The setup is as follows: a 2D square of elastic material of size L is subjected to the following boundary conditions: free slip on the sides, no slip at the bottom and free at the top. It has a density ρ and is placed is a gravity field $\boldsymbol{g} = -g\boldsymbol{e}_y$. For an isotropic elastic medium the stress tensor is given by:

$$\boldsymbol{\sigma} = \lambda(\boldsymbol{\nabla}\cdot\boldsymbol{u})\mathbf{1} + 2\mu\boldsymbol{\varepsilon}$$

where λ is the Lamé parameter and μ is the shear modulus. The displacement field is $\boldsymbol{u} = (0, u_y(y))$ because of symmetry reasons (we do not expect any of the dynamic quantities to depend on the x coordinate and also expect the horizontal displacement to be exactly zero). The velocity divergence is then $\boldsymbol{\nabla}\cdot\boldsymbol{u} = \partial u_y/\partial y$ and the strain tensor:

$$\boldsymbol{\varepsilon} = \left(\begin{array}{cc} 0 & 0 \\ 0 & \frac{\partial u_y}{\partial y} \end{array}\right)$$

so that the stress tensor is:

$$\boldsymbol{\sigma} = \left(\begin{array}{cc} \lambda\frac{\partial u_y}{\partial y} & 0 \\ 0 & (\lambda+2\mu)\frac{\partial u_y}{\partial y} \end{array}\right)$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{\sigma} = (\partial_x \quad \partial_y)\cdot\left(\begin{array}{cc} \lambda\frac{\partial u_y}{\partial y} & 0 \\ 0 & (\lambda+2\mu)\frac{\partial u_y}{\partial y} \end{array}\right) = \left(\begin{array}{c} 0 \\ (\lambda+2\mu)\frac{\partial^2 u_y}{\partial y^2} \end{array}\right) = \left(\begin{array}{c} 0 \\ \rho g \end{array}\right)$$

so that the vertical displacement is then given by:

$$u_y(y) = \frac{1}{2}\frac{\rho g}{\lambda+2\mu}y^2 + \alpha y + \beta$$

where α and β are two integration constants. We need now to use the two boundary conditions: the first one states that the displacement is zero at the bottom, i.e. $u_y(y=0)=0$ which immediately implies $\beta = 0$. The second states that the stress at the top is zero (free surface), which implies that $\partial u_y/\partial y(y=L) = 0$ which allows us to compute α. Finally:

$$u_y(y) = \frac{\rho g}{\lambda+2\mu}(\frac{y^2}{2} - Ly)$$

The pressure is given by

$$p = -(\lambda+\frac{2}{3}\mu)\boldsymbol{\nabla}\cdot\boldsymbol{u} = (\lambda+\frac{2}{3}\mu)\frac{\rho g}{\lambda+2\mu}(L-y) = \frac{\lambda+\frac{2}{3}\mu}{\lambda+2\mu}\rho g(L-y) = \frac{1+\frac{2\mu}{3\lambda}}{1+2\mu/\lambda}\rho g(L-y)$$

In the incompressible limit, the poisson ratio is $\nu \sim 0.5$. Materials are characterised by a finite Young's modulus E, which is related to ν and λ:

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \qquad \mu = \frac{E}{2(1+\nu)}$$

It is then clear that for incompressible parameters λ becomes infinite while μ remains finite. In that case the pressure then logically converges to the well known formula:

$$p = \rho g(L-y)$$