

# Time Series Anomaly Detection with Quantum Variational Rewinding

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★ [github.com/AgnostiqHQ/covalent](https://github.com/AgnostiqHQ/covalent)

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# PENNYLANE

(An open-source framework for QML and QC.)

```
import pennylane as qml

observables = [
    qml.Pauliz(0) @ qml.Pauliz(1),
    qml.Pauliz(0) @ qml.PauliX(1),
    qml.PauliX(0) @ qml.Pauliz(1),
    qml.PauliX(0) @ qml.PauliX(1),
]

dev = qml.device("default.qubit", wires=2, shots=8192)

@qml.qnode(dev)
def chsh_circuit(theta):
    # Prepare Bell state.
    qml.Hadamard(wires=0)
    qml.CNOT(wires=[0, 1])

    # Apply Y-rotation by angle `theta`.
    qml.RY(theta, wires=0)

    # Multiple returns to get all 4 expectation values.
    return [qml.expval(obs) for obs in observables]
```

observables as Python objects

a device that defines the execution backend

circuit definitions as Python functions

# Covalent.

- Parallelism
- Remote task execution
- Orchestration and management



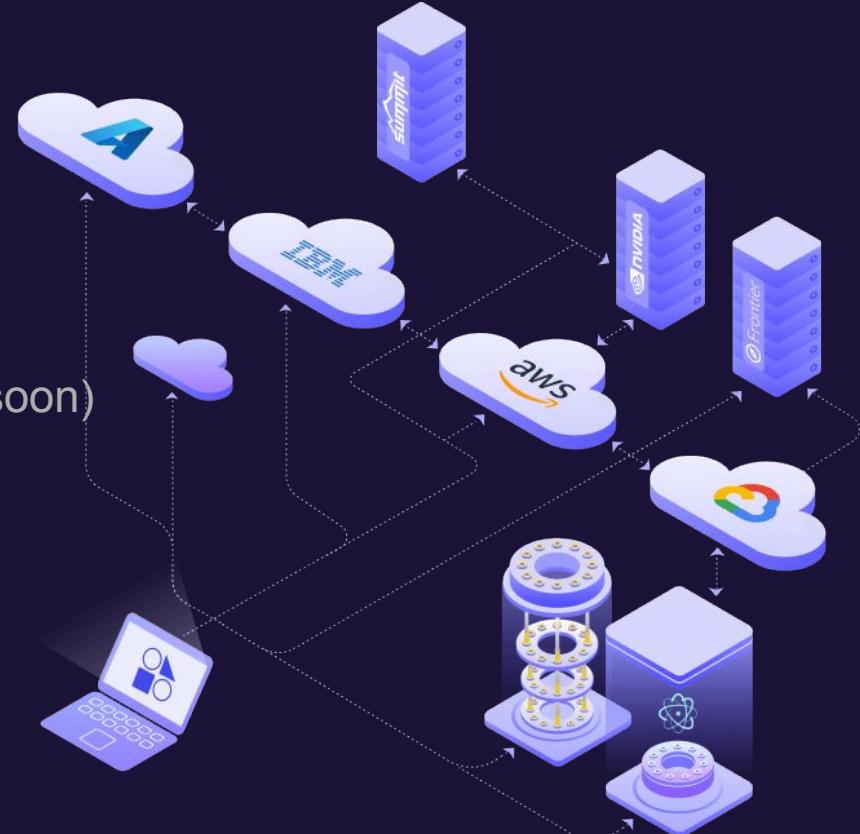
Ship your code anywhere...

