PROBLEM 1

a/b.

Based on the formula, the four moments are calculated as following:
Mean = $E(x)$
Variance = $E[(x-u)^2]$
Skewness = $E[(x-u)^3]$
Kurtosis = $E[(x-u)^4]$

MOMENTS	FORMULA				scipy Package
	biased		unbiased		
mean			$\frac{1}{n}\sum_i^n x_i$	1.049	1.049
variance	$\frac{1}{n}\sum_i^n (x_i - \mathrm{u})^2$	5.422	$\frac{1}{n-1}\sum_i^n (x_i - \mathrm{u})^2$	5.427	5.427
skewness	$\frac{1}{n}\sum_i^n (x_i - \mathrm{u})^3$	0.879	$\frac{n}{(n-1)(n-2)\sigma^{3/2}}\sum_i^n (x_i - \mathrm{u})^3$	0.881	0.881
kurtosis	$\frac{1}{n}\sum_i^n (x_i - \mathrm{u})^4$	26.070	$\frac{a}{\sigma^2}\sum_i^n (x_i - \mathrm{u})^4$	23.122	23.122

c.

To test whether the moments computed by the scipy package is biased, we used t-test to its variance, skewness, kurtosis function respectively.
Using np.randn to continually generate vector data which is normally distributed, then get the descriptive information of the calculated kurtosis of each set of data, based on which the t-value and p-value is calculated manually as 8.061350999910566e-05
And Built-in t-test p-value: 8.061350999923319e-0505 (at that trial)
So we can see that the variance, skewness and kurtosis produced by the scipy package is unbiased

PROBLEM 2

a.

The function for minimization on condition of normal distribution is:

$$ll = \sum_{i=1}^{n}[-\frac{1}{2}ln(\sigma^2 2\Pi) - \frac{1}{2}\left(\frac{x_i - u}{\sigma}\right)^4]$$

METHODS	Fitted_Function	Std_Error
OLS	y = 0.7753 * x + (-0.0874)	1.0126
MLE_n		1.0038

These two functions produced same beta, but the sigma for MLE method is smaller than the std_err of OLS, which resulted from the fact that the estimator of MLE not unbiased, so it is divided by a larger denominator.

b.

Using the scipy.stats.t.logpdf() function to generate the function for minimization

METHODS	Fitted_Function	Std_Error	R_squared
MLE_t	y = 0.6750 * x + (-0.0973)	0.8551	0.3397
MLE_n	y = 0.7753 * x + (-0.0874)	1.0038	0.3456

The t-squared under MLE for normal distribution assumption is higher than the t-distribution, so the formal one would be the better fit.

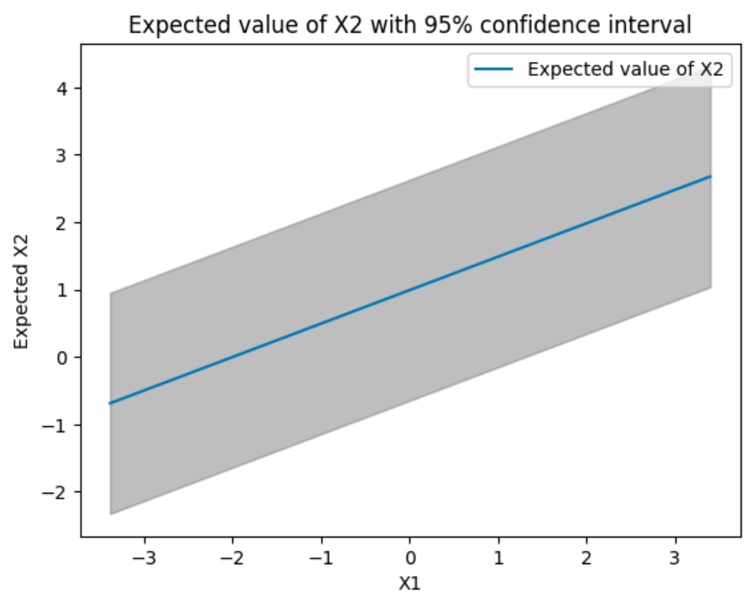
c.

The **mean** for X is [0.0010227 , 0.99024382]
The **covariance** for X is [[1.06977464, 0.53068455],
[0.53068455, 0.96147329]]

$$\bar{\mu} = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(a - \mu_2)$$

$$\bar{\Sigma} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}$$

Given the formula provided in class for conditional distribution, calculated the mean and variance for x2 on condition of each given x1 value.



PROBLEM 3

MODEL	Fitted_Function	AIC
AR(1)	`x_t = 0.2019 * x_t-1 + 2.1258`	1644.66
AR(2)	`x_t = (-0.3505) * x_t-2 + 0.2732 * x_t-1 + 2.1270`	1581.08
AR(3)	`x_t = 0.5047 * x_t-3 + (-0.4887) * x_t-2 + 0.4515 * x_t-1 + 2.1209`	**1436.66**
MA(1)	`x_t = 0.6434 * e_t-1 + 2.1236`	1567.40
MA(2)	`x_t = -0.2306 * e_t-2 + 0.4344 * e_t-1 + 2.1255`	1537.94
MA(3)	`x_t = -0.1531 * e_t-3 + -0.2286 * e_t-2 + 0.5582 * e_t-1 + 2.1259`	1536.87

AR(3) is the best among these models with the lowest AIC.