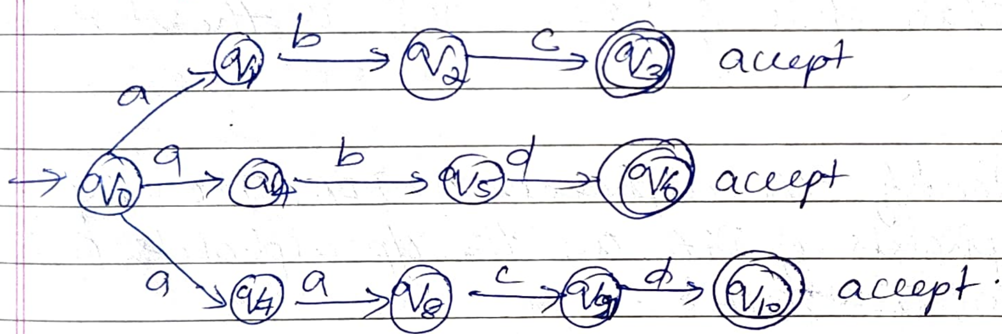
NON-DETERMINISTIC FINITE AUTOMATA (NFA)

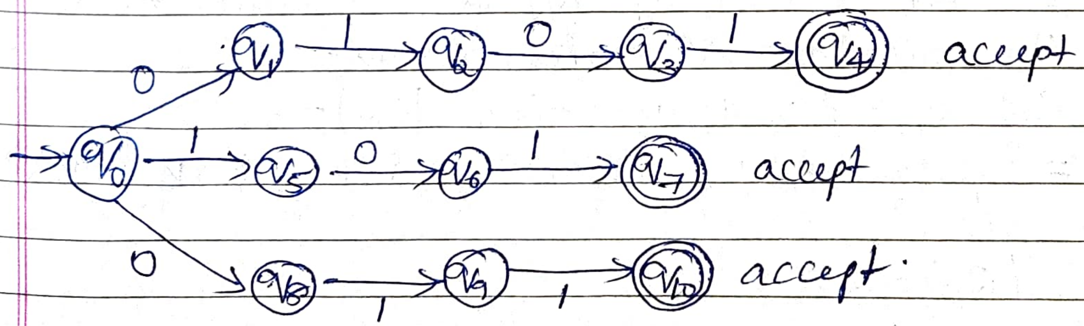
Defn: The NFA is a 5-tuple or quintuple
$M=(Q,\Sigma,\delta,q_0,F)$ where
M is the name of the m/c.
Q is non-empty, finite set of states
Σ is non-empty finite set of input alphabets.
$\delta : Q \times \Sigma$ to 2^Q i.e. δ is a transition function which is
or a mapping from $Q \times \Sigma$ to subset of 2^Q
$q_0 \in Q$ is the start state
$F \subseteq Q$ - Set of accepting states.

Ex Design an NFA to recognize the following set of strings abc, abd and aacd.

Soln



Q) Obtain NFA to recognize the following set of strings 0101, 101, 011



CONVERSION FROM NFA to DFA

- Subset construction
- Lazy evaluation

Subset Construction method :-

→ Describe the subset construction method procedure to convert an NFA into a DFA.

Soln: Given an NFA $M_N = (Q_N, \Sigma, \delta_N, q_0, F_N)$ which accepts the language $L(M_N)$, we can find an equivalent DFA $M_D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$ such that $L(M_D) = L(M_N)$

Step 1: Identify the start state of DFA. Since q_0 is the start state of NFA, $\{q_0\}$ is the start state of DFA.

Step 2: Identify the alphabets of DFA. The input alphabets of DFA are the i/p alphabets of NFA. So, $\Sigma = \{a, b\}$.

Step 3: Identify Q_D, which are the states of DFA. The set of subsets of Q_N will be the states of DFA Q_D. So, if Q_N has n states then Q_D will have 2^n states. For ex:
Let $Q_N = \{q_0, q_1, q_2\}$ then $|Q_N| = 3$.

$$Q_D = \{\underset{1}{\{\}}, \underset{2}{\{q_0\}}, \underset{3}{\{q_1\}}, \underset{4}{\{q_2\}}, \underset{5}{\{q_0, q_1\}}, \underset{6}{\{q_0, q_2\}}, \underset{7}{\{q_1, q_2\}}, \underset{8.}{\{q_0, q_1, q_2\}}\}$$

So, $|Q_D| = 8$.

In this ex, the no of states of NFA $= 3$. ∴ no of states of DFA $= 2^3 = 8$. If 'n' is the no of states of NFA, the no of states of DFA will be 2^n.

