# Report 4 - Financial Mathematics for Data Science

Luca Agosti - 2062889
(Dated: 03/04/2022)

The aim of this report is to study the Greeks's behaviour in respect to their parameters, in particular stock price, maturity time and volatility. The first part will be a calculation of the Greeks for a theoretical situation and in the second part we will apply them to a real example: a stock taken from the market. The third part will be about a comparison of the historical volatility to the implied one.

## FIRST PART

In this section we aim to study the Greeks of an European Call option of a stock which does not provide dividends, while fixing the strike price at $K = 100$ and the interest rate at 1%. The remaining parameters: volatility ($vol$), stock price ($S$) and maturity time ($T$) are not fixed and can vary in the following ranges:

- $vol \in \{5\%, 20\%, 75\%\}$
- $S$ from 60 to 140 with step 5
- $T \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 2, 3, 4, 5\}$ years

The Greeks calculation is performed in Excel using VBA code [1] and the results obtained can be visualized by plotting the Greeks's value in function of $T$ and $S$ for each one of them and for each volatility obtaining the graphs in fig.1.

From the graphs it is possible to notice immediately that $\Delta$, $\Gamma$ and $\Theta$ are considerably smoother when $T$ increases. This is because when the time to maturity is large hedging our portfolio requires less frequents trades than when $T$ is small. On the contrary $\nu$ and $\rho$ are increasing and also less smooth for large $T$. This behaviour is expected since the fact that for large maturities the effect of the volatility and interest rate is much greater than in a shorter span of time. The only unexpected graph is the one for $\Theta$ with volatility equal to 5%. We can see that in this case the Greek has a positive value of 0.002 instead of being always negative. By doing research I discovered that for an ITM European Call option on a non-dividend paying stock it is possible to have a positive value of $\Theta$ if the interest rate is sufficiently large [2]. Considering that this is exactly the case we are observing in the image I propose this phenomenon as a possible explanation. Lastly, a consideration about the volatility: it is possible to see how each and every one of the Greeks "opens up" when the volatility increases. This is a market reflection of the fact that for small values of $vol$ the probability of moving far ITM or OTM decreases and this implies that the graphs changes very rapidly around the ATM line, because it is near there that the stock will be the vast majority of the time, with much less variance than a situation with a greater volatility.

## SECOND PART

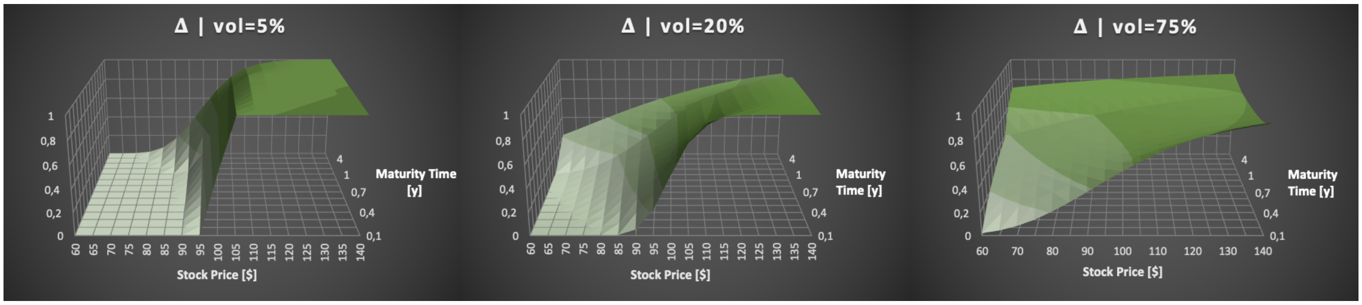
In the second part of the report we study the implied volatility and Greeks of the European Call options on a non-dividend paying stock. Finding such asset was not easy since the fact that we are using "Yahoo! Finance" as database and it does not declare the type of the option for any given stock. By doing research I discovered that the vast majority of the options traded on Indexes are European and with this knowledge I managed to find one with all the required criteria: the S&P500 Index (SPX) [3]. As a first step I graphed the value of the implied volatility indicated on "Yahoo! Finance" to see if it was possible to notice a general pattern for its behaviour. This graph is shown in fig.2. The ranges of the maturity time ($T$) and strike price ($K$) are:

