Chapter_04+05_Law_of_probability + Hypothesis tests

June 4, 2022

1 Law of probability

This chapter will explain the law of probability and give you some trading examples of it.

1.0.1 After this Chapter you will be able to:

- Compute and understand how to interprate the Bernoulli law
- Compute and understand how to interprate the Binomial law
- Compute and understand how to interprate the Poisson law
- Compute and understand how to interprate the Uniform law
- Compute and understand how to interprate the Exponential law
- Compute and understand how to interprate the Normal law

1.0.2 Exercises (Trading / Portfolio management):

- Compute the value at risk of a financial asset
- Analyze the stop loss and take profit

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Read our book: https://www.amazon.com/gp/product/B09HG18CYL

Quantreo's YouTube channel: https://www.youtube.com/channel/UCp7jckfiEglNf_Gj62VR0pw

[ ]:
```
!pip install yfinance
```

```
Collecting yfinance
  Downloading yfinance-0.1.67-py2.py3-none-any.whl (25 kB)
Requirement already satisfied: pandas>=0.24 in /usr/local/lib/python3.7/dist-
packages (from yfinance) (1.1.5)
Collecting lxml>=4.5.1
  Downloading lxml-4.7.1-cp37-cp37m-manylinux_2_17_x86_64.manylinux2014_x86_64.m
anylinux_2_24_x86_64.whl (6.4 MB)
     |                  | 6.4 MB 7.2 MB/s
Requirement already satisfied: numpy>=1.15 in
/usr/local/lib/python3.7/dist-packages (from yfinance) (1.19.5)
Requirement already satisfied: multitasking>=0.0.7 in
/usr/local/lib/python3.7/dist-packages (from yfinance) (0.0.10)
Requirement already satisfied: requests>=2.20 in /usr/local/lib/python3.7/dist-
packages (from yfinance) (2.23.0)
Requirement already satisfied: python-dateutil>=2.7.3 in
```

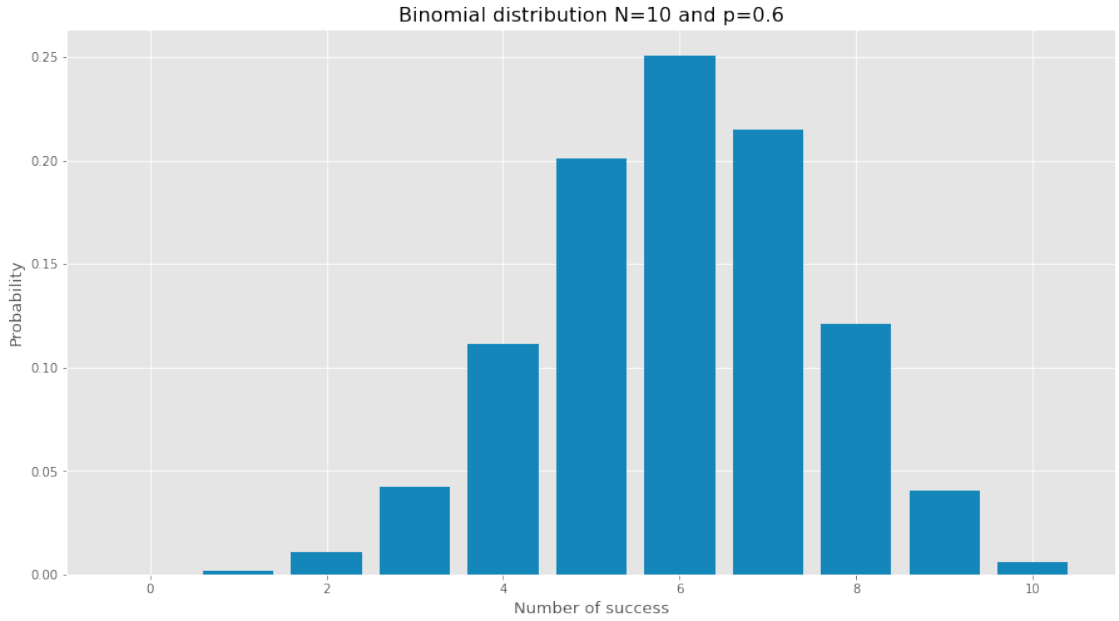
```
/usr/local/lib/python3.7/dist-packages (from pandas>=0.24->yfinance) (2.8.2)
Requirement already satisfied: pytz>=2017.2 in /usr/local/lib/python3.7/dist-
packages (from pandas>=0.24->yfinance) (2018.9)
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.7/dist-
packages (from python-dateutil>=2.7.3->pandas>=0.24->yfinance) (1.15.0)
Requirement already satisfied: urllib3!=1.25.0,!=1.25.1,<1.26,>=1.21.1 in
/usr/local/lib/python3.7/dist-packages (from requests>=2.20->yfinance) (1.24.3)
Requirement already satisfied: idna<3,>=2.5 in /usr/local/lib/python3.7/dist-
packages (from requests>=2.20->yfinance) (2.10)
Requirement already satisfied: certifi>=2017.4.17 in
/usr/local/lib/python3.7/dist-packages (from requests>=2.20->yfinance)
(2021.10.8)
Requirement already satisfied: chardet<4,>=3.0.2 in
/usr/local/lib/python3.7/dist-packages (from requests>=2.20->yfinance) (3.0.4)
Installing collected packages: lxml, yfinance
  Attempting uninstall: lxml
    Found existing installation: lxml 4.2.6
    Uninstalling lxml-4.2.6:
      Successfully uninstalled lxml-4.2.6
Successfully installed lxml-4.7.1 yfinance-0.1.67
```