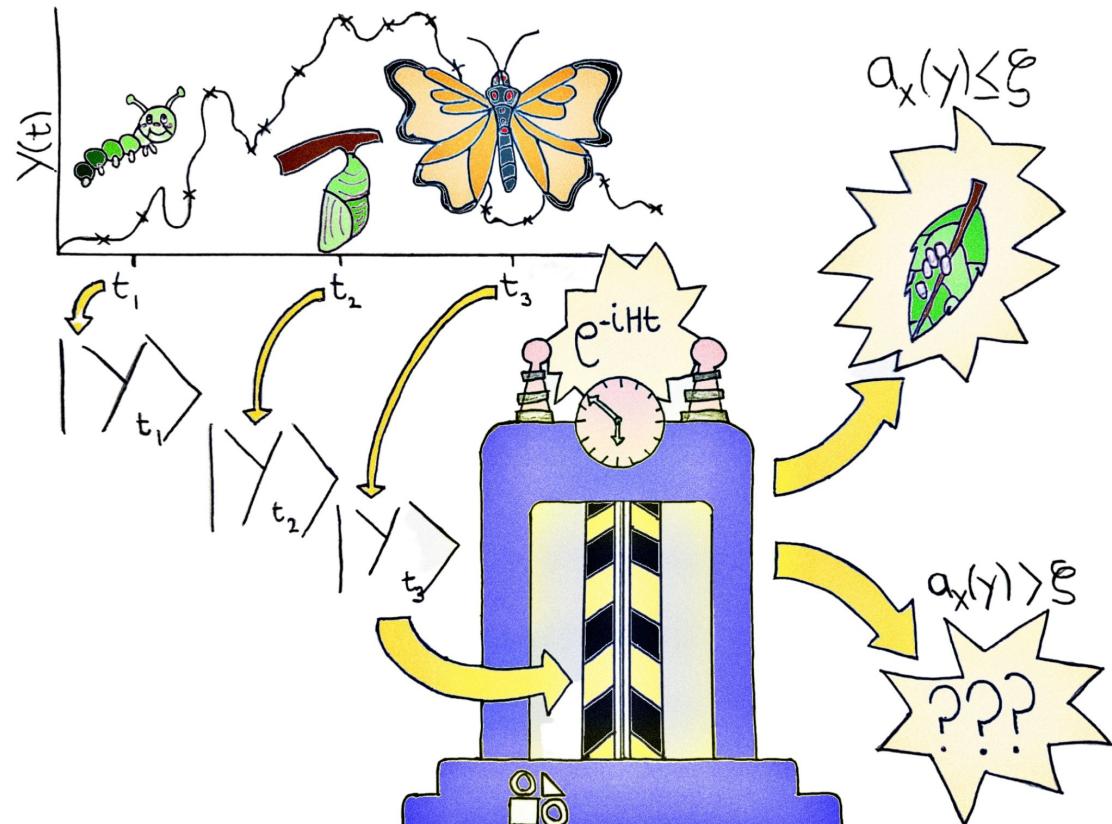
```
1 def cnn(x, y):
2     ...
3
4 def train(model):
5     tf.train(model)
6     return model
7
8 @qml.qnode(dev)
9 def qsvm(x, y):
10    ...
11
12 def workflow(inputs):
13     model1 = cnn(inputs[0], inputs[1])
14     model2 = qsvm(inputs[0], inputs[1])
15     return train(model1),train(model2)
16
17 print(workflow([1,2]))
```

```
1 @ct.electron(AWSBatchExecutor(vcpu=32))
2 def cnn(x, y):
3     ...
4
5 @ct.electron(SlurmExecutor(options))
6 def train(model):
7     tf.train(model)
8     return model
9
10 @ct.qelectron(QiskitExecutor(backend="ibmq_manila"))
11 @qml.qnode(dev)
12 def qsvm(x, y):
13     ...
14
15 @ct.lattice
16 def workflow(inputs):
17     model1 = cnn(inputs[0], inputs[1])
18     model2 = qsvm(inputs[0], inputs[1])
19     return train(model1),train(model2)
20
21 job_id = ct.dispatch(workflow)([1,2])
22 print(ct.get_result(job_id))
```

# Covalent Concepts Summary

1. Define workflow tasks (functions)
    - **@ct.electron** - classical
    - **@ct.qelectron** - quantum (coming)
  2. Define the workflow (a special function)
    - **@ct.lattice** - made up of electrons





# Let's talk about QML and Quantum Variational Rewinding

# Quantum Machine Learning

What is it ?

A computer program can be considered a learner if it improves its performance on a given set of *tasks (T)*, as measured by a *performance metric (P)*, with increasing *experience (E)*.

