Report 1 - Financial Mathematics for Data Science

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The aim of this report is the pricing of a 3 month and 6 month Call option using the (static) binomial model and a comparison with the corresponding market quote.

INTRODUCTION

The chosen stock for the option evaluation is "T-Mobile US, Inc. (TMUS)". The company, together with its subsidiaries, provides mobile communications services in the United States, Puerto Rico, and the United States Virgin Islands. The company offers voice, messaging, and data services to 108.7 million customers in the post-paid, prepaid, and wholesale markets. It also provides wireless devices, including smartphones, wearables, and tablets and other mobile communication devices, as well as wireless devices and accessories. In addition, the company offers services, devices, and accessories under the T-Mobile and Metro by T-Mobile brands through its owned and operated retail stores, T-Mobile app and customer care channels, and its websites. It also sells its devices to dealers and other third-party distributors for resale through independent third-party retail outlets and various third-party websites. As of December 31, 2021, it operated approximately 102,000 macro cell and 41,000 small cell/distributed antenna system sites. The company was founded in 1994 and is headquartered in Bellevue, Washington. What follows is a technical summary of some relevant information about TMUS.

Market Cap	155.62B
Enterprise Value	255.00B
Revenue	80.12B
Revenue per Share	64.24
Fiscal Year Ends	Dec 30 ,2021
Ex-Dividend Date	Apr 30, 2013
Last Split Date	Apr 30, 2013
Average Vol (3 month)	5.43M

Tab. I: Technical summary of TMUS

DATA GATHERING & PRICING PROCEDURE

For both the 3-month option and the 6-month one the procedure followed is the same. First of all the historic data of the stock is downloaded from "Yahoo! Finance" and exported into an Excel file where all the analysis calculation is done. Then, starting from this grid of raw data, the following steps are followed:

- From the "Adj. close" values compute the daily returns.

$$Return_t = \frac{Price_{t+1} - Price_t}{Price_t}$$

- Compute the daily volatility as the standard deviation of the daily returns vector.
- Obtain the yearly volatility from the daily one.

$$\sigma_{yearly} = \sigma_{daily}\sqrt{252}$$

- Compute the parameters u and $d = \frac{1}{u}$ of the binomial model from:

$$u = e^{\sigma_{yearly}\sqrt{T}}$$

  where T is the maturity period of the option expressed in years.

- Compute the capitalization factor r and discounting factor $\frac{1}{r}$ using simple compounding:

$$r = 1 + RT$$

  where R (the interest rate) is taken from "USD LIBOR interest rates - maturity 3 months" - Mar 11, 2022.

- Compute the risk neutral probability weight q.

$$q = \frac{r - d}{u - d}$$

- Compute the option price.

$$Call_{price} = \frac{1}{r}\left[q(Su - K, 0)^+ + (1 - q)(Sd - K, 0)^+\right]$$

  where S is the current market value of stock. The chosen strike price for this initial analysis is K=120, near the ATM price.

RESULTS

Before diving into the results discussion a few important remarks, the first one is that the calculations for this report were performed Mar 12, 2022 so every stat indicated in this work is updated only until that date. The second one is that, as we can see from table I, this stock does not provide dividends, so it is not necessary

Model Parameters		Daily Volatility		u Factor	Capitalisation factor	
S - 11/03/22	124,67	σ-daily	0,0221765	1,1924623	1,002065	
K - 17/06/22	120					
T [years]	0,25	Yearly Volatility		d Factor	Risk neutral probability weight	
R - 11/03/22	0,00826	σ-yearly	0,3520406	0,838601	q	0,46194381
Risk neutral price for the Call option - Model		Mid market price - 3 mo. Call option - K=120 - "Yahoo! Finance"				Difference
13,21		11,35				16,42%

Fig. 1: Summary of the parameters and results obtained for the pricing of a 3 month call option of the stock "TMUS"

