MAT2410(02) **응용수학 I(HW1)-answer** **Due Date(2020. 3/21)**

No : ______ 학과 : ______ 학번 : ______ 이름 : ______

1. 다음 함수의 도함수(derivative)를 구하시오.

   (1) $f(x) = e^{3x}$ $\qquad f'(x) = e^{3x} \cdot 3 = 3e^{3x}$

   (2) $f(x) = e^{-x/5}$ $\qquad f'(x) = e^{-x/5} \cdot (-\frac{1}{5}) = -\frac{1}{5}e^{-x/5}$

2. 다음 적분을 구하시오.

   (1) $\displaystyle\int_1^3 e^{-x/2}\,dx = \Big[-2e^{-x/2}\Big]_1^3 = -2e^{-3/2} - (-2e^{-1/2}) = -2e^{-3/2} + 2e^{-1/2}$

   (2) $\displaystyle\int_{-\infty}^x e^{-t}\,dt = \Big[-e^{-t}\Big]_{-\infty}^x = +\infty$(발산)

   특이적분(improper integral)이라 다음과 같이 계산해야하나 위의 방법으로 하셔도 됩니다.

   (2) $\displaystyle\int_{-\infty}^x e^{-t}\,dt = \lim_{u\to infty}\int_u^x e^{-t}\,dt = \lim_{u\to infty}\Big[-e^{-t}\Big]_u^x = \lim_{u\to infty}\Big(-e^{-x}+e^{-u}\Big) = +\infty$(발산)

3. 다음 함수의 매클로린 급수를 구하시오. (증명없이 결과만)

   (1) $\displaystyle f(x) = e^x = \sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n = \sum_{n=0}^{\infty}\frac{1}{n!}x^n = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$

   (2) $\displaystyle f(x) = e^{ax} = \sum_{n=0}^{\infty}\frac{1}{n!}(ax)^n = \sum_{n=0}^{\infty}\frac{a^n}{n!}x^n = 1 + ax + \frac{a^2}{2!}x^2 + \frac{a^3}{3!}x^3 + \cdots$

4. 다음을 구하시오.

   (1) $$\int_0^1\int_1^2 y(x+y)\,dy\,dx = \int_0^1\int_1^2 (xy+y^2)\,dy\,dx = \int_0^1\Big[\tfrac{x}{2}y^2+\tfrac{1}{3}y^3\Big]_1^2 dx = \int_0^1\Big(\Big(2x+\tfrac{8}{3}\Big)-\Big(\tfrac{x}{2}+\tfrac{1}{3}\Big)\Big)dx$$
   $$= \int_0^1\Big(\tfrac{3x}{2}+\tfrac{7}{3}\Big)dx = \Big[\tfrac{3x^2}{4}+\tfrac{7}{3}x\Big]_0^1 = \frac{3}{4}+\frac{7}{3}-0 = \frac{37}{12}$$

   (2) $\displaystyle\int_0^\infty e^{-x-2y}\,dy = \Big[-\frac{1}{2}e^{-x-2y}\Big]_0^\infty = 0 - \Big(-\tfrac{1}{2}e^{-x}\Big) = \frac{1}{2}e^{-x}$

   (3) $\displaystyle\int_0^\infty e^{-x-2y}\,dx = \Big[-e^{-x-2y}\Big]_0^\infty = 0 - \Big(-e^{-2y}\Big) = e^{-2y}$

5. $f(x,y) = 8xy$이고 영역 $D = \{(x,y)\in\mathbb{R}^2 \mid 0<x<y<1\}$ 일 때, 이중적분 $\iint_D f(x,y)\,dxdy$ 을 구하시오.

   Solution. 영역 D는 다음과 같이 표현된다.

   $D = \{(x,y)\in\mathbb{R}^2 \mid 0<x<1, x<y<1\}$ (Type I) $\qquad D = \{(x,y)\in\mathbb{R}^2 \mid 0<y<1, 0<x<y\}$ (Type II)

   따라서 주어진 이중적분값은 다음과 같다.

   $$(I):\quad \iint_D f(x,y)\,dxdy = \int_0^1\int_x^1 8xy\,dy\,dx = \int_0^1\Big[4xy^2\Big]_x^1 dx = \int_0^1\Big(4x-4x^3\Big)dx = \Big[2x^2-x^4\Big]_0^1 = 1$$

   $$(II):\quad \iint_D f(x,y)\,dxdy = \int_0^1\int_0^y 8xy\,dx\,dy = \int_0^1\Big[4x^2y\Big]_0^y dy = \int_0^1 4y^3\,dy = \Big[y^4\Big]_0^1 = 1$$