

1. Postulate that the solutions of the Dirac Equation have form of the plane wave and a spinor: $\psi(x^\mu) = u(p^\mu)e^{-i(Et - \vec{p} \cdot \vec{x})}$ and write out the DE for spinors.
2. Use the previous result and find four solutions of Dirac Equation for a particle with momentum \vec{p} and mass m .
3. Define the properties of Dirac spinors using their transformation properties i.e. apply \hat{P} and \hat{C} operator on any of u_i functions.
4. Show that for a particle/antiparticle with momentum $\vec{p} = (0,0,p)$ the u_1 and v_1 spinors represent spin up states and u_2 and v_2 spin down states.
5. Using the properties of the γ matrices and the definition of γ^5 , show that: $(\gamma^5)^2 = 1$, $\gamma^{5\dagger} = \gamma^5$, $\{\gamma^5, \gamma^\mu\} = 0$ where $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$.
6. Show that the chiral projection operators $P_L = \frac{1}{2}(1 - \gamma_5)$ and $P_R = \frac{1}{2}(1 + \gamma_5)$ satisfy: $P_L P_R = P_R P_L = 0$, $P_L + P_R = 1$, $P_L P_L = P_L$, $P_R P_R = P_R$, $P_L P_R = 0$. These conditions prove that these operators are projectors (applying one of them twice gives the same result as applying it once and that applying both of them results in the null state).