[ ]:
```
import numpy as np
import pandas as pd
import yfinance as yf
from scipy.stats import binom, poisson, expon
import matplotlib.pyplot as plt
import seaborn as sns
plt.style.use('ggplot')
```

[ ]:
```
# The code here will allow you to switch your graphics to dark mode for those␣
 ↪who choose to code in dark mode
import matplotlib.pyplot as plt

import matplotlib as mpl
from matplotlib import cycler
colors = cycler('color',
                ['#669FEE', '#66EE91', '#9988DD',
                 '#EECC55', '#88BB44', '#FFBBBB'])
plt.rc('figure', facecolor='#313233')
plt.rc('axes', facecolor="#313233", edgecolor='none',
       axisbelow=True, grid=True, prop_cycle=colors,
       labelcolor='gray')
plt.rc('grid', color='474A4A', linestyle='solid')
plt.rc('xtick', color='gray')
plt.rc('ytick', direction='out', color='gray')
plt.rc('legend', facecolor="#313233", edgecolor="#313233")
plt.rc("text", color="#C9C9C9")
```

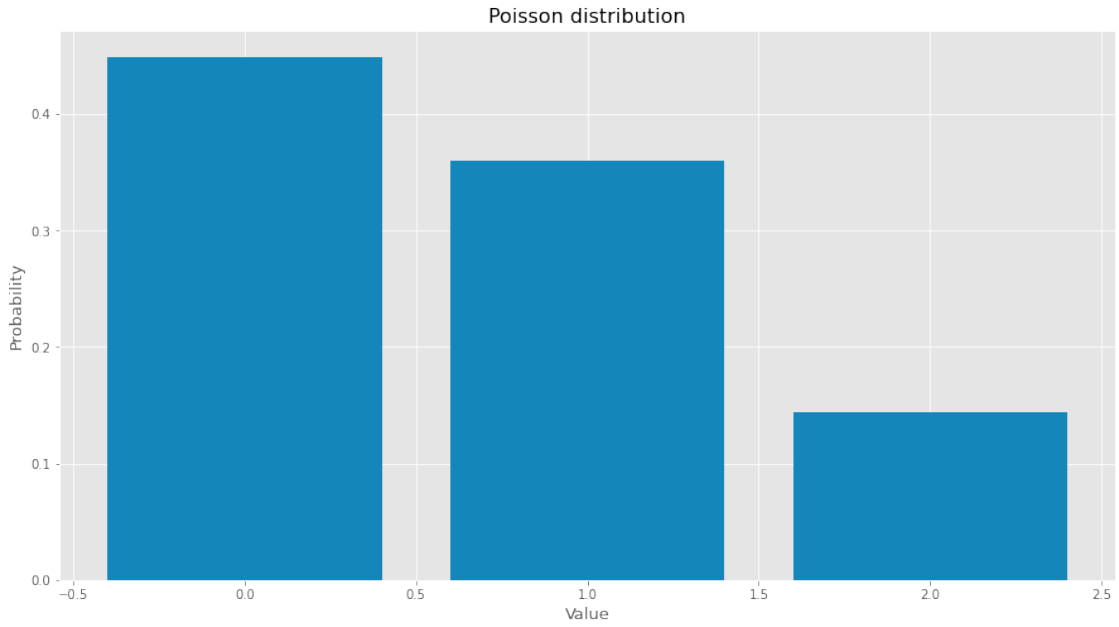
2 Binomial distribution

[ ]:
```
# ------------ Initialiaze distribution setting ---------------
# Number of iterations
n = 10

# Probability of success
p = 0.6


# ------------ Compute the probability ---------------
# List of possible number of success
nb_success = list(range(n + 1))

# list of pmf values
proba = [binom.pmf(b, n, p) for b in nb_success ]


# ------------ Plot the density ---------------
# Adapt the size
plt.figure(figsize=(15,8))

# Plot the distribution
plt.bar(nb_success, proba, color="#1586BA")

# Give x/y labels and a title
plt.title(f"Binomial distribution N={n} and p={p}", size=16)
plt.xlabel("Number of success", size=13)
plt.ylabel("Probability", size=13)

plt.show()
```



