

Computational Finance - Final Project

a.a. 2022-2023

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Objective: building a conservative portfolio of assets

Benchmark: an equity stock index for the US (or North America) market

Assets: split of the benchmark by supersectors

Overview



1. Introduction

- 1.1. Analysis of prices and returns
- 1.2. Correlation

2. Study on different constraints and moment types

- 2.1. Analysis with sample moments
 - 2.1.1. Unconstrained portfolio
 - 2.1.2. Portfolio without short selling
- 2.2. Analysis with equilibrium moments
 - 2.2.1. Unconstrained portfolio
 - 2.2.2. Portfolio without short selling
- 2.3. Comparison
- 2.4. Analysis considering a different time window

3. Building a conservative portfolio

- 3.1. Analysis with EWMA moments
 - 3.1.1. Portfolio without short selling
 - 3.1.2. Portfolio with upper bound
 - 3.1.3. Portfolio with group constraints
- 3.2. Analysis with sample moments
 - 3.2.1. Portfolio without short selling
 - 3.2.2. Portfolio with upper bound
 - 3.2.3. Portfolio with group constraints
- 3.3. Comparison

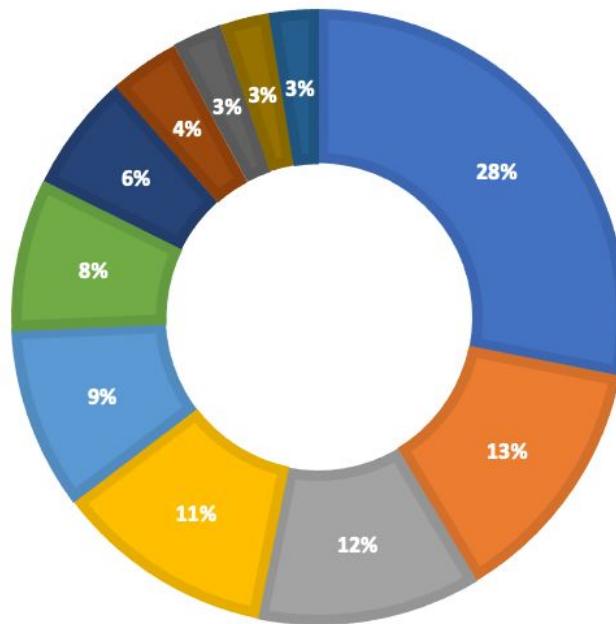
4. Conclusion

1. Introduction

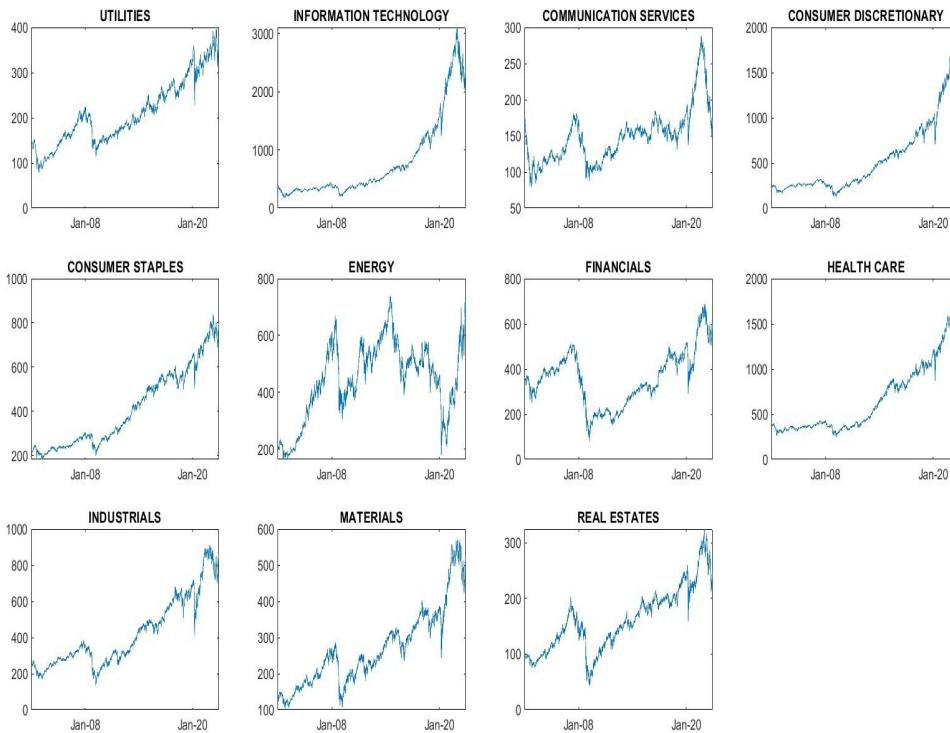
Prices and Returns

S&P 500

■ Information technology	■ Health care
■ Communication service	■ Industrials
■ Real estate	■ Materials
■ Consumer discretionary	■ Financials
■ Consumer staples	■ Energy
■ Utilities	

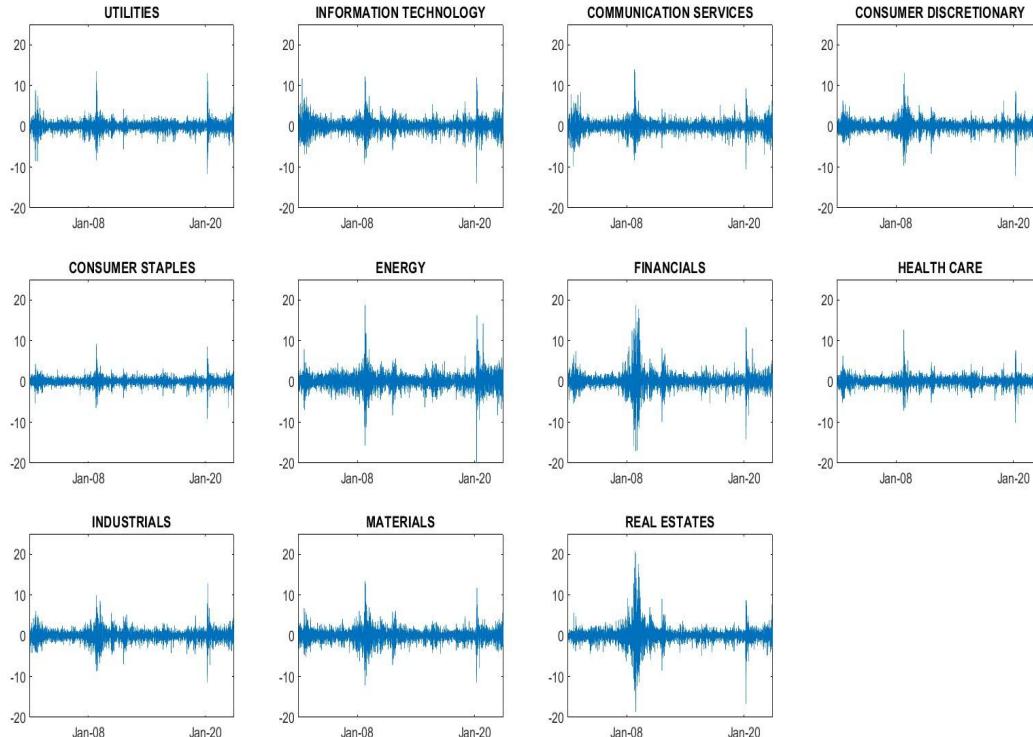


Asset prices



- **Steady increase of prices**
- **We notice the sudden drops during financial crises**

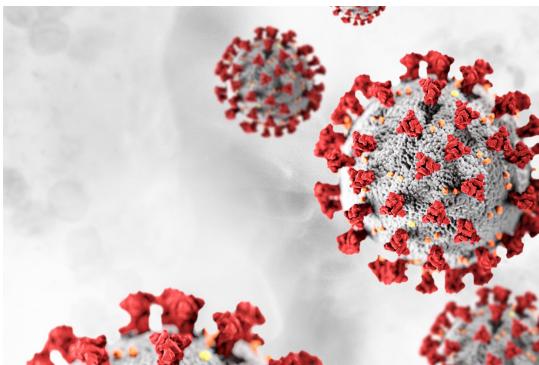
Supersectors' daily returns



- **High volatility during:**
 1. **2008 Financial crisis**
 2. **2020 COVID emergency**
- **Most volatile:**
 1. **Real Estate**
 2. **Energy**
- **Least volatile:**
 1. **Healthcare**
 2. **Consumer staples**

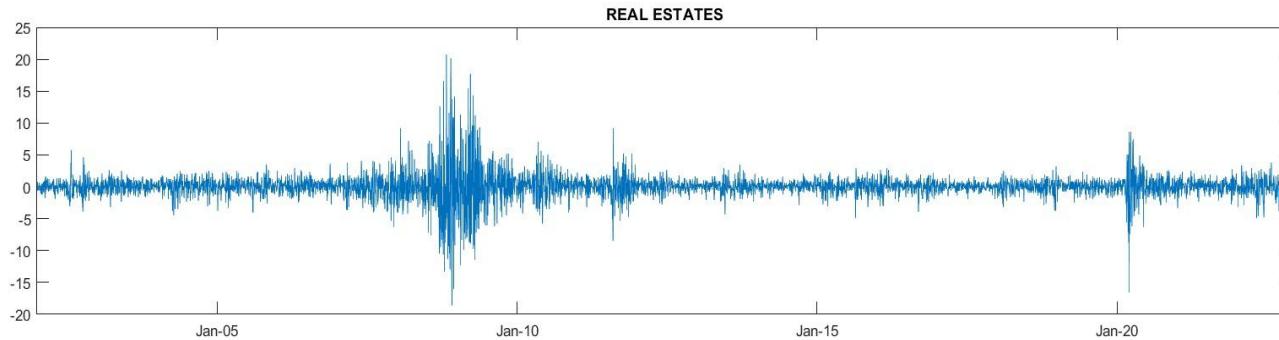
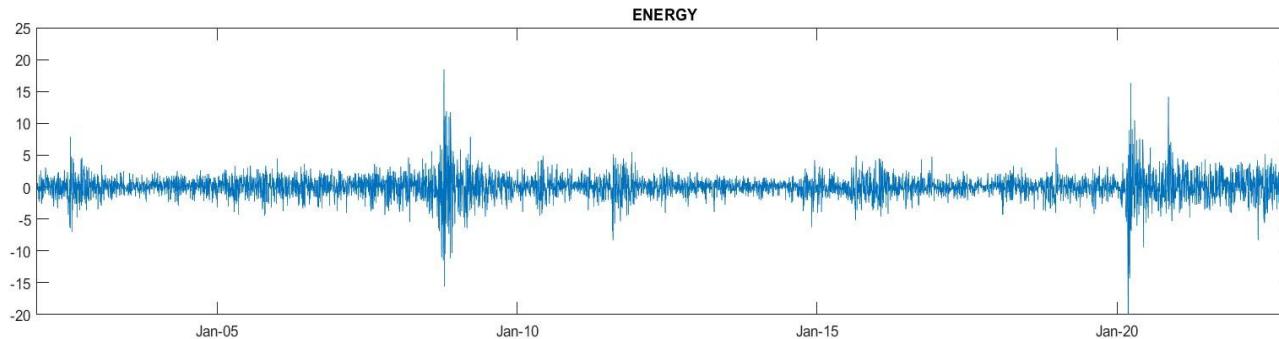
Why?

- Why did the 2008 financial crisis affect the real estate supersector?

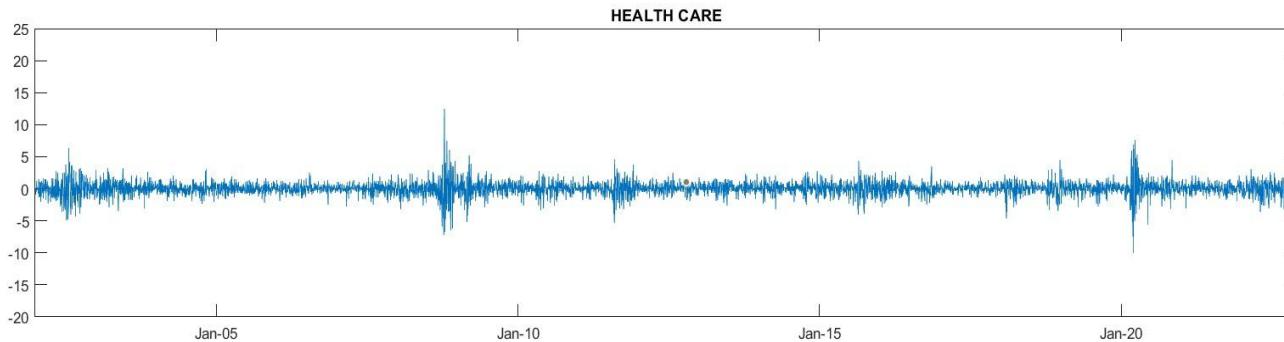
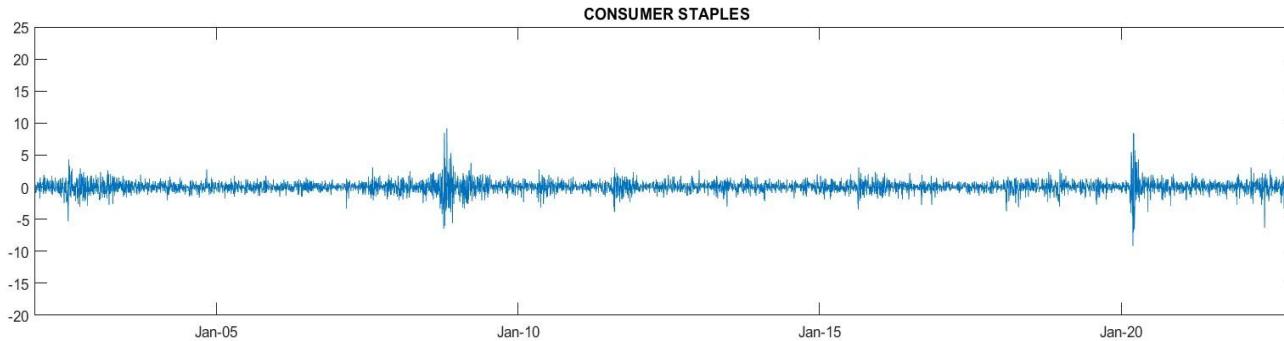


- Why did the 2020 COVID-19 emergency affect the energy sector?

