Solving the second order Linear ODE

- Overview

- Vector space
  - linear combination, linearly independent
  - set & space
  - basis
  - Fundamental Set of Solutions(FSS)

- Homogeneous linear ODE
  - reduction of order

- Homogeneous linear ODE with constant coefficients

Overview :

- For linear ODEs of second order

$$y'' + p(x)y' + q(x)y = r(x), \quad y(x_0) = K_0\,,\ y'(x_0) = K_1$$

(1) The homogeneous linear ODE

$$(H)\ y'' + p(x)y' + q(x)y = 0$$

has two linearly independent solutions $y_1(x)$ and $y_2(x)$.

(2) Superposition properities

(3) A particular solution
(4) The general solution
(5) Initial Value Problem

Homogeneous linear ODEs of second order

$$y'' + p(x)y' + q(x)y = 0 \qquad y(x_0) = K_0\,,\ y'(x_0) = K_1$$

(1) find two linearly independent solutions

(2) determine the general solution

(3) find constants with the initial condition

the general solutions forms a vector space with basis

Reduction of order

- How to obtain a basis if one solution is known?

Homogeneous linear ODEs with constant coefficients(1)

- Given DE $\quad y'' + ay' + by = 0$

Homogeneous linear ODEs with constant coefficients(2)

- case1

Homogeneous linear ODEs with constant coefficients(3)

- case2

Homogeneous linear ODEs with constant coefficients(4)

- case3

Euler-Cauchy equation

$$x^2y'' + axy' + by = 0$$

Existence and uniqueness of a solution to IVP

$y'' + p(x)y' + q(x)y = 0 \qquad y(x_0) = K_0\,,\; y'(x_0) = K_1$

Wornskian and linear independence of solutions(1)

- With y_1 and y_2 being the solutions of

$$y'' + p(x)y' + q(x)y = 0$$

- Wronski determinant(Wronskian) of y_1 and y_2 is defined by