The viscosity field μ is calculated as the harmonic average between a linear part μ_{lin} that depends on temperature only or on temperature and depth d , and a non-linear, plastic part μ_{plast} dependent on the strain rate:

$$\mu(T, z, \dot{\boldsymbol{\epsilon}}) = 2\left(\frac{1}{\mu_{\text{lin}}(T, z)} + \frac{1}{\mu_{\text{plast}}(\dot{\boldsymbol{\epsilon}})}\right)^{-1}. \tag{1}$$

The linear part is given by the linearized Arrhenius law (the so-called Frank-Kamenetskii approximation [**?**]):

$$\mu_{\text{lin}}(T, z) = \exp(-\gamma_T T + \gamma_z z), \tag{2}$$

where $\gamma_T = \ln(\Delta\mu_T)$ and $\gamma_z = \ln(\Delta\mu_z)$ are parameters controlling the total viscosity contrast due to temperature ($\Delta\mu_T$) and pressure ($\Delta\mu_z$). The non-linear part is given by [**?**]:

$$\mu_{\text{plast}}(\dot{\boldsymbol{\epsilon}}) = \mu^* + \frac{\sigma_Y}{\sqrt{\dot{\boldsymbol{\epsilon}} : \dot{\boldsymbol{\epsilon}}}}, \tag{3}$$

where μ^* is a constant representing the effective viscosity at high stresses [**?**] and σ_Y is the yield stress, also assumed to be constant. In 2-D, the denominator in the second term of equation (3) is given explicitly by

$$\sqrt{\dot{\boldsymbol{\epsilon}} : \dot{\boldsymbol{\epsilon}}} = \sqrt{\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}} = \sqrt{\left(\frac{\partial u_x}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)^2 + \left(\frac{\partial u_y}{\partial y}\right)^2}. \tag{4}$$

The viscoplastic flow law (equation 1) leads to linear viscous deformation at low stresses (equation (2)) and to plastic deformation for stresses that exceed σ_Y (equation (3)), with the decrease in viscosity limited by the choice of μ^* [**?**].

In all cases that we present, the domain is a two-dimensional square box. The mechanical boundary conditions are for all boundaries free-slip with no flux across, i.e. $\tau_{xy} = \tau_{yx} = 0$ and $\boldsymbol{u} \cdot \boldsymbol{n} = 0$, where $\boldsymbol{n}$ denotes the outward normal to the boundary. Concerning the energy equation, the bottom and top boundaries are isothermal, with the temperature T set to 1 and 0, respectively, while side-walls are assumed to be insulating, i.e. $\partial T/\partial x = 0$. The initial distribution of the temperature field is prescribed as follows:

$$T(x, y) = (1 - y) + A\cos(\pi x)\sin(\pi y), \tag{5}$$

where $A = 0.01$ is the amplitude of the initial perturbation.

In the following Table **??**, we list the benchmark cases according to the parameters used.

