1. (1). $\frac{1}{z(z^2+1)^2}$ 奇点: $0, \pm i$

极点: 0, 1级, $\pm i$, 2级.

(3). $\frac{1}{z^3-z^2-z+1}$ 奇点: ± 1

$= \frac{1}{z^2(z-1)-(z-1)}$. 极点: -1, 1级

$= \frac{1}{(z+1)(z-1)^2}$ 1, 2级.

(5). $\frac{z}{(1+z^2)(1+e^{\pi z})}$. 奇点: ni, $i\in Z$.

极点: i 2级

$-i$ 2级

其余, 1级

(7). $\frac{1}{z^2(e^z-1)}$ 奇点: 0, $2n\pi i$, $n\in Z$.

极点: 0, 3级, $2n\pi i$, 1级.

(9) $\frac{1}{\sin z^2}$ 奇点: 0, $\pm\sqrt{k\pi}$, $k\in Z$.

极点: 0: 2级.

$\pm\sqrt{k\pi}$, 1级.

3. $ch\frac{\pi i}{2} = \frac{1}{2}(e^{\frac{\pi}{2}i}+e^{-\frac{\pi}{2}i}) = 0$.

$\frac{d\,ch z}{dz}\Big|_{\frac{\pi i}{2}} = i \neq 0$

$\therefore z=\frac{\pi}{2}i$ 为 $ch z$ 1级零点.

6 (1) $\varphi(z)\psi(z)$:

$m+n$ 级零点. $m+n$ 级极点.

(2). $\frac{\varphi(z)}{\psi(z)}$:

$m-n$ 级零点. $n-m$ 级 极点.

(3). $\varphi(z)+\psi(z)$:

$\min\{m,n\}$ 级零点.

$\frac{-2e^{2z}-1+e^{2z}}{z^2} = \frac{-e^{2z}-1}{z^2}$

8. (2). $\frac{1-e^{2z}}{z^4}$, $z=0$ 为3级极点.

$Res[f(z),0] = \frac{1}{2}\lim_{z\to 0}\frac{d^2}{dz^2}\left(\frac{1-e^{2z}}{z}\right)$

$$\frac{df}{dz}=\frac{-2e^{2z}-1+e^{2z}}{z^2}=\frac{-e^{2z}-1}{z^2}$$

$$\frac{d^2f}{dz^2}=\frac{-2e^{2z}\cdot z^2+2z(-e^{2z}-1)}{z^4}$$

$$\mathrm{Res}[f(z),0]=-\frac{4}{3}$$

(3). $\frac{1+z^4}{(z^2+1)^3}$，$z=\pm i$ 为3级极点。

$$=\frac{1+z^4}{(z+i)^3(z-i)^3}$$

$$\mathrm{Res}[f(z),i]=\frac{1}{2}\lim_{z\to i}\frac{d^2}{dz^2}\frac{1+z^4}{(z+i)^3}=-\frac{3}{8}i$$

$$\mathrm{Res}[f(z),-i]=\frac{1}{2}\lim_{z\to -i}\frac{d^2}{dz^2}\frac{1+z^4}{(z-i)^3}=\frac{3}{8}i$$

(4). $\frac{z}{\cos z}$. $z=k\pi+\frac{\pi}{2}$，$k\in Z$ 为1级极点.

$$\mathrm{Res}[f(z),k\pi+\frac{\pi}{2}]=\lim_{z\to k\pi+\frac{\pi}{2}}\frac{z(z-k\pi-\frac{\pi}{2})}{\cos z}$$

$$=\lim_{z\to\cdots}\frac{2z-k\pi-\frac{\pi}{2}}{-\sin z}$$

$$=(-1)^{k+1}(k\pi+\frac{\pi}{2}),\quad k\in Z$$

(6). $z^2\sin\frac{1}{z}$ $z=0$ 为1级零点.

$$f(z)=z^2\cdot\left(\frac{1}{z}-\frac{1}{3!z^3}+\cdots\right).$$

$\therefore$ $z=0$ 为本性奇点

$$\mathrm{Res}[z^2\sin\frac{1}{z},0]=-\frac{1}{6}$$

(7). $\frac{1}{z\sin z}$ $z=0$ 为2级极点，$z=k\pi$ 为1级零点.

