

# Introduction to ODE

- Ordinary Differentiation Equations (ODE)
- General & Particular Solutions (singular solution)
- Direction fields
- Solving DE by computer

# 용어

- Ordinary(Partial) Differentiation Equations
  - Explicit(Implicit) form
  - Order of DE
  - Linear(Nonlinear) DE
  - Homogeneous(Nonhomogeneous) DE
  - Autonomous
- 
- General & Particular Solutions (singular solution)
  - Initial Value Problem(IVP), Boundary Value Problem(BVP)

# Examples (1)

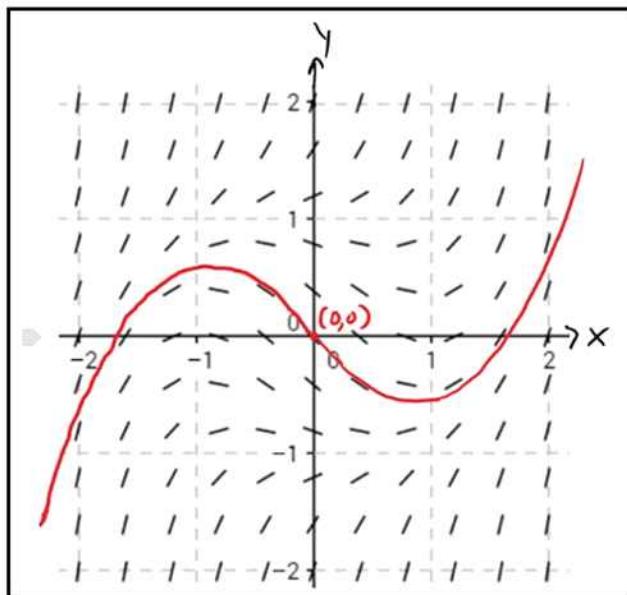
- $y' = -1$
  - $y'(x) + 2y(x) - 3 = 0$
  - $y'' + 3xy + 72 = 0$
  - $y'' + y' + y = 0$
  - $y''y' + \sin y + 2 = 0$
  - $(y')^2 - 3y = 0$
- 
- $2\frac{\partial y}{\partial x}(x, z) + 3\frac{\partial y}{\partial z}(x, z) - 2x = 0$
  - $\begin{cases} y'_1 + 2y_2 + 3 = 0 \\ y'_2 + 2y_1 + y_2 = 2 \end{cases}$

## Examples (2)

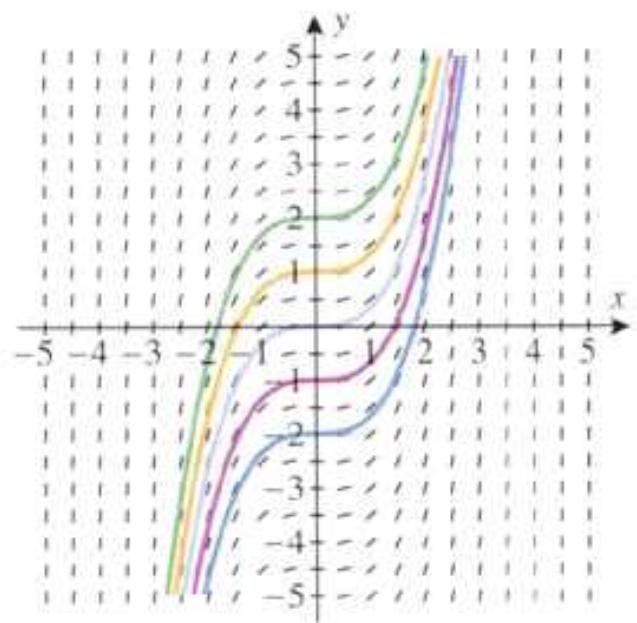
- $y'(x) = \cos x$
- $(y')^2 - x y' + y = 0$

# Geometric interpretation of $y' = f(x, y)$ (1)

- Direction field
- An idea of solving DE by computer
- Euler method
- $y' = x^2 + y^2 - 1, y(0) = 0$



$$y' = x^2, \quad y(0) = c$$



Direction field with integral curves

# Solving first order ODEs

- Separable ODEs
- Exact ODEs
  - Integrating factor(IF)
  - Reduction to exact
- Linear ODEs
  - Reduction to linear, Bernoulli equation