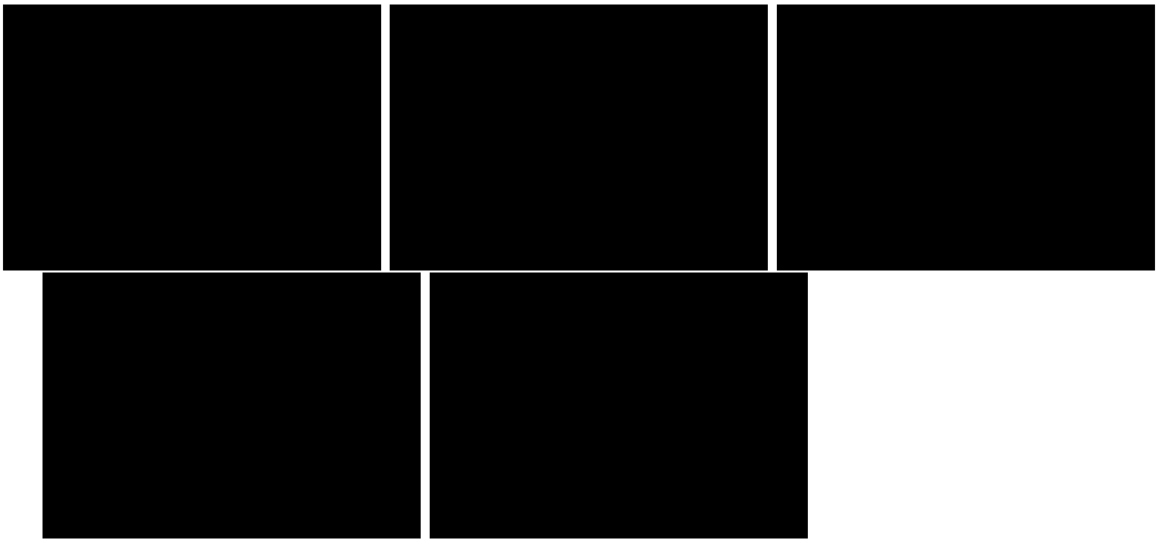
Case	Ra	$\Delta\mu_T$	$\Delta\mu_y$	μ^*	σ_Y	Convective regime
1	10^2	10^5	1	–	–	Stagnant lid
2	10^2	10^5	1	10^{-3}	1	Mobile lid
3	10^2	10^5	10	–	–	Stagnant lid
4	10^2	10^5	10	10^{-3}	1	Mobile lid
5a	10^2	10^5	10	10^{-3}	4	Periodic
5b	10^2	10^5	10	10^{-3}	3 – 5	Mobile lid – Periodic – Stagnant lid

Benchmark cases and corresponding parameters.

In Cases 1 and 3 the viscosity is directly calculated from equation (2), while for Cases 2, 4, 5a, and 5b, we used equation (1). For a given mesh resolution, Case 5b requires running simulations with yield stress varying between 3 and 5

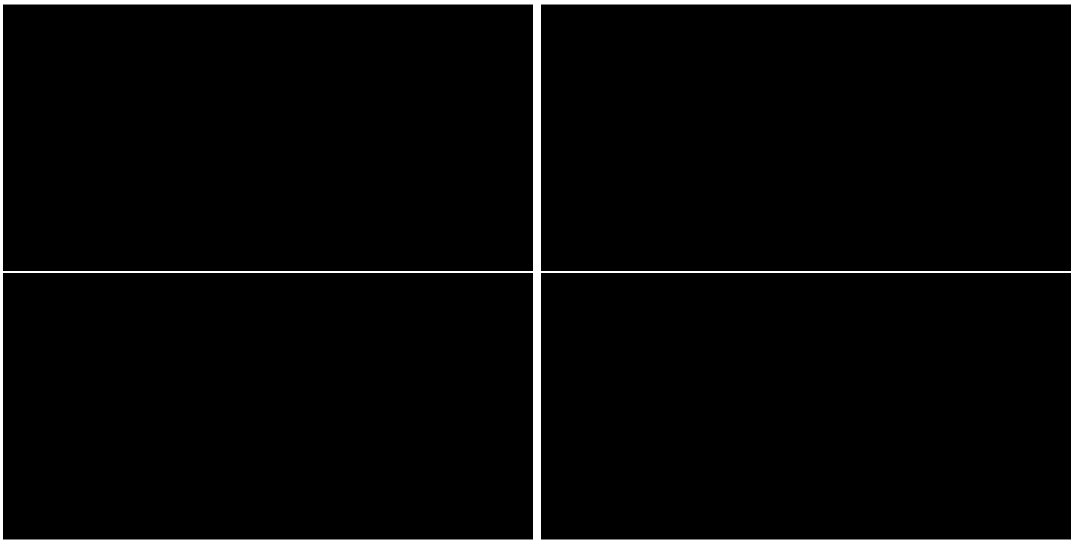
In all tests, the reference Rayleigh number is set at the surface ($y = 1$) to 10^2, and the viscosity contrast due to temperature $\Delta\mu_T$ is 10^5, implying therefore a maximum effective Rayleigh number of 10^7 for $T = 1$. Cases 3, 4, 5a, and 5b employ in addition a depth-dependent rheology with viscosity contrast $\Delta\mu_z = 10$. Cases 1 and 3 assume a linear viscous rheology that leads to a stagnant lid regime. Cases 2 and 4 assume a viscoplastic rheology that leads instead to a mobile lid regime. Case 5a also assumes a viscoplastic rheology but a higher yield stress, which ultimately causes the emergence of a strictly periodic regime. The setup of Case 5b is identical to that of Case 5a but the test consists in running several simulations using different yield stresses. Specifically, we varied σ_Y between 3 and 5 in increments of 0.1 in order to identify the values of the yield stress corresponding to the transition from mobile to periodic and from periodic to stagnant lid regime.