*experience(E)*



- Supervised data
- Unsupervised data

**Data**

*tasks(T)*



- Classification
- Regression
- Transcription
- Generation
- Anomaly Detection
- .....

**Model**

*performance metric(P)*



- Accuracy
- Error rate
- Regression Loss
- Score points (Games)
- ...

**Metric**

# Quantum Machine Learning

Quantum Variational Algorithms

Ritz Method  
(Variational method)

$$E_0 \leq \boxed{\frac{\langle \Psi(\theta) | \hat{H} | \Psi(\theta) \rangle}{\langle \Psi(\theta) | \Psi(\theta) \rangle}} := \langle \hat{O}(\theta) \rangle$$

Efficiently computed  
on quantum computer



Observables inform *quantum* performance metrics

# Time Series Anomaly Detection

Time series anomaly detection is a statistical problem that aims to identify data points in a time series that deviate significantly from the expected pattern or distribution.

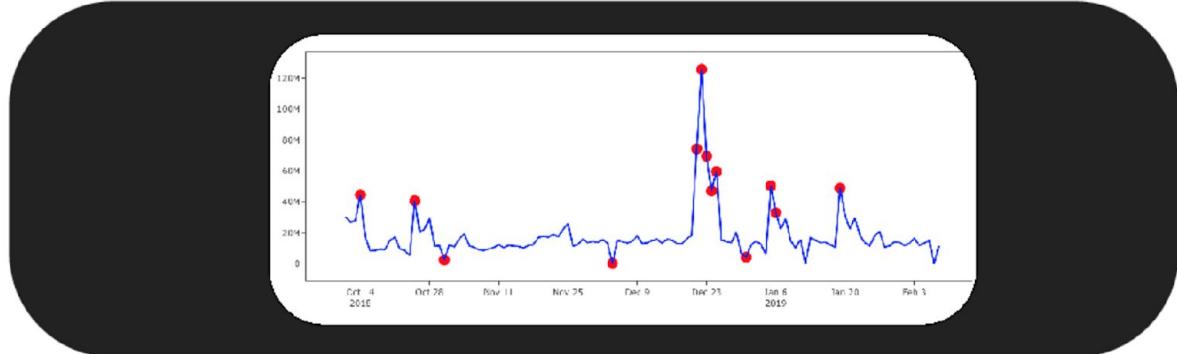
Intuition -

- Credit card fraud detection while (traveling ?)
- Single collider events - high energy physics
- Financial markets
- Natural events - weather, earthquakes etc...
- Heart rate
- Brain wave

