Report 5 - Financial Mathematics for Data Science

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The aim of this report is to study the Value at Risk (VaR) of an equibalanced portfolio of two stocks using various methods. In particular the historical parametric VaR, the historical simulated VaR and an application of Monte Carlo techniques.

FIRST PART - SETTING UP AN EQUIBALANCED PORTFOLIO

We aim to define an equibalanced portfolio constituted by two stocks, both quoted on the NYSE: T-Mobile (TMUS) and Boeing (BA). Both companies don't provide dividends and additional info about them can be found here [1] [2], which are also the sources for our historical data. The portfolio has an initial value of 10000$ divided evenly among the assets. The positions in both T-Mobile and Boeing is updated every day to preserve its equibalancing. We enter the market for a total of 6 months between Nov 3, 2021 and May 2, 2022. During this period the average daily return of the portfolio is -0,10% with a daily volatility of 1,88%. The final value of the portfolio is 8610$. In tab.I is reported a summary of the position.

	T-Mobile shares	Boeing shares	Portfolio value
Position open	41,00	23,43	10000
Position close	34,06	28,97	8610

Tab. I: Summary of the position

SECOND PART - CALCULATING THE PARAMETRIC VAR

From the daily returns of the stocks and the portfolio is possible to calculate the parametric VaR for each position and for the entire portfolio. This is done by calculating the daily volatilities and using them in the parametric VaR formula:

$$NdayVaR = 1dayVaR\sqrt{N} \tag{1}$$

where the 1-Day VaR depends on the confidence chosen and calculated from the corresponding quantile using the NORM.S function on Excel multiplied by the daily volatility. By following this procedure is possible to calculate the VaR for every horizon from 1 to 100 days. Obtaining the graphs in fig.1.

It can be immediately noticed that the sum of VaR of the stocks does not equal the VaR of the portfolio, precisely the sum is less than the latter for every horizon and every confidence level. This implies that there is a correlation less than 1 between the daily returns of T-Mobile and Boeing because by applying the known statistical result:

$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y} \tag{2}$$

this is exactly what we get (ρ represents the correlation between X and Y). We will have a confirmation of the sub-additivity of the VaR in the portfolio even in the next methods we will apply.

THIRD PART - RISKMETRICS EWMA VOLATILITY ESTIMATION

It is possible to derive the volatility necessary for the VaR using a different procedure than the daily one. We will use the EWMA volatility with optimal lambda equal to 0,94. The variance is calculated recursively using: [3]

$$\sigma_n^2(\text{EWMA}) = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2 \tag{3}$$

where u_{n-1}^2 are the squared daily returns at time $n-1$ and λ is a weight, chosen optimally as 0,94. From this procedure is possible to obtain an estimation of the VaR for the assets and the portfolio shown in fig.1. It is possible to notice again from the graph that we have sub-additivity for the VaR.

FOURTH PART - MONTE CARLO SIMULATION

Another way to compute the VaR is by using Monte Carlo simulations. Following the procedure indicated in the book "Hull" 9th edition pag.511 [4] is possible to implement a VBA code in excel [5] from which we can obtain the graphs for the VaR in respect to confidence level, horizon and number scenarios N per Monte Carlo simulation. In our case we chose $N = \{100, 1000, 10000\}$, unfortunately it was not possible to test a higher N because the PC used for the simulation was not powerful enough, or the algorithm implemented was not efficient enough, and already by testing $N = 50000$ Excel just gave in output an error. It also necessary to give an important remark, the implementation of the Monte Carlo simulation requires a risk-free interest rate. This risk-free rate has to be equal to the LIBOR overnight interest for every horizon. This is because the VaR for horizons greater

