4.(2) $\overline{z_1 \pm z_2} = \overline{z_1} \pm \overline{z_2}$

证明：设 $z_1 = a_1 + b_1 i$，$z_2 = a_2 + b_2 i$

$\overline{z_1 + z_2} = a_1 + a_2 - (b_1 + b_2) i = \overline{z_1} + \overline{z_2}$

同理：$\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$

$\Rightarrow \overline{z_1 \pm z_2} = \overline{z_1} \pm \overline{z_2}$

(3). $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$

证明：设 $z_1 = a_1 + b_1 i$，$z_2 = a_2 + b_2 i$

$z_1 z_2 = a_1 a_2 - b_1 b_2 + (a_1 b_2 + a_2 b_1) i$

$\overline{z_1 z_2} = a_1 a_2 - b_1 b_2 - (a_1 b_2 + a_2 b_1) i$

$\overline{z_1}\,\overline{z_2} = a_1 a_2 - b_1 b_2 - (a_1 b_2 + a_2 b_1) i$

$\Rightarrow \overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$

(4) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$，$z_2 \neq 0$

证明：设 $z_1 = a_1 + b_1 i$，$z_2 = a_2 + b_2 i$

由(3)可知，$\overline{\left(\frac{z_1}{z_2}\right)} = \overline{z_1 \cdot z_2^{-1}} = \overline{z_1} \cdot \overline{z_2^{-1}}$

即证 $\overline{z_2^{-1}} = \overline{z_2}^{-1}$

$\overline{z_2^{-1}} = \overline{\frac{1}{a_2 + b_2 i}} = \frac{a_2 - b_2 i}{a_2^2 + b_2^2} = \overline{z_2}^{-1}$

6. $|z^n + a| \leq |z^n| + |a|$，当且仅当 z^n 与 a 共线

$\leq 1 + |a|$

8.(3). $\arg z = \frac{\pi}{3}$　$z = 2\left(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right) = 2e^{\frac{\pi}{3} i}$

(4)

由余弦定理 $|z| = \sqrt{2 - 2\cos\varphi}$

$\arg z = \frac{\pi}{2} - \frac{\varphi}{2}$

$z = \sqrt{\frac{\pi}{2} - \frac{\varphi}{2}}\left(\cos\left(\frac{\pi}{2} - \frac{\varphi}{2}\right) + i \sin\left(\frac{\pi}{2} - \frac{\varphi}{2}\right)\right)$

$= \sqrt{\frac{\pi}{2} - \frac{\varphi}{2}}\, e^{\left(\frac{\pi}{2} - \frac{\varphi}{2}\right) i}$

(5) $z = \frac{2i}{-1+i} = \frac{2i(-1-i)}{2} = 1 - i = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right)\right)$

$= \sqrt{2}\, e^{-\frac{\pi}{4} i}$

(6). $z = \frac{\cos 10\varphi + i \sin 10\varphi}{\cos 9\varphi - i \sin 9\varphi} = \cos 19\varphi + i \sin 19\varphi$

$= e^{19\varphi i}$

14. (1) $(\sqrt{3}-i)^5=(2e^{-\frac{\pi}{6}i})^5=32e^{-\frac{5\pi}{6}i}$

(2) $(1+i)^6=(\sqrt{2}e^{\frac{\pi}{4}i})^6=8e^{\frac{3}{2}\pi i}$

(3). 得 $z^3=i$ $\quad 3\theta=2k\pi.$

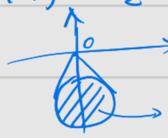
$\Rightarrow \theta=\frac{2}{3}k\pi.$

$\Rightarrow z=e^{\frac{2}{3}k\pi i}$

(4). $\mathrm{Arg}(1-i)=-\frac{\pi}{4}+2k\pi.$

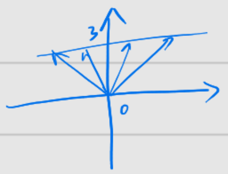
$(1-i)^{\frac{1}{3}}=2^{\frac{1}{6}}e^{(-\frac{\pi}{12}+\frac{2}{3}k\pi)i}$

21. (1)

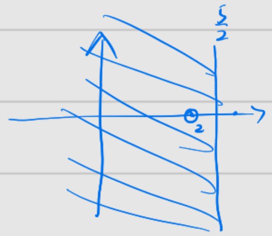


除阴影的区域.

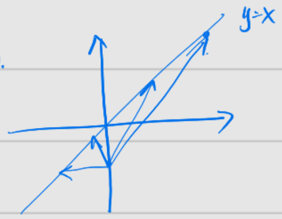
(4).



(6).



(8).

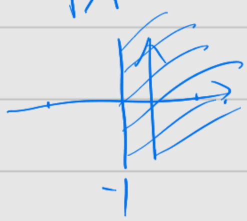


22. (1). 一、二象限. 无界. 单连通

(3).

无界 单连通

(5)



无界 单连通

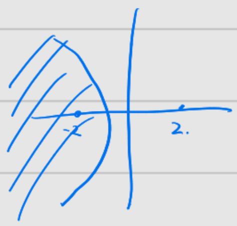
(7).

$(x-1)^2+y^2\le 16(x+1)^2+16y^2$

$x^2-2x+1+y^2\le 16x^2+32x+16+16y^2$

$15x^2+34x+15y^2+15\ge 0.$

(9).



25. (1) $z = t + t i$.

$y = x$.

(2). $x = a\cos t$, $y = b\sin t$.

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(3). $x = t$, $y = \frac{1}{t}$

$xy = 1$.

(4). $x = t^2$, $y = \frac{1}{t^2}$.

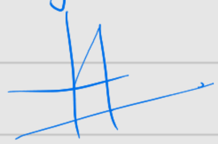
$xy = 1$. $(x, y > 0)$

26. (1). $x^2 + y^2 = \frac{1}{4}$.

(2). $z = a + ai$

$w = \frac{1}{a+ai} = \frac{a-ai}{2a^2} = \frac{1-i}{2a}$

$x + y = 0$.

(3).  $y = -1$

(4).  $w = \frac{1}{1+\sin\theta + i\cos\theta}$

$= \frac{1+\sin\theta - 2i\cos\theta}{2}$.

$(x-1)^2 + \frac{y^2}{4} = 1$

27. (1) $w_1 = -i$

$w_2 = 2i(1+i) = -2+2i$

$w_3 = 8i$

(2) $0 < \arg w < \pi$.

29. ?