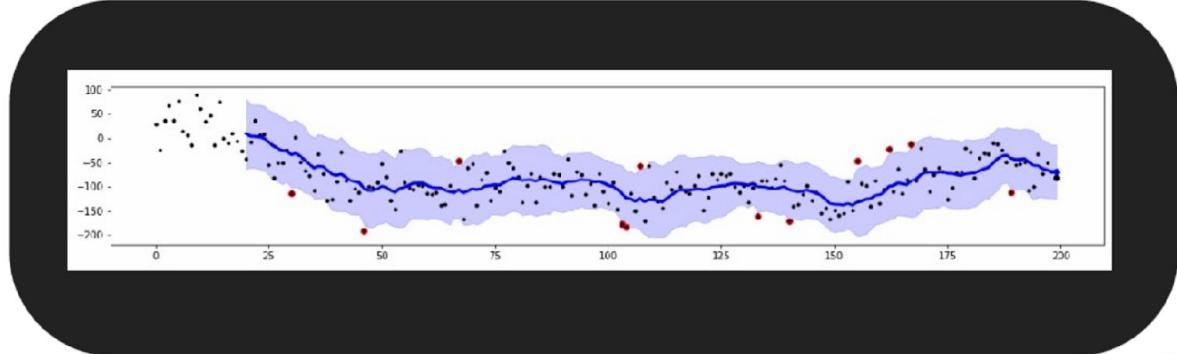
# Time Series Anomaly Detection

*Examples*

Obvious



Not-so obvious



# Time Series Anomaly Detection

## Data

$$x_{t_1}, x_{t_2}, \dots x_{t_n}$$

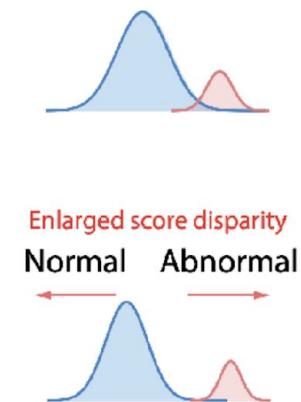
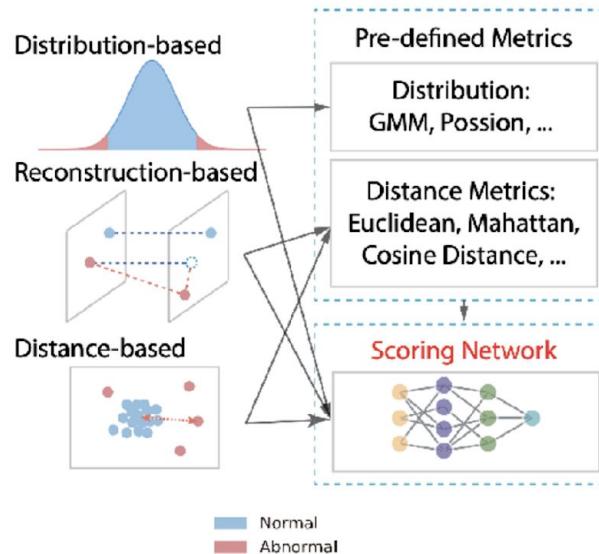
## Representation Learning

$$\mathcal{R}(x_{t_1}, x_{t_2}, \dots x_{t_n})$$

## Scores (Measure)

$$f(\mathcal{R}(x_{t_1}, x_{t_2}, \dots x_{t_n})) \rightarrow \mathbb{R}$$

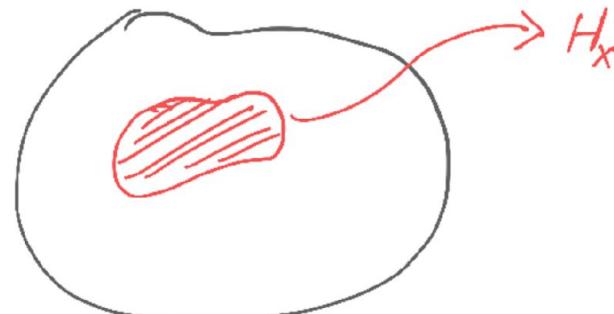
- Stock data
- Experimental data
- Weather data
- Sequential data



# Quantum Variational Rewinding

**Problem :** Given set of *normal* time series  $X : \{x^{(i)}(t)\}$ , learn a model  $\hat{a}_X$ , which scores the *anomalous* nature of unseen time series  $\bar{x}(t)$

**Hypothesis:** For a given set  $X$  there exists a subspace of Hamiltonians  $H_X$ , that can time devolve each quantum embedded normal  $|x^{(i)}(t)\rangle$  to an unique state  $|o_X\rangle$

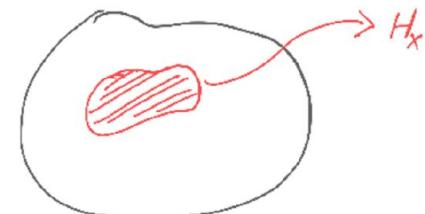


# Quantum Variational Rewinding

**Hypothesis:** For a given set  $X$  there exists a subspace of Hamiltonians  $H_X$ , that can time devolve each quantum embedded normal  $|x^{(i)}(t)\rangle$  to an unique state  $|o_X\rangle$