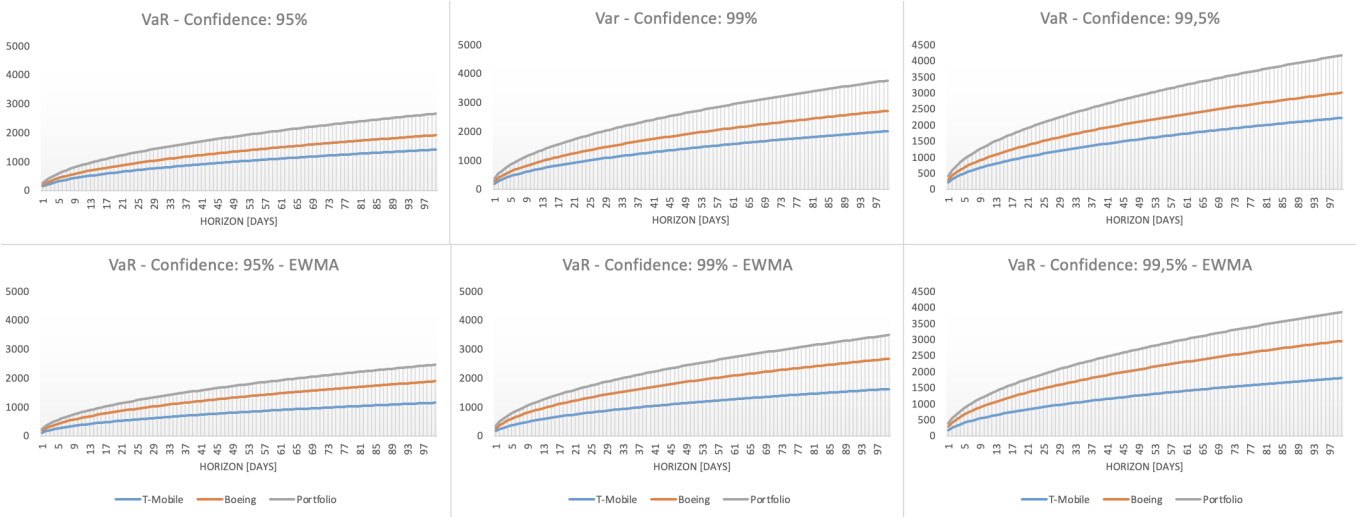

Fig. 1: VaR for different confidence levels in respect to horizon calculated using the parametric method and the EWMA method for the volatility - From left to right we have T-Mobile, Boeing and at the end the Portfolio

than 1 day is calculated from the 1-Day VaR multiplied by the square root of the horizon, so we are actually always calculating the 1-Day VaR and than extending it. Applying the LIBOR 3-month interest to an horizon of 3 months would result in an underestimation of the VaR. While having care of these remarks is possible to do the Monte Carlo simulation and obtain the graphs in fig.2.

FIFTH PART - HISTORICAL VAR

Another method to estimate the VaR of our position is with the historical volatility. We will only calculate the 1-month and 3-month VaR because the 1 day VaR of the historical method is calculated in the same way as the historical simulation method, explained in the next chapter of this report. The 6-month VaR cannot be computed because we will need another 6 months of data for that. Now lets explain the procedure: we will calculate the vectors of 1-month and 3-month returns from our data using the same formula for the daily returns but considering the 21st and 63rd day respectively after the one considered rather than the day after. From this vectors is possible to compute the 1-month volatility and the 3-month one from which is possible to estimate the VaR using the same formula adopted in the second part of this report. The results of this calculation are summarized in the tables II and III.

We can see that the historical VaR tends to underestimate in respect to what we see in the graphs of the other methods. We will now see the historical simulation method for calculating the VaR which has a greater accuracy.

Confidence	T-Mobile [$]	Boeing [$]	Portfolio [$]
95%	588	596	843
99%	831	843	1191
99,5%	921	933	1319

Tab. II: Historical VaR for horizon time equal to 1 month

Confidence	T-Mobile [$]	Boeing [$]	Portfolio [$]
95%	829	894	939
99%	1248	1599	2022
99,5%	1421	1945	2695

Tab. III: Historical VaR for horizon time equal to 3 month

SIXTH PART - HISTORICAL SIMULATION VAR

For this method we start from the distribution of the daily returns of our assets and portfolio. We now simulate 125 scenarios (1 for each daily return) following the procedure in pag.497 of "The Hull" [4]. We estimate the 1-Day VaR at confidence level X% as the limit of the $(1 - X)$ percentile of the corresponding distribution and we know that from the 1-Day VaR we can obtain the VaR for any horizon, in our case 1,21,63 and 126 days. All these results are shown in fig.3.

