1. 沿下列路线计算 $\int_0^{3+i} z^2 dz$

(1) 自原点至 $3+i$ 直线段.

$x=3t$, $y=t$.

原式 $= \int_0^1 (3t+it)^2 d(3t+it)$

$= (3+i)^3 \times \frac{1}{3} = 6+\frac{26}{3}i$

(2) 原式 $= \int_0^1 (3t)^2 d(3t) + i\int_0^1 (3+it)^2 d(3+it) = 6+\frac{26}{3}i$

(3). 原式 $= \int_0^1 (it)^2 d(it) + \int_0^3 (t+i)^2 d(t+i) = 6+\frac{26}{3}i$.

2. $y=x$: 原式 $= \int_0^1 (t^2+it)\, d(t+it)$.

$= (1+i)\left(\int_0^1 t^2 dt + i\int_0^1 t\,dt\right)$

$= (1+i)\left(\frac{1}{3}+\frac{1}{2}i\right)$

$= -\frac{1}{6}+\frac{5}{6}i$

$y=x^2$: 原式 $= \int_0^1 (t^2+it^2)\, d(t^2+it^2) = -\frac{1}{6}+\frac{5}{6}i$

4. $\oint_C \bar{z}\, dz \oint_C \frac{1}{z} dz = \oint_C \frac{1}{e^{i\theta}} de^{i\theta} = \oint_C i\, d\theta = 2\pi i$

$z=e^{i\theta}$

6. (2). $z^2+2z+4 \neq 0$ ∴ 原函数解析 ∴ $\oint_C \frac{dz}{z^2+2z+4} = 0$.

(4). $\oint_C \frac{dz}{z-\frac{i}{2}} = 2\pi i$

(6). 原式 $= \oint_C \frac{\frac{1}{z+2}}{z-\frac{i}{2}} dz = 2\pi i \cdot \frac{1}{z+2}\Big|_{z=\frac{i}{2}} = \frac{4\pi i}{4+i}$

7. (1) $\oint_C \frac{e^z}{z-2} dz = 2\pi i \cdot e^z\big|_{z=2} = 2\pi e^2 i$.

(3). $\oint_C \frac{e^{iz} dz}{z^2+1} = 2\pi i \cdot e^{iz}\big|_{z=i} = e^{-1} 2\pi i$

(5). 原函数在 $|z|=r<1$ 上解析 原式 $=0$.

(7). $\oint_C \frac{dz}{(z^2+1)(z^2+4)} = 2\pi i \cdot \frac{1}{z^2+4}\Big|_{z=i} = \frac{2}{3}\pi i$

(9) $\cos z_0 \times 2\pi i = \oint \frac{\sin z}{(z-z_0)^2} dz$

原式 $=0$.

8. (1) $\int_{-\pi i}^{3\pi i} e^{2z}dz$ (3) $\int_{-\pi i}^{\pi i}\sin^2 z\,dz$ (4). $\int_0^1 z\sin z\,dz$ (5) $\int_0^i (z-i)e^{-z}dz$.

$z: y=it.$

原式 $=\int_{-\pi}^{3\pi} e^{2ti}\,d\,it$

$= i\int_{-\pi}^{3\pi} e^{2i\cdot t}dt$

$=\frac{1}{2}\left(e^{6\pi i}-e^{-2\pi i}\right).$

(3) $z: y=it$

原式 $=\int_{-\pi}^{\pi}\sin^2 it\,d(it)$

$=i\int_{-\pi}^{\pi}\sin^2 it\,dt.$

$=$

(4). $=-\int_0^1 z\,d\cos z.$

$=-z\cos z|_0^1-\int_0^1\cos z\,dz.$

$=-\cos 1+\sin 1$

(5) $=\int_0^i(-z+i)\,de^{-z}$

$=e^{-z}(-z+i)|_0^i+\int_0^i e^{-z}dz$

$=i-e^{i}=i-\cos 1+i\sin 1$

9. (1) 原式 $=4\times 2\pi i+3\times 2\pi i=14\pi i$

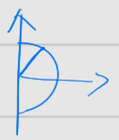
(3) 原式 $=0.$

(5). 原式 $=\begin{cases} e^{a}2\pi i, & |a|<1\\ 0, & |a|\geq 1\end{cases}$

10. $\frac{1}{z^2}$在C上解析

∴ $\oint_C \frac{1}{z^2}dz=0.$

12. $\int_0^z \frac{1}{1+\xi^2}d\xi=\int_{C_1+C_2}\frac{1}{1+z^2}dz$



$=\int_0^1\frac{1}{1+t^2}dt+\int_0^\theta \frac{1}{2}di\theta$

$=\frac{1}{4}\pi+\frac{1}{2}\theta i$

15. 当z_0在C_1内: 原式 $=\frac{1}{2\pi i}\times\oint_{C_1}\frac{z^2dz}{z-z_0}=z_0^2$

······ C_2 ······ $=\frac{1}{2\pi i}\times\oint_{C_2}\frac{\sin z\,dz}{z-z_0}=\sin z_0.$

19. $\oint_C\frac{f'(z)}{f(z)}=0.$

∵ $f(z)$解析

∴ $f'(z)$解析.

21. $\oint_C\frac{f(z)}{(z-z_0)^2}=2\pi i f'(z_0)=\oint_C\frac{f'(z)}{z-z_0}dz$ 证明完毕

30 (1). $u=(x-y)(x^2+4xy+y^2)$

$\frac{\partial u}{\partial x}=x^2+4xy+y^2+(x-y)(2x+4y)=x^2+4xy+y^2+2x^2-4y^2+2xy=3x^2+6xy-3y^2$

$v=3x^2y+3xy^2-y^3+g(x)$

$$\frac{\partial u}{\partial y} = -x^2-4xy-y^2+(x-y)(4x+2y) = -x^2-4xy-y^2+4x^2-2y^2-2xy = 3x^2-6xy-3y^2$$

$$\frac{\partial v}{\partial x} = 6xy+3y^2+g'(x)$$

$$\therefore g'(x) = 3x^2 \quad g(x) = x^3+c.$$

$$f(z) = (x-y)(x^2+4xy+y^2) + i(3x^2y+3xy^2-y^3+x^3+c)$$

$$= z^3 + i(3z^2+3z+z^3+c)$$

(3) $u = 2(x-1)y$, $f(2) = -i$

$$\frac{\partial u}{\partial x} = 2y \qquad \frac{\partial u}{\partial y} = 2x-2$$

$$v = y^2+g(x). \quad \frac{\partial v}{\partial x} = g'(x) \quad g(x) = 2x-x^2+c.$$

$$f(z) = 2(x-1)y+i(y^2+2x-x^2+c).$$

$$= 2(z-1)+i(2z-z^2+c)$$

$$\because f(2) = -i$$

$$\therefore 2+ic = -i \quad c = -\frac{2+i}{i} = 2i+1$$

$$f(z) = 2z-4+i(2z-z^2+1)$$

31. $v = e^{px}\sin y.$

$$\frac{\partial v}{\partial x} = pe^{px}\sin y \quad \frac{\partial^2 v}{\partial x^2} = p^2e^{px}\sin y$$

$$\frac{\partial v}{\partial y} = e^{px}\cos y \quad \frac{\partial^2 v}{\partial y^2} = -e^{px}\sin y$$

$$\therefore (p^2-1)e^{px}\sin y = 0$$

$$p = \pm 1$$