Before you proceed further, please read :
`http://en.wikipedia.org/wiki/Gravity_anomaly`
`http://en.wikipedia.org/wiki/Gravimeter`

Let us consider a vertical domain $Lx \times Ly$ where $L_x = 1000km$ and $Ly = 500km$. This domain is discretised by means of a grid which counts $nnp = nnx \times nny$ nodes. This grid then counts $nel = nelx \times nely = (nnx-1) \times (nny-1)$ cells. The horizontal spacing between nodes is hx and the vertical spacing is hy.

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Assume that this domain is filled with a rock type which mass density is given by $\rho_{medium} = 3000kg/m^3$, and that there is a circular inclusion of another rock type ($\rho_{sphere} = 3200kg/m^3$) at location ($xsphere$, $ysphere$) of radius $rsphere$. The density in the system is then given by

$$\rho(x,y) = \begin{cases} \rho_{sphere} \text{ inside the circle} \\ \rho_{medium} \text{ outside the circle} \end{cases}$$

Let us now assume that we place $nsurf$ gravimeters at the surface of the model. These are placed equidistantly between coordinates $x = 0$ and coordinates $x = Lx$. We will use the arrays $xsurf$ and $ysurf$ to store the coordinates of these locations. The spacing between the gravimeters is $\delta_x = Lx/(nsurf - 1)$.

At any given point (x_i, y_i) in a 2D space, one can show that the gravity anomaly due to the presence of a circular inclusion can be computed as follows:

$$g(x_i, y_i) = 2\pi \mathcal{G} (\rho_{sphere} - \rho_0) R^2 \frac{y_i - y_{sphere}}{(x_i - x_{sphere})^2 + (y_i - y_{sphere})^2} \tag{1}$$

where r_{sphere} is the radius of the inclusion, (x_{sphere}, y_{sphere}) are the coordinates of the center of the inclusion, and ρ_0 is a reference density.

However, the general formula to compute the gravity anomaly at a given point (x_i, y_i) in space due to a density anomaly of any shape is given by:

$$g(x_i, y_i) = 2\mathcal{G} \int\int_\Omega \frac{\Delta\rho(x,y)(y - y_i)}{(x - x_i)^2 + (y - y_i)^2} dxdy \tag{2}$$

where Ω is the area of the domain on which the integration is to be carried out. Furthermore the density anomaly can be written : $\Delta\rho(x, y) = \rho(x, y) - \rho_0$. We can then carry out the integration for each cell and sum their contributions:

$$g(x_i, y_i) = 2\mathcal{G} \sum_{ic=1}^{nel} \int\int_{\Omega_e} \frac{(\rho(x,y) - \rho_0)(y - y_i)}{(x - x_i)^2 + (y - y_i)^2} dxdy \tag{3}$$

where Ω_e is now the area of a single cell. Finally, one can assume the density to be constant within each cell so that $\rho(x, y) \to \rho(ic)$ and $\int\int_{\Omega_e} dxdy \to hx \times hy$ and then

$$g(x_i, y_i) = 2\mathcal{G} \sum_{ic=1}^{nel} \frac{(\rho(ic) - \rho_0)(y(ic) - y_i)}{(x(ic) - x_i)^2 + (y(ic) - y_i)^2} s_x s_y \tag{4}$$

We will then use the array $gsurf$ to store the value of the gravity anomaly measured at each gravimeter at the surface.

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To go further

- explore the effect of the size of the inclusion on the gravity profile.
- explore the effect of the ρ_0 value.
- explore the effect of the grid resolution.
- measure the time that is required to complete task 9 by means of the *cpu_time* subroutine (google it). How does this time vary with nsurf ? how does it vary when the grid resolution is doubled ?
- Assume now that $\rho_2 < \rho_1$. What does the gravity profile look like ?
- what happens when the gravimeters are no more at the surface of the Earth but in a satellite ?
- if you really can't get enough, redo the whole exercise in 3D...