- $T \in \{1, 2, 3, 6, 9, 24\}$ months
- $K$ from 4000 to 5500 with step 100

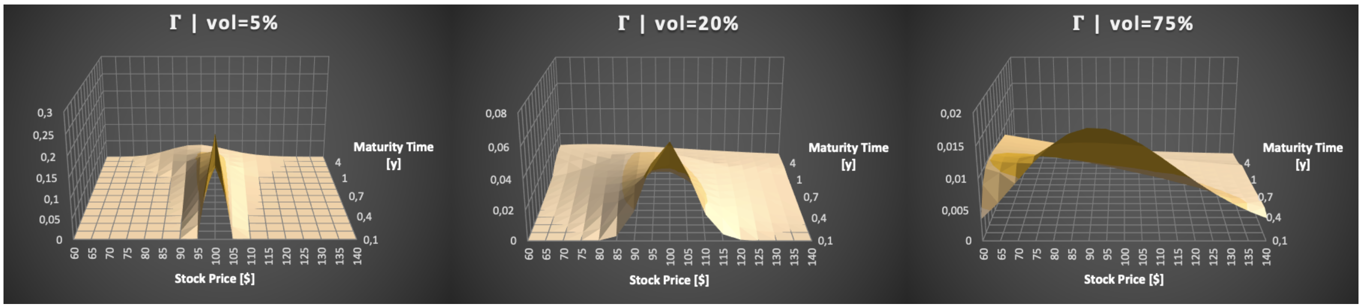
unfortunately the data for $T = 12$ months had very poor quality, maybe because it is less traded than the other maturity, and therefore I preferred to exclude that maturity from the analysis. Instead I added the data for $T = 24$ months to have at least one $T$ above one year. From the implied volatility graph is possible to see the characteristic volatility smile for small $T$ and is also possible to notice the flattening of the surface as the maturity increases. This behaviour is expected since the fact that by increasing $T$ the Black-Scholes formula tends to approximate better and better the real market price and therefore we expect to see the flat volatility curve predicted by the model.
In addiction to the implied volatility is also possible to study the Greeks of the S&P500 index. By varying $T$ and $K$ in the ranges described before and by fixing the stock price to $S = 4545, 86$: the market value at the time of writing Apr 03, 2022, is possible to obtain the Greeks. We consider as volatility values the ones reported in fig.2. The interest rate ($r$) values are extrapolated from Bloomberg Apr 01, 2022:

It is now possible to plot the Greeks for the S&P500 Call option, obtaining the graphs shown in fig.3.

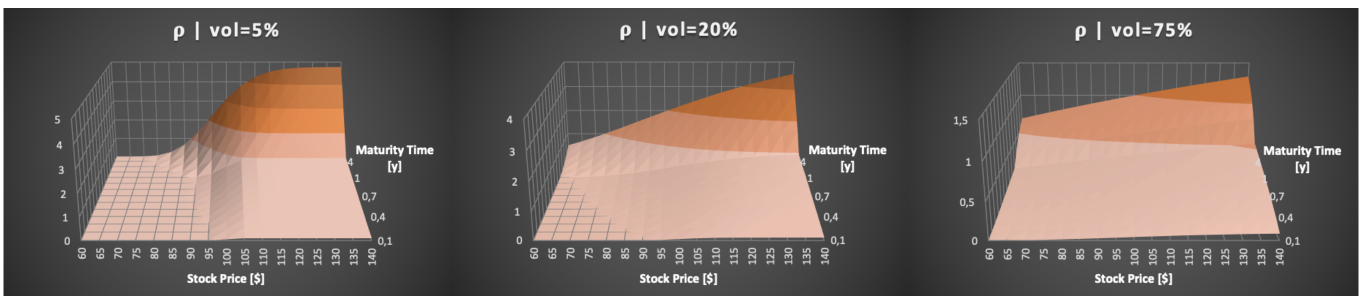
The graphs show that each and every one of the Greeks has a familiar behaviour, very similar to the one shown