Model Parameters		Daily Volatility		u Factor	Capitalisation factor	
S - 11/03/22	124,67	σ-daily	0,01676649	1,1874467	1,004710708	
K - 19/08/22	120					
T [years]	0,417	Yearly Volatility		d Factor	Risk neutral probability weight	
R - 11/03/22	0,0113057	σ-yearly	0,26615974	0,84214306	q	0,470796217
Risk neutral price for the Call option - Model		Mid market price - 5 mo. Call option - K=120 - "Yahoo! Finance"				Difference
13,14		13,75				-4,45%

Fig. 2: Summary of the parameters and results obtained for the pricing of a 5 month call option of the stock "TMUS"

to account for them in the option price estimation.
The results of this preliminary analysis are shown in figure 1 and 2. Unfortunately the stock "TMUS" does not offer a 6-month call option but rather a 5-month one. I adjusted the parameter T accordingly, from 0.5 to 0.417.

We can compare the price estimation of the model with the one offered by the market considering the "Mid market price" of the option, obtained as the average of the offered "Bid" and "Ask" prices. We can than compute the difference between them to observe that the model utilized in this analysis is overpricing the studied options by more than 16% for the 3 mo. and underpricing the 5 mo. by 4%. This results are satisfying considering the simplicity of our model, but it still brought me to study more closely the price behavior in function of K. In figure 3 and 4 are shown the prices of all the 3mo. and 5mo. call options offered with a strike price between 55 and 165.

From the values of the difference between the estimated price and the market one we can see that the model is overpricing the 3mo. option in an interval around the ATM value, from $K = 115$ to $K = 140$ and underpricing when far ITM or OTM. The 5mo. option instead is being underpriced for every value of K. We can also see that for both options the extreme OTM calls are being priced zero dollars by the model instead of the small, but not null, price that they are being offered at. This implies that the current model cannot be used in that price region, but that is understandable considering its simplicity. Also for the 3mo. option the underpricing problem isn't nearly as big as for the 5mo. one, this brings me to believe that the model is failing to grasp some aspects of the pricing problem related to how much time in the future the estimation is done for. In conclusion just an observation on the behaviour of the "Difference" values, it is noticeable that the gap between the binomial model and the mid market price is following a trend similar to a sine-like function damped by an exponential.

Analysis for multiple strike prices K		
K - 17/06/22	Price Call option - Model	Mid market price - 3 mo. Call option - K=120 - "Yahoo! Finance"	Difference
55	69,78	69,73	0,08%
65	59,80	60,23	-0,71%
70	54,81	56,05	-2,20%
75	49,82	50,15	-0,65%
80	44,83	44,24	1,34%
85	39,85	39,50	0,87%
90	34,86	36,30	-3,98%
95	29,87	30,35	-1,60%
100	24,88	27,03	-7,97%
105	20,13	22,65	-11,13%
110	17,82	18,60	-4,17%
115	15,52	14,75	5,21%
120	13,21	11,35	16,42%
125	10,91	8,58	27,14%
130	8,60	6,13	40,36%
135	6,30	4,33	45,48%
140	3,99	3,06	30,53%
145	1,69	2,05	-17,60%
150	0,00	1,39	-100,00%
155	0,00	0,91	-100,00%
160	0,00	0,56	-100,00%
165	0,00	0,42	-100,00%

Fig. 3: Analysis for multiple strike prices - 3 month call option

Analysis for multiple strike prices K		
K - 19/08/22	Price Call option - Model	Mid market price - 5 mo. Call option - K=120 - "Yahoo! Finance"	Difference
55	69,93	70,40	-0,67%
80	45,05	46,95	-4,06%
85	40,07	42,20	-5,05%
90	35,09	37,23	-5,74%
95	30,12	30,20	-0,28%
100	25,14	28,45	-11,64%
105	20,17	24,43	-17,45%
110	17,82	20,45	-12,84%
115	15,48	17,08	-9,36%
120	13,14	13,75	-4,45%
125	10,80	10,98	-1,68%
130	8,45	8,70	-2,84%
135	6,11	6,63	-7,84%
140	3,77	5,05	-25,41%
145	1,42	3,83	-62,82%
150	0,00	2,76	-100,00%
155	0,00	2,08	-100,00%
160	0,00	1,70	-100,00%
165	0,00	1,05	-100,00%

Fig. 4: Analysis for multiple strike prices - 5 month call option