Focus on the two most unstable assets



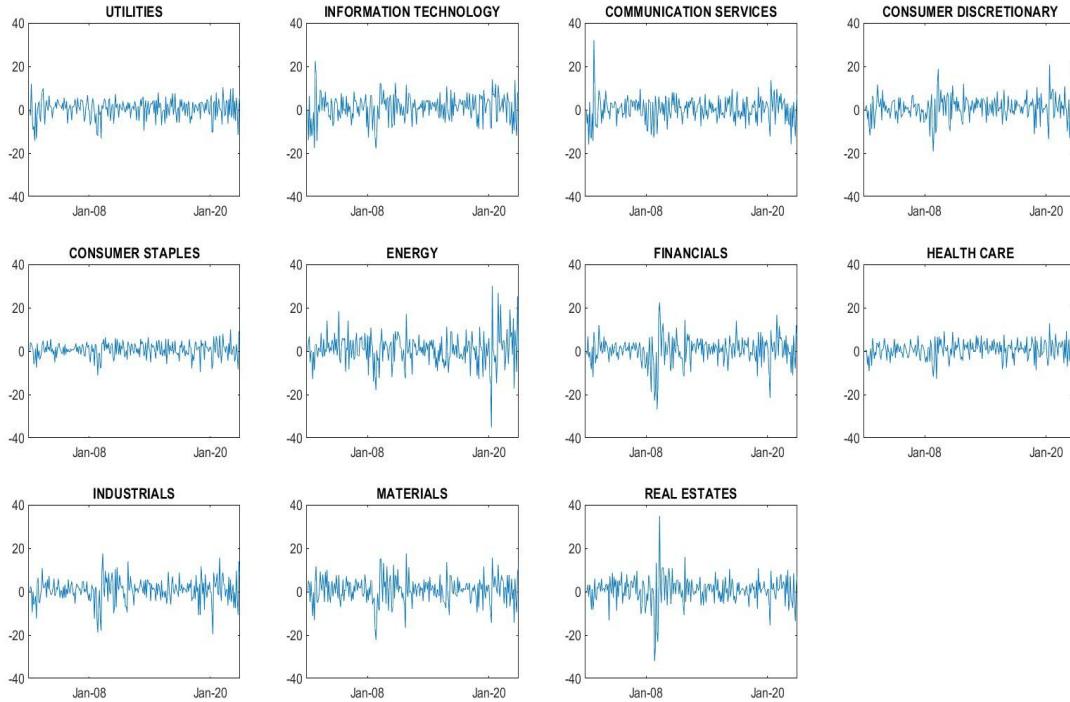
Focus on the two most stable assets



Daily returns analysis

	Mean	Median	StDev	Min	Max	Skew	Kurt	
S&P500 ES UTILITIES	0.02	0.04	1.22	-11.54	13.52	0.25	18.56	● Positive means
S&P500 ES INFO TECHNOLOGY	0.05	0.05	1.52	-13.91	12.14	0.11	10.26	
S&P500 ES COMM. SVS	0.01	0.00	1.38	-10.44	13.80	0.15	11.79	● Skewness value is around zero
S&P500 ES CONSUMER DISCRETIONARY	0.04	0.04	1.37	-12.08	13.10	-0.10	10.96	
S&P500 ES CONSUMER STAPLES	0.03	0.02	0.91	-9.24	9.24	-0.04	16.06	
S&P500 ES ENERGY	0.04	0.00	1.80	-20.08	18.48	-0.22	16.31	
S&P500 ES FINANCIALS	0.03	0.00	1.87	-17.01	18.77	0.34	20.66	● High Kurtosis
S&P500 ES HEALTH CARE	0.03	0.03	1.09	-9.99	12.43	-0.02	12.71	
S&P500 ES INDUSTRIALS	0.03	0.03	1.34	-11.45	12.75	-0.21	11.51	
S&P500 ES MATERIALS	0.04	0.03	1.50	-12.13	13.28	-0.18	10.97	
S&P500 ES REAL ESTATE	0.03	0.04	1.89	-18.47	20.74	0.42	23.31	

Monthly returns



- **Less notable trends in the monthly returns compared to the daily ones**

Monthly returns analysis

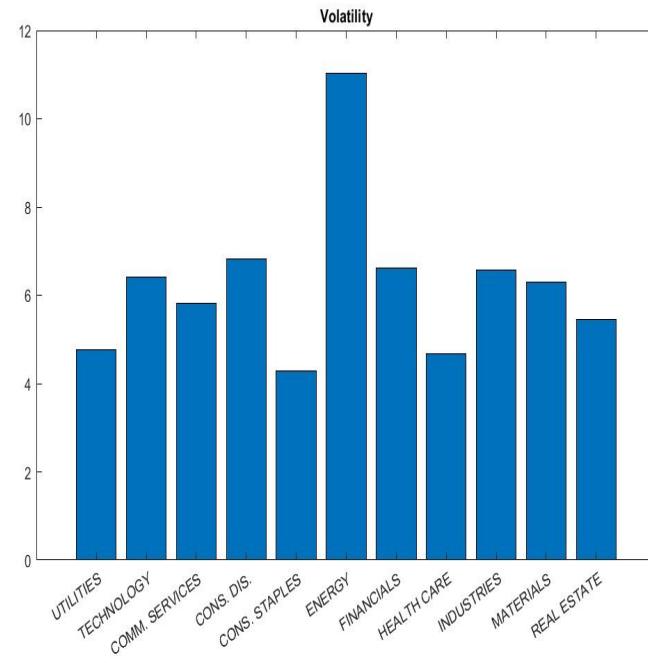
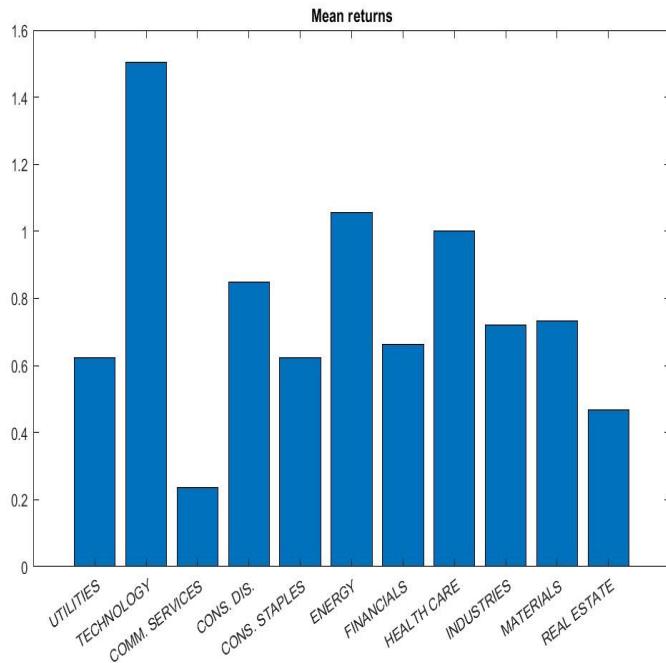
	Mean	Median	StDev	Min	Max	Skew	Kurt
S&P500 ES UTILITIES	0.48	0.98	4.37	-14.15	11.91	-0.70	3.97
S&P500 ES INFO TECHNOLOGY	0.93	1.57	5.93	-17.81	22.29	-0.30	3.85
S&P500 ES COMM. SVS	0.20	0.42	5.54	-15.94	31.87	0.27	6.88
S&P500 ES CONSUMER DISCRETIONARY	0.77	0.75	5.38	-19.27	20.51	-0.06	4.70
S&P500 ES CONSUMER STAPLES	0.57	0.82	3.35	-11.08	9.95	-0.46	3.67
S&P500 ES ENERGY	0.75	1.03	7.29	-34.97	29.66	-0.03	6.57
S&P500 ES FINANCIALS	0.42	1.13	6.25	-26.55	22.17	-0.68	6.03
S&P500 ES HEALTH CARE	0.65	0.86	3.93	-12.84	12.50	-0.31	3.46
S&P500 ES INDUSTRIALS	0.64	0.98	5.36	-19.29	17.72	-0.50	4.90
S&P500 ES MATERIALS	0.71	0.87	5.86	-22.18	17.60	-0.30	4.20
S&P500 ES REAL ESTATE	0.59	0.99	6.31	-32.09	34.77	-0.59	10.04

- **Higher means**
- **Negative Skewness**
- **Kurtosis higher than 3 but lower in respect to the daily returns**

2. Study on different constraints and moment types

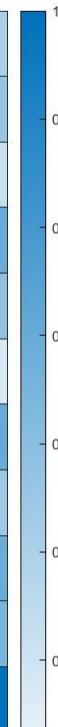
Focus on last 5 years

Mean returns & Volatility

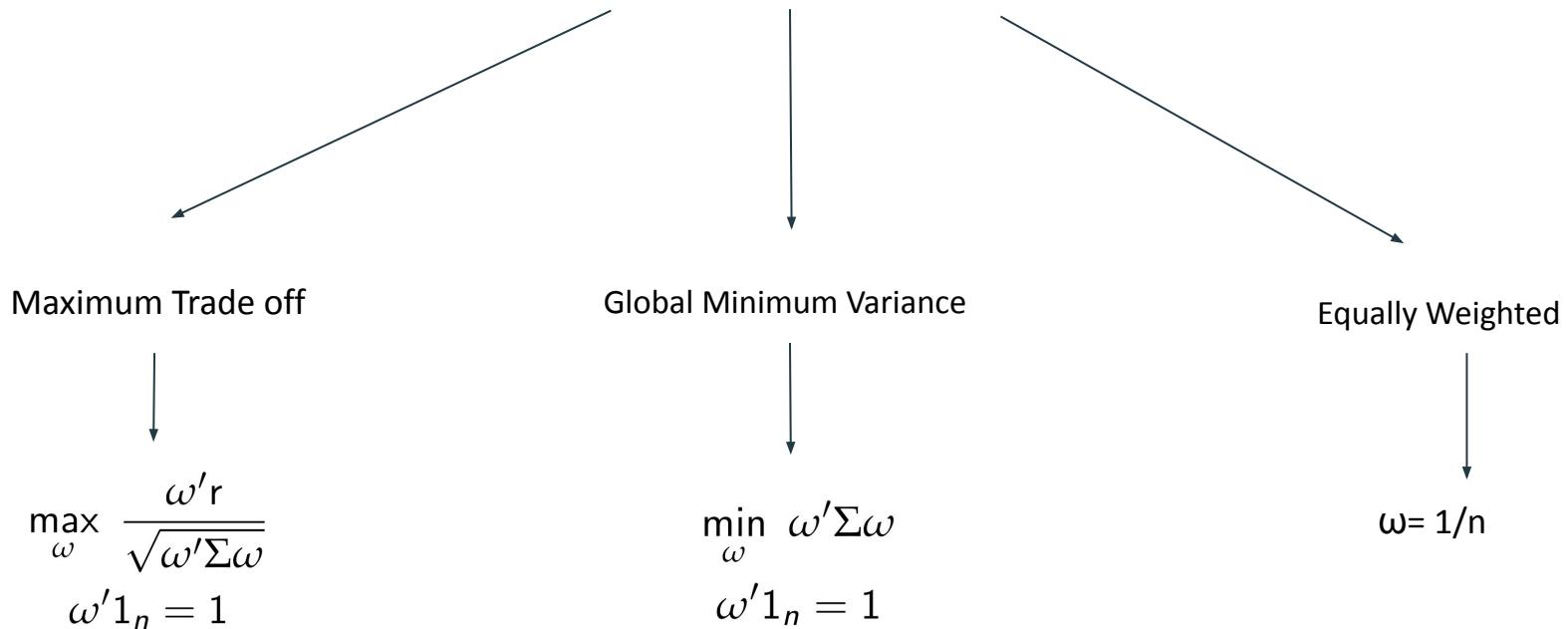


Correlation Matrix between returns

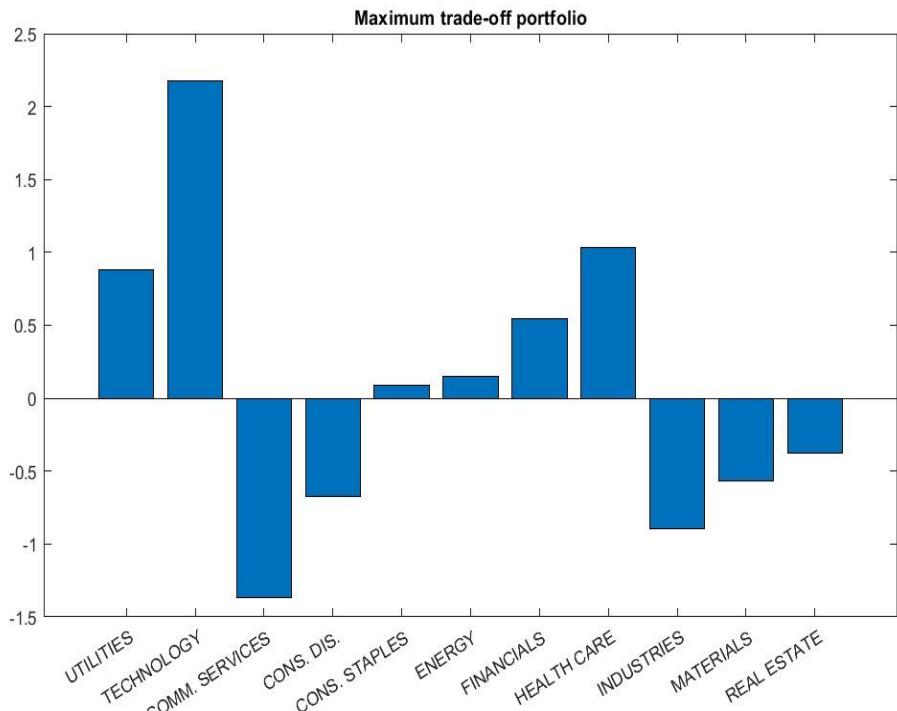
Correlation matrix											
	UTI	TEC	C&S	DIS	STP	ERG	FIN	HEA	IND	MAT	REA
UTI	1	0.4066	0.467	0.4006	0.5842	0.3335	0.3554	0.4659	0.4295	0.4132	0.5065
TEC	0.4066	1	0.6454	0.8154	0.5539	0.4681	0.6782	0.6297	0.7617	0.7373	0.5625
C&S	0.467	0.6454	1	0.6022	0.5491	0.4242	0.5086	0.521	0.5984	0.5503	0.4195
DIS	0.4006	0.8154	0.6022	1	0.6316	0.4963	0.7722	0.6666	0.8327	0.7849	0.6937
STP	0.5842	0.5539	0.5491	0.6316	1	0.4096	0.6103	0.6799	0.689	0.6076	0.583
ERG	0.3335	0.4681	0.4242	0.4963	0.4096	1	0.5586	0.4279	0.6216	0.652	0.371
FIN	0.3554	0.6782	0.5086	0.7722	0.6103	0.5586	1	0.6454	0.8554	0.7623	0.6977
HEA	0.4659	0.6297	0.521	0.6666	0.6799	0.4279	0.6454	1	0.6935	0.647	0.5445
IND	0.4295	0.7617	0.5984	0.8327	0.689	0.6216	0.8554	0.6935	1	0.869	0.6854
MAT	0.4132	0.7373	0.5503	0.7849	0.6076	0.652	0.7623	0.647	0.869	1	0.6442
REA	0.5065	0.5625	0.4195	0.6937	0.583	0.371	0.6977	0.5445	0.6854	0.6442	1



