten days fixed down-and-out and up-and-out call option. This is a particular type of double barrier option which is a normal call option unless the underlying asset price observed during these ten days crosses the predetermined barriers level L and H respectively, in which case the option ceases to exist. For this option we must simulate the underlying asset price for each fixing date in order to check for the crossing of the barriers. Assuming the asset price follows GBM (Geometric Brownian Motion), the simulation of the asset price takes the usual form:

$$S_{t+\Delta t} = S_t \exp(v\Delta t + \sigma\sqrt{\Delta t z})$$

Where we assume Δt is one day and z is a standard normal random variable as usual. Monte Carlo simulation proceeds in exactly the same way as for a standard option, except that at each time step we check whether the asset price has crossed the barriers level L and H respectively. If so then we terminate the simulation of that path and the pay-off for that path is determined to be zero.

We price an at-the-money, non-resetting, zero rebate, double barrier call option with the current asset price at 65 and volatility of 30%. The risk-free interest rate is assumed to be at 5% per annum, the asset pays zero dividend yield per annum, and an upper knock-out barrier of 70 and a lower knock-out barrier of 60. The simulation has 10 time steps and run for 1000 to 10000 simulations; $X = 65$, $T = 10/365 = 0.0274$, $S = 65$, $r = 0.05$, $N = 10$, $\sigma = 0.30$, $L = 60$, $H = 70$, $\delta = 0$, *for the example we use* $M = 1000$ the above figure illustrates the numerical results for the simulation of the path for $j = 1000$.

Firstly, the constants; Δt *(dt)*, $v\Delta t$ *(nudt)*, $\sigma\sqrt{\Delta t}\,(sigsdt)$ are precomputed;

$$\Delta t = \frac{T}{N} \times \frac{10}{365} = \frac{1}{10} \times \frac{10}{365} = 0.00274$$

$$nudt = (r - \delta - \frac{1}{2}\sigma^2)\Delta t = (0.05 - 0 - 0.5 \times 0.3^2) \times 0.00274 = 0.0000137$$

$$sigsdt = \sigma\sqrt{\Delta t} = 0.3\sqrt{0.00274} = 0.0157$$

Then for each simulation j=1 to M = 1000, S_t is initialised to S = 65 and BARRIER_CROSSED = FALSE which indicates that the barrier has not yet been crossed. Then for each time step i = 1 to N = 10, S_t is simulated and the crossing of the barriers is checked. For example for j = 1000 and i = 1 we have

$$S_t = S_t \times \exp(nudt + sigsdt \times \varepsilon) = 65 \times \exp(0.0000137 + 0.0157 \times 0.58384) = 65.5994$$

$L < S_t < H$, therefore BARRIER_CROSSED is FALSE and the loop continues. For i = 6 we have

$$S_t = S_t \times \exp(nudt + sigsdt \times \varepsilon) = 65 \times \exp(0.0000137 + 0.0157 \times -5.8384) = 59.3076$$

$S_t < L < H$, therefore BARRIER_CROSSED = TRUE and the loop terminates.

For $j = 1000$ we have BARRIER_CROSSED = TRUE therefore: $C_t = 0$

The sum of the values of C_t = max(0,St-X) = max(0,65.5994-65) = 0.5994 or C_t = max(0,St-X) = max(0,62.6789-65) = 0 are accumulated in *sum_CT*. The estimate of the option value is the given by

$$call\_value = \frac{sum\_CT}{M} \times \exp(-r \times T) = \frac{786.4009}{1000} \times \exp(-0.05 \times 0.0274) = 0.78579 \times 0.9986 = 0.7853$$

an extract part from the Monte Carlo VBA code

```
rnmut = (r - q - 0.5 * sigma ^ 2) * dt
sigt = sigma * Sqr(dt)
sum = 0
try_no = sim10000
k = 0

' for each simulation do this
For i = 1 To try_no
   St = S
   barrier = 0
   payoff = 0

   'for each step do the following
   For j = 1 To step
      'randns = Application.NormInv(Rnd(), dyr, adjsd)
      randns = Application.NormSInv(Rnd)
      'share price for call option
      St = St * Exp(rnmut + randns * sigt)
      'payoff for option
      If (St >= H) Then
         barrier = 1
         Exit For
      End If
      If (St <= L) Then
         barrier = 1
         Exit For
      End If
    Next j

   If barrier = 1 Then
      payoff = 0
   Else
      payoff = Application.Max(iopt * (St - X), 0)
   End If

   sum = sum + payoff
   k = k + 1
   If i = sim1000 Then

      If k = 1000 Then
      opv1000 = Exp(-r * tyr) * sum / sim1000
      i = 0
      sum = 0
      End If
   End If
```

