1. (1) A skew-symmetric matrix satisfies $A^T = -A$. In 3 by 3 case, show that the determinant must be zero.

   (2) How about a 4 by 4 skew-symmetric matrix? Give a 4 by 4 skew-symmetric matrix with $\det A \neq 0$.

2. Let $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$.

Find the determinant of A^{-1} and $A - \lambda I$. For which values of λ is $A - \lambda I$ a singular matrix?

3. By applying row operations to produce an upper triangular matrix, compute

$$\det \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} \text{ and } \det \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

4. Find the cofactor matrix C of $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ and compare AC^T with A^{-1}.