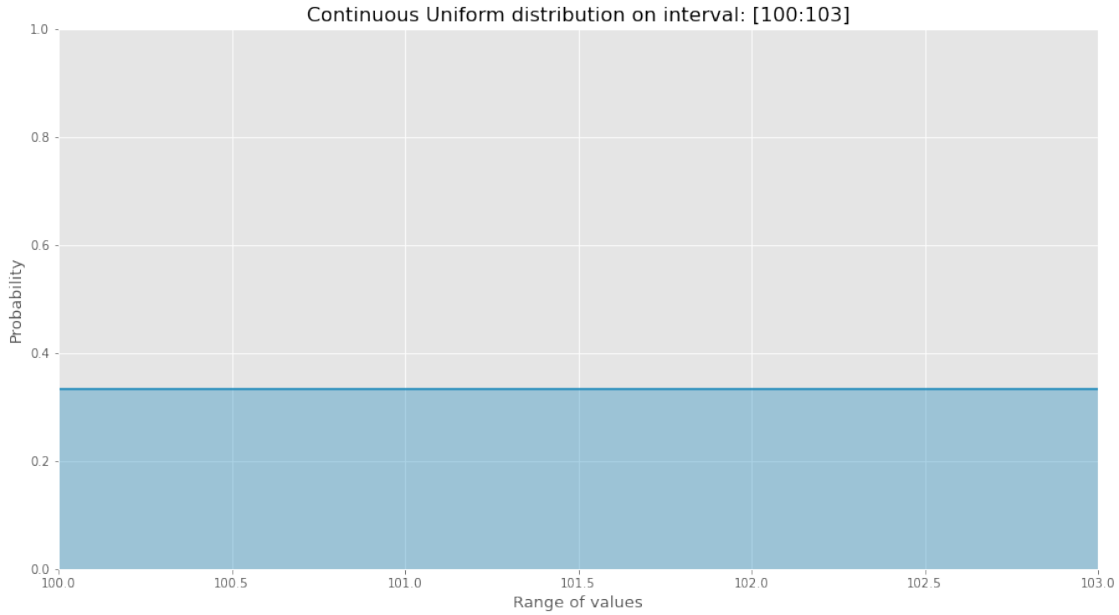
3 Poisson Distribution

[ ]:
```
# ------------ Simulate Poisson Process ---------------
# Set lambda parameter
l = 0.8

# Find the discrete values which cover the domain
x = np.arange(poisson.ppf(0.01, l),
              poisson.ppf(0.99, l))

# Compute the proba of each value
proba = poisson.pmf(x, l)

# ------------ Plot the density ---------------
# Adapt the size
plt.figure(figsize=(15,8))

# Plot the distribution
plt.bar(x, proba, color="#1586BA")

# Give x/y labels and a title
plt.title(f"Poisson distribution", size=16)
plt.xlabel("Value", size=13)
plt.ylabel("Probability", size=13)

plt.show()
```