Portfolio

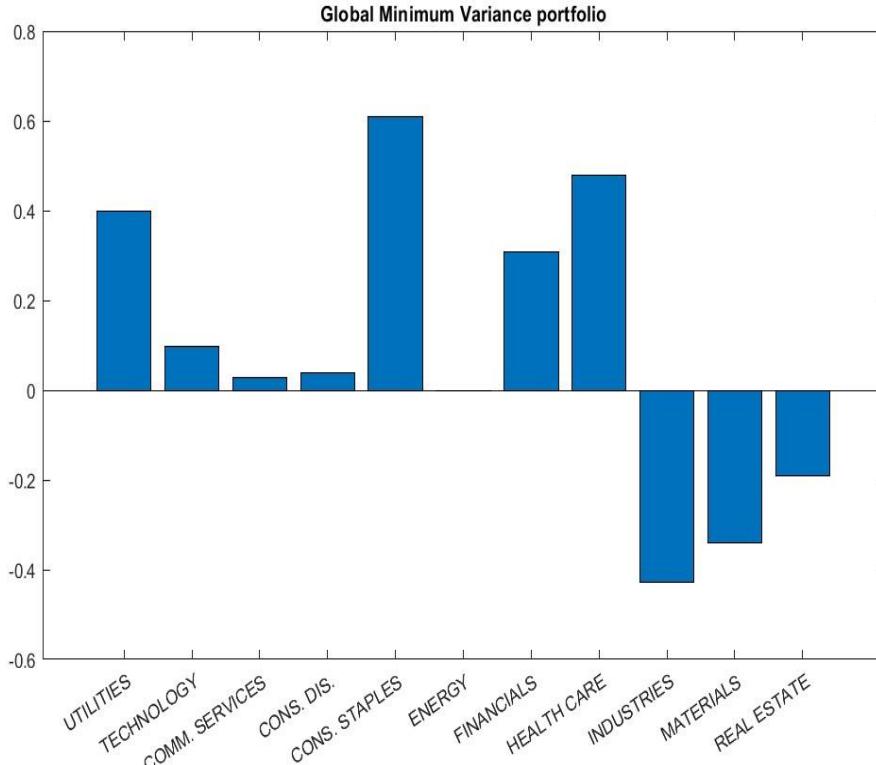


Maximum trade-off portfolio (MS or TAN)



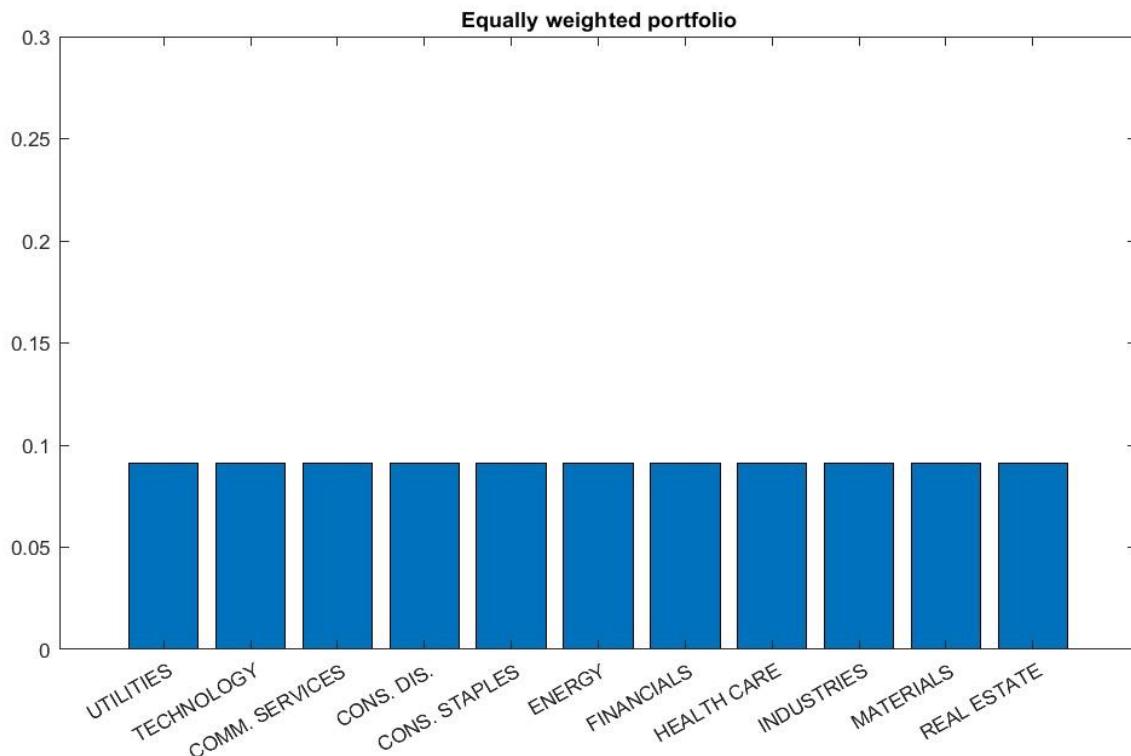
- We are investing in the assets with the highest expected returns and shorting the ones with a lower value, maximizing the MS portfolios condition

Global Minimum Variance portfolio (GMV)

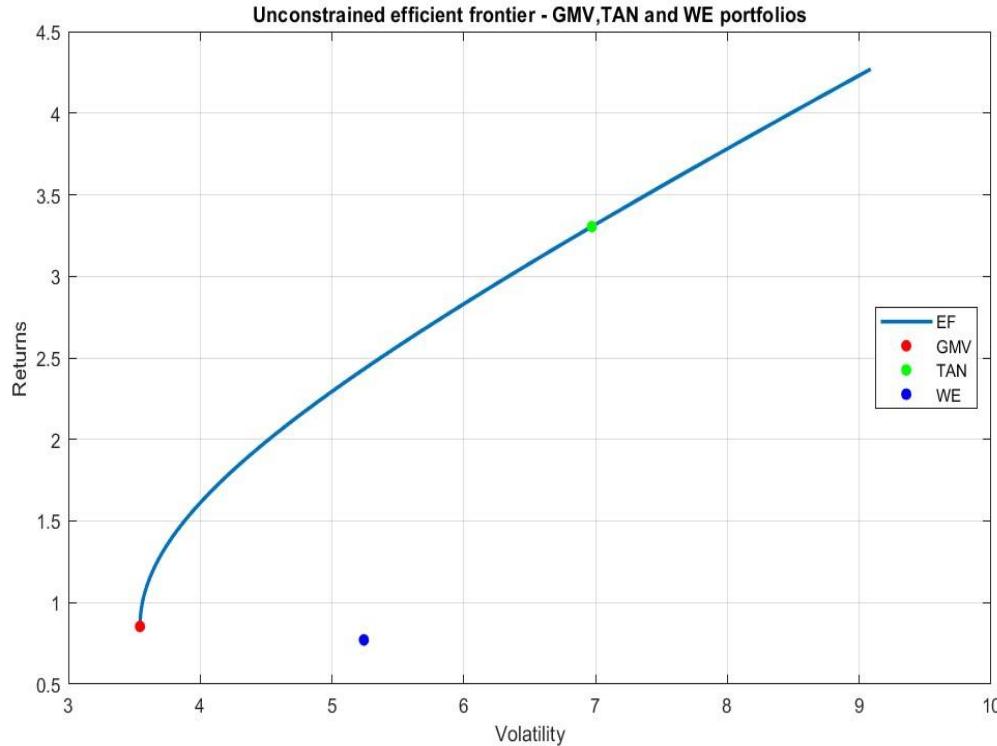


- We are investing in the assets with the lowest volatility value

Equally Weighted portfolio (WE)



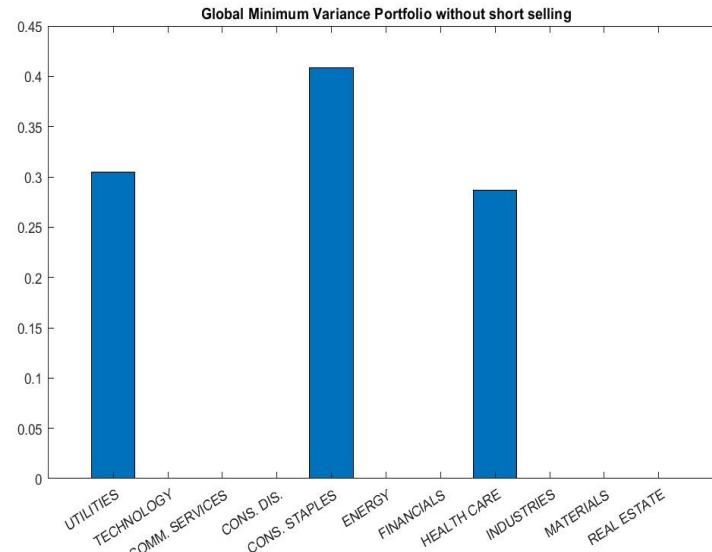
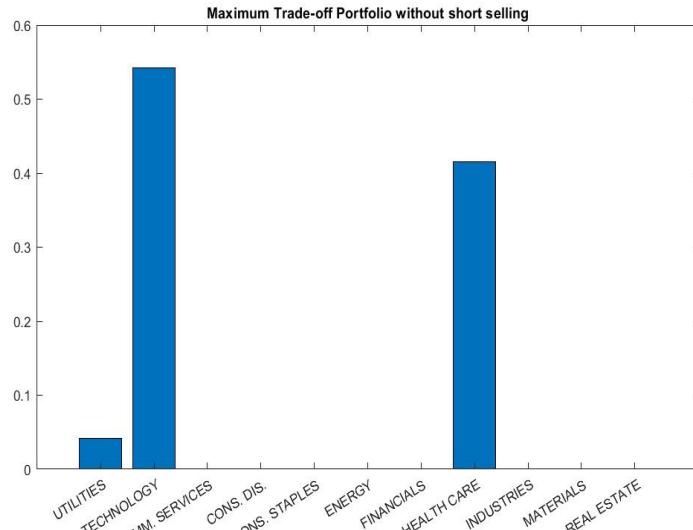
Unconstrained efficient frontier



- **The EW portfolio is performing very poorly when compared to the ones in the efficient frontier (EF)**

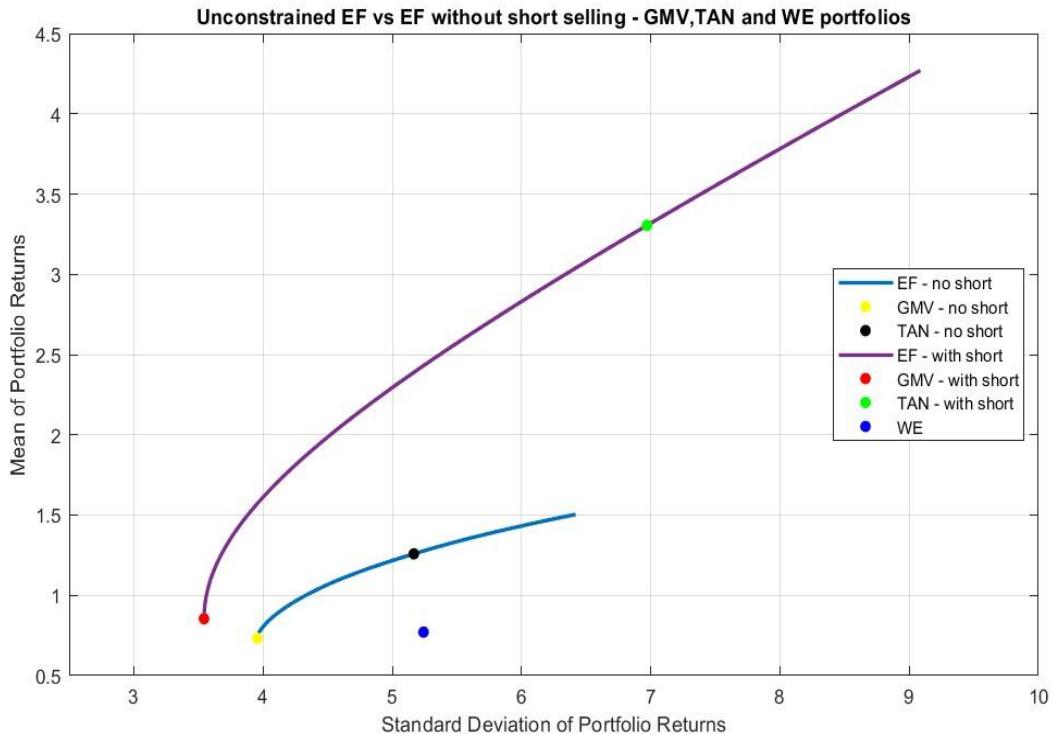
Positive weights (No short selling)

Maximum Trade-off and GMV Portfolios - No short selling



- With the additional constraint the MS and GMV portfolios are investing only in a few assets