$$\mathrm{Res}[f(z),0]=\frac{1}{2}\lim_{z\to 0}\frac{d}{dz}\left(\frac{z}{\sin z}\right).$$

$$=\frac{1}{2}\lim_{z\to 0}\frac{\sin z-z\cos z}{\sin^2 z}$$

$$=0.$$

$$\mathrm{Res}[f(z),k\pi]=\lim_{z\to k\pi}\frac{1}{k\pi\cos k\pi}=\frac{(-1)^k}{k\pi}.$$

9.(2) $\oint_{|z|=2} \frac{e^{2z}}{(z-1)^2} dz$

$= 2\pi i \cdot \text{Res}\left[\frac{e^{2z}}{(z-1)^2}, 1\right]$

$= 2\pi i \times (e^{2z})' \Big|_{z=1} = 4\pi i e^2$

(3). $\oint_{|z|=\frac{3}{2}} \frac{1-\cos z}{z^m} dz$

$f(z) = \frac{1}{z^m}\left[\frac{1}{2!}z^2 - \frac{1}{4!}z^4 + \cdots\right]$

$\therefore m$ 为偶时，$f(0)$ 存在.

$m=2n+1$ 时，

$\text{Res}\left[\frac{1-\cos z}{z^m}, 0\right] = \frac{(-1)^{n+1}}{(2n)!}$.

$\therefore$ 原式 $= \begin{cases} 2\pi i \frac{(-1)^{n+1}}{(2n)!}, & m=2n+1,\ n \in N \\ 0, & \text{else}. \end{cases}$

(4). $\oint_{|z|=3} \tan \pi z \, dz$

$f(z) = \frac{\sin \pi z}{\cos \pi z}$.

$\text{Res}[f(z), k+\frac{1}{2}] = -\frac{1}{\pi}$

$\therefore \oint_{|z|} f(z) dz = 2\pi i \times 6 \times (-\frac{1}{\pi}) = -12i$

10.(1) $e^z = 1 + z + \frac{1}{2!}z^2 + \cdots$

$e^{\frac{1}{z^2}} = 1 + \frac{1}{z^2} + \frac{1}{2!}\frac{1}{z^4} + \cdots$

$\text{Res}[e^{\frac{1}{z^2}}, \infty] = 0$.

(2) $\cos z - \sin z = 1 - z - \frac{1}{2!}z^2 + \frac{1}{3!}z^3 + \cdots$

本性奇点. $\text{Res}[f(z), 0] = 0$.

11 (1). $f(z) = \frac{e^z}{z^2-1}$

$\text{Res}[f(z), -1] = -\frac{1}{2}e^{-1}$

$\text{Res}[f(z), 1] = \frac{1}{2}e$

$\text{Res}[f(z), \infty] = \frac{1}{2}e^{-1} - \frac{1}{2}e$.

(2). $f(z) = \dfrac{1}{z(z+1)^4(z-4)}$.

$\mathrm{Res}[f(z), \infty] = -\mathrm{Res}\left[\frac{1}{z^2} f\left(\frac{1}{z}\right), 0\right] = 0$.

12 (1). 原式 $= -2\pi i\, \mathrm{Res}[f(z), \infty]$

$= 2\pi i\, \mathrm{Res}\left[\frac{1}{z^2} \cdot f(z), 0\right]$.

$= 2\pi i$.

(3). 原式 $= -\mathrm{Res}[f(z), \infty]\, 2\pi i$

$f(z) = z^n \dfrac{1}{1+\frac{1}{z^n}} = z^n\left(1 - \frac{1}{z^n} + \frac{1}{z^{2n}} - \cdots\right)$.

$= z^n - 1 + \dfrac{1}{z^n} - \dfrac{1}{z^{2n}} \cdots$

∴ 原式 $= \begin{cases} -2\pi i, & n=1 \\ 0, & \text{else}. \end{cases}$