2.(1) 求 $f(z)=x^2-iy$. 在何处可导, 何处解析.

$\frac{\partial u}{\partial x}=2x$ $\frac{\partial u}{\partial y}=0$ $\frac{\partial v}{\partial x}=0$ $\frac{\partial v}{\partial y}=-1$

$\therefore x=-\frac{1}{2}, y\in R$ 时可导

(2). $f(z)=xy^2+ixy$

$\frac{\partial u}{\partial x}=y^2$ $\frac{\partial u}{\partial y}=2xy$ $\frac{\partial v}{\partial x}=y$ $\frac{\partial v}{\partial y}=x$

$\begin{cases} x=y^2 \\ 2xy=-x \end{cases}$ $y=-\frac{1}{2}, x=\frac{1}{4}$

$\therefore$ 仅在 $z=\frac{1}{4}-\frac{1}{2}i$ 可导, 不解析.

3.(3). $\frac{1}{z^2-1}$ (4). $\frac{az+b}{cz+d}$

令 $z^2-1=0$. $z=1$

$\therefore$ 除 $z=1$ 外解析.

$f'=\frac{-2z}{(z^2-1)^2}$

① 若 $\frac{a}{c}=\frac{b}{d}$ 则 R 上解析 $f'=0$.

② 若 $c=0$. R上解析. $f'=\frac{a}{d}$

③ $d=0$ 除 $z=0$ 外解析 $f=\frac{a}{c}+\frac{b}{cz}$ $f'=-\frac{b}{cz^2}$

④ $c\neq 0, d\neq 0, \frac{a}{c}\neq\frac{b}{d}$. 除 $z=-\frac{d}{c}$ 外解析

$f=\frac{\frac{a}{c}(cz+d)+b-\frac{ad}{c}}{cz+d}=\frac{a}{c}+\frac{b-\frac{ad}{c}}{cz+d}$

$f'=\frac{-b+\frac{ad}{c}}{(cz+d)^2}$

4.(1) $\frac{z+1}{z(z^2+1)}$

令 $z(z^2+1)=0$ $z=0$ 或 i

(2). $\frac{z-2}{(z+1)^2(z^2+1)}$

令 $(z+1)^2(z^2+1)=0$. $z=-1$ 或 i

8. $my^3+nx^2y+i(x^3+lxy^2)$

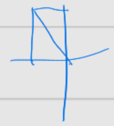
$\frac{\partial u}{\partial x}=2nxy$ $\frac{\partial u}{\partial y}=3my^2+nx^2$

$\frac{\partial v}{\partial x}=3x^2+ly^2$ $\frac{\partial v}{\partial y}=2lxy$.

$\Rightarrow \begin{cases} 2nxy=2lxy. \\ 3x^2+ly^2+3my^2+nx^2=0. \end{cases}$ 恒成立.

$\therefore$ $n=-3$ $l=-3$. $m=1$

15. $\mathrm{Ln}(-i) = \ln 1 - \frac{\pi}{2} i = -\frac{\pi}{2} i$



$\mathrm{Ln}(-3+4i) = \ln 5 + \arctan\frac{-3}{4}\, i$

18. $e^{1-i\frac{\pi}{2}} = e^{1}\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right) = -ei$

$\exp[(1-i\pi)/4] = e^{\frac{1}{4}}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right) = e^{\frac{1}{4}}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i\right).$

$3^{i} = e^{\ln 3 \cdot i} = \cos\ln 3 + i\sin\ln 3.$

$(1+i)^{i} = e^{i\,\mathrm{Ln}(1+i)} = e^{i+\frac{\pi}{4}i} = \cos\left(1+\frac{\pi}{4}\right) + i\sin\left(1+\frac{\pi}{4}\right).$