Comparison between unconstrained EF vs EF without short selling

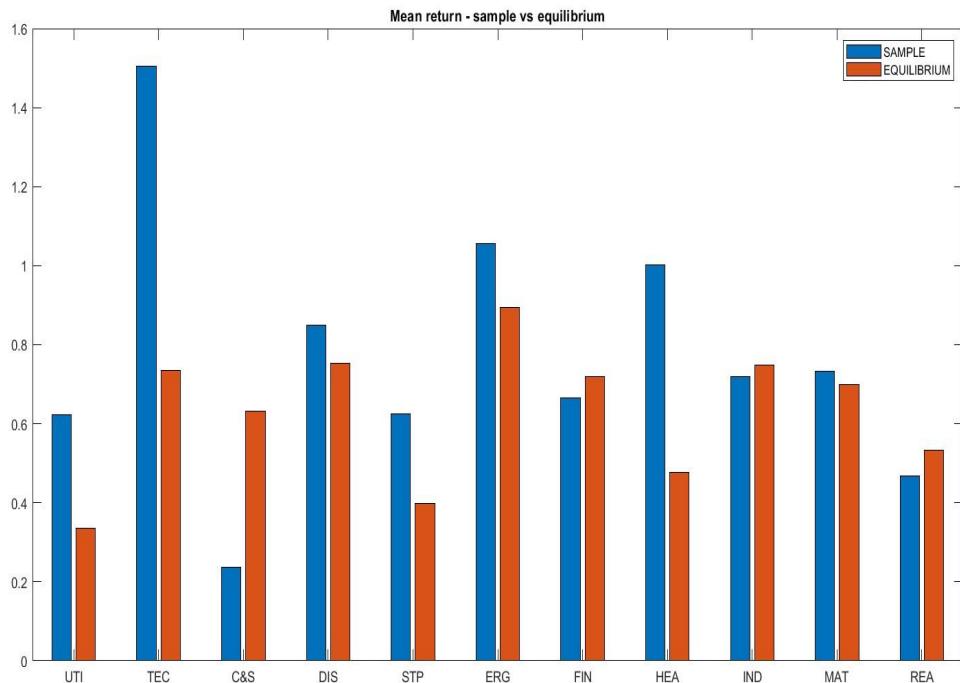


- **The additional constraint moves the new EF to the right**
- **The new EF is shorter in respect to the unconstrained one because the positive weights condition decreases the number of possible portfolios**

Equilibrium moments

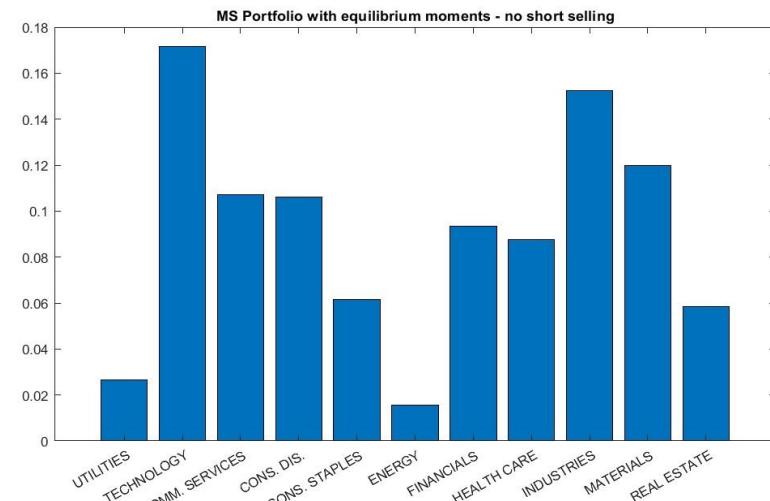
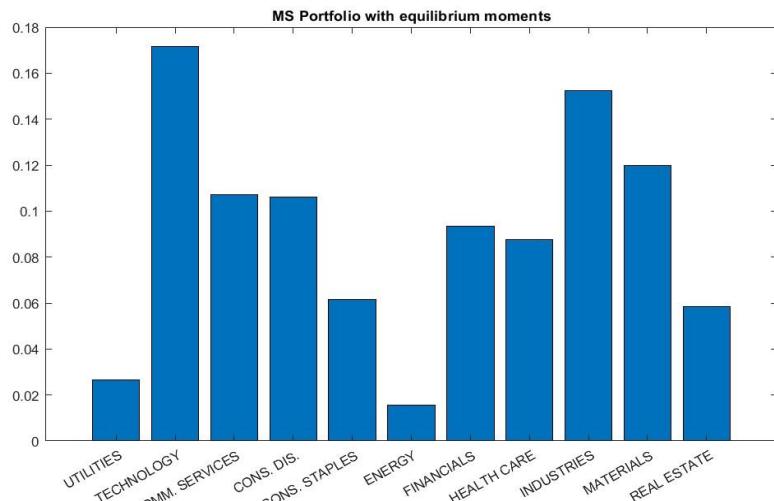
$$r_{i,t} = \alpha_i + \beta_i (r_t^m) + \varepsilon_t$$

Comparison of mean returns - Sample vs Equilibrium moments



- We notice that the new moments calculation procedure makes the returns more “balanced” compared to the sample method

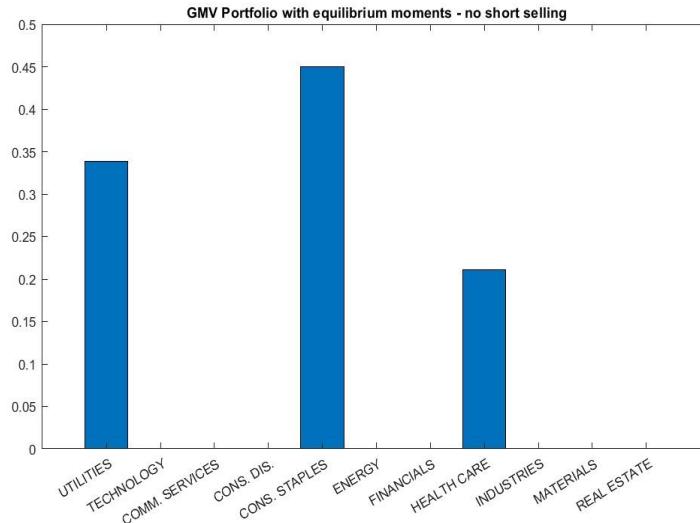
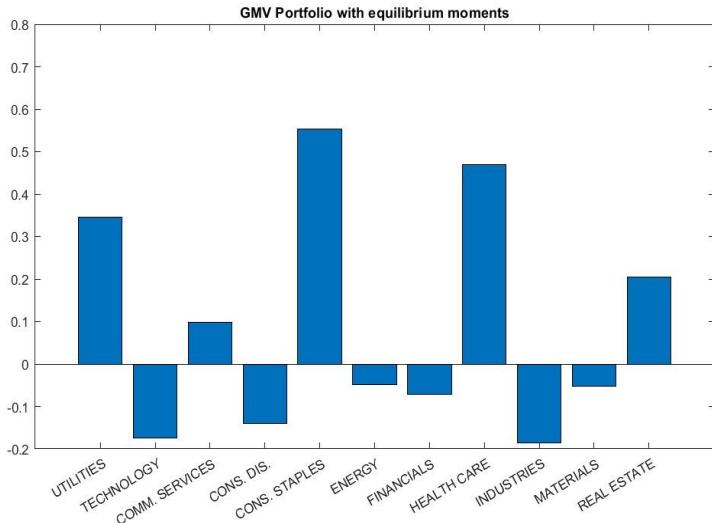
MS Portfolio with equilibrium moments - Unconstrained Vs Positive weights



- We notice that the two graphs are identical because the more balanced returns make the unconstrained weights already positive, without the need of adding other conditions

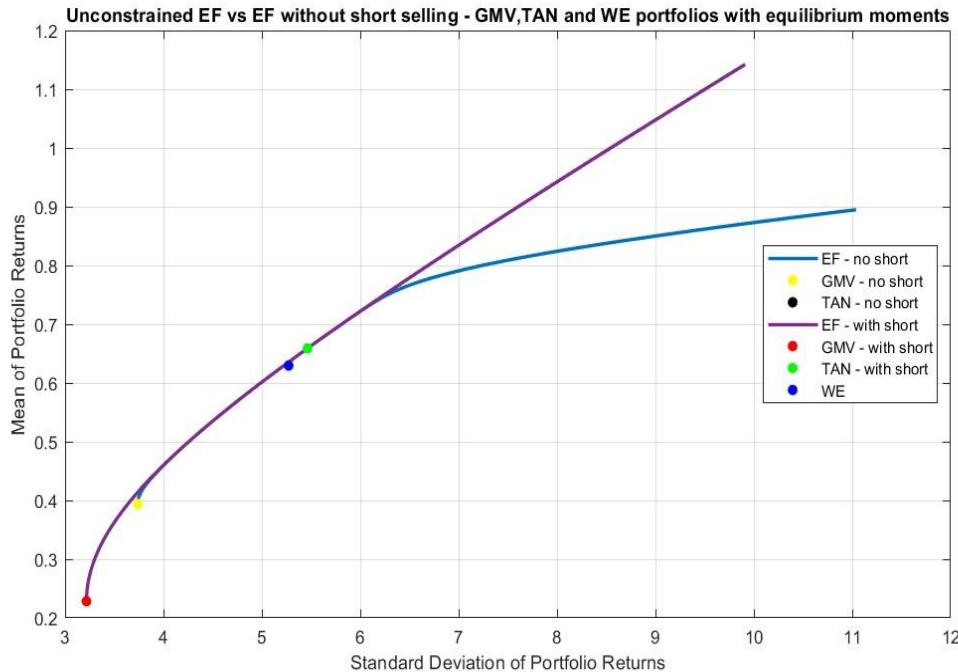
GMV Portfolio with equilibrium moments

- Unconstrained Vs Positive weights



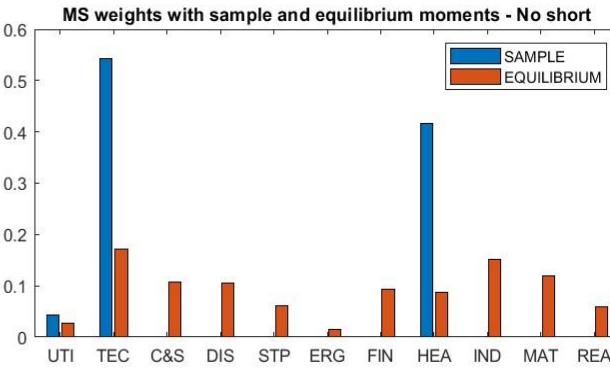
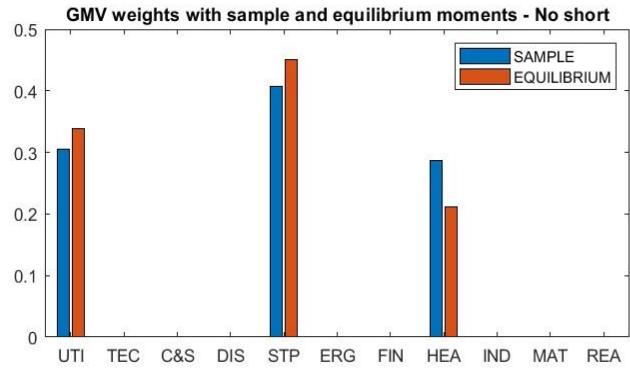
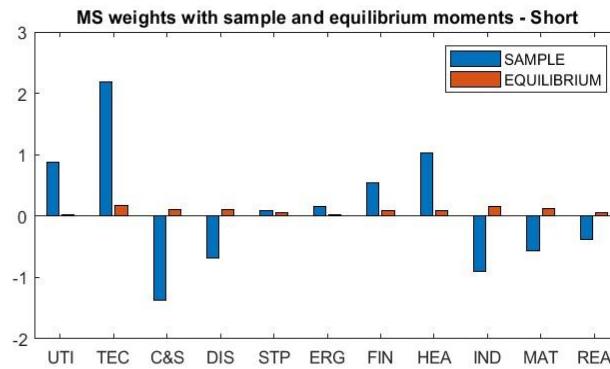
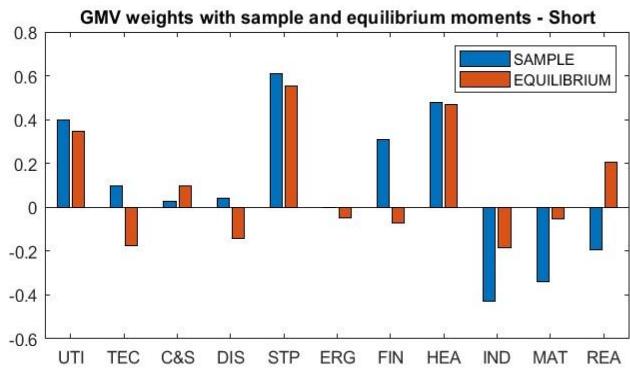
- For the GMV portfolio we still have negative weights in the unconstrained case and in the positive weights graph we only invest in the three least volatile assets

Unconstrained EF vs EF without short selling - Equilibrium moments



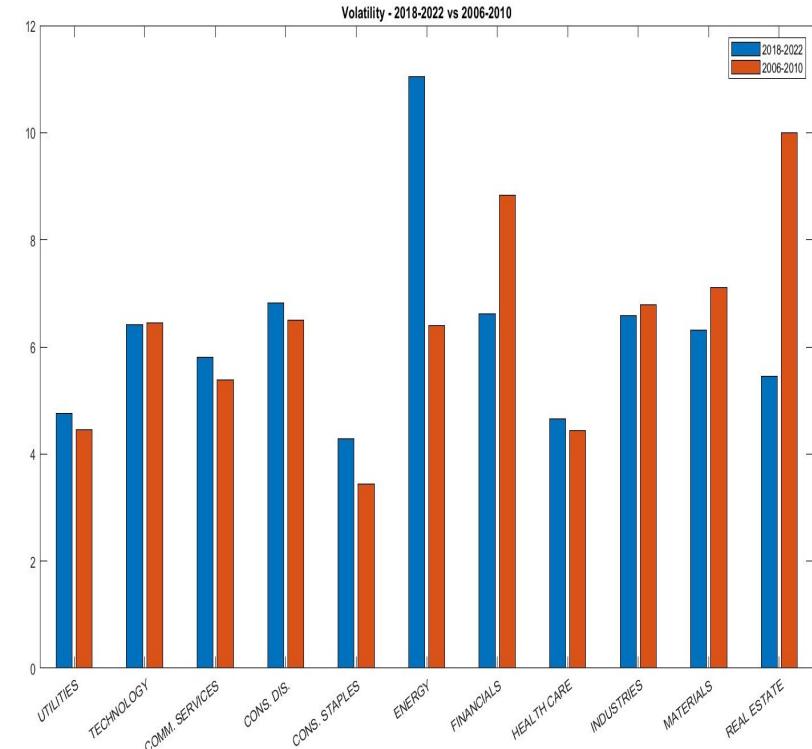
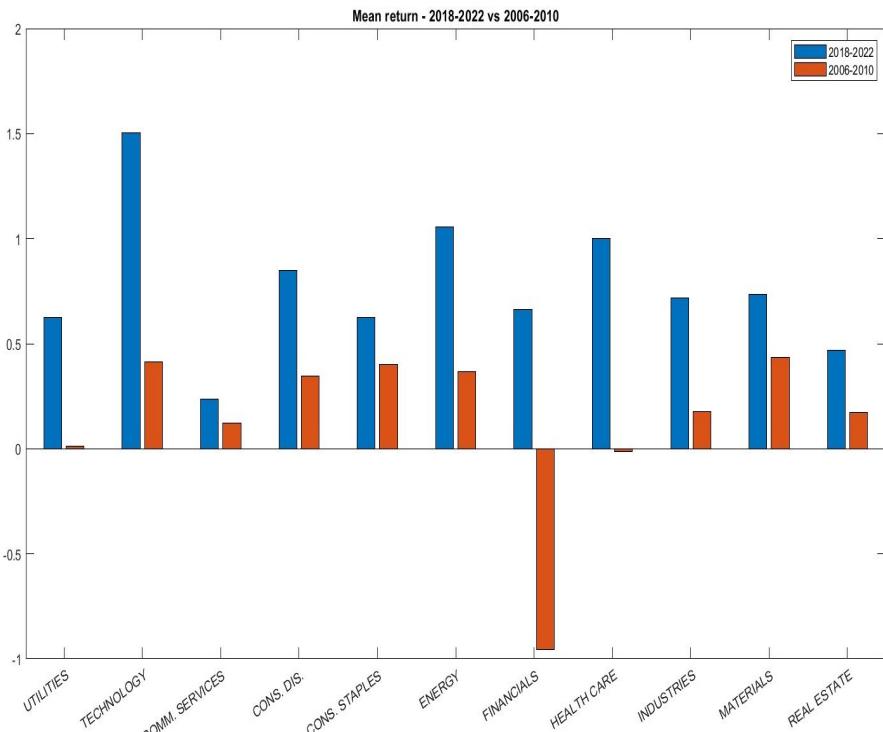
- The two EF are similar to each other until the unconstrained one starts to make use of negative weights, at that point they diverge