We run the Monte Carlo simulation 10 times for 1000 to 10000 simulations and get the average option value of 0.76816 which is slightly different from the binomial tree option value of 0.71445, we also observe a option value outfit of 0.71640 at 2000 simulations which is very close to the binomial tree option value.
Monte Carlo simulation is well suited to valuing path-dependent options and options with many stochastic variables. This has its disadvantages such as lack of speed and the disability to handle American-style derivatives easily, but there are two ways it can be adapted to handle them. The first involves a least-square analysis to relate the value of continuing (i.e. not exercising) to the values of relevant variables. The second involves parametrizing the early exercise boundary and determining it iteratively by working back from the end of the option towards the beginning.
Trees can be used to value many types of barrier options, but the convergence of the option value to the correct value as the number of time steps is increased tends to be slow. One approach for improving convergence is to arrange the geometry of the tree so that nodes always lie on the barriers. Another is to use an interpolation schema to adjust for the fact that the barrier being assumed by the tree is different from the true barrier. A third is to design the tree so that it provides a finer representation of movements in the underlying asset price near the barrier.

(b) **What rationale would justify the use of this type of exotic option strategy by the client.**

Barrier options are one of the most popular types of exotic options traded over-the-counter on stocks, stock indices, foreign currencies, commodities and interest rates. In addition to the popularity of OTCs, several types of barrier options are traded on securities exchanges. Examples of exchange-traded barrier options is double-barrier knock-out call warrants on the Australian stock index, Ordinaries Index, introduced in 1998 by the Australian Stock Exchange (ASE).

There are three primary reasons to use double-barrier knock-out call options rather than standard call options. Firstly, double-barrier knock-out call options may more closely match investor beliefs about the future behaviour of the asset. By buying a double-barrier knock-out call options, one can eliminate paying for those scenarios one feels are unlikely. Secondly, double-barrier knock-out call options premiums are generally lower than those of standard call options since an additional condition has to be met for the option holder to receive the payoff (e.g., the lower and the upper barriers not reached for double-barrier knock-out call options). The premium discount afforded by the barriers provision can be substantial, especially when volatility is high. Thirdly, double-barrier knock-out call options may match hedging needs more closely in certain situations. We can envision a situation where the hedger does not need his option to hedge any longer if the asset crosses a certain barrier level.

Despite their popularity, double-barrier knock-out call options contract designs have a number of disadvantages affecting both option buyers and sellers. The double-barrier knock-out call options buyer stands to lose his entire option position due to a short-term price spike through the barriers. Moreover, an obvious conflict of interest exists between barrier option dealers and their clients, leading to the possibility of short-term market manipulation and increased volatility around popular barrier levels (see Hsu (1997) and Taleb (1997) for illuminating discussions of "barrier event" risk and "liquidity holes" in the currency markets). At the same time, the option seller has to cope with serious hedging difficulties near the barriers. The hedging problems with the double-barrier knock-out call options are well documented in the literature and are discussed in Section (c) of this paper.

(c) **Describe briefly the hedging that the writer of the option could expect to face.**

Consider an option hedger who sold a standard European call. Assume the standard perfect markets assumptions hold and the underlying asset follows geometric Brownian motion. Then, according to the Black and Scholes (1973) argument, the hedger should execute a dynamic delta-hedging strategy by continuously trading in the underlying risky asset and the risk-free bond. The balance of the hedger's trading account will perfectly offset the liability on the short option position for all terminal values of the underlying price. For a standard European call, the hedge ratio or delta $\Delta = \Delta(S, t)$ is a continuous function of S and always lies inside the interval [0, 1].

The situation is more complicated for barrier options. First, the barrier option delta is discontinuous at the barrier for all times to maturity. For a double-barrier knock-out call, it is positive near the lower barrier and negative near the upper barrier. To hedge close to the upper barrier, the hedger needs to take a short position in the underlying asset. As the underlying goes up and the barrier is crossed, the entire short hedge position has to be liquidated at once on a stop-loss order. The execution of a large stop-loss order has a risk of "slippage" (buying back the underlying asset at a price greater than the barrier level). This adds to the cost of hedging barrier options and is reflected in wider bid–ask spreads for these OTC contracts. Moreover, in the currency markets, it sometimes happens that many barrier option positions with closely placed barriers exist in the market ("densely mined market"). Consequently, all barrier option writers place their stop-loss buy orders around the same barrier levels to cover their short hedges. When the market rallies through the barrier, all stop-loss orders get triggered and, due to liquidity limitations, this results in a further rally ("liquidity hole") and poor execution ("slippage"). See Taleb (1997) for a detailed discussion of these phenomena of "mined markets" and "liquidity holes".