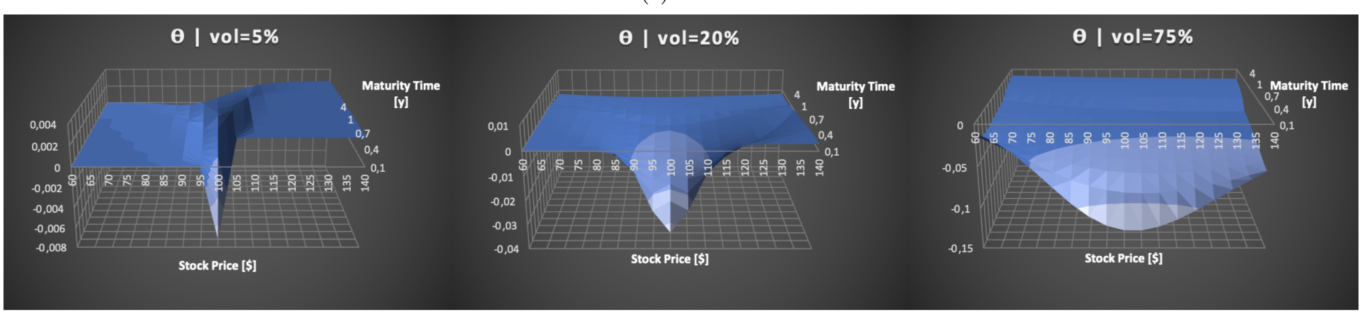

(a) Delta



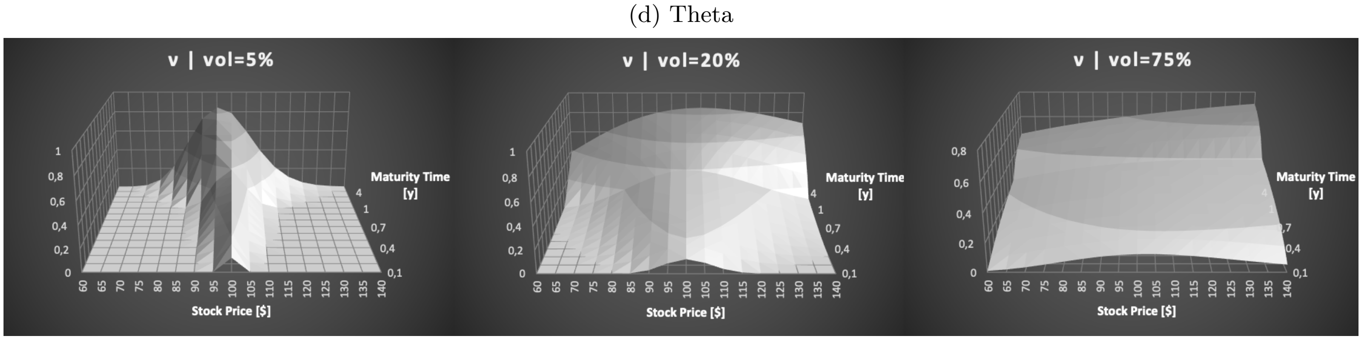

(b) Gamma




(c) Rho




(d) Theta




(e) Vega

Fig. 1: Greeks for the European Call option with $K = 100$ and $r = 1\%$

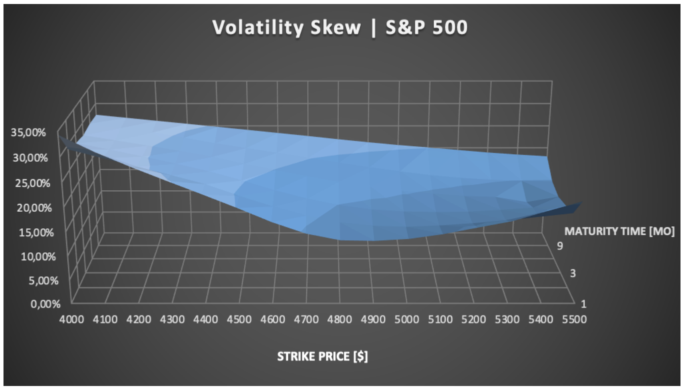



Fig. 2: Implied volatility skew for the S&P500 index

| r [%] | T [mo] |
|---|---|
| 0,96167 | 1 |
| 0,97770 | 2 |
| 0,97236 | 3 |
| 1,36804 | 6 |
| 1,73292 | 9 |
| 2,65335 | 24 |

Tab. I: Interest rates (US zone)

in fig.1 for the theoretical example in the first part. The only notable thing is that the $\Gamma$ graph is asymmetric, implying that for the S&P500 index is more urgent to modify the portfolio for Call options just OTM than for the ITM ones. In the other graphs we can see the smoothness patterns already observed and discussed in the First Part.

## THIRD PART

In the third and last part of this report we compute the historical volatility of the S&P500 and use it to price an ATM Call option for the same maturities chosen in the second part. We will then compare the prices obtained using the Black-Scholes formula with the historical