Linear Algebra(1)

Matrices & Vectors
Linear System, Gauss Elimination
Vector Space, Subspace

Determinant of a matrix
Inverse matrix, Cramer’s Rule

Linear Algebra(2)

Orthogonality

Eigenvalues & Eigenvectors

Similarity transformation

Diagonalization

Quadratic form

System of linear equations, Gauss elimination

- existence and uniqueness of solution
  - 선형연립방정식의 해의 존재성에 대한 해석 방법 (2차원, 3차원)
- elementary row operation
- Gauss elimination
  - 선형연립방정식의 해 구하기
- echelon form, invertible, nonsingular
- matrix inversion(inversion algorithm)

Linear System, Coefficient Matrix, Augmented Matrix

- System of Linear Equations

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

<u>행렬방정식 표현</u>

$$\begin{cases} x_1 - x_2 + x_3 = 0 \\ \qquad x_2 - 4x_3 = 4 \\ -4x_1 + 5x_2 + 9x_3 = -9 \\ \\ A\mathbf{x} = \mathbf{b} \end{cases}$$

계수행렬

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -4 \\ -4 & 5 & 9 \end{bmatrix}$$

첨가행렬

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ -4 & 5 & 9 & -9 \end{bmatrix}$$

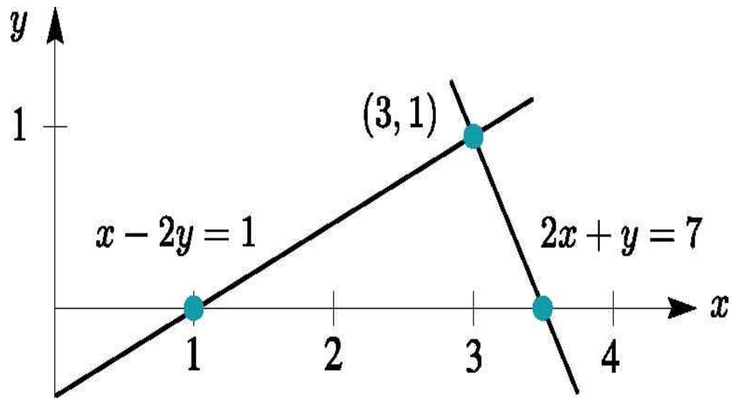
<u>첨가계수행렬 표현</u>

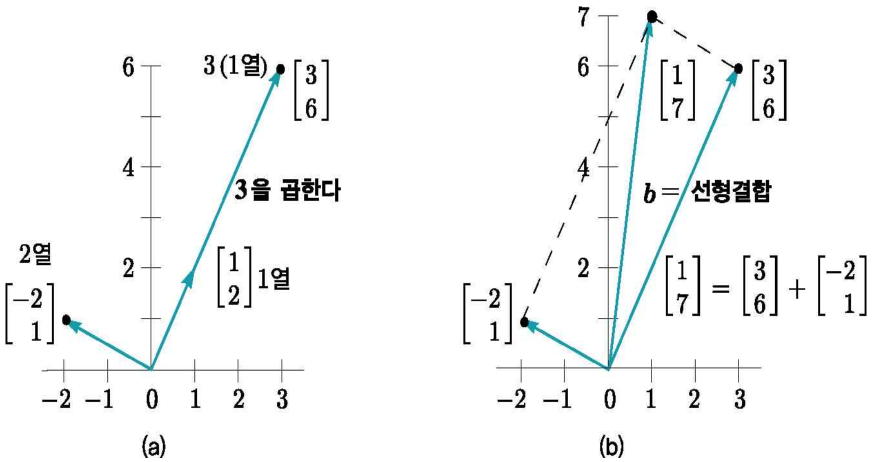
$$\left[\, \left[A \mid \mathbf{b}\right] \right.$$

Linear System, Geometric Interpretation

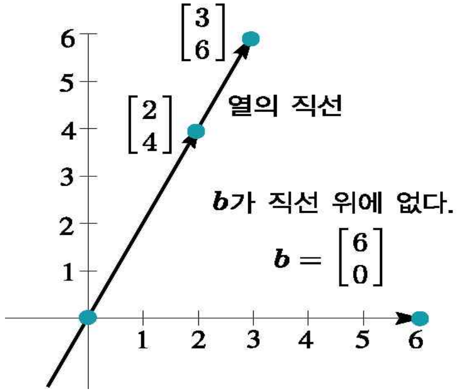
$$\begin{cases} x - 2y = 1 \\ 2x + y = 7 \end{cases}$$

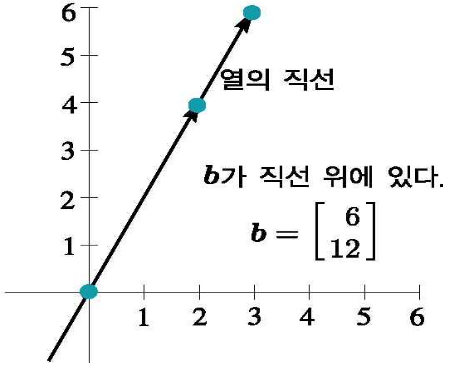
$$x\begin{bmatrix}1\\2\end{bmatrix} + y\begin{bmatrix}-2\\1\end{bmatrix} = \begin{bmatrix}1\\7\end{bmatrix}$$