Each  $|x(t)\rangle$  is a quantum state evolved to a time  $t$  as generated by an *unknown embedding Hamiltonian*  $\hat{H}_E \in H_X$

$$|x(t)\rangle = e^{-it\hat{H}_E} |o_X\rangle$$



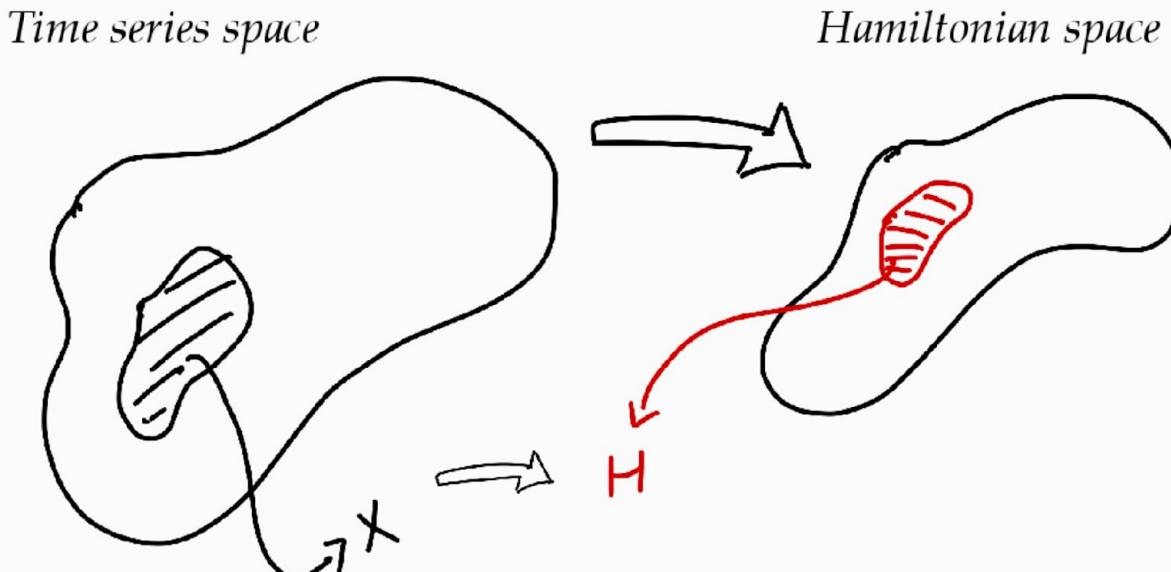
If one is able to learn the set  $H_X$ , then for new data  $|\bar{x}\rangle$ , anomaly score is given by

$$\hat{a}_X(\bar{x}(t)) = \mathbb{E}_{\hat{H} \sim H_X} \left[ \langle \bar{x}(t) | e^{-it\hat{H}} | o_X \rangle \right]$$

# Quantum Variational Rewinding

## *Intuition*

Learnt the representation of the time series in the latent space of Hamiltonian

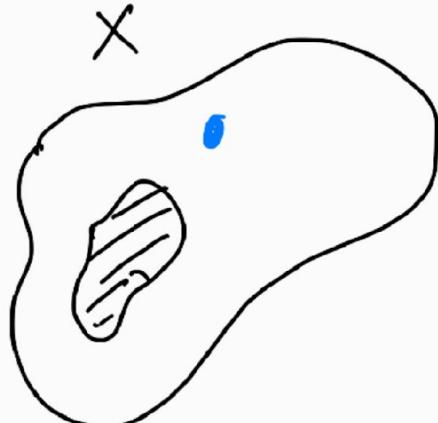


# Quantum Variational Rewinding

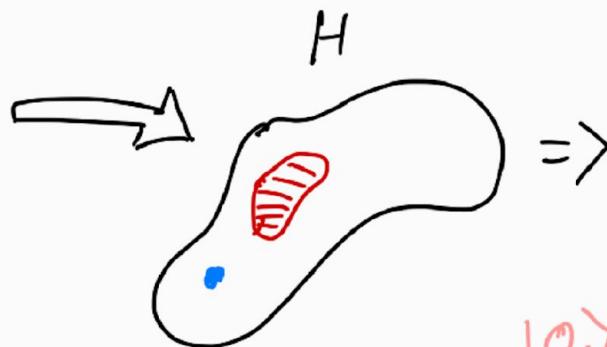
## Intuition

How does anomalous unseen data behave ?