Summing Up

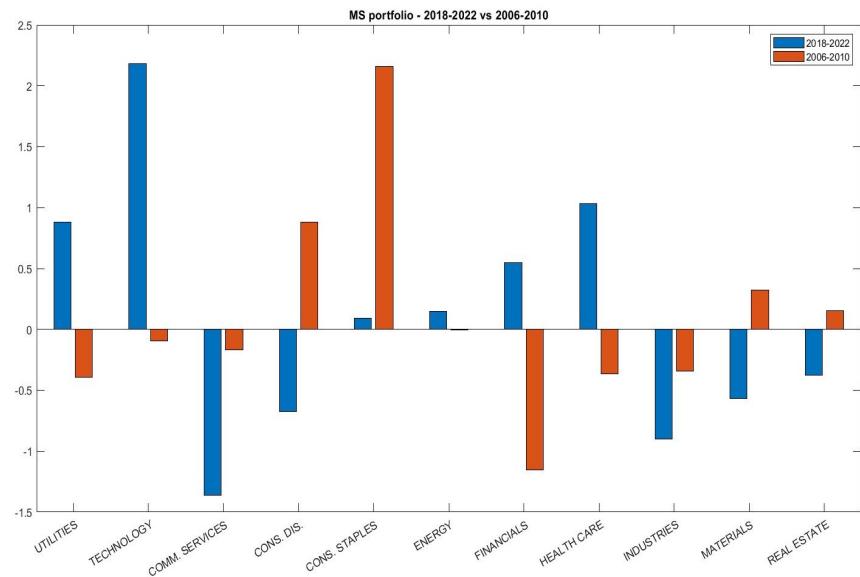
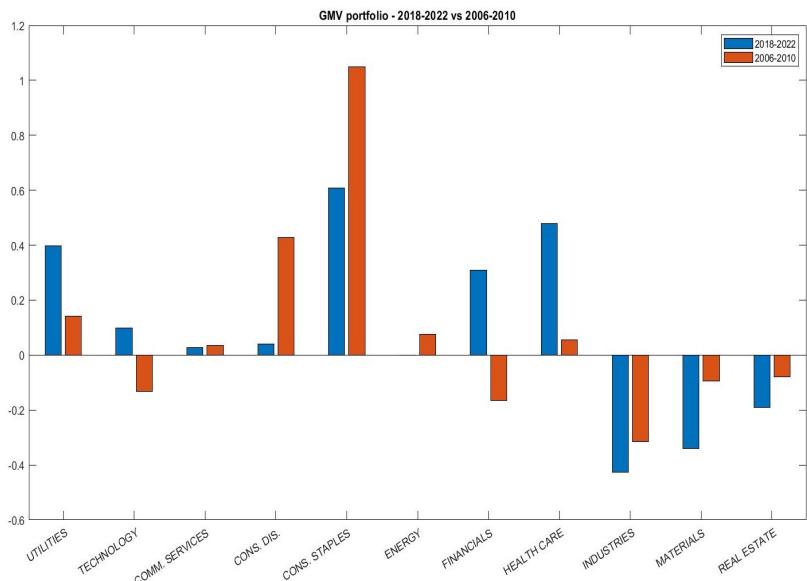


Focus on the period 2006-2010

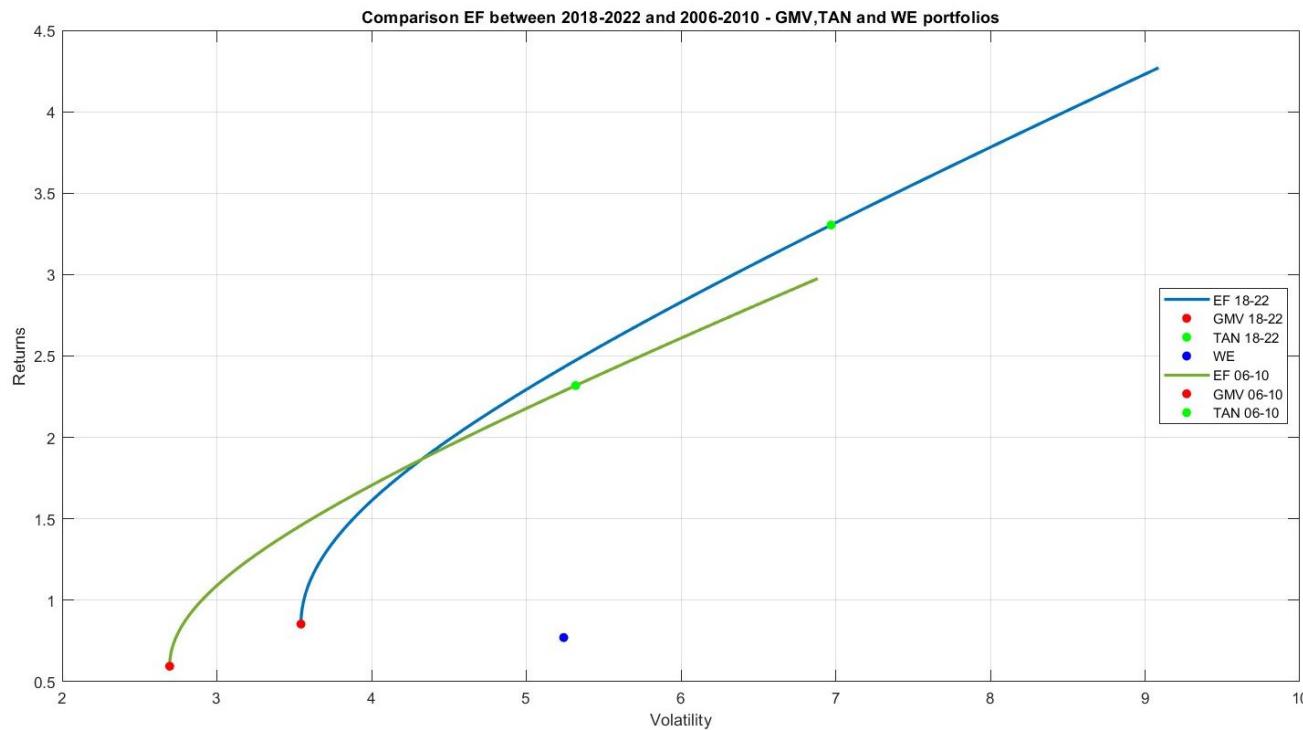
Mean returns & Volatility



GMV and TAN portfolio



Comparison EF between 2018-2022 and 2006-2010

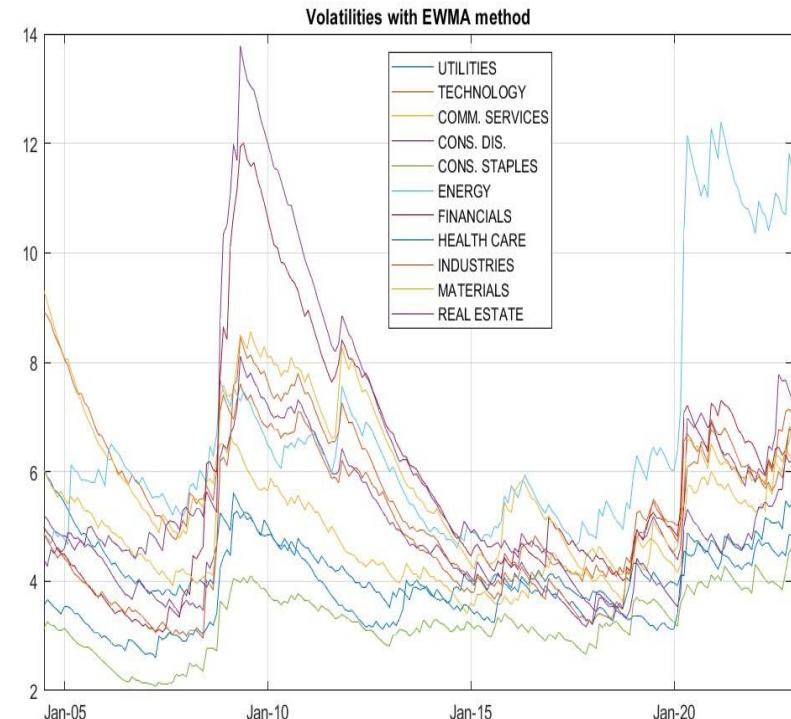
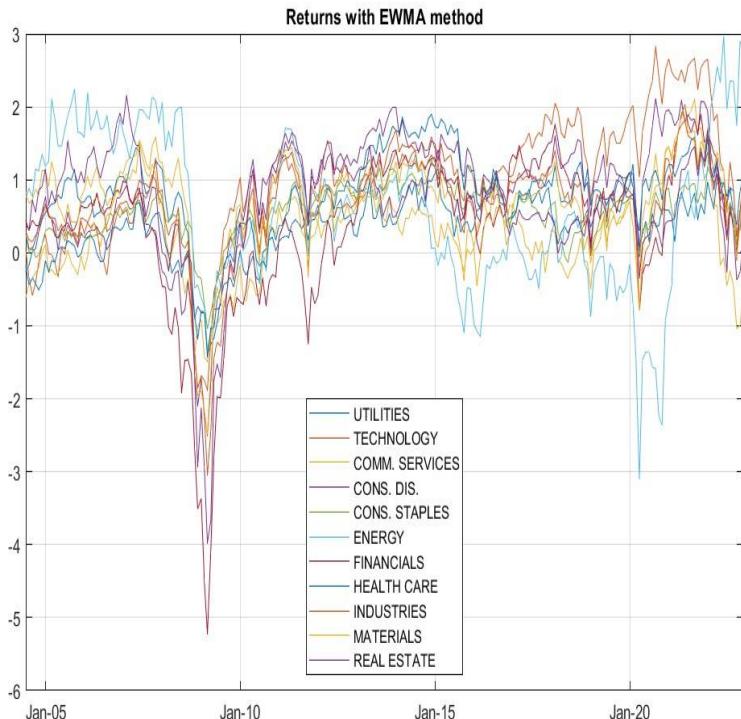


