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Calculating non-dimensional stability functions for the momentum and heat transport in the convective atmospheric boundary layer based on Cabauw database

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1 Introduction

The aim of the exercise is to use the Monin-Obukhow theory for calculating non-dimensional stability functions for momentum and heat transport in the convective boundary layer in the atmosphere. The data measured in the Cabauw meteorological station is analyzed. The Obukhov length and the dimensionless height parameter are calculated. Stability functions for the momentum and heat transport are calculated and plotted against the non-dimensional height parameter. Results are compared with predictions of the standard Monin-Obukhov theory.

2 Atmospheric Boundary Layer

The atmospheric boundary layer (ABL) is a layer in the atmosphere, with depth of the order of 1 km above Earth's surface. It is directly influenced by the presence of Earth's surface and responds to surface forcings with a timescale of about an hour or less (Stull, 1988). Its representation is crucial for numerical weather predictions as well as climate projections. One of the key characteristics of the ABL over land is its diurnal variation. The ABL takes two forms, a typical day-time state which is convective and a typical night-time state, which is stable. The day-time convective state is called a mixed layer. Its growth is tied to solar heating of the ground. It develops about half an hour after the sunrise and grows in depth during the day. It is characterized by intense mixing, with thermals of warm air rising from the ground. About half an hour before the sunset, thermals cease to form and turbulence decays. A neutrally stratified residual layer is formed. In the night, the bottom of the residual layer is transformed into a stable layer, with reduced turbulence. The diurnal evolution of the ABL is presented in figure 1. This exercise focuses on the convective, day-time regime of the ABL.

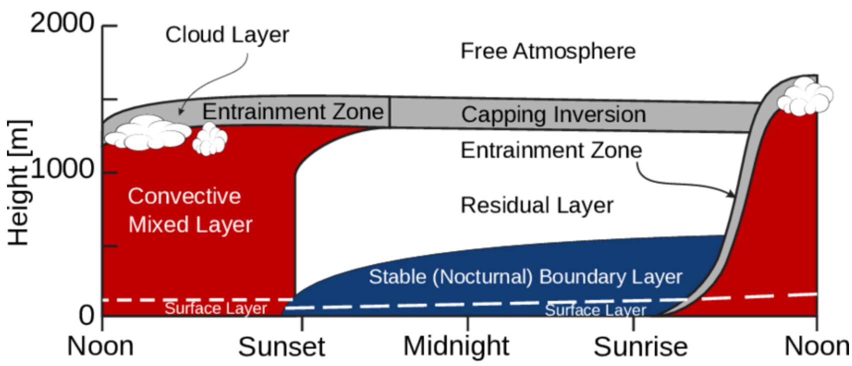

Figure 1: The diurnal cycle of the atmospheric boundary layer over land (Allaerts, 2016).

3 Turbulent fluxes

The meteorology of the boundary layer often concerns fluxes of mass, heat, moisture and momentum. Flux is defined as the transfer of a quantity per unit area per unit time. In most of the boundary layer, the vertical velocity is approximately equal to zero and thus vertical advective fluxes are negligible, but vertical turbulent fluxes are significant. Turbulent fluxes of momentum τ and turbulent fluxes of heat H are

$$\tau = -\rho\langle u_i' u_j' \rangle, \tag{1}$$

$$H = c_p \rho \langle u_i' \theta' \rangle, \tag{2}$$

where ρ is the air density, c_p is the specific heat capacity of the air at a constant pressure, u'_i and u'_j are turbulent velocity components (calculated with Reynolds decomposition) and θ' is the turbulent part of the potential temperature. The kinematic form of fluxes is often used. Kinematic turbulent fluxes of momentum are $\langle u'_i u'_j \rangle$. Kinematic turbulent fluxes of heat are $\langle u'_i \theta' \rangle$. This exercise concerns vertical turbulent fluxes which are measured at the Earth's surface.