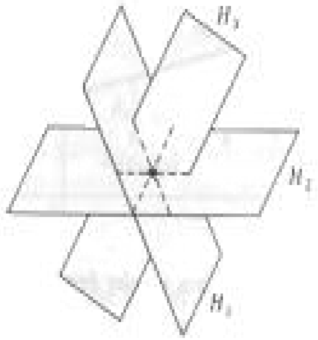
$$\begin{cases} 2x + 3y = b_1 \\ 4x + 6y = b_2 \end{cases} \qquad x\begin{bmatrix} 2 \\ 4 \end{bmatrix} + y\begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$



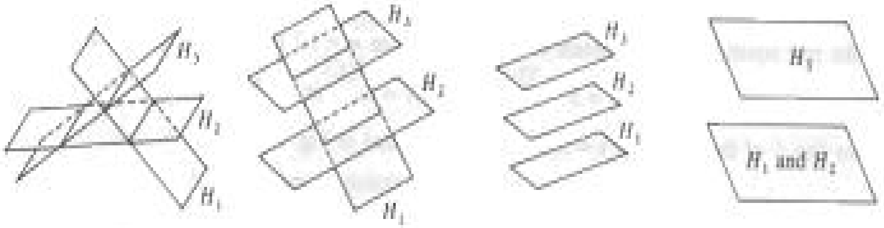


$$\begin{cases} x + 2y + 3z = 6 \\ 2x + 5y + 2z = 4 \\ 6x - 3y + \ z = 2 \end{cases}$$

$$x\begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix} + y\begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix} + z\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$$

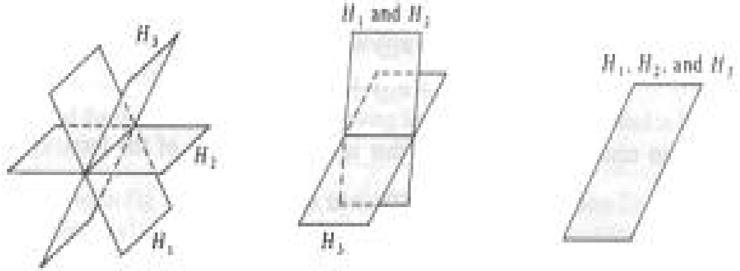


(a) Unique solution



(i) (ii) (iii) (iv)

(b) No solutions



(i) (ii) (iii)

(c) Infinite number of solutions

How to find the solution?

- Back Substitution

$$\begin{cases} x - y + z = 0 \\ 10y + 25z = 90 \\ -95\, z = -190 \end{cases}$$

$$\begin{cases} 2x + 5y = 2 \\ -4x + 3y = -30 \end{cases}$$

- Elementary Row Operation

- 두 식의 위치교환
- 한 식에 0이 아닌 상수 곱하기
- 한 식을 상수배해서 다른 식에 더하기

Gauss Elimination(unique solution)

$$\begin{cases} x + 2y + 3z = 6 \\ 2x + 5y + 2z = 4 \\ 6x - 3y + \ z = 2 \end{cases}$$

Gauss Elimination(infinitely many solution)

$$\begin{cases} 3.0x + 2.0y + 2.0z - 5.0w = 8.0 \\ 0.6x + 1.5y + 1.5z - 5.4w = 2.7 \\ 1.2x - 0.3y - 0.3z + 2.4w = 2.1 \end{cases}$$

Gauss Elimination(no solution)

$$\begin{cases} 3x + 2y + z = 3 \\ 2x + y + z = 0 \\ 6x + 2y + 4z = 6 \end{cases}$$

Row Echelon form

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \blacklozenge & \# & \# & \# & \# \\ 0 & 0 & \blacklozenge & \# & \# \\ 0 & 0 & 0 & \blacklozenge & \# \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

◆ : leading entry of a row
: arbitrary numbers

$$\begin{bmatrix} c_{11} & c_{12} & \cdots & \cdots & \cdots & c_{1n} \\ 0 & c_{22} & \cdots & \cdots & \cdots & c_{2n} \\ 0 & 0 & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & c_{rr} & \cdots & c_{rn} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of Matrix, Linear independent vectors

- linear combination of vectors
- linearly independence of a set of vectors

- rank of a matrix : 행렬의 행벡터 중 선형독립인 행벡터의 최대 수

$$
\begin{matrix}
3 & 0 & 2 & 2 \\
-6 & 42 & 24 & 54 \\
21 & -21 & 0 & -15
\end{matrix}
$$

Properites of Rank

- elementrary row operation에 의해 변형된 행렬들은 모두 같은 rank를 갖는다.

- 어떤 행렬의 선형독립인 행벡터의 최대수와 열벡터의 최대수는 같다.