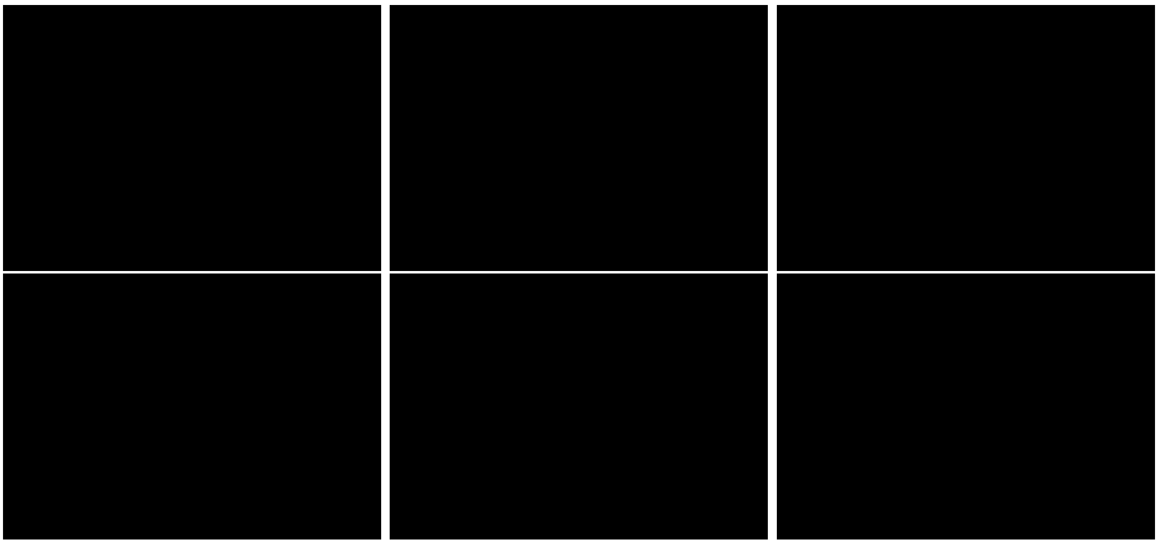
0.0.1 Case 0: Newtonian case, a la Blankenbach et al., 1989



