Elements of Logic

Logical Language

Definitions: *A triangle is a polygon with three sides.*
Constraints: *Parents are older than their children.*
Partial Information: *Abby likes one of Cody or Dana.*

Logical Reasoning

Model Checking - truth tables, logic grids
Symbolic Manipulation - formula transformations, proofs

"Metalevel" Concepts and Analysis

Validity, Contingency, Unsatisfiability
Equivalence, Entailment, Consistency
Soundness, Completeness, Decidability

23

Rules of Inference

A *rule of inference* is a reasoning pattern consisting of some premises and some conclusions.

In other words, if we believe the premises, a rule of inference tells us that we should also believe the conclusions.

Symbolic manipulation rather than model checking.

24

Sample Rule of Inference

All of Abby's friends are Bess's friends.
All of Bess's friends are Cody's friends.
***Therefore**, all of Abby's friends are Cody's friends.*

25

Sample Rule of Inference

All Accords are Hondas.
All Hondas are Japanese.
***Therefore**, all Accords are Japanese.*

26

General Rule of Inference

All *x are y.*
All *y are z.*
Therefore, ***all*** *x are z.*

27

Unsound Rule of Inference

All x are y.
Some y are z.
Therefore, *some x are z.*

No! No!! No!!!

28

Using Unsound Rule of Inference

All Toyotas are Japanese cars.
Some Japanese cars are made in America.
***Therefore**, some Toyotas are made in America.*

Sometimes produces a result that *happens* to be true.

29

Using Unsound Rule of Inference

All Toyotas are cars.
Some cars are Porsches.
***Therefore**, some Toyotas are Porsches.*

Sometimes produces a result that *happens* to be false.

30

Provability

A set of premises *logically entails* a conclusion if and only if every world that satisfies the premises satisfies the conclusion.

A conclusion is *provable* from a set of premises if and only if there is a finite sequence of sentences in which every element is either a premise or the result of applying a *sound* rule of inference to earlier members in the sequence.

31

Soundness and Completeness

As we shall see, for well-behaved logics, logical entailment and provability are identical - a set of premises **logically entails** a conclusion *if and only if* the conclusion is **provable** from the premises.

This is a very big deal.

32

Deduction

A rule of inference is *sound* if and only if the conclusion is true whenever the premises are true.

The application of sound rules of inference is called *deduction*.

33

Induction

Induction is reasoning from the specific to the general.

I have seen 1000 black ravens.
I have never seen a raven that is not black.
Therefore, every raven is black.

Induction is not necessarily sound (but it can be useful).

34

Induction versus Deduction

Induction is the basis for **Science** (and machine learning)
Deduction is the subject matter of **Logic**.

Science aspires to discover / propose **new** knowledge.
Logic aspires to apply and/or analyze **existing** knowledge.

35

Multiple Logics

Propositional Logic (logical operators)

If *it is raining* ***and*** *it is cold,* ***then*** *the ground is wet.*

Relational Logic (variables and quantifiers)

If ***x*** *is a parent of* ***y****, then* ***x*** *is older than* ***y****.*

Functional Logic (compound terms)

$\{a, b\}$ *is a subset of* $\{a, b, c\}$.

36