3. Building conservative portfolios

with sample moments and EWMA method

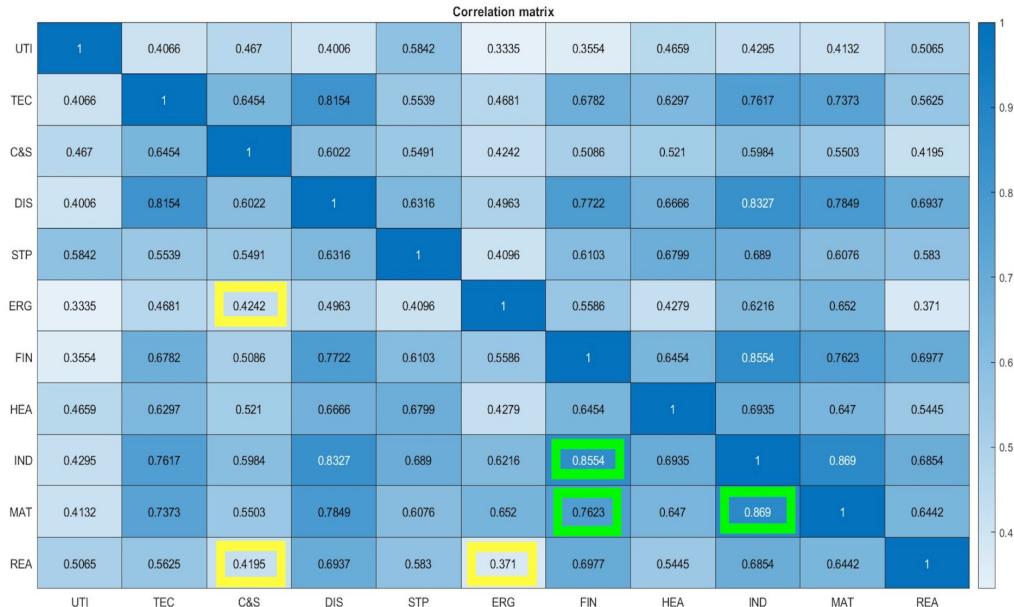
EWMA method

Daily returns and Volatility with EWMA moments

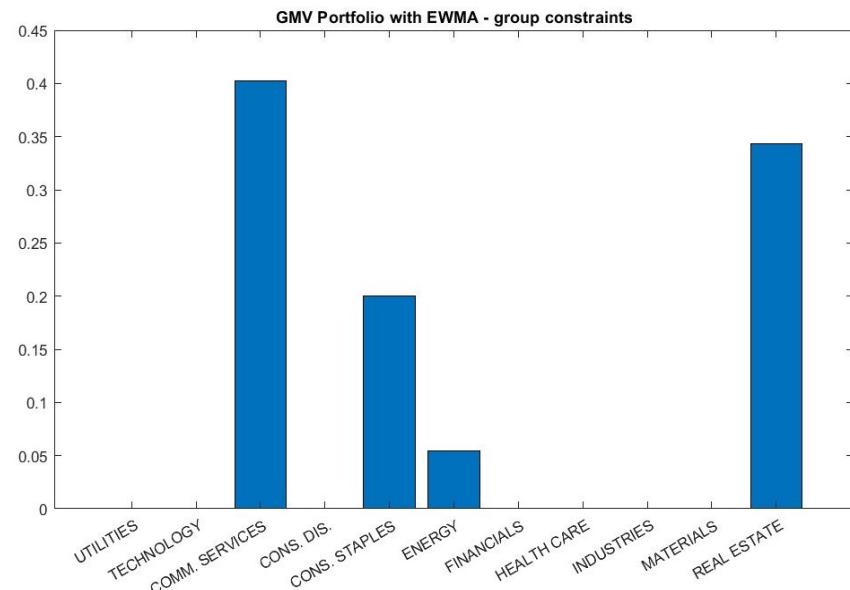
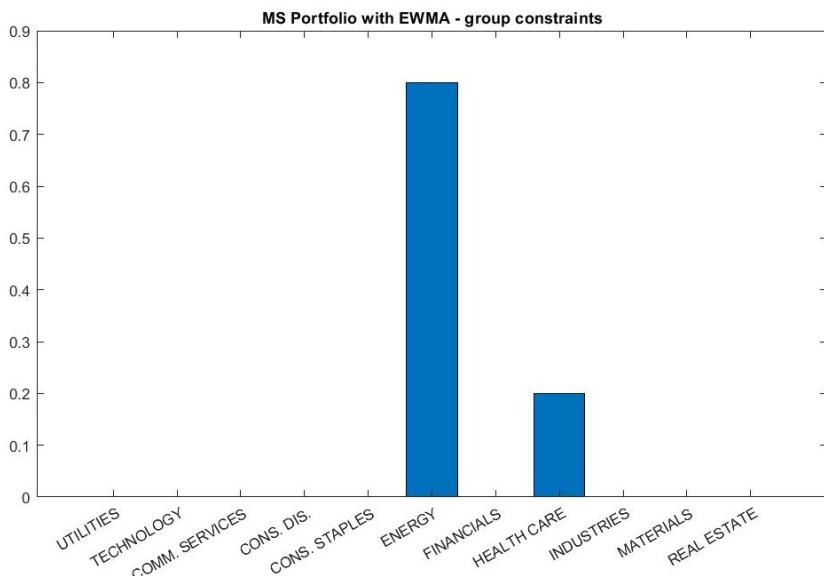


Diversification (group constraints)

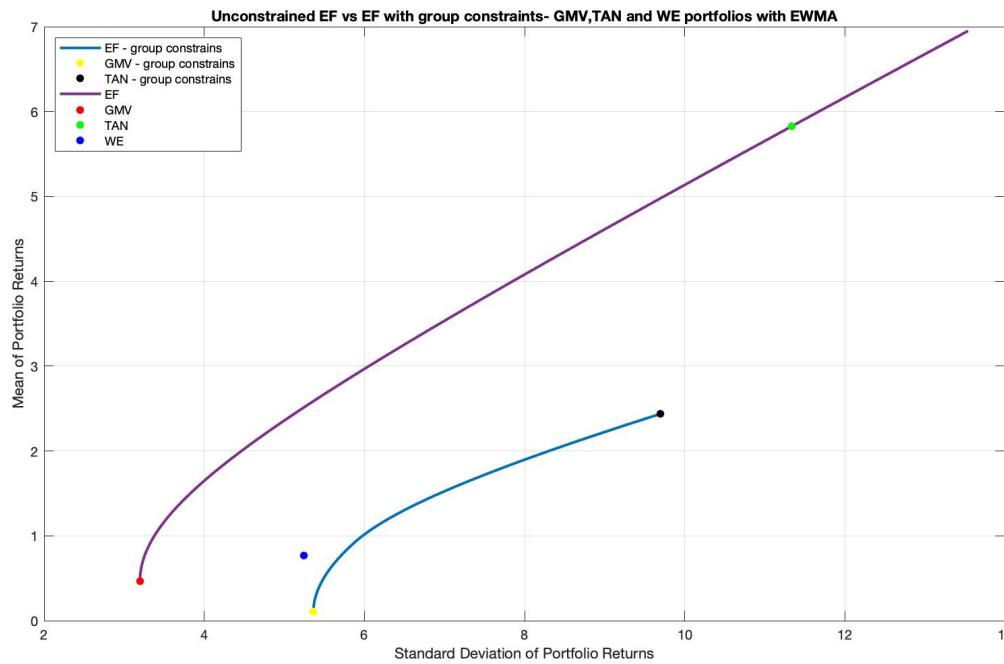
- Invest from 0.7 to 1 in Comm. services, Cons. staples and Real Estate
- Don't invest in materials, industrials and finance
- Invest at most 0.3 in the other assets



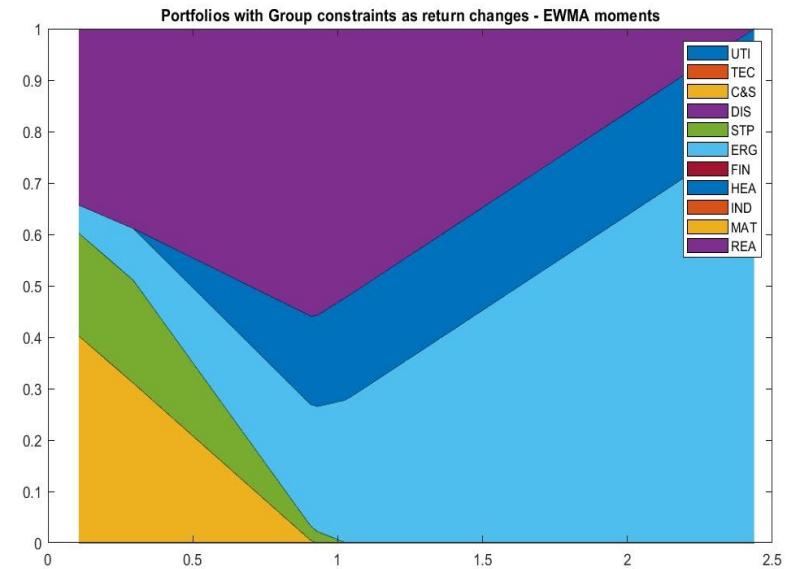
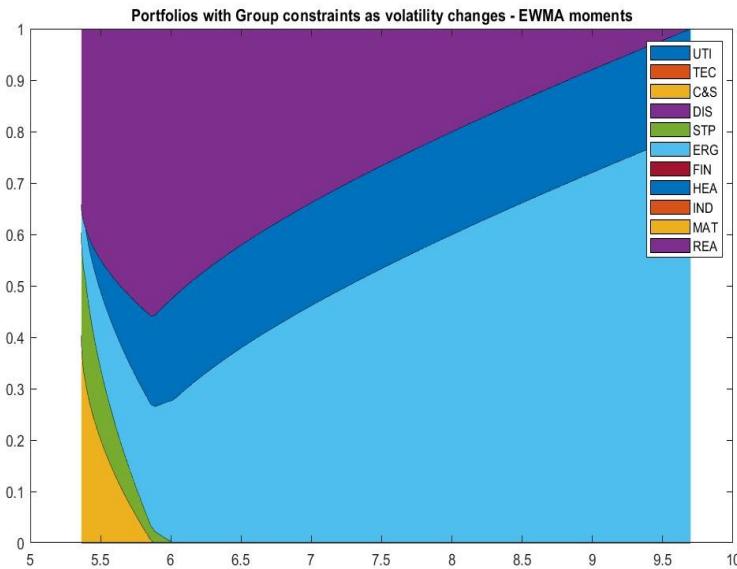
MS and GMV portfolio with group constraints - EWMA moments



Unconstrained EF vs EF with group constraints - EWMA moments

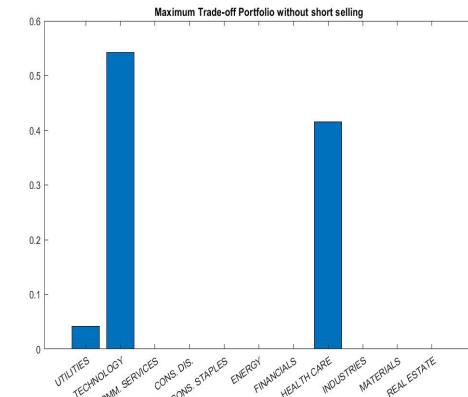
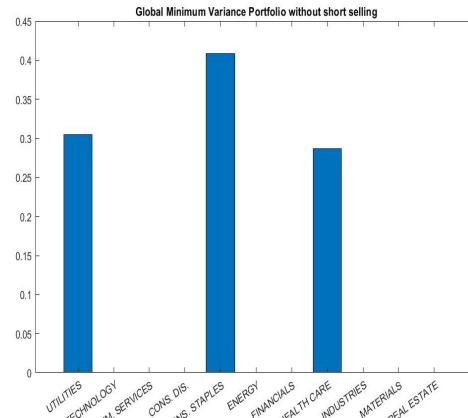


Portfolios with Group constraints - EWMA moments

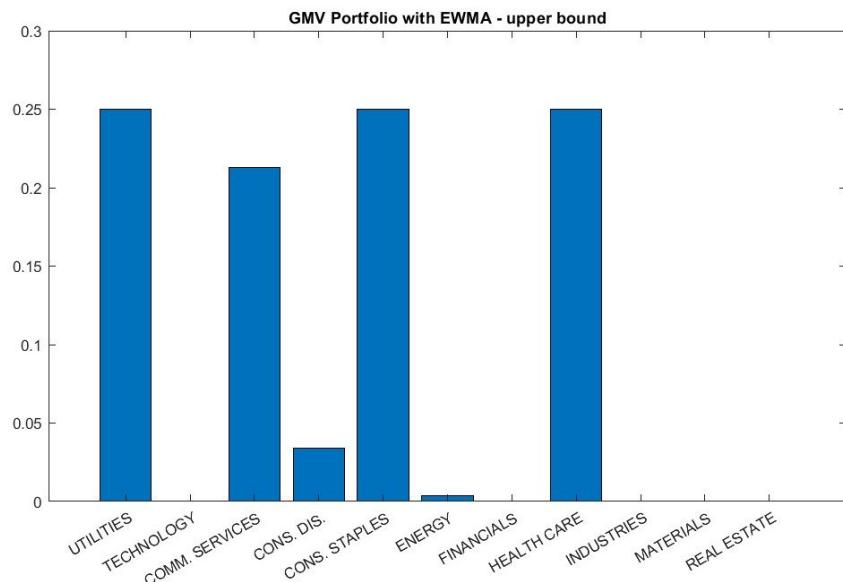
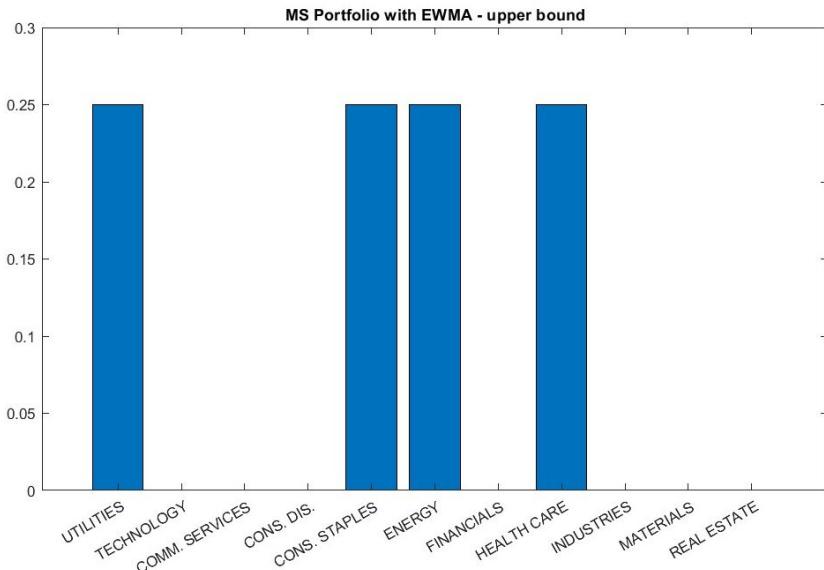


Upper bound

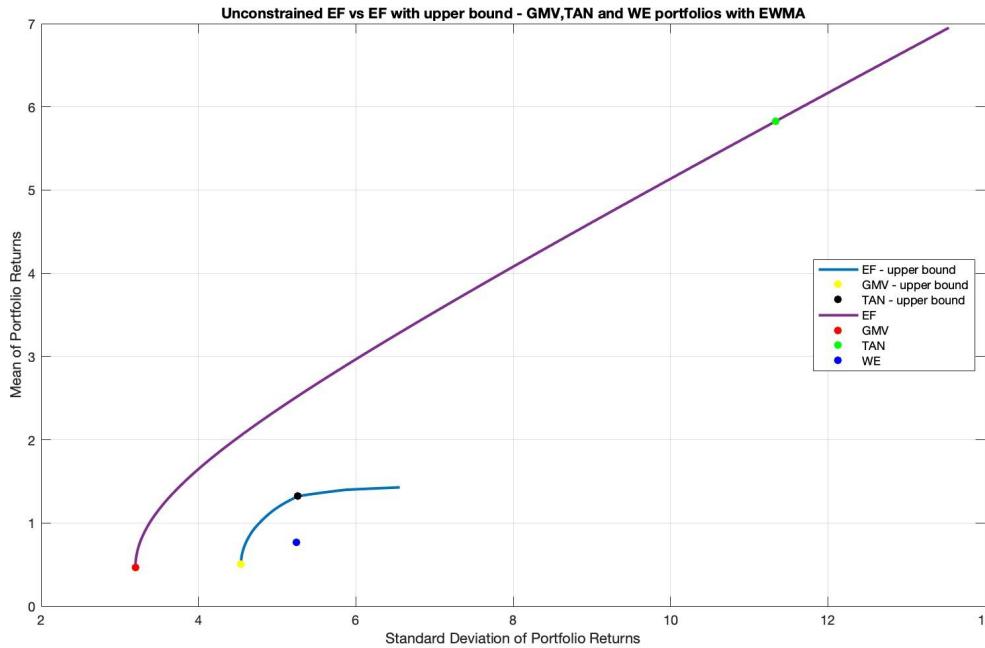
- We would like to add an upper bound to portfolios like the ones to the right
- The upper bound promotes diversification
- The upper limit can't be too high and neither too low
- We want to avoid to invest too much in one single asset
- We chose upper bound equal to 0.25