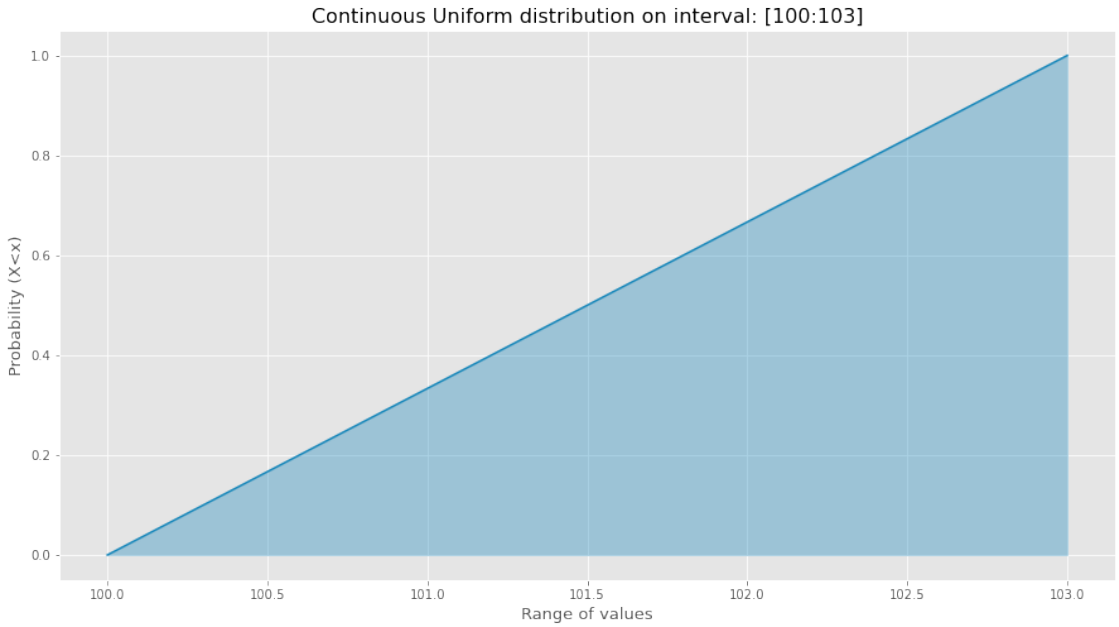


4 Continuous Uniform Law

[ ]:
```
# ------------ Initialiaze distribution setting ---------------
a = 100
b = 103

# ------------ Plot the density ---------------
# Adapt the size
plt.figure(figsize=(15,8))

# Plot the distribution
plt.axhline(1/(b-a), color="#1586BA")
plt.fill_between([a,b],0, 1/(b-a), color="#1586BA", alpha=0.35)

# Give x/y labels and a title
plt.title(f"Continuous Uniform distribution on interval: [{a}:{b}]", size=16)
plt.xlabel("Range of values", size=13)
plt.ylabel("Probability", size=13)

#
plt.ylim([0,1])
plt.xlim([a,b])


plt.show()
```



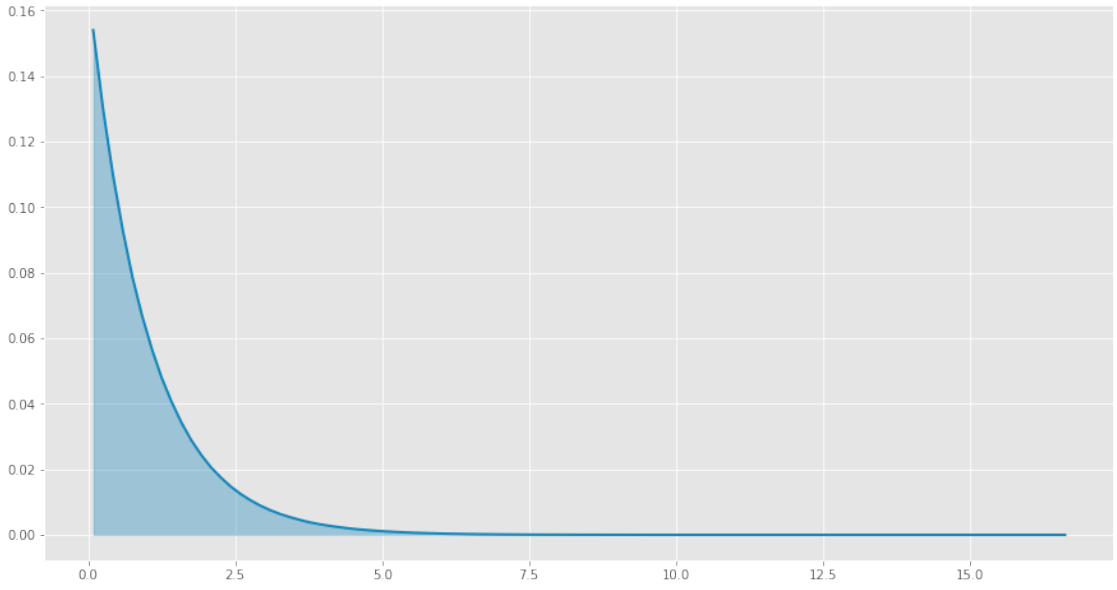
[ ]:
```
# ------------ Initialiaze distribution setting ---------------
a = 100
b = 103

domain = np.linspace(a,b,50)
p = [(x-a)/(b-a) for x in domain]
# ------------ Plot the density ---------------
# Adapt the size
plt.figure(figsize=(15,8))

# Plot the distribution
plt.plot(domain, p,color="#1586BA")
plt.fill_between(domain,0, p, color="#1586BA", alpha=0.35)

# Give x/y labels and a title
plt.title(f"Continuous Uniform distribution on interval: [{a}:{b}]", size=16)
plt.xlabel("Range of values", size=13)
plt.ylabel("Probability (X<x)", size=13)

#
# plt.ylim([0,1])
# plt.xlim([a,b])


plt.show()
```