Second, a potentially more serious problem with barrier options that knock out in-the-money (*reverse knock-out options*), such as double-barrier knock-out call options, is that their delta is *unbounded* as expiration approaches and the underlying price nears the barrier. For up-and-out and double barrier calls, delta tends to $-\infty$ as $S \rightarrow U$ and $t \rightarrow T$, and the hedger is forced to take progressively larger short positions in the underlying. In practice, arbitrarily large short positions are not acceptable, and the Black–Scholes hedging argument essentially breaks down for reverse knock-out options.

Furthermore, consider what happens if there are any imperfections, such as discrete rather than continuous trading, transaction costs, or volatility misspecification, that introduce hedging errors. The hedging errors can explode for those sample paths that are near the barrier around expiration. For example, even small, but positive, proportional transaction costs applied to progressively larger positions will accumulate to increasingly large amounts.

Another method of hedging is the static hedging (Carr, Ellis and Gupta, 1997; Derman, Ergener and Kani, 1995) of barrier options with portfolios of vanilla calls and puts is an alternative to dynamic delta-hedging.

Consider a call option that has two barriers, so that the call knocks out if either barrier is hit. We assume that the initial forward price and strike are both between the two barriers of L=60 and H=70 respectively. There is a parity relation between this double knock-out call (O^2C) and a double knock in call (I^2C), which knocks in if either barrier is hit:

$$O^2C(X,L,H) = C(X) - I^2C(X,L,H),$$

where X is the strike, L is the lower barrier, and H is the higher barrier. We will again focus on replicating the payoffs of a double knock out call using static portfolios of standard options.

On its surface, a double knock out call $O^2C(X,L,H)$ appears to be a combination of a down-and-out call option $DOC(X,L)$ and an up-and-out call option $UOC(X,H)$. The payoff of the $O^2C(X,L,H)$ is zero if either barrier is hit and the standard call payoff at expiry if neither barrier is hit. A portfolio of a call knocking out at the lower barrier and a call knocking out at the higher barrier would give these payoffs, so long as the knock out of one option also knocked out the other. This additional specification is necessary as otherwise the surviving option contributes value at the other's barrier.

To construct the replicating portfolio for the $O^2C(X,L,H)$, we begin by purchasing a standard call $C(X)$ to provide the desired payoff at expiry. We will then attempt to zero out the value at each barrier separately. If we knew in advance that the forward price reaches the lower barrier L before it reaches the higher barrier H, then our analysis of a down- and-out call implies that the value of the call $C(X)$ can be nullified along the barrier L by initially selling XL^{-1} , puts struck at L^2X^{-1} . Thus, the replicating portfolio under this assumption would be:

$$O^2C(X,L,H) \approx C(X) - XL^{-1} P(L^2X^{-1}),. \qquad (1)$$

Alternatively, if we knew in advance that the forward price reaches the higher barrier H first, then from equation

$$UOC(X,H) = C(X) – XH^{-1}C(H^2X^{-1}) – (H – X)[2BC(H) + H^{-1} C(H)], H>X,F \qquad (2)$$

UOC – an up-and-out call
BC – a binary call

the replicating portfolio would instead be:

$$O^2C(X,L,H) \approx C(X) - XH^{-1}C(H^2X^{-1})-(H-K)[2BC(H)+H^{-1}C(H)], \qquad (3)$$

Because we don't know in advance which barrier will be hit first, we try combining the two portfolios:

$$\begin{aligned} O^2C(X,L,H) &\approx C(X) - DIC(X,L) - UIC(X,H) \\ &= C(X) — XL^{-1}P(L^2X^{-1}) – XH^{-1}C(H^2X^{-1}) - (H - X)[2BC(H) + H^{-1}C(H)], \end{aligned} \qquad (4)$$

The problem with this portfolio is that each written-in call contributes (negative) value at the other's barrier. For example, if the forward price reaches H first, then the down-and-in call option $DIC(X,L) = XL^{-1}P(L^2X^{-1})$ contributes (negative) value along H. Thus, we need to add securities to the portfolio in an effort to zero out value along each barrier. Along the barrier H,the negative influence of the XL^{-1} puts struck at L^2X^{-1} can be offset by buying LH^{-1} calls struck at H^2XL^{-2}. To cancel the negative influence of the $UIC(X,H)$ along the barrier L, we will need to extend Put-Call Symmetry *(PCS)* to binary calls.

