

# IOITC 2016 TST Day 2

## Alice and Bracket Sequences

Annoyed that Alice is very good at permutations, Dori decided to challenge her with her forte - bracket sequences. There are  $n$  different types of matching brackets available and Dori has one pair of each type with her. Denote the brackets that Dori has by  $A_1, B_1, \dots, A_n, B_n$ . Here,  $A_i$  is the  $i^{th}$  opening bracket and  $B_i$  is the corresponding closing bracket. Now, she considers Proper Bracket Sequences (PBSes) with these bracket pairs.

Formally, we define PBSes as follows. For every  $i$ , the sequence " $A_i, B_i$ " is a PBS. For two PBSes  $X$  and  $Y$ , the sequence " $X, Y$ " which is a concatenation of the two, is also a PBS. Finally, if  $X$  is a PBS, then " $A_i, X, B_i$ " is a PBS for all  $i$ . That is, enclosing a PBS within a pair of brackets results in a PBS.

Suppose  $A_1 = \{, B_1 = \}$ ,  $A_2 = [, B_2 = ]$ ,  $A_3 = ($  and  $B_3 = )$ , then the sequence " $()$ " is a PBS. " $[]$ " is a PBS sequence. " $()[]$ " is a concatenation of two PBSes, and hence is also a PBS. " $\{()[]\}$ " is a PBS because a PBS is enclosed within  $\{\}$ .

We are only interested in PBSes which have exactly one pair of brackets of each of the  $n$  types.

Dori defines something known as the indexing permutation. For a given PBS  $X$ , the indexing permutation of  $X$  is  $p_1, q_1, p_2, q_2, \dots, p_n, q_n$  where  $p_i$  denotes the index of  $A_i$  in  $X$  and  $q_i$  denotes the index of  $B_i$  in  $X$ . Here, the indices of  $X$  run from 1 to  $2n$ . So,  $p_1, q_1, \dots, p_n, q_n$  is a permutation of  $1, 2, \dots, 2n$ .

Suppose  $A_1 = \{, B_1 = \}$ ,  $A_2 = [, B_2 = ]$ ,  $A_3 = ($  and  $B_3 = )$ , " $\{()[]\}$ " is a PBS and its indexing permutation would be 1 6 4 5 2 3.

Finally, Dori defines the rank of an indexing permutation as follows:

Take all the indexing permutations of PBSes achievable using  $A_1, B_1, \dots, A_n, B_n$  and sort them in lexicographic order. The rank of an indexing permutation is defined to be one more than the number of indexing permutations that are lexicographically smaller than it. Note that indexing permutations are defined only for PBSes. Dori gives Alice the indexing permutation of a PBS and asks her the rank of this indexing permutation among all indexing permutations of this size. This time, Alice is truly stumped. Please help her find the rank and since the answer can be too large, print it modulo  $10^9 + 7$ .

### Note:

- For any positive integer  $m$ , a permutation  $p_1, p_2, \dots, p_m$  is lexicographically smaller than  $q_1, q_2, \dots, q_m$  if there exists an integer  $i < m$  such that  $p_1 = q_1, \dots, p_i = q_i$  and  $p_{i+1} < q_{i+1}$
- For any positive integers  $a, b$  such that  $b$  divides  $a$ , we have  $(\frac{a}{b})\%(10^9 + 7) = (a * b^{10^9+5})\%(10^9 + 7)$ .

### Input

The first line contains  $n$ .

The next  $n$  lines contain two integers each, the opening and closing indices of the  $i^{th}$  bracket: ie.  $p_i$  and  $q_i$ .

### Output

Output the rank of the bracket sequence modulo  $10^9 + 7$ .

### Test Data

#### Subtask 1 (9 Points):

- $1 \leq n \leq 50$

**Subtask 2 (23 Points):**

- $1 \leq n \leq 500$

**Subtask 2 (68 Points):**

- $1 \leq n \leq 2000$

**Sample Input1**

```
2
3 4
1 2
```

**Sample Output1**

```
4
```

**Sample Input2**

```
2
1 2
3 4
```

**Sample Output2**

```
1
```

**Sample Input3**

```
3
1 4
5 6
2 3
```

**Sample Output3**

```
6
```

**Limits**

Time: 4 seconds

Memory: 512 MB