5 Exponential distribution

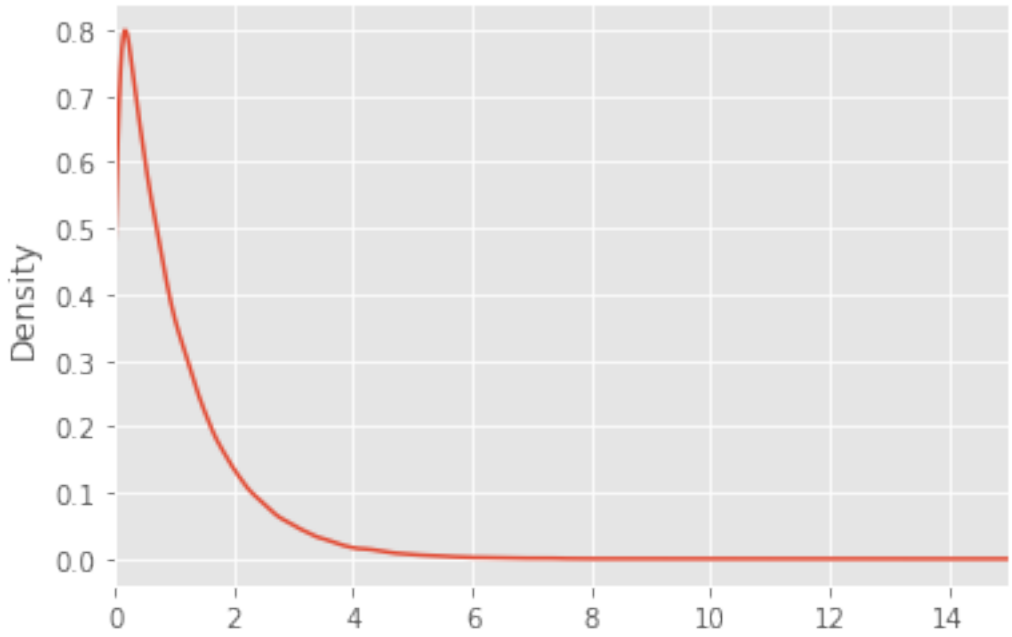
https://stackoverflow.com/questions/15415455/plotting-probability-density-function-by-sample-with-matplotlib

[ ]:
```
import numpy as np
from scipy.interpolate import UnivariateSpline
from matplotlib import pyplot as plt

N = 10000000
n = N//100000
s = np.random.exponential(scale=1, size=(N,)) # generate your data sample with␣
 ↪N elements
p, x = np.histogram(s, bins=n) # bin it into n = N//10 bins
x = x[:-1] + (x[1] - x[0])/2   # convert bin edges to centers
f = UnivariateSpline(x, p, s=n)
plt.figure(figsize=(15,8))
plt.plot(x, f(x)/N, color="#1586BA", linewidth=2)
plt.fill_between(x,0, f(x)/N, color="#1586BA", alpha=0.35)

plt.show()
```



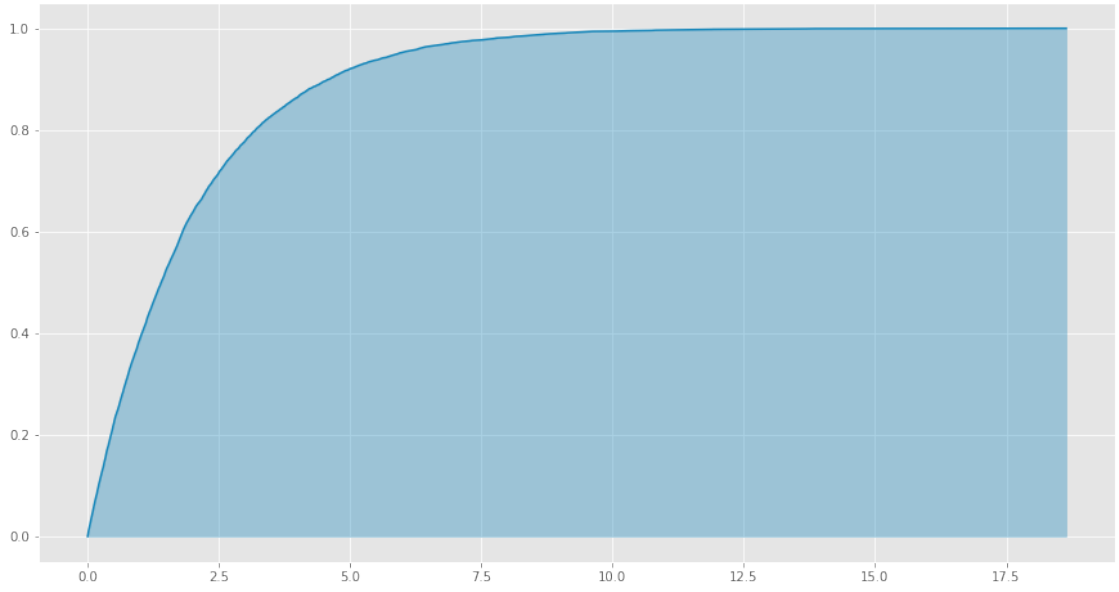
[ ]:
```
# ------------ Simulate realization coming from a exponential law␣
↪----------------
arr = np.random.exponential(scale=1, size=(100000,))
df = pd.DataFrame(arr, columns=["sim"]).sort_values(by="sim", ascending=True)
df["probability"] = 1/len(df)

df["sim"].plot(kind="kde", xlim=[0,15])
```