volatility to the ones indicated on "Yahoo! Finance" for the same option. Unfortunately Yahoo! does not provide the historical daily prices for my index, so I used the Nasdaq database to gather my data [4]. After importing the historical data on Excel it was possible to compute the daily volatilities as the standard deviation of each daily returns vector, one for each maturity studied, in my case the maturities were the same as the second part, so the ones in tab.I. From the daily volatilities I computed the yearly ones for each $T$, I concluded by applying the Blach-Scholes formula using them with strike price ATM $K = 4550$ and interest rate $r$ taken from Bloomberg and indicated in I. The results of this calculations are shown in II with the market price taken from "Yahoo! Finance" the Apr 03, 2022 as a comparison.

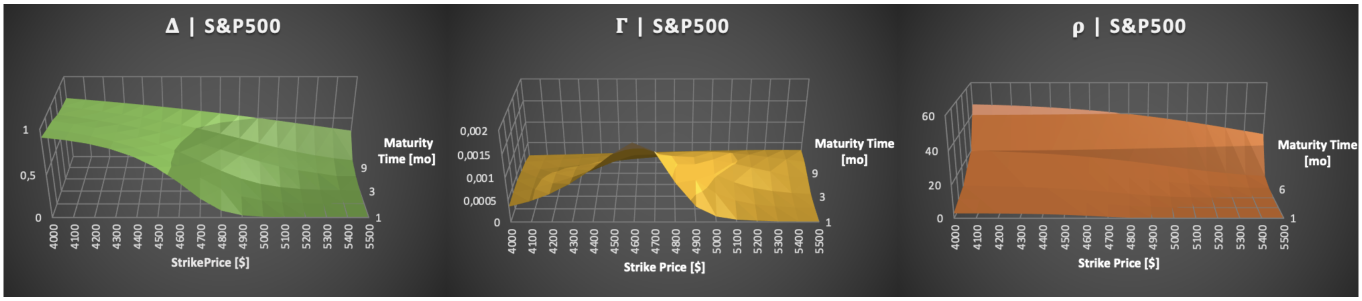
| T [mo] | B&S | Market | Error [%] |
|---|---|---|---|
| 1 | 118,27 | 72,93 | 62,17 |
| 2 | 168,29 | 110,00 | 52,99 |
| 3 | 196,75 | 154,36 | 27,46 |
| 6 | 243,72 | 239,83 | 1,62 |
| 9 | 278,17 | 323,00 | -14,88 |
| 24 | 582,61 | 525,83 | 10,80 |