Step 4: Identify the final states of DFA. If $\{q_i, q_j -- q_k\}$ is a state in Q_D, then $\{q_i, q_j -- q_k\}$ will be the final state of DFA, provided one of $q_i, q_j -- q_k$ is the final state of NFA.

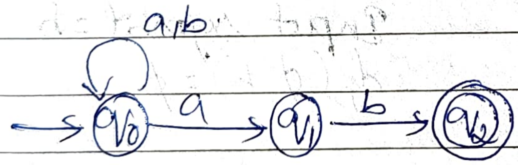
Step 5: Identify the transitions (δ_D) of DFA. For each state, $\{q_i, q_j -- q_k\}$ in Q_D and for each input symbol a in Σ, the transition can be obtained as shown below:

$$\delta_D = (\{q_i, q_j -- q_k\}, a) = \delta_N(q_i, a) \cup \delta_N(q_j, a) \cup -- \delta_N(q_k, a)$$

Thus DFA can be obtained using subset construction method.

* Obtain the DFA for the following NFA using subset construction method

Soln:



accept

Soln: The transition table for the above DFA is:

δ	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	ϕ	$\{q_2\}$
$* q_2$	ϕ	ϕ

Step 1:- Identify the start state of DFA. Since q_0 is the start state of NFA, $\{q_0\}$ is the start state of DFA.

Step 2: Identify the alphabets of DFA. The i/p alphabets of NFA are the i/p alphabets of DFA. So, $\Sigma = \{a, b\}$

Step 3: Identify Q_D which are the states of DFA.
Here, $Q_N = \{q_0, q_1, q_2\}$. Its subsets are the states of DFA. So, states of DFA are:
$Q_D = \{\phi, \{q_0\}, \{q_1\}, \{q_2\}, \{q_0, q_1\} \{q_0, q_2\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$

Step 4: Identify the final states of DFA. Since q_2 is the final state of NFA, wherever q_2 is present, the corresponding set is the final state of DFA. So,
$F_D = \{\{q_2\}, \{q_0, q_2\} \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$

Step 5: Identify the transitions (i.e δ_D) of DFA.
Obtain the transitions for each of the states of Q_D obtained in step 3 as shown below:

For state ϕ:-
Input symbol = a
$\delta_D(\phi, a) = \phi$

Input symbol = b
$\delta_D(\phi, b) = \phi$

For state $\{q_0\}$:
I/p sym = a
$\delta_D(\{q_0\}, a) = \{q_0, q_1\}$

I/p sym = b
$\delta_D(\{q_0\}, b) = \{q_0\}$

For state $\{q_1\}$:
I/p sym = a
$\delta_D(\{q_1\}, a) = \phi$

I/p sym = b.
$\delta_D(\{q_1\}, b) = \{q_2\}$

For state $\{q_2\}$:
I/p sym = a
$\delta_D(\{q_2\}, a) = \phi$

I/p sym = b
$\delta_D(\{q_2\}, b) = \phi$

For state $\{q_0, q_1\}$:

I/p sym = a

$\delta_D(\{q_0, q_1\}, a) = \delta_N(\{q_0, q_1\}, a)$
$= \delta_N(q_0, a) \cup \delta_N(q_1, a)$
$= \{q_0, q_1\} \cup \phi$
$= \{q_0, q_1\}$

I/p sym = b

$\delta_D(\{q_0, q_1\}, b) = \delta_N(\{q_0, q_1\}, b)$
$= \delta_N(q_0, b) \cup \delta_N(q_1, b)$
$= \{q_0\} \cup \{q_2\}$
$= \{q_0, q_2\}$

For state $\{q_0, q_2\}$

I/p sym = a

$\delta_D(\{q_0, q_2\}, a) = \delta_N(\{q_0, q_2\}, a)$
$= \delta_N(q_0, a) \cup \delta_N(q_2, a)$
$= \{q_0, q_1\} \cup \phi$
$= \{q_0, q_1\}$

I/p sym = b

$\delta_D(\{q_0, q_2\}, b) = \delta_N(\{q_0, q_2\}, b)$
$= \delta_N(q_0, b) \cup \delta_N(q_2, b)$
$= \{q_0\} \cup \phi = \{q_0\}$

For state $\{q_1, q_2\}$:

