

Introduction to Probability, Statistics and Data Handling	Random Variables and Probability Distribution
Tutorial 3	

- Let  $X$  denote the number of boys in a randomly selected three-child family. Assuming that boys and girls are equally likely, construct the probability distribution of  $X$ .
- Robin Hood has three arrows and plans to hunt for a boar in Scherwood Forest. There is 0.7 probability that he hits. He shoots until he hits or exhausts his weapon. Find the probability distribution and expected value of the number of his shots.
- You apply for a job in a factory that claims that the average salary is above 3,500€. With a sad surprise you see that your first earning is 2,000€. Do you have a strong proof that the ad was a fake? Construct a probability distribution for a random variable that describes salary for a staff that consist of 100 workers with 2,000€, 10 managers that earn 10,000€ and two directors with 50,000€. What parameter should you rather asked for when applying for a job?
- Five thousand lottery tickets are sold for \$1 each. One ticket will win \$1,000, two tickets will win \$500 each, and ten tickets will win \$100 each. Let  $X$  denote the net gain from the purchase of a randomly selected ticket.
  - Construct the probability distribution of  $X$
  - Compute the expected value  $E(X)$  of  $X$ . Interpret its meaning.
  - Compute the standard deviation  $\sigma$  of  $X$ .
- A waiter at the restaurant starts shifts with 0 dollars. Every guest gives him 5 or 10 dollars as a tip, with 5 dollar tip being 2 times more probable than 10 dollar one. One day the waiter served 4 guests. Calculate :
  - probability of waiter getting less than 30 dollars in tip.
  - expected sum of tips after 4 guests.
- The amount of time, in minutes, that a person must wait for a bus is uniformly distributed between zero and 15 minutes, inclusive.
  - What is the probability that a person waits fewer than 12.5 minutes?
  - On the average, how long must a person wait? Find the mean,  $\mu$ , and the standard deviation,  $\sigma$ .
  - Ninety percent of the time, the time a person must wait falls below what value?
- Given that the random variable  $X$  has density function:

$$f(x) = \begin{cases} 2x, & 0 < x < a \\ 0, & \text{otherwise} \end{cases}$$

- Determine  $a$ .
  - Find:  $P\left(\frac{1}{2} < X < \frac{3}{4}\right)$  and  $P\left(-\frac{1}{2} < X < \frac{1}{2}\right)$ .
- Determine the parameter  $\lambda$  in PDF given by the formula:  $f(x) = \lambda e^{-\lambda x}$ . Calculate the probabilities in the interval of  $x$   $\left(\frac{1}{2} < X < 5\right)$  and  $\lambda = 2$ .