[ ]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x7f34f2b84f50>
```



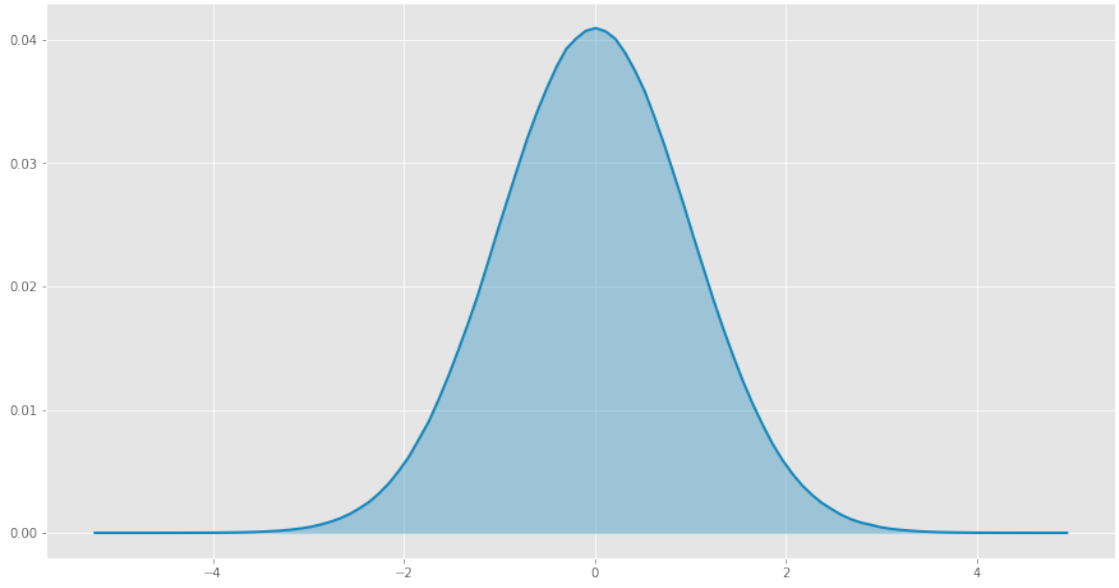
[ ]:
```
arr = np.random.exponential(scale=2, size=(10000,))
df = pd.DataFrame(arr, columns=["sim"]).sort_values(by="sim", ascending=True)
df["probability"] = 1/len(df)

plt.figure(figsize=(15,8))
plt.plot(df["sim"], df["probability"].cumsum(), color="#1586BA")
plt.fill_between(df["sim"],0, df["probability"].cumsum(), color="#1586BA",␣
↪alpha=0.35)
```

[ ]:
```
<matplotlib.collections.PolyCollection at 0x7f34ea990490>
```



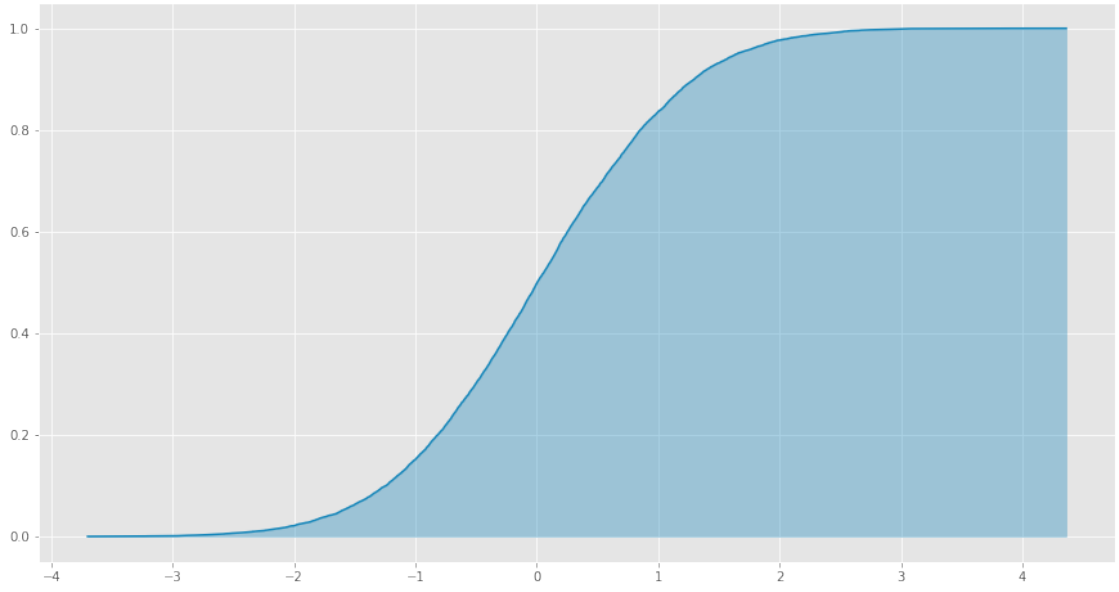
6 The normal distribution

[ ]:
```
import numpy as np
from scipy.interpolate import UnivariateSpline
from matplotlib import pyplot as plt