Tab. II: Comparison between the Call prices obtained with the B&S formula and from the market with relative error

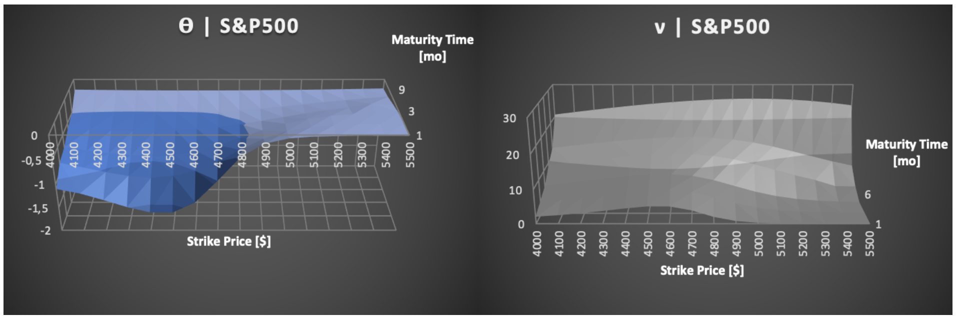
From the tab.II it is immediately obvious that the two prices are very different, with an extremely large relative error for five out of the six maturities studied. The reason of this discrepancy could be in the difference between volatilises. In graph 4 is plotted the difference between the historical volatility and the implied one for each maturity studied.

One of the possible explanation of these discrepancies is an Over/Under valuing of the option. Large differences between volatilities could be the indication of an arbitrage opportunity as a consequence of the mispricing of the contract. However the difference is so large that is more probable that the gap between prices is just a remark of the fact that the Black-Scholes model is just too simple to take into account all the possible factors that goes into the market pricing of the option. We can only use that model to have an approximation at order zero, but not to reliably predict every price for each possible maturity.

[1] "https://investexcel.net/black-scholes-greeks-vba/," .
[2] J. C. Hull, "Futures and other derivatives," (2015).
[3] "https://finance.yahoo.com/quote/%5espx/options/," .
[4] "https://www.nasdaq.com/market-activity/index/spx/historical," .




(a) Delta, Gamma and Rho




(b) Theta and Vega

Fig. 3: Greeks for the S&P500 Call option

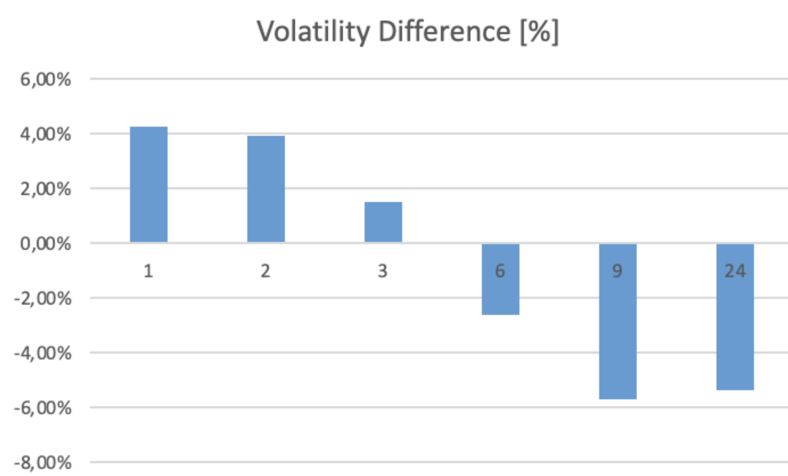



Fig. 4: Difference between the historical volatility and the implied one as a function of the maturity time in months