I/p sym = a

$\delta_D(\{q_1, q_2\}, a) = \delta_N(\{q_1, q_2\}, a)$
$= \delta_N(q_1, a) \cup \delta_N(q_2, a)$
$= \phi \cup \phi = \phi$

i/p sym = b

$\hat{\delta}_D(\{q_1, q_2\}, b) = \delta_N(\{q_1, q_2\}, b)$
$= \delta_N(q_1, b) \cup \delta_N(q_2, b)$
$= \{q_2\} \cup \phi$
$= \{q_2\}$

For state $\{q_0, q_1, q_2\}$

i/p sym = a:

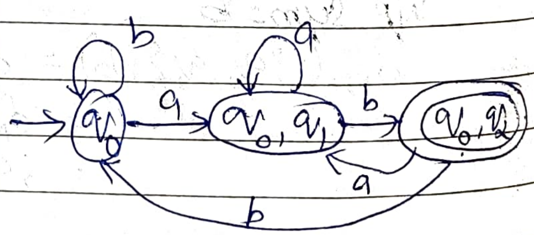
$\hat{\delta}_D(\{q_0, q_1, q_2\}, a) = \delta_N(\{q_0, q_1, q_2\}, a)$
$= \delta_N(q_0, a) \cup \delta_N(q_1, a) \cup \delta_N(q_2, a)$
$= \{q_0, q_1\} \cup \phi \cup \phi$
$= \{q_0, q_1\}$

i/p sym = b

$\hat{\delta}_D(\{q_0, q_1, q_2\}, b) = \delta_N(\{q_0, q_1, q_2\}, b)$
$= \delta_N(q_0, b) \cup \delta_N(q_1, b) \cup \delta_N(q_2, b)$
$= \{q_0\} \cup \{q_2\} \cup \phi$
$= \{q_0, q_2\}$

Now, all the above transitions can be represented using the transition table as shown below:

	δ	a	b
1	ϕ	ϕ	ϕ
2	$\rightarrow \{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
3	$\{q_1\}$	ϕ	$\{q_2\}$
4	$* \{q_2\}$	ϕ	ϕ
5	$* \{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
6	$* \{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0\}$
7	$* \{q_1, q_2\}$	ϕ	$\{q_2\}$
8	$* \{q_0, q_1, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



The states $\{q_1\}, \{q_2\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}$ are not reachable from the start state. Hence can be eliminated.

Disadvantage of Subset Construction method:-

The time complexity is exponential i.e $\Sigma \times 2^n$. It takes a very long time to construct the table.

Conversion from NFA to DFA:-

LAZY EVALUATION METHOD;

Given an NFA $M_N = (Q_N, \Sigma, \delta_N, q_0, F_N)$ which accepts the language $L(M_N)$, we can find an equivalent DFA $M_D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$ such that $L(M_D) = L(M_N)$

Step 1; Identify the start state of DFA. Since q_0 is the start state of NFA, $\{q_0\}$ is the start state of DFA.

Step 2; Identify the alphabets of DFA. The input alphabets of DFA are the input alphabets of NFA.
So, $\Sigma = \{a, b\}$.

Step 3; Identify the transitions (δ_D) of DFA. For each state $\{q_i, q_j \ldots q_k\}$ in Q_D and for each input symbol a in Σ, the transition can be obtained as shown below:

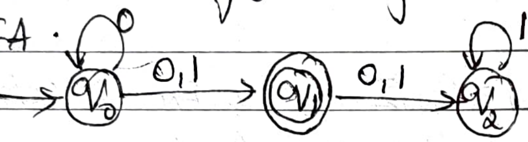
$$\delta_D(\{q_i, q_j, \ldots q_k\}, a) = \delta_N(q_i, a) \cup \delta_N(q_j, a) \cup \ldots \delta_N(q_k, a)$$
$$= [q_l, q_m \ldots q_n] \text{ say.}$$

- Add the state $[q_l, q_m, \ldots q_n]$ to Q_D, if its not already in Q_D
- Add the transition from $[q_i, q_j, \ldots q_k]$ to $[q_l, q_m \ldots q_n]$ on the input a.
- Step 3 has to be repeated for each state that is added to Q_D.

Step 4: Identify the final states of DFA. If $\{q_i, q_j - - - q_k\}$ is a state in Q_D and if one of $q_i, q_j - - - q_k$ is the final state of NFA, then $\{q_i, q_j - - - q_k\}$ will be the final state of DFA.

Problems:

1) Convert the following NFA to its equivalent DFA.



Soln:- The transition table for the above DFA is

δ_N	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$* q_1$	$\{q_2\}$	$\{q_2\}$
q_2	ϕ	$\{q_2\}$

Step 1: Identify the start state of DFA. Since q_0 is the start state of NFA, $\{q_0\}$ is the start state of DFA.

