Report 2 - Financial Mathematics for Data Science

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The aim of this report is to deduce the discount rate given by the market to a chosen stock using a box spread and then combine it with the "Call-Put Parity" to obtain the implied dividend yield for that particular stock.

INTRODUCTION

The chosen stock for the evaluation is "Verizon Communications Inc. (VZ)", quoted on the NYSE. Through its subsidiaries, Verizon offers communications, technology, information, and entertainment products and services to consumers, businesses, and governmental entities worldwide. Its Consumer segment provides postpaid and prepaid service plans; internet access on notebook computers and tablets; wireless equipment, including smartphones and other handsets; and wireless-enabled internet devices, such as tablets, and other wireless-enabled connected devices comprising smart watches. It also provides residential fixed connectivity solutions, such as internet, video, and voice services; and sells network access to mobile virtual network operators. As of December 31, 2021, it had approximately 115 million wireless retail connections, 7 million wireline broadband connections, and 4 million Fios video connections. The company's Business segment provides network connectivity products, including private networking, private cloud connectivity, virtual and software defined networking, and internet access services; and internet protocol-based voice and video services, unified communications and collaboration tools, and customer contact center solutions. This segment also offers a suite of management and data security services; domestic and global voice and data solutions, such as voice calling, messaging services, conferencing, contact center solutions, and private line and data access networks; customer premises equipment; installation, maintenance, and site services; and Internet of Things products and services. The company was formerly known as Bell Atlantic Corporation and changed its name to Verizon Communications Inc. in June 2000. Verizon Communications Inc. was incorporated in 1983 and is headquartered in New York, New York.
The tab.I shows a quick summary of some relevant stock data.

Market Cap	215,26B
Enterprise Value	388,97B
Revenue	133,61B
Revenue per Share	32,21
Fiscal Year Ends	Dec 30 ,2021
Ex-Dividend Date	Apr 06, 2022
Last Split Date	Jul 01, 2010
Average Vol (3 month)	23,7M

Tab. I: Technical summary of VZ

Maturity	Discount factor
1 mo	1,0465
3 mo	1,0235
5 mo	1,0445
10 mo	1,0415

Tab. II: Discount factor in respect to maturity time

PROCEDURE FOLLOWED AND RESULTS

Estimation of the discount factor from the market

The first step of the procedure is to compute the discount factor $D(0,T)$ of the stock VZ. This can be done by building a "Box Spread". This type of spread can be built by buying two ITM options, a Put and a Call, and by shorting another Put and Call OTM. If the strike prices chosen ITM and OTM, called K_1 and K_2 (with $K_1 < K_2$) are the same for the Put and Call the payoff of the Box has a very simple form and is equal to $K_2 - K_1$. Knowing the prices of all the option involved in the trade allows us to estimate the discount rate of the chosen stock from the market itself as:

$$D_(0,T) = \frac{C_{K_1} - C_{K_2} + P_{K_2} - P_{K_1}}{K_2 - K_1}$$

where $C_{K_1}, C_{K_2}, P_{K_1}, P_{K_2}$ are the prices of the corresponding option at the indicated strike price. By doing this analysis in Excel and for multiple maturity times T it is possible to obtain the tab.II of discount factor estimations after fixing the values of the strike prices $K_1 = 45,00$ and $K_2 = 55,00$.

Estimation of the implicit dividend yield

The deduction of the discount factor from the market allows us to estimate the amount of future dividend that the company associated with the VZ stock is likely to give. This is possible by exploiting the "Call-Put Parity" equation for the price of a given future for the chosen stock. Doing so allows us to write the following:

Maturity	Impl. Dividend Yield (q)
1 mo	-18,14%
3 mo	-2,14%
5 mo	-2,66%
10 mo	-0,60%

Tab. III: Implicit dividend yield by the stock VZ

$$C_{T,K} - P_{T,K} = D(0,T)(F(0,T) - K)$$

where K is the strike price for the Call and Put option ATM, in this case $K = 51$ for $T = 1mo$ and $K = 50$ for the other maturity times. $F(0,T)$ is the strike option of the studied future contract and in our case we can write it as:

$$F(0,T) = \frac{S_0}{D(0,T)} - Div(T)$$

where $S_0 = 51,28$ is the current stock price as of Mar 26, 2022 and $Div(T)$ is the amount of dividends given by VZ in the maturity time T. We can estimate the latter as:

$$Div(T) = S_0 e^{-qT}$$

where q is the implicit dividend yield until time T. Therefore we can estimate it as: (from "The Hull" pag.376)

$$q = -\frac{1}{T} log\left(\frac{C_K - P_K + KD(0,T)}{S_0})\right)$$

From tab.III is possible to comprehend the dividend structure of the stock VZ for the maturities $T = 1, 3, 5, 10$ months.

COMMENT ON THE MODEL UTILIZED AND CONCLUSIONS

From tab.III it is immediately apparent one thing: the dividend yield is negative for every studied maturity. This is somewhat unexpected, since the fact that the dividends usually are a positive sum of money given by the stock company to strengthen its image on the market and reward the loyalty of its shareholders. Our negative values are the product of the bad model implementation that we have chosen. There are mainly two big problems that brought us to this result and both of them have a common root that will be explained in the following sections.

- The first problem is that the Box spread implemented in the first part of this report is designed to reflect the market discount rate on the asset only if it is built with European Options. Since the fact that our main financial database for this analysis is "Yahoo! Finance" the majority of the stock there represented are American options and so does VZ. A box built with American options will be overpriced compared to an European one and this implies that our study of the discount rate $D(0,T)$ overestimates it.

- The second problem is that, just as the Box Spread, also the "Call-Put Parity" exploited in the second part of this report is based upon the assumption that all the traded options are European. For the reasons already mentioned in the latter point we know that VZ is not one of them and therefore this implies that there could be an error in the results given by the analysis.

The combined effect of applying both of this methods to the wrong kind of option is likely the cause of the negative implied divided yield obtained in the analysis. This could be solved by adopting a more sophisticated financial database from "Yahoo! Finance", which could have a simpler way to tell apart and research European options instead of American ones.