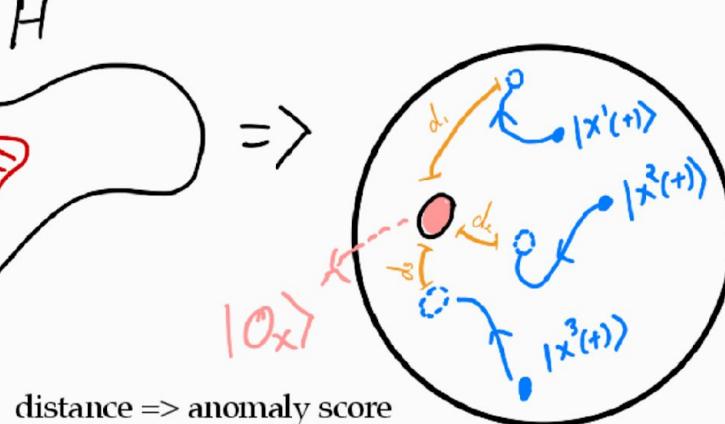
Time series space



Hamiltonian space



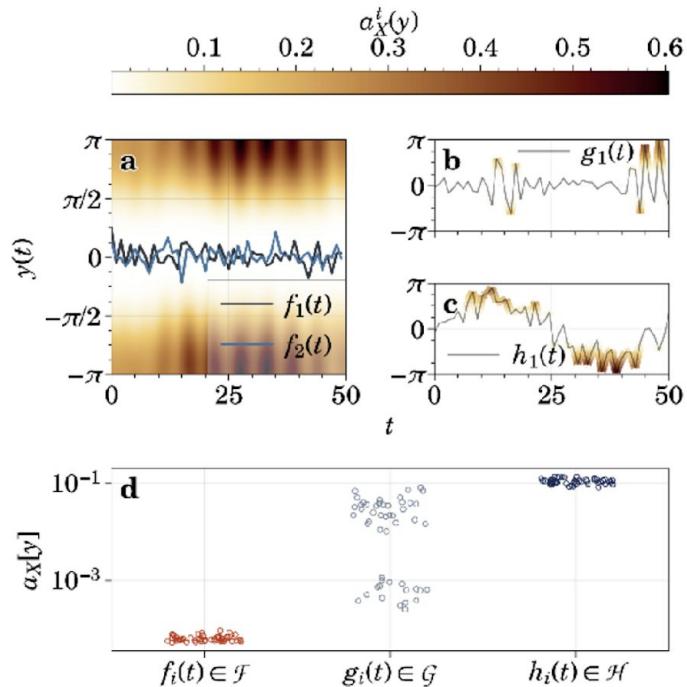
Time devolution using  $H$



distance => anomaly score

# Quantum Variational Rewinding

## Results



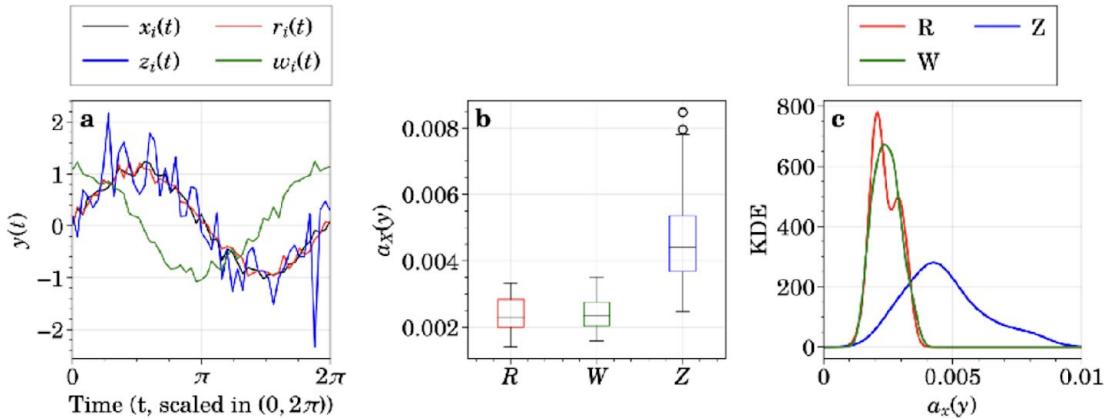
QVR applied to synthetically generated univariate training and testing data. (a) A heat-map of the time-resolved anomaly score  $a_X^t(y)$ . Two testing signals  $f_1(t), f_2(t) \in \mathcal{F}$  (generated using the same mechanism as the training data) are over-plotted and are seen in the region of low  $a_X^t(y)$ . (b-c) Example signals from the test sets  $\mathcal{G}$  and  $\mathcal{H}$  respectively, colored according to  $a_X^t(y)$ .  $\mathcal{G}$  contains noisy signals with randomly inserted *anomaly spikes* and  $\mathcal{H}$  contains noisy  $\sin(t)$  signals. (d) The anomaly scores  $a_X[y]$  for each time series in the sets  $\mathcal{F}, \mathcal{G}$  and  $\mathcal{H}$ . The y-axis has a logarithmic scale.