So $Q_D = \{\{q_0\}\}$

Step 2: Identify the alphabets of DFA. The input alphabets of NFA are the input alphabets of DFA. So, $\Sigma = \{0,1\}$.

Step 3: Identify the transitions $\{\delta_D\}$ of DFA. Start from the start state q_0 and find the transitions as shown below:

For state $\{q_0\}$:

I/p sym = 0

$\delta_D(\{q_0\}, 0) = \delta_N(\{q_0\}, 0)$
$= \{q_0, q_1\}$

I/p sym = 1

$\delta_D(\{q_0\}, 1) = \delta_N(\{q_0\}, 1)$
$= \{q_1\}$

The two states $\{q_0, q_1\}$ and $\{q_1\}$ are not in Q_D.
So, add them to Q_D.
$\therefore Q_D = \{\{q_0\}, \{q_0, q_1\}, \{q_1\}\}$

Consider the state $\{q_0, q_1\}$:
i/p sym = 0.
$\delta_D(\{q_0, q_1\}, 0) = \delta_N(\{q_0, q_1\}, 0)$
$= \delta_N(q_0, 0) \cup \delta_N(q_1, 0)$
$= \{q_0, q_1\} \cup \{q_2\}$
$= \{q_0, q_1, q_2\}$

i/p sym = 1
$\delta_D(\{q_0, q_1\}, 1) = \delta_N(\{q_0, q_1\}, 1)$
$= \delta_N(q_0, 1) \cup \delta_N(q_1, 1)$
$= \{q_1\} \cup \{q_2\}$
$= \{q_1, q_2\}$

These two states are added to Q_D.
$\therefore Q_D = \{\{q_0\}, \{q_0, q_1\}, \{q_1\}, \{q_0, q_1, q_2\}, \{q_1, q_2\}\}$

For state q_1:
i/p sym = 0
$\delta_D(\{q_1\}, 0) = \delta_N(\{q_1\}, 0)$
$= \{q_2\}$

i/p sym = 1
$\delta_D(\{q_1\}, 1) = \delta_N(\{q_1\}, 1)$
$= \{q_2\}$

Add these states to Q_D.
$\therefore Q_D = \{\{q_0\}, \{q_0, q_1\}, \{q_1\}, \{q_0, q_1, q_2\}, \{q_1, q_2\}, \{q_2\}\}$

For state $\{q_0, q_1, q_2\}$
i/p sym = 0.
$\delta_D(\{q_0, q_1, q_2\}, 0) = \delta_N(\{q_0, q_1, q_2\}, 0)$
$= \delta_N(q_0, 0) \cup \delta_N(q_1, 0) \cup \delta_N(q_2, 0)$
$= \{q_0, q_1\} \cup \{q_2\} \cup \emptyset$
$= \{q_0, q_1, q_2\}$

I/P sym = 1

$\delta_D(\{q_0, q_1, q_2\}, 1) = \delta_N(\{q_0, q_1, q_2\}, 1)$

$= \delta_N(q_0, 1) \cup \delta_N(q_1, 1) \cup \delta_N(q_2, 1)$

$= \{q_1\} \cup \{q_2\} \cup \{q_2\}$

$= \{q_1, q_2\}$

These two states are already in Q_D. So, ignore.

For state $\{q_1, q_2\}$:

I/P sym = 0

$\delta_D(\{q_1, q_2\}, 0) = \delta_N(\{q_1, q_2\}, 0)$

$= \delta_N(q_1, 0) \cup \delta_N(q_2, 0)$

$= \{q_2\} \cup \phi$

$= \{q_2\}$

I/P sym = 1

$\delta_D(\{q_1, q_2\}, 1) = \delta_N(\{q_1, q_2\}, 1)$

$= \delta_N(q_1, 1) \cup \delta_N(q_2, 1)$

$= \{q_2\} \cup \{q_2\}$

$= \{q_2\}$

This state is already in Q_D. So, ignore.