For an example a binary call (put) is a cash-or-nothing option that pays \$1 if the stock price is above (below) a strike price X, and zero otherwise. Similarly, a gap call (put) is an asset-or-nothing option that pays the stock price \$65 if it is above (below) a strike price X, and zero otherwise. The following parity relations are easily shown:

$$GC(X) = X.BC(X) + C(X), \qquad GP(X) = X.BP(X) - P(X).$$

Since binary options may be synthesized out of standard options, these parity relations imply that the same is true for gap options.

BINARY PUT-CALL Symmetry: *Given frictionless markets, no arbitrage, zero drift, and deterministic volatility, the following relationships hold:*

$$X^{1/2}BC(X) = GP(H)H^{-1/2} \qquad H^{1/2}BP(H) = GC(X)\, X^{-1/2}, \tag{5}$$

where the geometric mean of the binary call strike X and the binary put strike H is the forward price F:

$$(XH)^{1/2} = F.$$

With this result, we can cancel the negative influence of the *UIC(X,H)* in equation (4) along the barrier *L*. Thus, our first layer approximation for the double knock out call value is:

$$\begin{aligned} O^2C(X,L,H) = {} & C(X) - L^{-1}(XP(L^2X^{-1}) - HP(L^2XH^{-2})) - H^{-1}(XC(H^2X^{-1}) - LC(H^2KL^{-2})) \\ & -(H- X)[2BC(H) + H^{-1}C(H) - 2L^{-1}GP(L^2H) - L^{-1}P(L^2H^{-1})]. \end{aligned} \tag{6}$$

Although equation (6) is a better approximation than equation (4), the added options still contribute value at the other's barrier. Thus, we need to continue to subtract or add options, noting that each additional layer of hedge at one barrier creates an error at the other barrier. As a result, the replicating portfolio for a double knock-out call can be written as an infinite sum:

$$\begin{aligned} O^2C(X,L,H) = {} & C(X) - \sum_{n=0}^{\infty}[L^{-1}(HL^{-1})^n(XP(L^2X^{-1}(LH^{-1})^{2n}) - HP(X(LH^{-1})^{2(n+1)})) \\ & + H^{-1}(LH^{-1})^n(XC(H^2X^{-1}(HL^{-1})^{2n}) - LC(X(HL^{-1})^{2(n+1)})) + 2(H-X)(HL^{-1})^n \\ & \times [BC(CH(HL^{-1})^{2n}) - L^{-1}GP(L(LH^{-1})^{2n+1})] \\ & + (H- X)[H^{-1}(LH^{-1})^n\, C(H(HL^{-1})^{2n}) - L^{-1}(HL^{-1})^n\, P(L(LH^{-1})^{2n+1})]] \end{aligned} \tag{7}$$

Note that the options in the infinite sum are all initially out-of-the-money. Furthermore, as n increases, the number of options held and the options' moneyness both decrease exponentially. As a result, for large n, the options' contribution to the infinite sum becomes minimal.
In general, bringing in the barriers of a double knock out call reduces both its value and the number of options needed to achieve a given accuracy.

Toft and Xuan (1998) study the performance of static hedging for up-and-out calls. They study static hedges proposed by Derman, Ergener and Kani (1995) that replicate an up-and-out call with a portfolio of vanilla calls of different maturities. They find that performance of static hedges, as with dynamic hedging schemes, is extremely sensitive to the discontinuity in payoff on the barrier. They conclude: "In general, options with large discontinuous payoffs on the barrier are very difficult to delta hedge because the option's delta changes rapidly when the barrier is hit. It appears that a static hedge performs poorly in exactly the same situations as those where delta hedges are inadequate." Thus, reverse knock-out options with "hard" barriers are difficult to hedge not only dynamically, but statically as well.

I conclude this section with the quote from Tompkins (1997): "Therefore, it is not surprising that these products tend to trade in the Over the Counter market at prices which are approximately double the theoretical price.

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