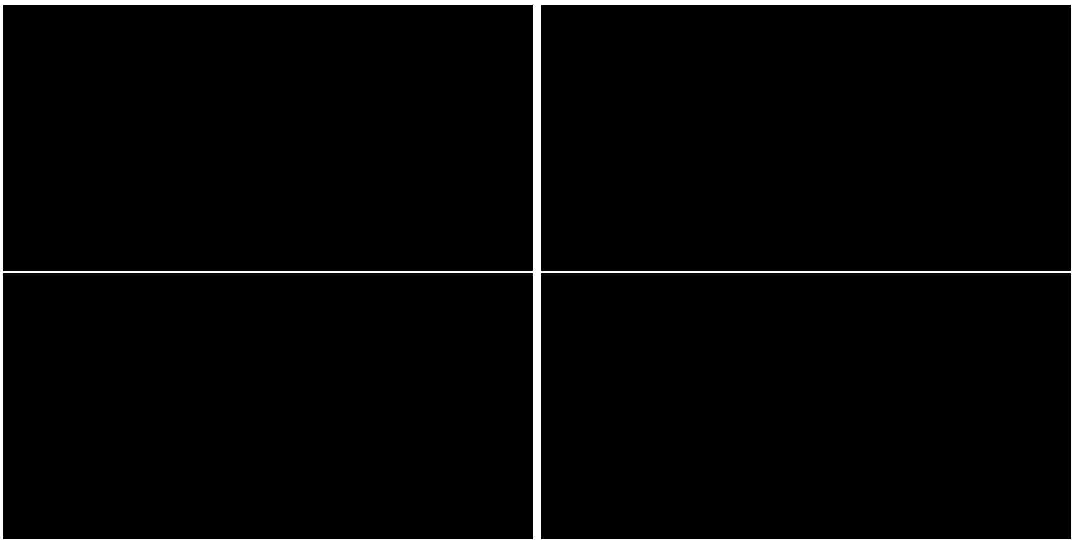
0.0.2 Case 1

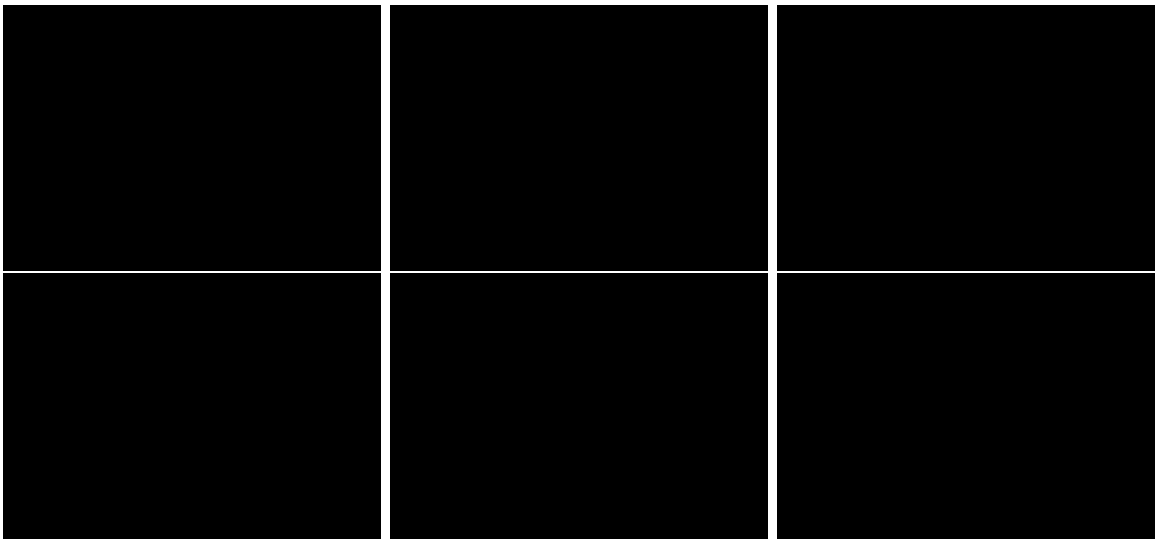
In this case $\mu^\star = 0$ and $\sigma_Y = 0$ so that μ_{plast} can be discarded. The CFL number is set to 0.5 and the viscosity is given by $\mu(T, z, \dot{\boldsymbol{\epsilon}}) = \mu_{\text{lin}}(T, z)$. And since $\Delta\mu_z = 1$ then $\gamma_z = 0$ so that $\mu_{\text{lin}}(T, z) = \exp(-\gamma_T T)$