For state $\{q_2\}$:

I/P sym = 0

$\delta_D(\{q_2\}, 0) = \delta_N(\{q_2\}, 0)$

$= \phi$

I/P sym = 1

$\delta_D(\{q_2\}, 1) = \delta_N(\{q_2\}, 1)$

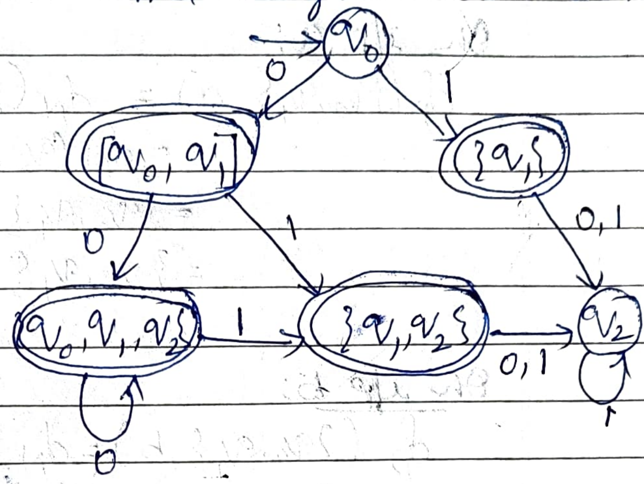
$= \{q_2\}$

This state is already in Q_D. So ignore

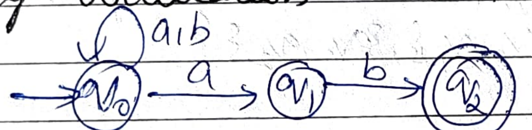
$Q_D = \{\{q_0\}, \{q_0, q_1\}, \{q_1\}, \{q_0, q_1, q_2\}, \{q_1, q_2\}, \{q_2\}\}$

The final transition table and diagram are shown:

δ	0	1
$\rightarrow\{q_0\}$	$\{q_0, q_1\}$	$\{q_1\}$
$*\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
$*\{q_1\}$	$\{q_2\}$	$\{q_2\}$
$*\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
$*\{q_1, q_2\}$	$\{q_2\}$	$\{q_2\}$
$\{q_2\}$	ϕ	$\{q_2\}$



2) Obtain the DFA for the following NFA using lazy evaluation method.



Step 1:

Soln. Identify the start state of DFA. Since q_0 is the start state of NFA, $\{q_0\}$ is the start state of DFA.

So, $Q_D = \{\{q_0\}\}$

Step 2: Identify the alphabets of DFA. The i/p alphabets of NFA are the i/p alphabets of DFA. So, $\Sigma = \{a, b\}$

Step 3: Identify the transitions δ_D of DFA.

Consider the state $\{q_0\}$

On i/p a:

$\delta_D(\{q_0\}, a) = \delta_N(\{q_0\}, a)$
$= \{q_0, q_1\}$

On i/p b:

$\delta_D(\{q_0\}, b) = \delta_N(\{q_0\}, b)$
$= \{q_0\}$

Since these ~~two~~ states are not in Q_D, add them

$\therefore Q_D = \{\{q_0\}, \{q_0, q_1\}\}$

Consider the state $\{q_0, q_1\}$

On i/p a:

$\delta_D(\{q_0, q_1\}, a) = \delta_N(\{q_0, q_1\}, a)$

$= \delta_N(q_0, a) \cup \delta_N(q_1, a)$

$= \{q_0, q_1\} \cup \phi$

$= \{q_0, q_1\}$

On i/p b:

$\delta_D(\{q_0, q_1\}, b) = \delta_N(\{q_0, q_1\}, b)$

$= \delta_N(q_0, b), \cup \delta_N(q_1, b)$

$= \{q_0\} \cup \{q_2\}$

$= \{q_0, q_2\}$

Add $\{q_0, q_2\}$ to Q_D since it is not present.

$\therefore Q_D = \{\{q_0\}, \{q_0, q_1\}, \{q_0, q_2\}\}$

Consider the state $\{q_0, q_2\}$:

On i/p a:

$\delta_D(\{q_0, q_2\}, a) = \delta_N(\{q_0, q_2\}, a)$

$= \delta_N(q_0, a) \cup \delta_N(q_2, a)$

$= \{q_0, q_1\} \cup \phi$

$= \{q_0, q_1\}$

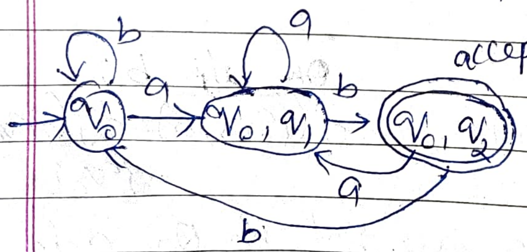
On i/p b:

$\delta_D(\{q_0, q_2\}, b) = \delta_N(\{q_0, q_2\}, b)$

$= \delta_N(q_0, b) \cup \delta_N(q_2, b)$

$= \{q_0\} \cup \phi = \{q_0\}$

$\therefore Q_D = \{\{q_0\}, \{q_0, q_1\}, \{q_0, q_2\}\}$



	a	b
$\rightarrow\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$*\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0\}$