

This fieldstone was developed in collaboration with Lukas van de Wiel.

The domain is an annulus with inner radius R_1 and outer radius R_2 . It is filled with a single elastic material characterised by a Young's modulus E and a Poisson ratio ν , a density ρ_0 . The gravity $\mathbf{g} = -g_0 \mathbf{e}_r$ is pointing towards the center of the domain.

2μ is missing!

Given these assumptions, the momentum Stokes equations in the annulus are [?]

$$\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{v_r}{r^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} - \frac{\partial p}{\partial r} = \rho g_r \quad (1)$$

$$\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial v_\theta}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2} - \frac{1}{r} \frac{\partial p}{\partial \theta} = 0 \quad (2)$$

Equations (1) and (2) are the momentum equations in polar coordinates. The problem at hand is axisymmetric so that the tangential component of the displacement vector v_θ is assumed to be zero as well as all terms containing ∂_θ . Under these assumptions the second equation is automatically fulfilled. Of the first one remain the following terms:

$$\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} - \frac{\partial p}{\partial r} = \rho g_r$$

As we have seen before we have

$$p = -\lambda \nabla \cdot \mathbf{v} = -\lambda \left(\frac{1}{r} \frac{\partial(r v_r)}{\partial r} \right)$$

Inserting this expression into the first equation yields:

$$\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} = \frac{\rho g_0}{2\mu + \lambda}$$

We now look at the boundary conditions. On the inner boundary we prescribe $v_r(r = R_1) = 0$ while free surface boundary conditions are prescribed on the outer boundary, i.e. $\boldsymbol{\sigma} \cdot \mathbf{n} = 0$ (i.e. there is no force applied on the surface).

The components of the strain tensor are

$$\varepsilon_{rr} = \frac{\partial v_r}{\partial r} \quad (3)$$

$$\varepsilon_{\theta\theta} = \frac{v_r}{r} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} = \frac{v_r}{r} \quad (4)$$

$$\varepsilon_{r\theta} = \frac{1}{2} \left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right) = 0 \quad (5)$$

The stress tensor is given by

$$\boldsymbol{\sigma} = -p \mathbf{1} + 2\mu \boldsymbol{\varepsilon} = \lambda (\nabla \cdot \mathbf{v}) \mathbf{1} + 2\mu \boldsymbol{\varepsilon}$$

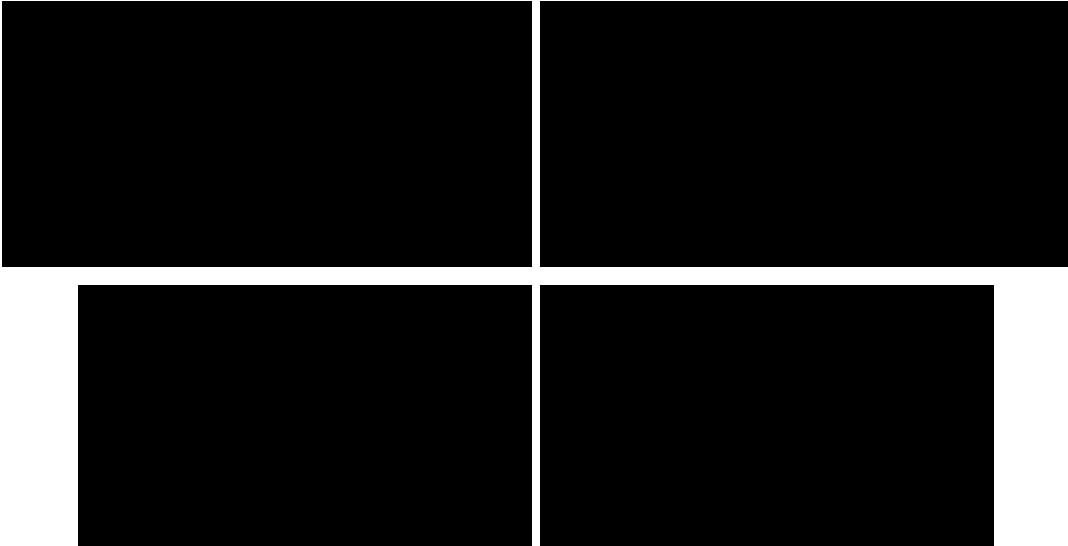
which is a diagonal tensor so that the free surface condition is:

$$\sigma_{rr} = (2\mu + \lambda) \frac{\partial v_r}{\partial r} + \lambda \frac{v_r}{r} = 0 \quad r = R_2$$

The solution is then

$$v_r(r) = \frac{\rho g_0}{3(2\mu + \lambda)} r^2 + br + \frac{c}{r}$$

where b, c are determined by means of the boundary conditions.



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