# Separable ODEs (1)

## Separable ODEs (2)

- reduction to separable form

$$(1) y' = f\left(\frac{y}{x}\right) \text{ 꼴 일 때}$$

$$(2) \text{ 치환 이용 } ax + by + c = v$$

## Separable ODEs (3)

$$(1) \quad 9yy' + 4x = 0$$

$$(2) \quad y' = 1 + y^2$$

$$(3) \quad y' = k y$$

$$(4) \quad y' = -\frac{y}{x}$$

$$(5) \quad 2xyy' = y^2 - x^2$$

$$(6) \quad (2x - 4y + 5)y' + (x - 2y + 3) = 0$$

# Exact ODEs (1)

- Given DE  $M(x,y) + N(x,y)y' = 0$

If there exists a function  $u(x,y)$  such that

then  $u(x,y) = c$  is a general solution to the DE.

The DE is called "exact DE"

## Exact ODEs (2)

- How to check if the given DE is exact?

## Exact ODEs (3)

- How to solve the exact DE?

$$(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$$

## Exact ODEs (4)

$$\sin x \cosh y \, dx - \cos x \sinh y \, dy = 0$$

$$-y \, dx + x \, dy = 0$$

## Exact ODEs (5)

- Integrating factor(IF)
- reduction to exact form

$$(e^{x+y} + ye^y)dx + (xe^y - 1) dy = 0$$

# Linear ODEs

- homogeneous linear & non-homogeneous linear

$$y' + p(x)y = r(x)$$

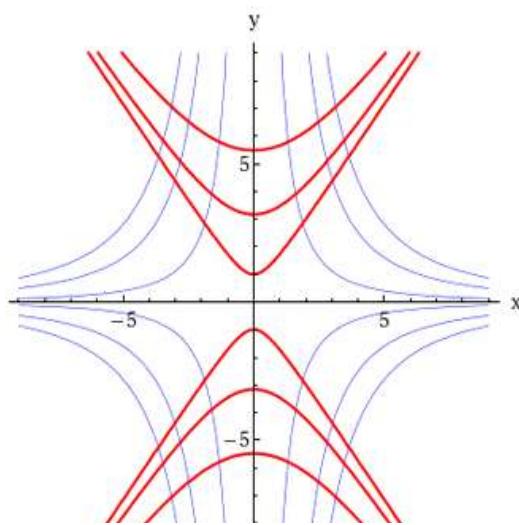
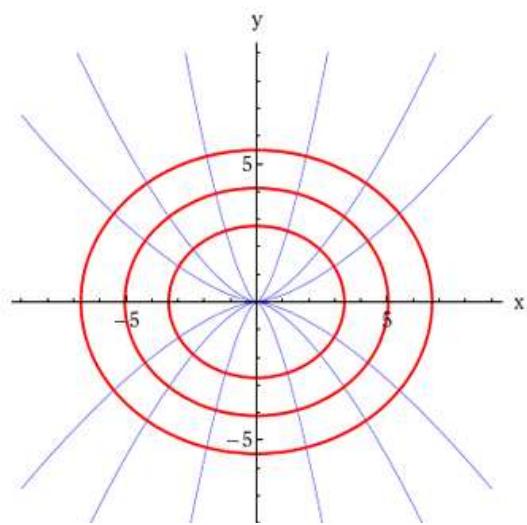
# Bernoulli DE

- homogeneous linear & non-homogeneous linear

$$y' + p(x)y = g(x) y^\alpha$$

$$y' = Ay - By^2, \quad A, B > 0$$

# Orthogonal trajectories of curves



# Existence & Uniqueness of Solutions for IVP

- Existence Theorem

For given IVP  $y' = f(x,y)$ ,  $y(x_0) = y_0$  , if  $f(x,y)$  is continuous and bounded such that  $|f(x,y)| \leq K$ , in the region  $R : |x - x_0| < a, |y - y_0| < b$  .

Then the IVP has at least one solution  $y(x)$  on the interval  $|x - x_0| < \alpha$ ,  $\alpha = \min(a, \frac{b}{K})$

- Uniqueness Theorem

# Existence & Uniqueness of Solutions for IVP

$$y' = y^{\frac{1}{3}}, \quad y(0) = 0$$

$$y' = 1 + y^2, \quad y(0) = 0$$