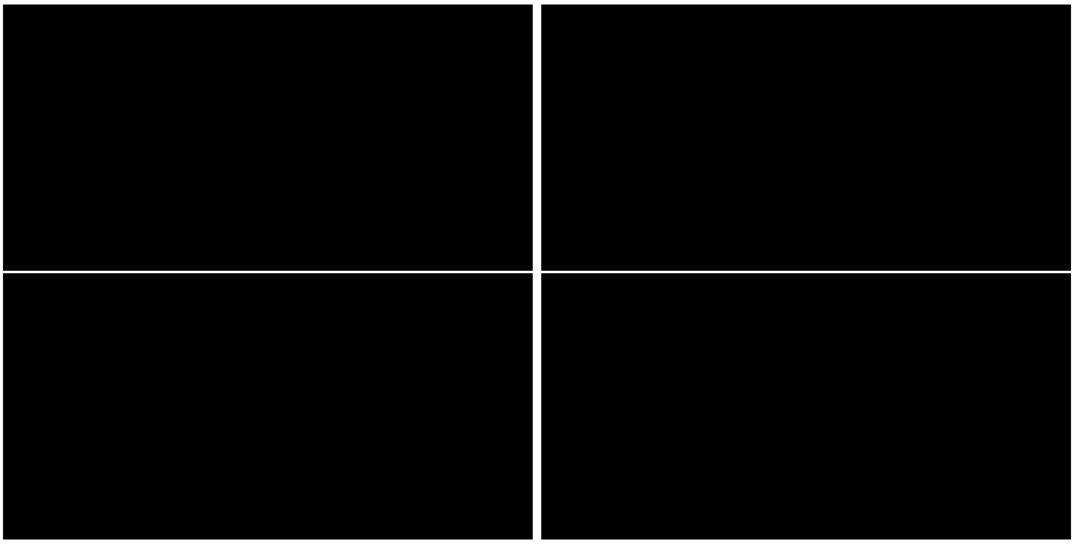


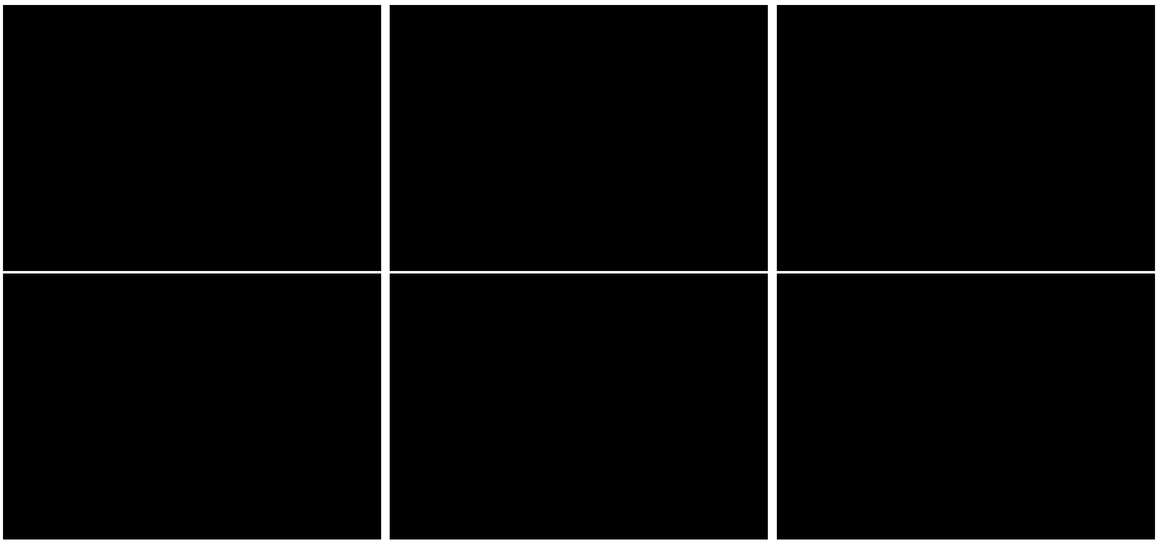
0.0.3 Case 2