CONCLUSIONS

In this report we have seen different methods and approaches to calculate the VaR of an equibalanced portfolio constituted by shares in the stocks T-Mobile and Boeing. It is important to notice that for each method

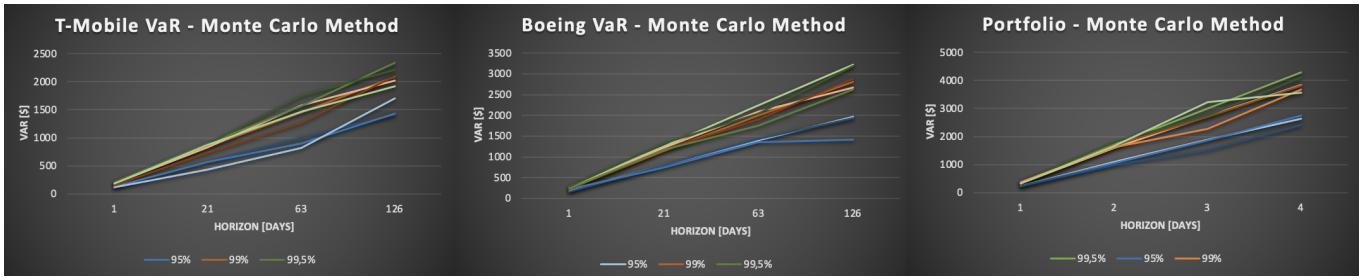

Fig. 2: Graphs for VaR in respect to horizon at times $T = \{1, 21, 63, 126\}$ days. Different colors means different confidence level and the darker the color the higher the number of scenarios used for the Monte Carlo simulation where $N = \{100, 1000, 10000\}$

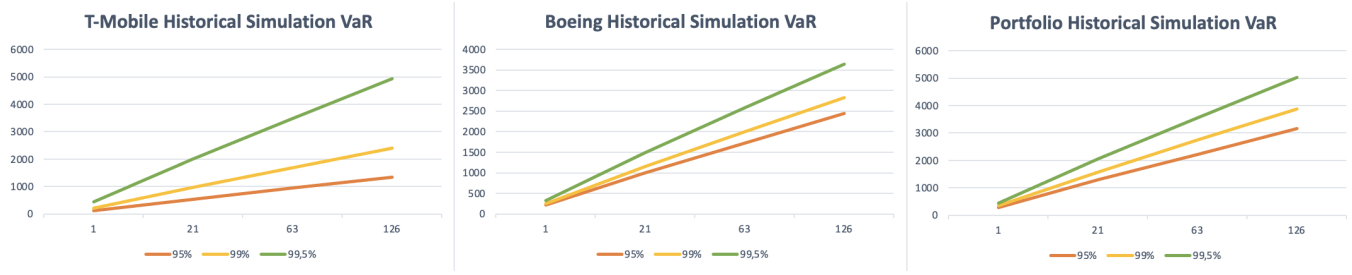

Fig. 3: Historical simulated VaR for our portfolio and its assets in respect to time for 3 confidence levels.

implemented its possibly to see from the graph that the VaR of the stocks sums up in a sub-additive way in respect to the VaR of the portfolio, for every confidence level chosen, implying the correlation between the stocks cited in the second part of the report.

[1] "https://it.finance.yahoo.com/quote/tmus/," .

[2] "https://it.finance.yahoo.com/quote/ba?p=ba.tsrc=fin-srch," .

[3] "https://www.investopedia.com/articles/07/ewma.asp," .

[4] J. C. Hull, "Futures and other derivatives," (2015).

[5] "https://investexcel.net/calculate-value-at-risk-with-monte-carlo-simulation/," .