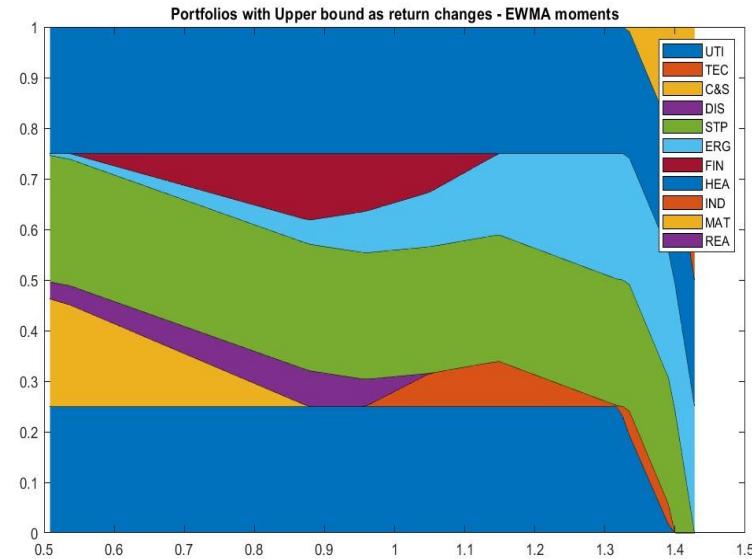
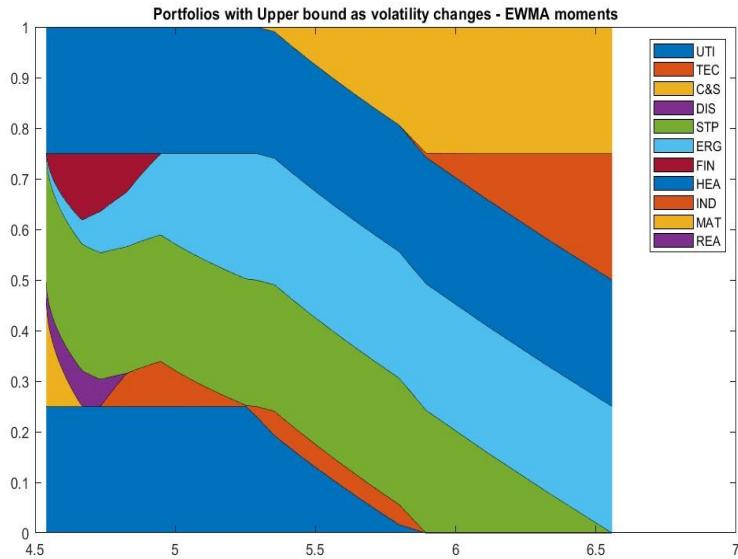
MS and GMV portfolios with upper bound - EWMA moments



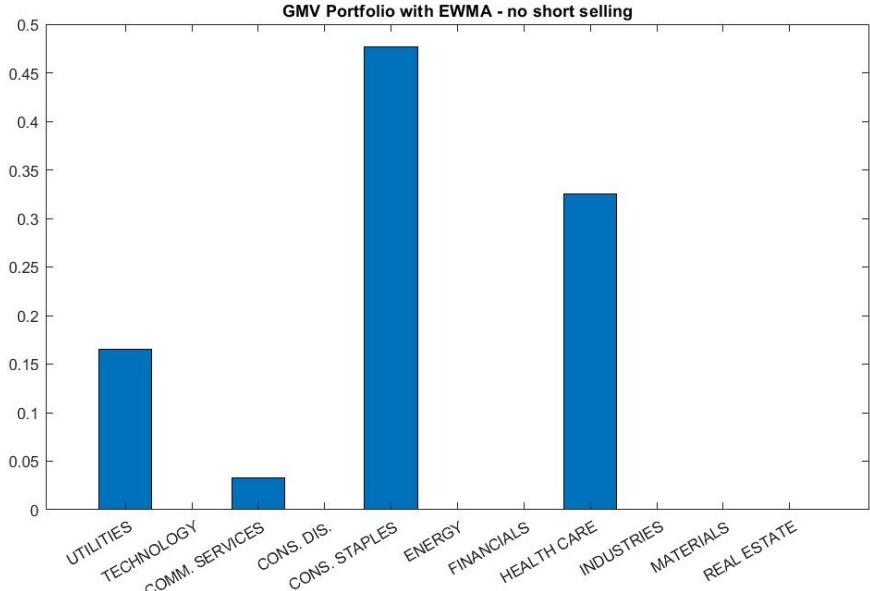
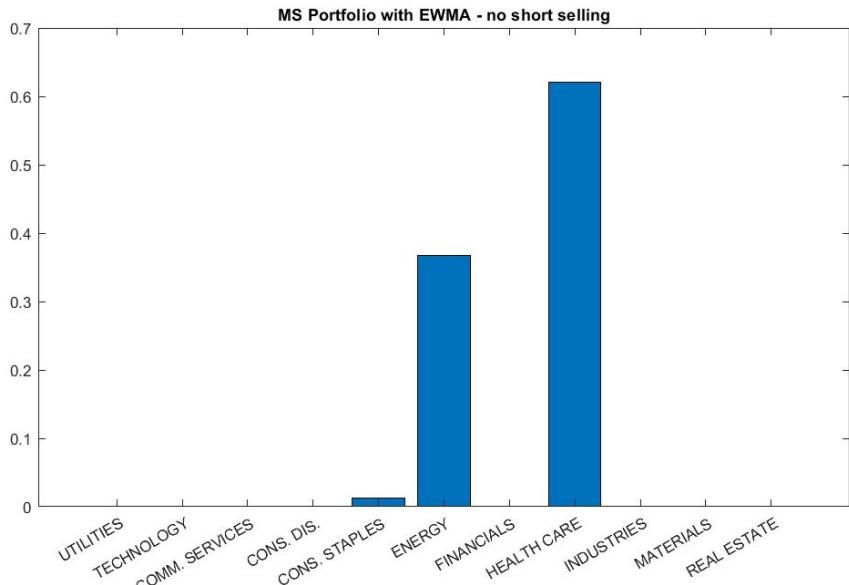
Unconstrained EF versus EF with upper bound - EWMA moments



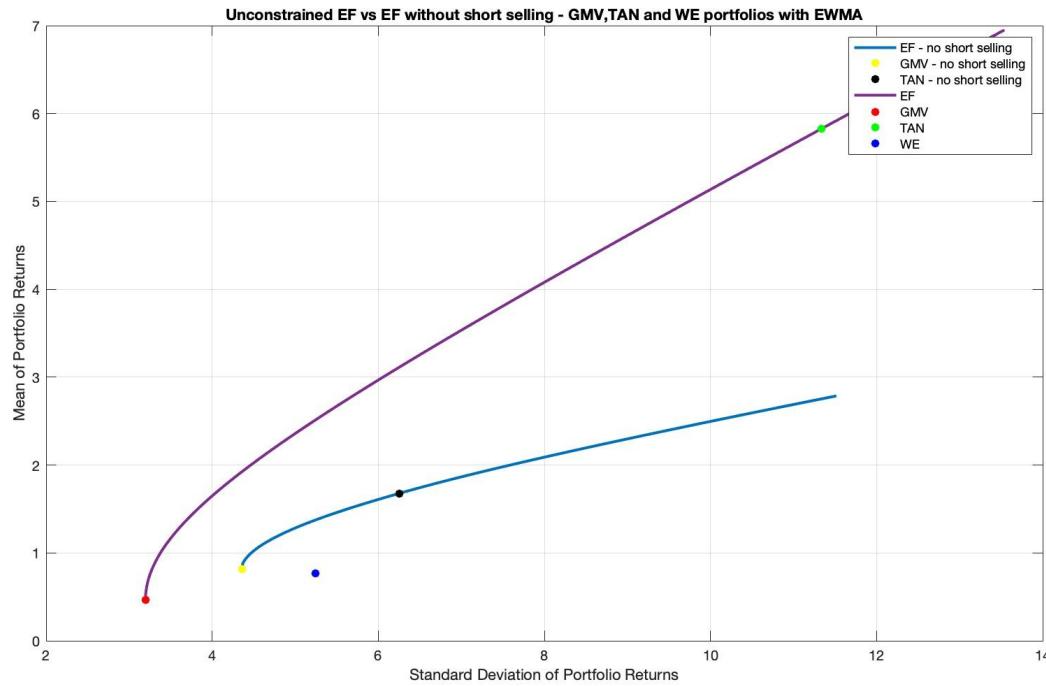
Portfolios with upper bound - EWMA moments



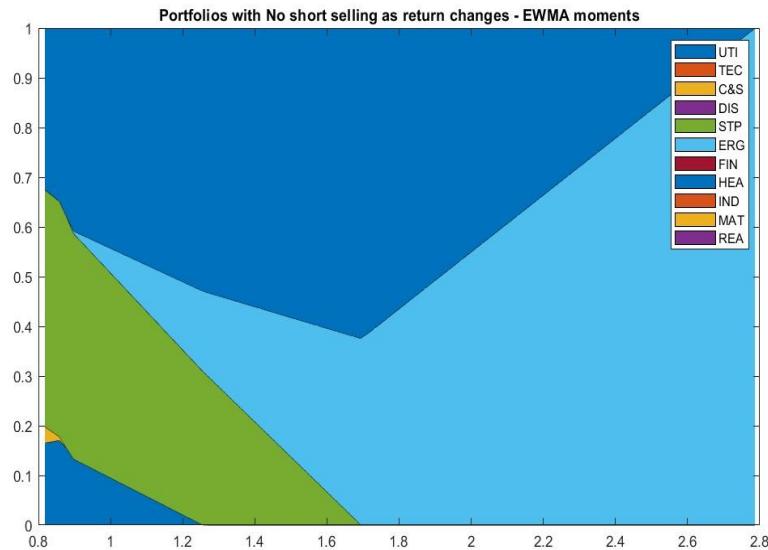
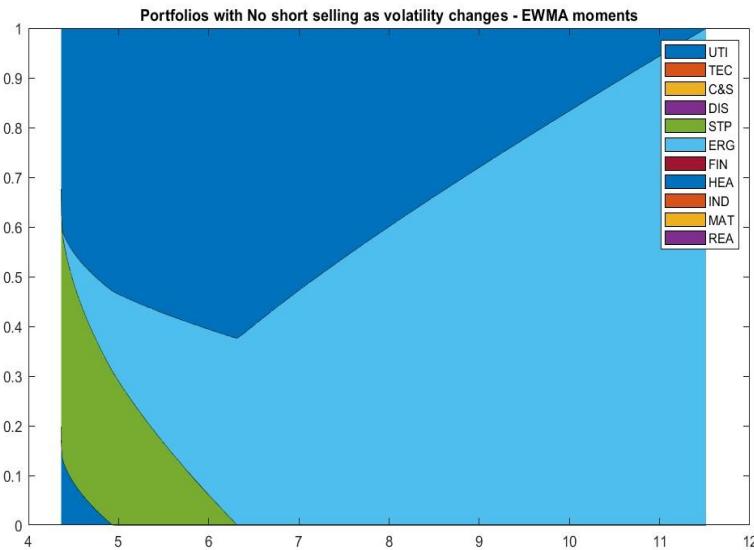
MS and GMV portfolio without short selling - EWMA moments



