Nonhomogeneous linear ODEs of second order

$y'' + p(x)y' + q(x)y = r(x) \qquad y(x_0) = K_0\,,\ y'(x_0) = K_1$

$$y'' + 2y' + 101y = 10.4\, e^{x} \qquad y(0) = 1.1\,,\ y'(0) = -\,0.9$$

Solution by undetermined coefficient method

- candidate for $y_p(x)$ in $y'' + p(x)y' + q(x)y = r(x)$

Term in $r(x)$	Choice for $y_p(x)$
$ke^{\gamma x}$	$Ce^{\gamma x}$
$kx^n \ (n = 0, 1, \cdots)$	$K_n x^n + K_{n-1}x^{n-1} + \cdots + K_1 x + K_0$
$k \cos \omega x$ $k \sin \omega x$	$K \cos \omega x + M \sin \omega x$
$ke^{\alpha x} \cos \omega x$ $ke^{\alpha x} \sin \omega x$	$e^{\alpha x}(K \cos \omega x + M \sin \omega x)$

- If the candidate for $y_p(x)$ happens to be a solution of (H), then multiply $y_p(x)$ by x (or x^2)

1 (any constant)	A
$5x+7$	$Ax+B$
$3x^2-2$	Ax^2+Bx+C
$\sin 4x$	$A\cos 4x+B\sin 4x$
$\cos 4x$	$A\cos 4x+B\sin 4x$
e^{5x}	Ae^{5x}
$(9x-2)e^{5x}$	$(Ax+B)e^{5x}$
x^2e^{5x}	$(Ax^2+Bx+C)e^{5x}$
$e^{3x}\sin 4x$	$Ae^{3x}\cos 4x+Be^{3x}\sin 4x$
$5x^2\sin 4x$	$(Ax^2+Bx+C)\cos 4x+(E^2+Fx+G)\sin 4x$
$xe^{3x}\cos 4x$	$(Ax+B)e^{3x}\cos 4x+(Cx+E)e^{3x}\sin 4x$
$(5x+7)+\sin 4x$	$(Ax+B)+(C\cos 4x+D\sin 4x)$

$$y'' + 4y = 8\,x^2$$

$$y'' - 3y' + 2y = e^{x}$$

$$y'' + 2y' + y = e^{-x}$$

$$y'' + 2y' + 5y = 1.25\ e^{0.5x} + 40\ cos4x\ - 55\ sin4x$$

$$y'' + 2y' + 5y = 1.25\ e^{0.5x} + 40\ cos2x$$

$$y'' + 2y' + 5y = 1.25\ e^{0.5x} + 40e^{-x} cos2x$$

Solution by variation of parameters

- Find a particular solution $y_p(x)$ to $y'' + p(x)y' + q(x)y = r(x)$