4 Monin-Obukhov theory

A useful scaling parameter for the ABL is the Obukhov length L:

$$L = -\frac{|\langle u'w' \rangle_s|^{3/2}}{k \langle w'b' \rangle_s} = -\frac{\theta^* |\langle u'w' \rangle_s|^{3/2}}{k\ g\ \langle \theta' w' \rangle_s}, \tag{3}$$

where $k \approx 0.4$ is the von Karman constant, $\langle u'w' \rangle_s$ is the vertical kinematic momentum flux at the surface, $\langle w'b' \rangle_s = g \langle \theta' w' \rangle_s / \theta^*$ is the buoyancy flux at the surface, where $\langle \theta' w' \rangle_s$ is the kinematic heat flux at the surface, g is the gravitational constant and θ^* is the average surface temperature. The Obukhov length is proportional to the height above the surface at which buoyant factors first dominate over shear production of turbulence (Stull, 1988). The Obukhov length is useful for determining stability of the surface layer. $L < 0$ corresponds to a positive buouancy flux, which indicates that the layer is statically unstable. $L > 0$ corresponds to a negative buoyancy flux, indicating that the layer is statically stable. For neutral conditions, the buoyancy flux is close to zero and thus the absolute value of L is very large. Smaller $|L|$ corresponds to larger deviations from neutral conditions. This exercise is focused on the unstable, convective regime ($L < 0$).

A dimensionless height parameter ζ can be introduced, such that

$$\zeta = \frac{z}{L}, \tag{4}$$

where z is the height above the surface.

The Monin–Obukhov similarity theory describes non-dimensionalized gradients of the mean horizontal flow velocity and the mean potential temperature in the surface layer under non-neutral conditions, as functions of the dimensionless height parameter. The dimensionless wind shear ϕ_m and the dimensionless lapse rate ϕ_h are introduced:

$$\phi_m = \frac{d\bar{U}}{dz} \frac{z\ k}{|\langle u'w' \rangle_s|^{1/2}}, \tag{5}$$

$$\phi_h = \frac{d\bar{\theta}}{dz} \frac{z\ k}{|\langle \theta' w' \rangle_s|} |\langle u'w' \rangle|_s^{1/2}, \tag{6}$$

where $d\bar{U}/dz$ is the gradient of the horizontal mean flow velocity and $d\bar{\theta}/dz$ is the gradient of the mean potential temperature.

The following universal functions for ϕ_m and ϕ_h can be determined based on experimental data (Businger et al., 1971):

$$\phi_m(\zeta) = (1 - 15\zeta)^{-1/4} \quad \text{for } -2 < \zeta < 0, \tag{7}$$

$$\phi_m(\zeta) = 1 + 4.7\zeta \quad \text{for } 0 < \zeta < 1, \tag{8}$$

$$\phi_h(\zeta) = 0.74(1 - 9\zeta)^{-1/2} \quad \text{for } -2 < \zeta < 0, \tag{9}$$

$$\phi_h(\zeta) = 0.74 + 4.7\zeta \quad \text{for } 0 < \zeta < 1. \tag{10}$$

5 The Cabauw station

The Cabauw station is located in a rural area in the western part of The Netherlands, as described on the website of the observatory ("Ruisdael Observatory", 2024). On the station there is a mast of the height 213 m, on which instruments for remote sensing and in-situ measurements are installed. It allows for measuring vertical profiles of aerosol properties, clouds, humidity, temperature and wind velocity, as well as total column measurements and measurements of turbulent surface fluxes.

For the purpose of this exercise, the measured mean wind velocity, mean temperature and mean surface momentum and heat fluxes are found in the Cabauw database. Data from the whole month of August 2022 is analyzed.

The mean temperature and the mean wind velocity are measured on 7 different altitudes along the mast, but in this exercise only data from altitudes of 10 m and 20 m are considered. The measurements of mean temperature and velocity, as well as mean turbulent fluxes at the surface are done every 10 minutes.

It is assumed in this exercise that the potential temperature is the same as the temperature, because measurements are done relatively close to the surface.

6 Results

The dimensionless height parameter ζ, the dimensionless wind shear ϕ_m and the dimensionless lapse rate ϕ_h are calculated from the Caubauw data. It is assumed that gradients $d\bar{U}/dz$ and $d\bar{\theta}/dz$ are approximately equal to $\Delta\bar{U}/\Delta z$ and $\Delta\bar{\theta}/\Delta z$, where $\Delta\bar{U}$ and $\Delta\bar{\theta}$ are differences between measurements at heights of 20 m and 10 m. Figures 2 and 3 show the results, restricted to the range of ζ for which universal functions (equations 7, 8, 9, 10) are defined. In addition, the plotted values of ϕ_m and ϕ_h are restricted to the range between -5 and 10. It is worth noting that many data points lie outside of the presented ranges. The universal functions are plotted alongside the data.