<https://arxiv.org/abs/2210.16438>

# Quantum Variational Rewinding

## Results



Experiments with noisy univariate sinusoids. (a) an example time series from each of the sets  $X$ ,  $R$ ,  $W$  and  $Z$ . (b) box plots of the anomaly scores obtained from each testing set. (c) The Kernel Density Estimate (KDE) on the distribution of anomaly scores for each of the testing sets.

1.  $R = \{r_i : i = 1, \dots, 50\}$  where each  $r_i$  is defined as the  $x_i$  in the training set, but are not identical to the training set since  $c_i$  are random.

2.  $W = \{w_i : i = 1, \dots, 50\}$  where each  $w_i$  has the form  $w_i = (\cos(0), \cos(\frac{2\pi}{50}), \cos(\frac{4\pi}{50}), \dots, \cos(2\pi) + (\xi_i, \dots, \xi_i))$  where  $\xi_i \sim \mathcal{N}(0.1, 0.1)$ .

3.  $Z = \{z_i : i = 1, \dots, 50\}$  where each  $z_i$  has the form  $z_i = (\sin(0), \sin(\frac{2\pi}{50}), \sin(\frac{4\pi}{50}), \dots, \sin(2\pi) + (\xi_i, \dots, \xi_i))$  where  $\xi_i \sim \mathcal{N}(0.1, 0.5)$ .



<https://arxiv.org/abs/2210.16438>

Baker, Jack S., et al. "Quantum Variational Rewinding for Time Series Anomaly Detection." arXiv preprint arXiv:2210.16438 (2022).

# Notebook: Quantum Variational Rewinding



[https://github.com/AgnostiqHQ/tutorials\\_covalent\\_qsite\\_2023](https://github.com/AgnostiqHQ/tutorials_covalent_qsite_2023)

## Quantum Variational Rewinding

Authors: Jack Stephen Baker, Santosh Kumar Radha

Revised by: Ara G.

This notebook is based on our QVR demo for PennyLane: [Quantum detection of time series anomalies](#)

## Background

A general time series  $\mathbf{y}$  can be described as a sequence of  $p$ -many observations of a process/system arranged in chronological order:

$$\mathbf{y} := (\mathbf{y}_t : t \in T), \quad T :=$$

! NOTE: We drop the bold-face for  $\mathbf{y}$  beyond this point.

## Goal

The goal of QVR and many other (classical) machine learning algorithms for time series anomaly detection is to determine a threshold  $\zeta \in \mathbb{R}$  (which is defined analogously to  $y$  in the above), from which the anomaly score function was learnt. When passed a general time series  $\mathbf{y}$ , then, for an unseen time series  $y$  and a threshold  $\zeta \in \mathbb{R}$ , the series is said to be anomalous should  $a_X(y) > \zeta$ , and normal otherwise.