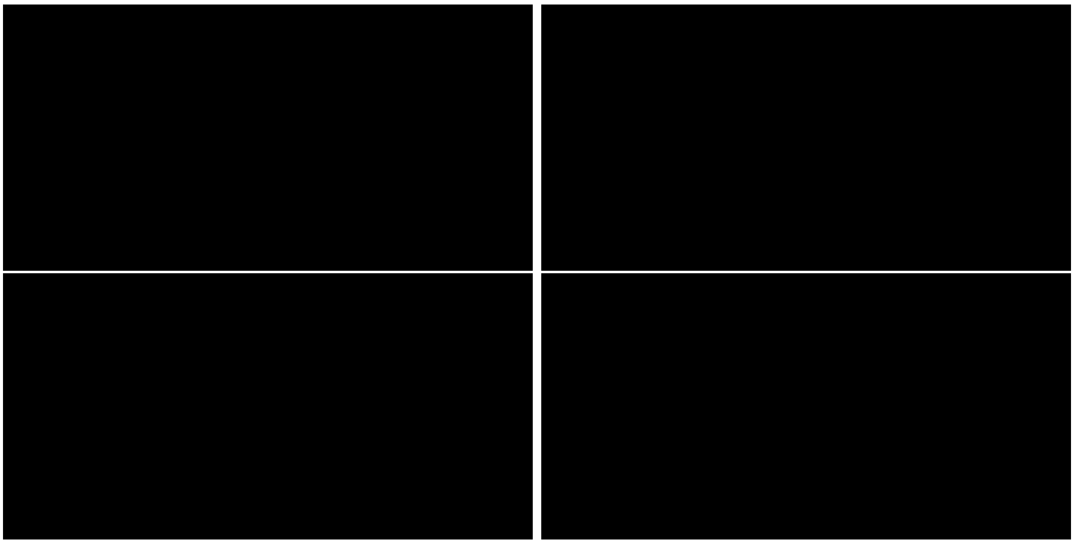


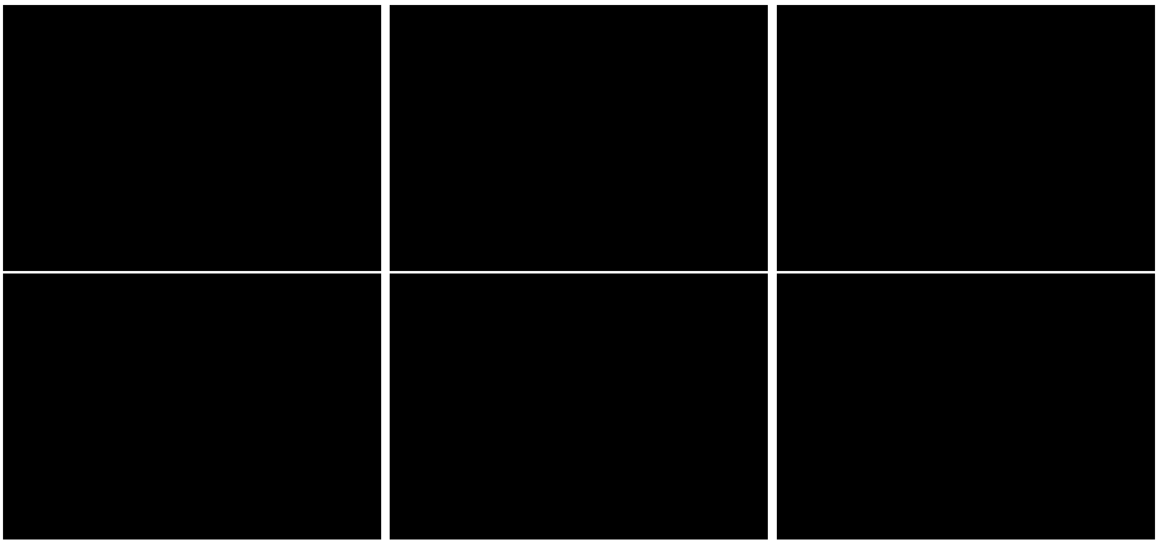
0.0.4 Case 3





0.0.5 Case 4





0.0.6 Case 5