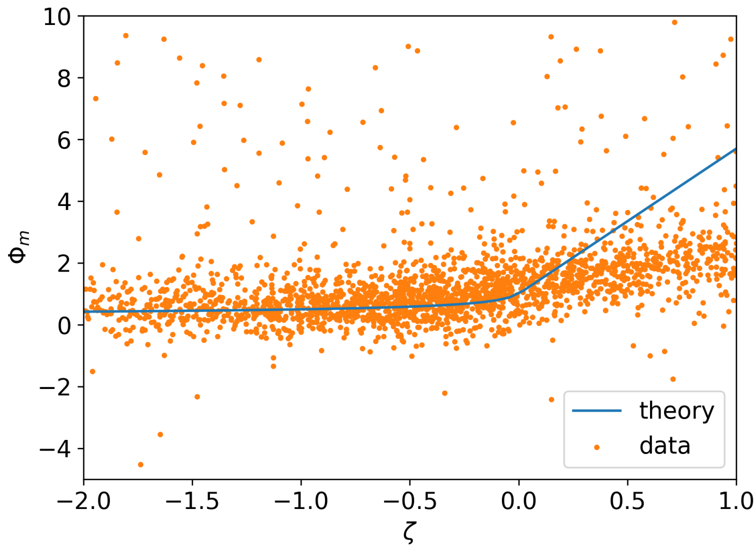

Figure 2: The dimensionless wind shear ϕ_m as a function of the dimensionless height parameter ζ, calculated from the Cabauw data (orange dots) and the universal function (the blue line).

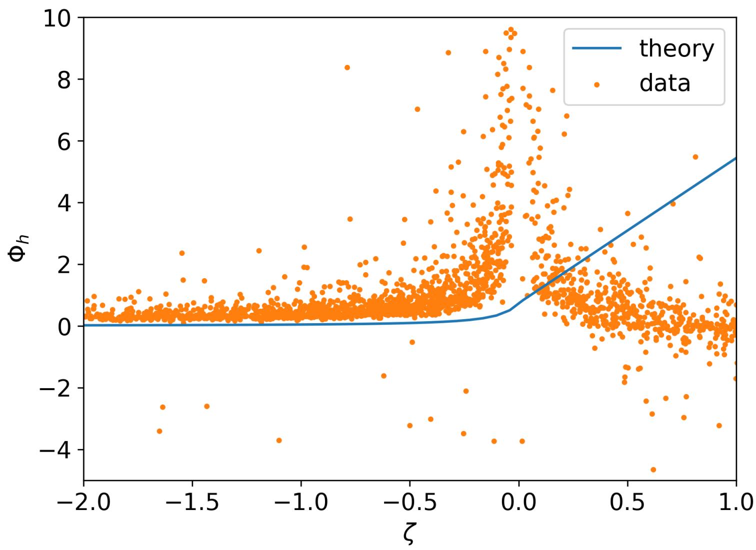

Figure 3: The dimensionless lapse rate ϕ_h as a function of the dimensionless height parameter ζ, calculated from the Cabauw data (orange dots) and the universal function (the blue line).

Figures 4 and 5 show the same data as figures 2 and 3, but restricted to the convective regime ($\zeta < 0$).

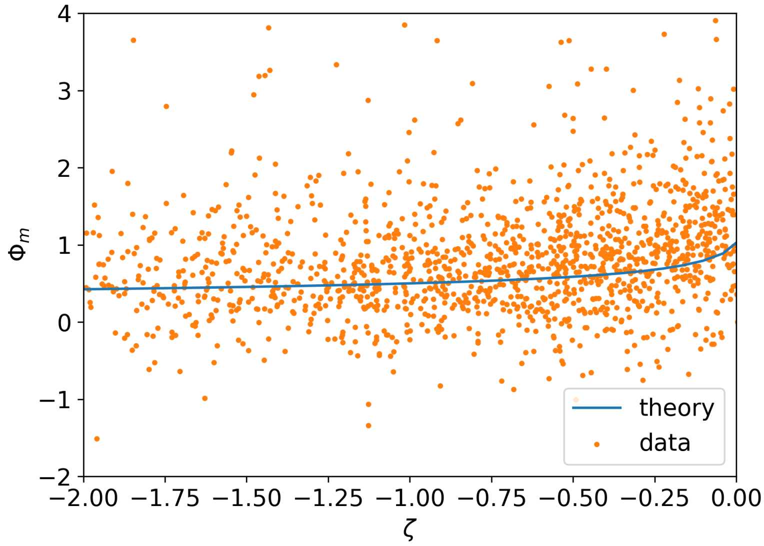

Figure 4: The same as figure 2, but restricted to the convective regime.