Unconstrained EF vs EF without short selling - EWMA moments

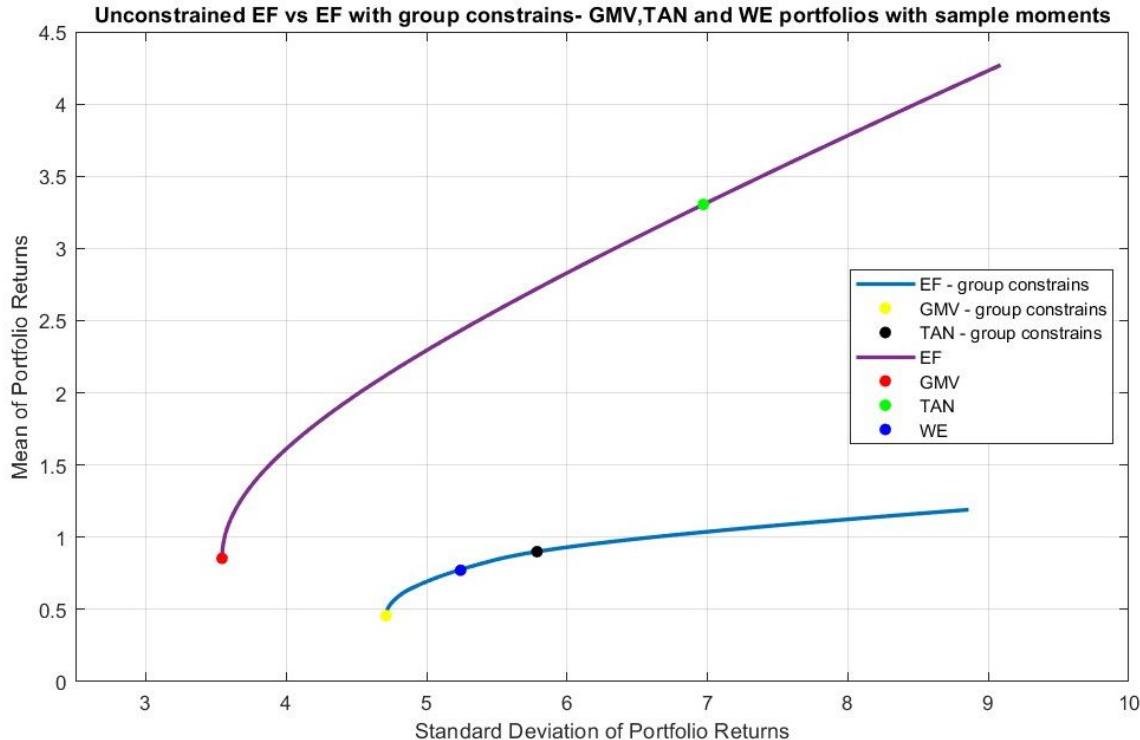


Portfolios without short selling - EWMA moments



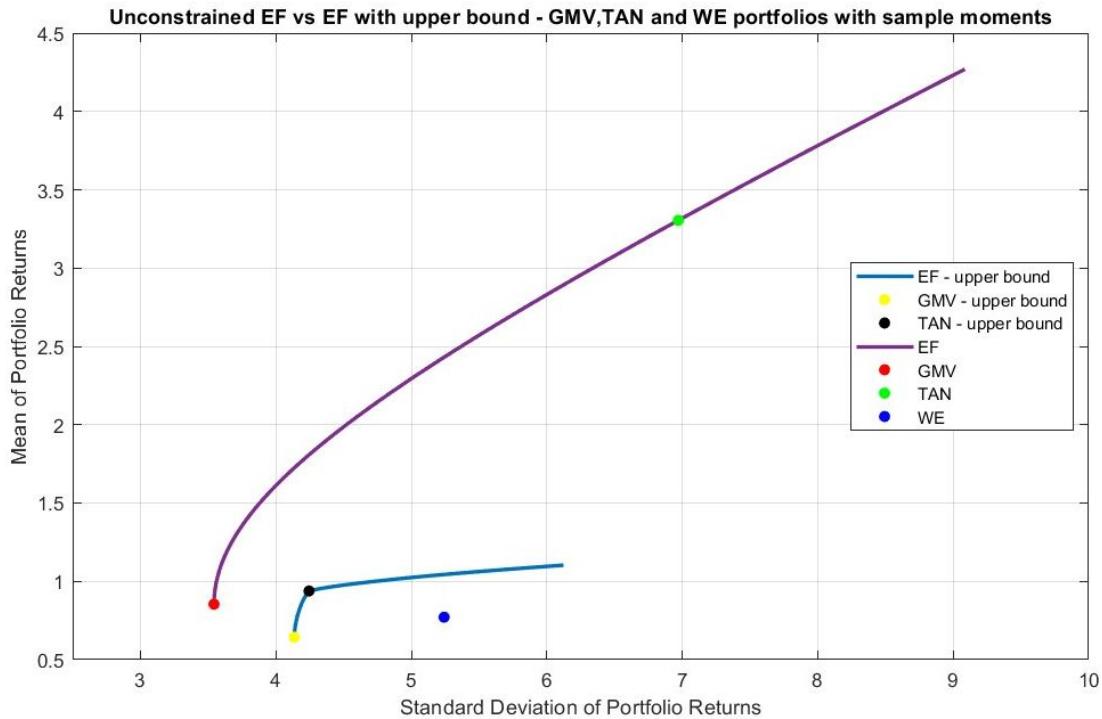
Sample means method

Unconstrained EF vs EF with group constraints - Sample moments

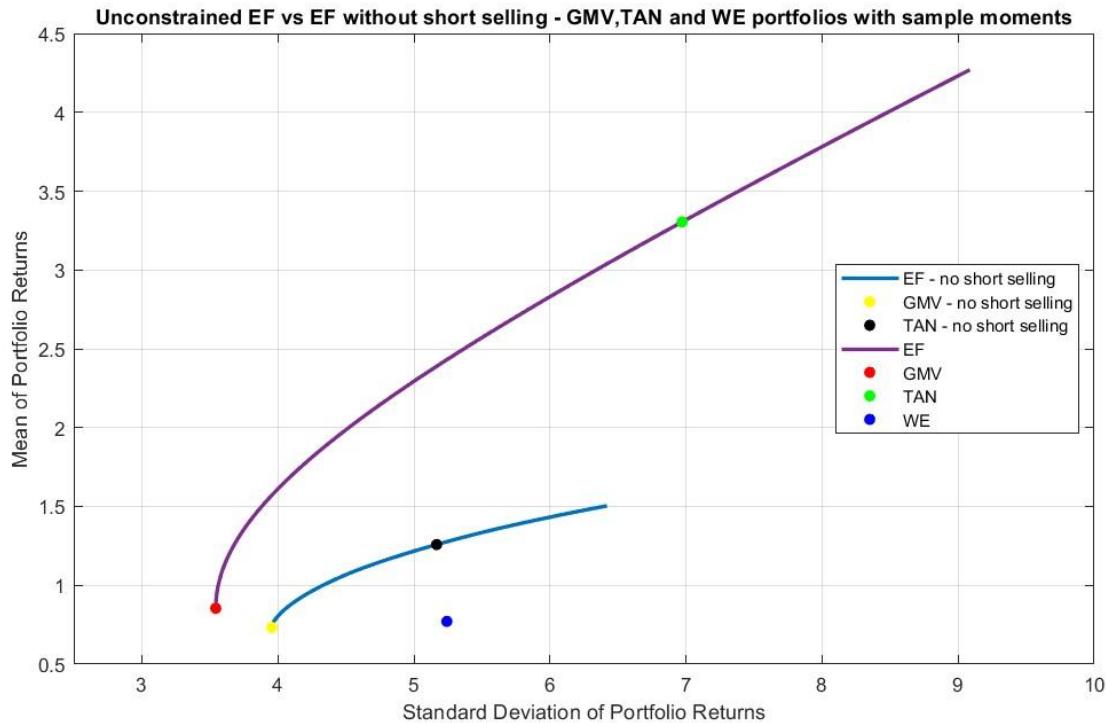


- The EF is much lower and shifted to the right when compared to the unconstrained one

Unconstrained EF vs EF with upper bound - Sample moments

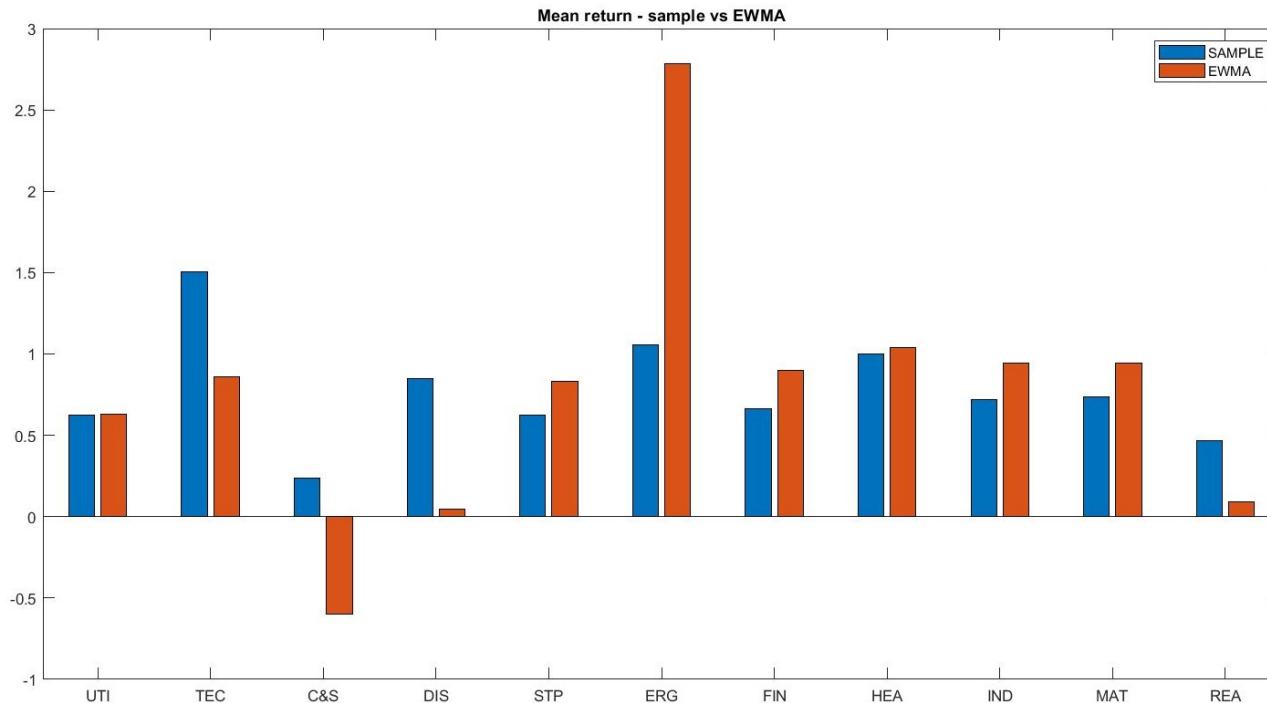


Unconstrained EF vs EF without short selling - Sample moments

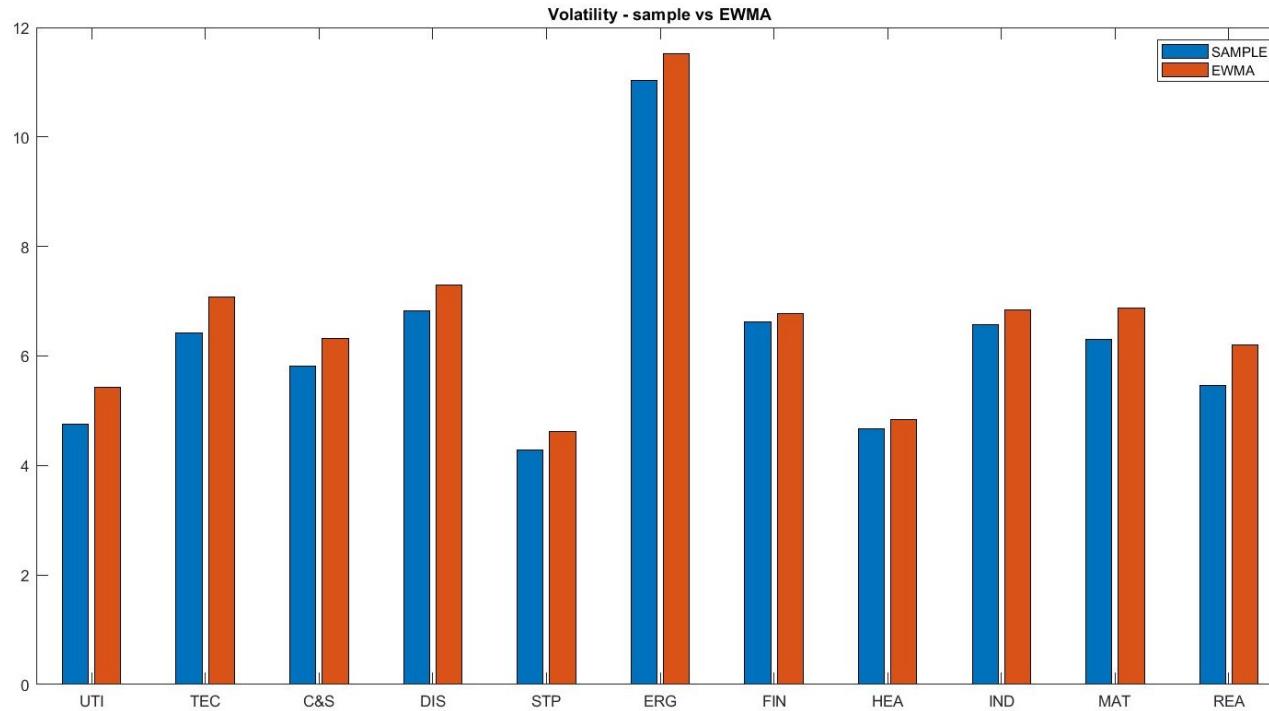


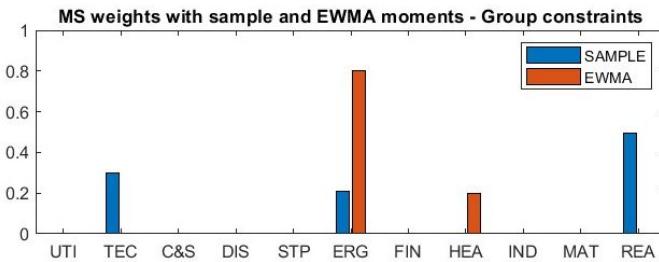
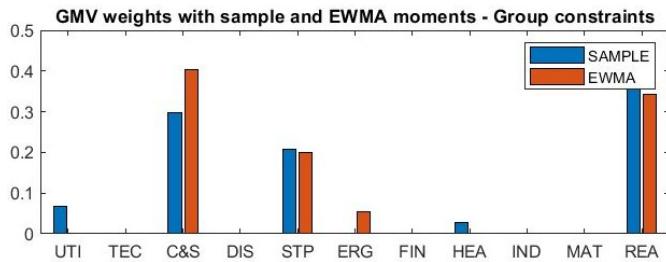
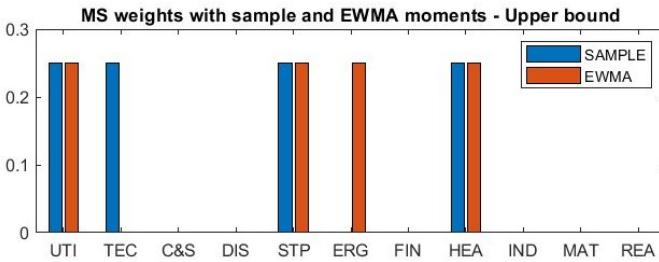
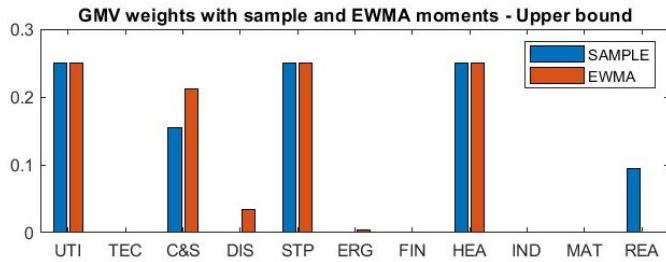
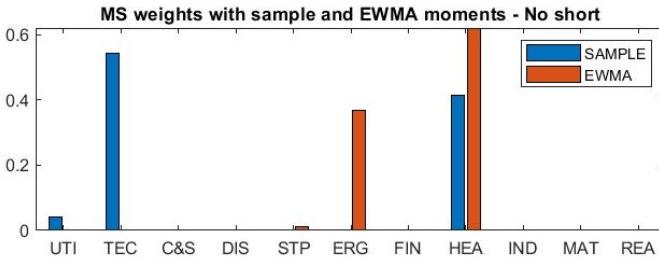
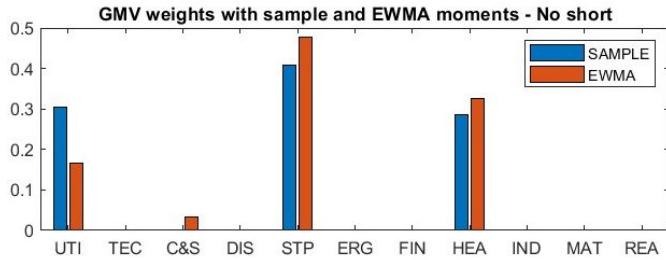
Comparison between sample means and EWMA methods

Mean returns: Sample vs EWMA



Volatility: Sample moments versus EWMA





Conclusions

- Considering that the aim of our analysis is to find an efficient portfolio while being extremely risk averse we choose as our final configuration the unconstrained GMV calculated using the sample means method.
- Is worth noting that when adding a constraint the new EF is obviously worse than the unconstrained case. Our tools cannot handle the portfolio diversity resulting from the addition of the upper bound. For a definitive answer about the best portfolio configuration we should perform a deeper analysis integrating more parameters that can valorize the addition of constraints in a better way.

Thank you for your attention!