N = 10000000
n = N//100000
s = np.random.normal(scale=1, size=(N,)) # generate your data sample with N␣
 ↪elements
p, x = np.histogram(s, bins=n) # bin it into n = N//10 bins
x = x[:-1] + (x[1] - x[0])/2   # convert bin edges to centers
f = UnivariateSpline(x, p, s=n)
plt.figure(figsize=(15,8))
plt.plot(x, f(x)/N, color="#1586BA", linewidth=2)
plt.fill_between(x,0, f(x)/N, color="#1586BA", alpha=0.35)

plt.show()
```



[ ]:
```
#plt.plot(np.random.exponential(scale=2, size=(1000)))

arr = np.random.normal(scale=1, size=(10000,))
df = pd.DataFrame(arr, columns=["sim"]).sort_values(by="sim", ascending=True)
df["probability"] = 1/len(df)

plt.figure(figsize=(15,8))
plt.plot(df["sim"], df["probability"].cumsum(), color="#1586BA")
plt.fill_between(df["sim"],0, df["probability"].cumsum(), color="#1586BA",␣
↪alpha=0.35)
```

[ ]:
```
<matplotlib.collections.PolyCollection at 0x7f34ea5ed2d0>
```



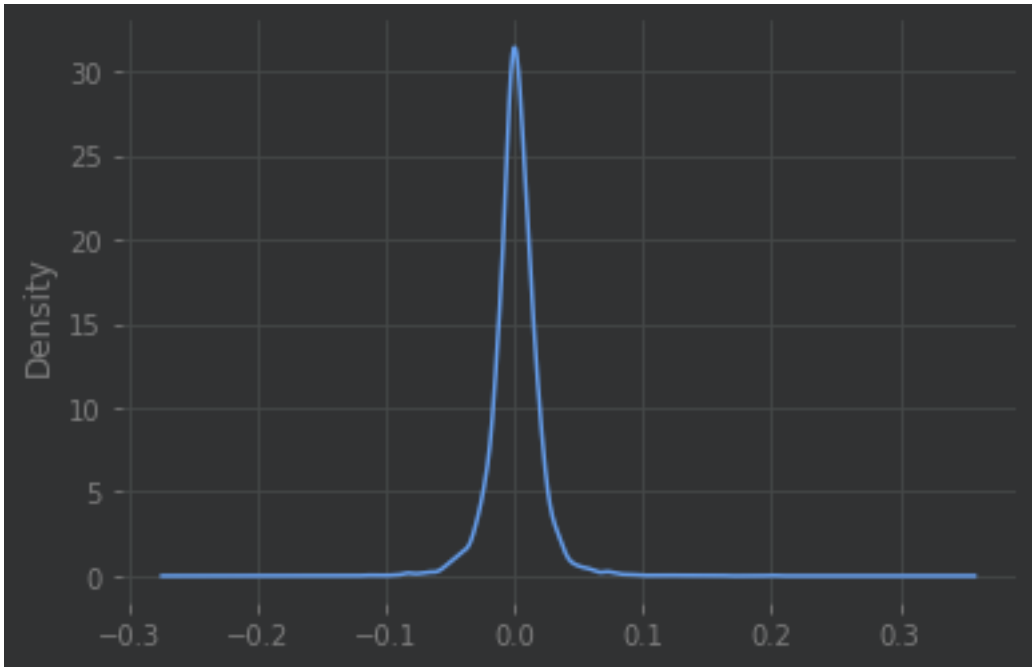
7 Exercise: Compute the value at risk of a financial asset

[ ]:
```
df = yf.download("GOOG").pct_change(1).dropna()
```

```
[**********100%**********]  1 of 1 completed
```

[ ]:
```
df["Close"].plot(kind="kde")
```

[ ]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x7ff5f862b1d0>
```



[ ]:
```
VaR = np.quantile(df["Adj Close"], 0.05)*100
print(f"Value at risk: {'%.2f' % VaR} %")
```

```
Value at risk: -2.71 %
```

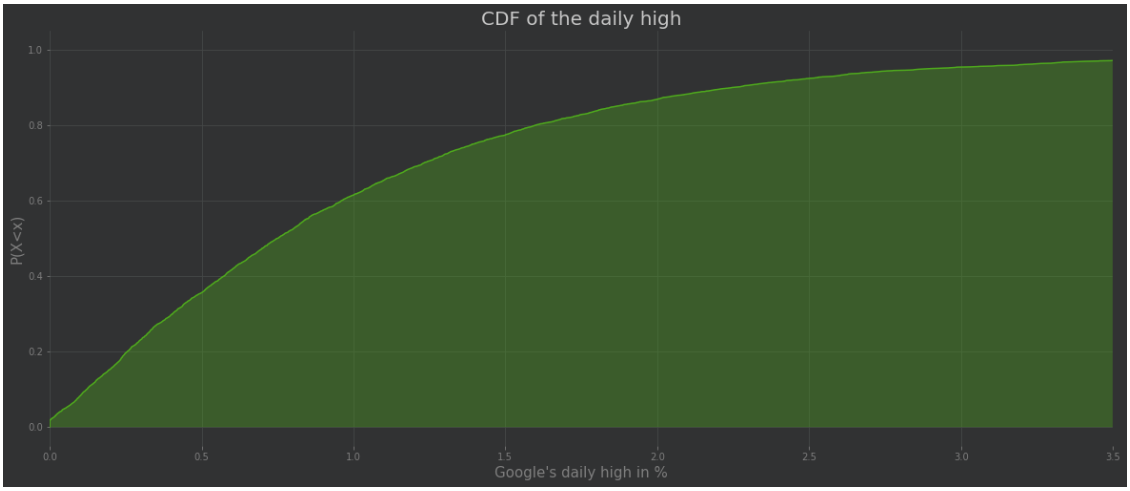
8 Exercise: Analyze the stop loss and take profit

