PROBLEM 1
Assume that:

$$r_t \sim N(0, \sigma^2)$$
$$P_{t-1} = m$$
$$E(r_t) = 0$$

1. **Classical Brownian Motion**

$$E(P_t) = P_{t-1} + E(r_t)$$
$$= m$$

$$SD(P_t) = SD(r_t + P_{t-1})$$
$$= \sigma$$

2. **Arithmetic Return System**

$$E(P_t) = E(P_{t-1}(r_t + 1))$$
$$= P_{t-1} + E(r_t) \cdot P_{t-1}$$
$$= m$$

$$SD(P_t) = SD(P_{t-1}(r_t + 1))$$
$$= SD(r_t) \cdot P_{t-1}$$
$$= m\sigma$$

3. **Log Return or Geometric Brownian Motio**

$$E(P_t) = E(P_{t-1} \cdot e^{r_t})$$
$$= m \cdot e^{\frac{1}{2}\sigma^2}$$

$$SD(P_t) = m \cdot \sqrt{e^{\sigma^2} - 1}$$

When using python to simulate the distribution, the result meets my expectation (set sigma_return = 1, p_t-1 = 100)

```
simulation(10000, 'CBM') #[99.99807992497432, 0.9983846342220718]
simulation(10000, 'ARS') #[98.99937707258351, 101.87680798705381]
simulation(10000, 'LR')  #[166.83023248105218, 220.90530738215818]
```

PROBLEM 2

Normal Distribution
95% VaR: 16.225807942951008

Normal Distribution with an Exponentially Weighted variance(0.94)
95% VaR: 9.01339396

MLE fitted T distribution
95% VaR: -12.531381097807715

Fitted AR(1) model
95% VaR: -0.5362

Historic Simulation
95% VaR: 11.8128

PROBLEM 3

1. **EWM-DELTA NORMAL**

Portfolio A:

```
Delta Normal
Current Portfolio Value: 1089316.15994
Current Portfolio VaR: 15206.390928062503
```

Portfolio B:

```
Delta Normal
Current Portfolio Value: 574542.4051499999
Current Portfolio VaR: 7741.25096360738
```

Portfolio C:

```
Delta Normal
Current Portfolio Value: 1387409.5075200002
Current Portfolio VaR: 17877.73301652641
```

So the total VaR is 40825.37490819629

2. **DELTA NORMAL**

Portfolio A:

```
Delta Normal
Current Portfolio Value: 1089316.15994
Current Portfolio VaR: 21023.358694467683
```

Portfolio B:

```
Delta Normal
Current Portfolio Value: 574542.4051499999
Current Portfolio VaR: 11958.18284195713
```

Portfolio C:

```
Delta Normal
Current Portfolio Value: 1387409.5075200002
Current Portfolio VaR: 26528.567057577176
```

So the total VaR is 59510.108594001984

3. MC Normal

Portfolio A:

```
MC Normal
Current Portfolio Value: 1089316.15994
Current Portfolio VaR: 20909.341125770938
```

Portfolio B:

```
MC Normal
Current Portfolio Value: 574542.4051499999
Current Portfolio VaR: 11956.031727552763
```

Portfolio C:

```
MC Normal
Current Portfolio Value: 1387409.5075200002
Current Portfolio VaR: 26859.609962853137
```

So the total VaR is 59724.98281617684

4. Historical

Portfolio A:

```
Historical VaR
Current Portfolio Value: 1089316.15994
Current Portfolio VaR: 17065.300954190083
```

Portfolio B:

```
Historical VaR
Current Portfolio Value: 574542.4051499999
Current Portfolio VaR: 10983.463846970699
```

Portfolio C:

```
Historical VaR
Current Portfolio Value: 1387409.5075200002
Current Portfolio VaR: 22186.51922579715
```

So the total VaR is 50235.28402695793

The reason that I choose Historical model is that It doesn’t depend on the assumption of either linear or normal distribution. From calculating the skewness of the stock data, we can see that it is not proper to assume its normal distribution.