1 Inverzna matrica

Primer 1 *Izračunati inverznu matricu matrice* $A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 3 & 3 & 5 \\ 2 & 1 & 4 & 3 \\ 4 & 3 & 7 & 6 \end{bmatrix}$.

Rešenje. Da bismo izračunali inverznu matricu matrice A dovoljno je rešiti po X matričnu jednačinu $AX = E$, jer će tada biti $X = A^{-1}$. Dakle, treba rešiti sledeću matričnu jednačinu:

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 3 & 3 & 5 \\ 2 & 1 & 4 & 3 \\ 4 & 3 & 7 & 6 \end{bmatrix} \cdot \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Ako strpljivo raspišemo proizvod matrica na levoj strani, dobijamo da treba simultano rešiti četiri sistema jednačina koji svi imaju istu matricu sistema:

$$\begin{array}{rcl@{\qquad}rcl}
x_{11} + x_{21} + 2x_{31} + 2x_{41} & = & 1 & x_{12} + x_{22} + 2x_{32} + 2x_{42} & = & 0 \\
2x_{11} + 3x_{21} + 3x_{31} + 5x_{41} & = & 0 & 2x_{12} + 3x_{22} + 3x_{32} + 5x_{42} & = & 1 \\
2x_{11} + x_{21} + 4x_{31} + 3x_{41} & = & 0 & 2x_{12} + x_{22} + 4x_{32} + 3x_{42} & = & 0 \\
4x_{11} + 3x_{21} + 7x_{31} + 6x_{41} & = & 0 & 4x_{12} + 3x_{22} + 7x_{32} + 6x_{42} & = & 0
\end{array}$$

$$\begin{array}{rcl@{\qquad}rcl}
x_{13} + x_{23} + 2x_{33} + 2x_{43} & = & 0 & x_{14} + x_{24} + 2x_{34} + 2x_{44} & = & 0 \\
2x_{13} + 3x_{23} + 3x_{33} + 5x_{43} & = & 0 & 2x_{14} + 3x_{24} + 3x_{34} + 5x_{44} & = & 0 \\
2x_{13} + x_{23} + 4x_{33} + 3x_{43} & = & 1 & 2x_{14} + x_{24} + 4x_{34} + 3x_{44} & = & 0 \\
4x_{13} + 3x_{23} + 7x_{33} + 6x_{43} & = & 0 & 4x_{14} + 3x_{24} + 7x_{34} + 6x_{44} & = & 1
\end{array}$$

Primenićemo Gaus-Žordanovu eliminaciju:

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 2 & 2 & 1 & 0 & 0 & 0 \\ 2 & 3 & 3 & 5 & 0 & 1 & 0 & 0 \\ 2 & 1 & 4 & 3 & 0 & 0 & 1 & 0 \\ 4 & 3 & 7 & 6 & 0 & 0 & 0 & 1 \end{array}\right] \sim \ldots \sim \left[\begin{array}{cccc|rrrr} 1 & 0 & 0 & 0 & -7 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 4 & -1 & -3 & 1 \\ 0 & 0 & 1 & 0 & 4 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 2 & -1 \end{array}\right].$$

Prema tome, $A^{-1} = \begin{bmatrix} -7 & 1 & 1 & 1 \\ 4 & -1 & -3 & 1 \\ 4 & -1 & -1 & 0 \\ -2 & 1 & 2 & -1 \end{bmatrix}$.