

# Linear Algebra(1)

Matrices & Vectors

Linear System, Gauss Elimination

Vector Space, Subspace

Determinant of a matrix

Inverse matrix, Cramer's Rule

# Linear Algebra(2)

Orthogonality

Eigenvalues & Eigenvectors

Similarity transformation

Diagonalization

Quadratic form

# Eigenvalues & Eigenvectors

- eigenvalues and eigenvectors
- symmetric, skew-symmetric, and orthogonal matrices
- similar matrices
- similarity transformation
- diagonalizations
- quadratic form

# Eigenvalue and eigenvector of a matrix (1)

- For a square matrix  $A \in R^{n \times n}$ ,

# Eigenvalue and eigenvector of a matrix (2)

- characteristic polynomial
- algebraic multiplicity
- geometric multiplicity

## Eigenvalue and eigenvector of a matrix (3)

- For a matrix  $A \in R^{n \times n}$ , eigenvalue of  $A$  may be a complex number.
- eigenspace of eigenvalue  $\lambda$  : set of eigenvectors for  $\lambda$

# Examples

- Find eigenvalues and eigenvectors of  $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

- $A = \begin{bmatrix} 0.7 & 0.1 & 0 \\ 0.2 & 0.9 & 0.2 \\ 0.1 & 0 & 0.8 \end{bmatrix}$

# Symmetric, skew-symmetric, and orthogonal matrices (1)

- symmetric matrix
- skew-symmetric matrix
- orthogonal matrix

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## Symmetric, skew-symmetric, and orthogonal matrices (2)

- (1) Eigenvalue of symmetric matrix is real.
- (2) Eigenvalue of skew-symmetric matrix is pure imaginary or zero.
- (3) Eigenvalue of orthogonal matrix has magnitude 1.

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# Orthogonal transformation

- $y = Ax$ ,  $A$  is orthogonal