Linear Algebra(1)

Matrices & Vectors

Linear System, Gauss Elimination

Vector Space, Subspace

Determinant of a matrix

Inverse matrix, Cramer’s Rule

Linear Algebra(2)

Orthogonality

Eigenvalues & Eigenvectors

Similarity transformation

Diagonalization

Quadratic form

Determinants

- determinant of a matrix
- properties of determinant
- Cramer's rule
- inverse of a matrix

  - computing inverse(Gauss-Jordan elimination)
  - formular for the inverse
  - properties of inverse and nonsingular matrices

Determinant of a matrix

- For a square matrix $A \in R^{n\times n}$,

Elementary row operation & determinant

- $det(P_{ij}A) \quad = \; -det(A)$

  $det(E_{ij}A) \quad = \; det(A)$

  $det(D_i(k)A) \quad = \; k \; det(A)$

- 두 행이 비례관계인 행이 존재하는 행렬의 행렬식은 0
- 0 만으로 이루어진 행이 존재하는 행렬의 행렬식은 0
- 삼각행렬의 행렬식은 대각성분의 곱
- 두 행렬의 곱의 행렬식은 각 행렬의 행렬식의 곱

- $det(E_{ij}A) \quad = \; det(A)$

Cramer's Rule

Cramer's Rule for Three Equations in Three Unknowns

The solution to the system

$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}$$

is given by $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$, and $z = \frac{D_z}{D}$, where

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \qquad D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},$$

$$D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \qquad D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix},$$

provided that $D \neq 0$.

Inverse of a matrix

- For a square matrix $A \in R^{n \times n}$, the inverse of A is a matrix B such that

$$AB = I \quad \text{and} \quad BA = I$$

and we denote B by A^{-1} .

- A^{-1} exists (A is nonsingular)

  rank$(A) = n$

  $det\ (A) \neq 0$

Gauss-Jordan elimination

A formula for the inverse

$$\mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|}\mathbf{C}^{\mathrm{T}} = \frac{1}{|\mathbf{A}|}\begin{pmatrix} \mathbf{C}_{11} & \mathbf{C}_{21} & \cdots & \mathbf{C}_{n1} \\ \mathbf{C}_{12} & \mathbf{C}_{22} & \cdots & \mathbf{C}_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_{1n} & \mathbf{C}_{2n} & \cdots & \mathbf{C}_{nn} \end{pmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Properties about nonsingular matrix and determinant

- inverse of 'a diagonal matrix'

- $(AB)^{-1} = B^{-1}A^{-1}$

  $(A^{-1})^{-1} = A$

  $det(AB) = det(A)\,det(B) = det(BA)$

- if A is singular, AB and BA are singular.
- if A is nonsingular, $AB = 0$ implies $B = 0$.