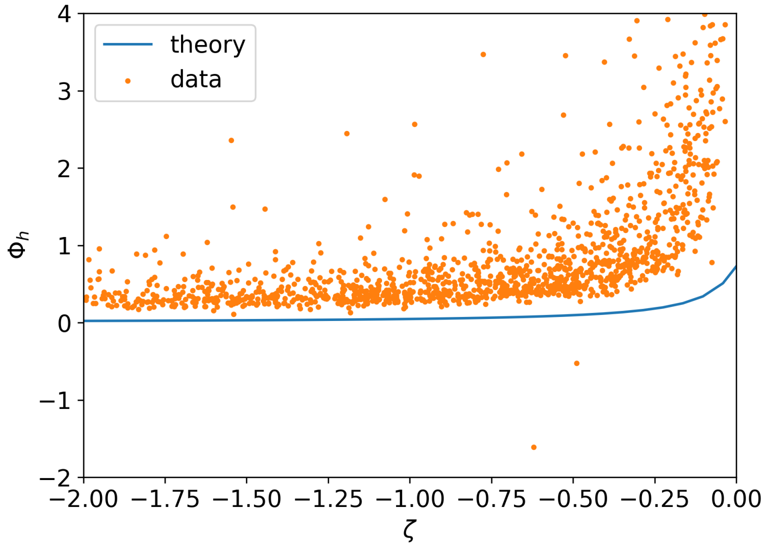

Figure 5: The same as figure 3, but restricted to the convective regime.

In the convective regime, dimensionless gradients of velocity and temperature calculated from the Cabauw data are roughly in agreement with universal functions. For ϕ_m, most of the data points are located closely around the theoretical curve. For ϕ_h, most of the data points are slightly above the theoretical curve, thus the dimensionless gradient of temperature is larger than the one predicted by the Monin-Obukhov theory. In the region close to $\zeta = 0$, most of the data points for ϕ_h deviate significantly from the theoretical curve.

In the stable regime ($\zeta > 0$), deviations from universal functions are much more significant than in the convective regime. For the dimensionless lapse rate, there is a peak in $\zeta \approx 0$, which differs significantly from the prediction of the Monin-Obukhov theory. This peak can be explained by the fact that $\zeta \approx 0$ (very large $|L|$) corresponds to neutral conditions, for which the buoyancy flux is close to zero. The momentum flux may also be close to zero. Thus, uncertainties of measurements of momentum and heat fluxes may be of the same order as the fluxes. This results in calculating a ratio of two very small numbers with relatively large uncertainties in equation 6, which in turn results in unrealistic values of ϕ_h.

Data points which are outside of ranges presented in figures 2 and 3 can be considered outliers and can be results of measurement errors. In addition, the data in the Cabauw station is measured every 10 minutes, which can be insufficient to obtain convergence, because turbulent fluctuations with longer timescales are not detected. This may be important especially for the heat flux. The measured $|\langle \theta' w' \rangle_s|$ may be smaller than its actual value, because fluctuations with long time periods are not measured. This can cause the calculated values of ϕ_h to be larger than the universal function.

7 Conclusions

The Obukhov length, the dimensionless height parameter, the dimensionless wind shear and the dimensionless lapse rate were calculated from the data measured in the Caubauw station in August 2022. The dimensionless wind shear and the dimensionless lapse rate were plotted as functions of the height parameter and compared with universal functions from the Monin-Obukhov theory. It was found that in the convective regime, the data is mostly in agreement with the Monin-Obukhov theory (with the exception of the the height parameter close to zero, where the dimensionless lapse rate deviates from the theory). In the stable regime, the data differs significantly from predictions of the Monin-Obukhov theory.

References

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