## Embedding

The first step for doing all of this *quantumly* is to generate a sequence  $\mathcal{S} := (|x_t\rangle : t \in T)$  of  $n$ -qubit quantum states corresponding to the state evolved to a time  $t$ , as generated by an *unknown embedding Hamiltonian*  $H_E$ . That is, each element of  $\mathcal{S}$  is defined by  $|x_t\rangle = e^{-iH_E t}|x\rangle$  for some feature map (see the [PennyLane embedding templates](#) for efficient quantum circuits for doing so). Next, we operate on each  $|x_t\rangle$  with a parameterized  $e^{-iH(\alpha, \gamma)t}$  operator to prepare the states

$$|\mathbf{x}_t, \boldsymbol{\alpha}, \boldsymbol{\gamma}\rangle := e^{-iH(\boldsymbol{\alpha}, \boldsymbol{\gamma})t} |x_t\rangle,$$

where we write  $e^{-iH(\boldsymbol{\alpha}, \boldsymbol{\gamma})t}$  as an eigendecomposition



[pennylane.ai](https://pennylane.ai)

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Demos / Quantum Machine Learning / Quantum Detection Of Time Series Anomalies

# Quantum detection of time series anomalies

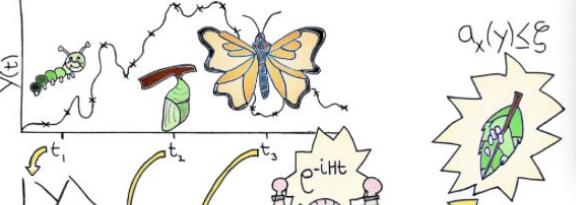
Jack Stephen Baker Santosh Kumar Radha

Published February 6, 2023. Last updated February 6, 2023.

Systems producing observable characteristics which evolve with time are almost everywhere we look. The ten as day turns to night, stock markets fluctuate and the bacteria colony living in the coffee cup to your right, which you would clean yesterday, is slowly growing (seriously, clean it). In many situations, it is important to know whether something is starting to behave abnormally. For example, if the pressure inside a nuclear fission reactor starts violently fluctuating, it is important to be alerted of that. The task of identifying such temporally abnormal behaviour is known as time series anomaly detection and is well known in machine learning circles.

In this tutorial, we take a stab at time series anomaly detection using the *Quantum Variational Rewinding* algorithm proposed by Baker, Horowitz, Radha et. al (2022) [1] – a quantum machine learning algorithm for gate model quantum computers. QVR leverages the power of unitary time evolution/devolution operators to learn a model of normal time series data. Given a new (i.e., unseen in training) time series, the normal model produces a value that, beyond a threshold, indicates anomalous behaviour. In this tutorial, we'll be showing you how all of this works, combining elements from Codecademy and PyTorch.

Before getting into the technical details of the algorithm, let's get a high-level overview with the help of the cartoon below:



<https://arxiv.org/abs/2210.16438>

arxiv.org

## Quantum variational rewinding for time series anomaly detection

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(Dated: November 4, 2022)

Electron dynamics, financial markets and nuclear fission reactors, though seemingly unrelated, all produce observable characteristics evolving with time. Within this broad scope, departures from normal temporal behavior range from academically interesting to potentially catastrophic. New algorithms for time series anomaly detection (TAD) are therefore certainly in demand. With the advent of newly accessible quantum processing units (QPUs), exploring a quantum approach to TAD is now relevant and is the topic of this work. Our approach - *Quantum Variational Rewinding*, or, QVR - trains a family of parameterized unitary time-devolution operators to cluster normal time series instances encoded within quantum states. Unseen time series are assigned an anomaly score based upon their distance from the cluster center, which, beyond a given threshold, classifies anomalous behaviour. After a first demonstration with a simple and didactic case, QVR is used to study the real problem of identifying anomalous behavior in cryptocurrency market data. Finally, multivariate time series from the cryptocurrency use case are studied using IBM's Falcon r5.11H family of quantum computing hardware QPUs, where anomalies are sought out using results from banding

# All contributors are welcome!

🔗 [github.com/AgnostiqHQ/covalent/issues](https://github.com/AgnostiqHQ/covalent/issues)

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<input type="checkbox"/> <a href="#">Fix MNIST tutorial image</a> <a href="#">good first issue</a> <a href="#">help wanted</a>	#1422 opened on Nov 22, 2022 by cjao					<a href="#">1</a>

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Thanks for listening 😊