[ ]:
```
# Import the data
df = yf.download("GOOG")

# Compute variation Max and Min by day
df["high"] = (df["High"] - df["Open"])/df["Open"]
df["low"] = (df["Low"] - df["Open"])/df["Open"]
```

```
[**********************100%**********************]  1 of 1 completed
```

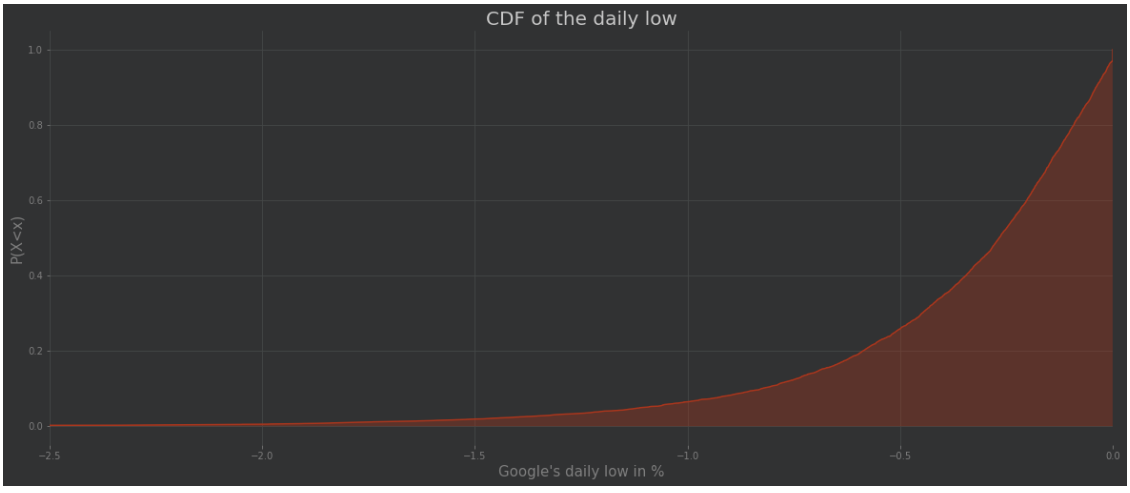
[ ]:
```
# -------- Prepare the data ----------
# Remove missing values and order them
dfh = df[["high"]].dropna().sort_values(by="high", ascending=True)

# Weight for each value
dfh["probability"] = 1/len(dfh)


# -------- Plot the graph ----------
# Adapt the size
```

```
plt.figure(figsize=(20,8))

# Put a ylabel
plt.ylabel("P(X<x)", size=15)

# Put a xlabel
plt.xlabel("Google's daily high in %", size=15)

# Put a title
plt.title("CDF of the daily high", size=20)

# Plot the line of the CDF
plt.plot(dfh["high"]*100, dfh["probability"].cumsum(), color="#50AB1E")

# Plot the area of the CDF
plt.fill_between(dfh["high"]*100,0, dfh["probability"].cumsum(),␣
↪color="#50AB1E", alpha=0.35)

# Define x limits
plt.xlim([0,3.5])

# Plot the graph
plt.show()
```



[ ]:
```
# -------- Prepare the data ----------

# Remove missing values and order them
dfl = df[["low"]].dropna().sort_values(by="low", ascending=True)

# Weight for each value
```

```
dfl["probability"] = 1/len(dfl)


# -------- Plot the graph ----------
# Adapt the size
plt.figure(figsize=(20,8))

# Put a ylabel
plt.ylabel("P(X<x)", size=15)

# Put a xlabel
plt.xlabel("Google's daily low in %", size=15)

# Put a title
plt.title("CDF of the daily low", size=20)

# Plot the line of the CDF
plt.plot(dfl["low"]*100, dfl["probability"].cumsum(), color="#AB371E")

# Plot the area of the CDF
plt.fill_between(dfl["low"]*100,0, dfl["probability"].cumsum(),␣
↪color="#AB371E", alpha=0.35)

# Define x limits
plt.xlim([-2.5, 0])

# Plot the graph
plt.show()
```



[ ]: