Black Money and Demonetisation*

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Abstract

On 8th November 2016, Government of India announced the surprise 'Demonetisation' of ₹500 (US $7.70) and ₹1000 (US $15) bank notes replacing them with new notes. The government claimed that this action would curtail the shadow economy and crack down the use of illicit and counterfeit cash to fund illegal activity. The sudden nature of the announcement and the prolonged cash shortages in the weeks that followed - created significant disruption throughout the Indian economy. In this paper, we build a theoretical framework to explain some of the stylised facts of demonetisation in India and characterize it's implications on the Stationary monetary equilibrium of the economy. The paper closely explains the tradeoff faced by the agents with regards to holding black money and evading taxes on one hand and getting heavily penalised if caught by the auditors on the other. The model also explains how money laundering naturally emerges when the government compels agents to reveal their true taxable incomes via demonetisation.

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1 Introduction

Economies all across the world, have been facing (in varying degrees), the existence of 'Black Economy'/ 'Shadow Economy' and taking measures to mitigate the issues it entails - black money, counterfeit currency, terrorism, and tax evasion to name a few. One of the most common manifestations of black economy is when individuals underreport their incomes or money holdings in order to evade taxes. The money that individuals use in transactions and for consumption but that is not reported for tax purposes is what we shall define as *'Black Money'* in this paper.

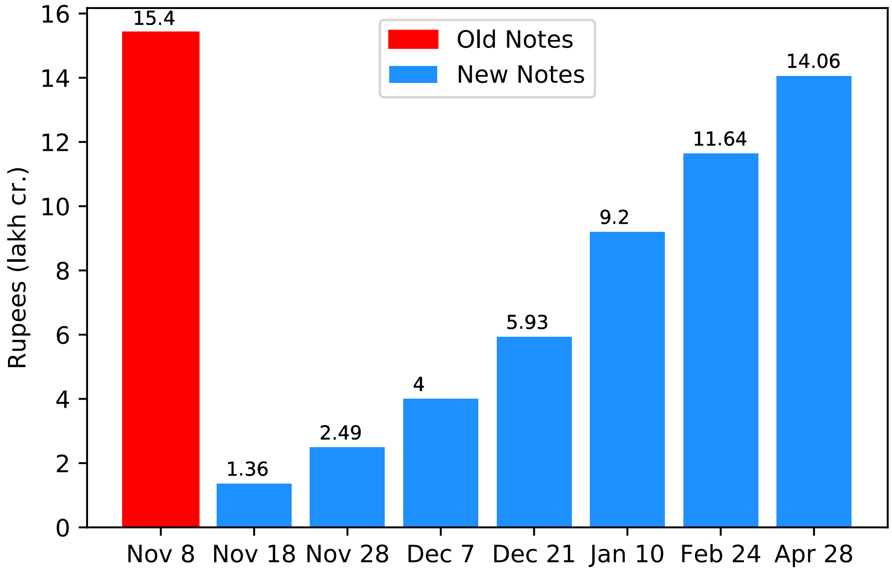
Historically, many economies have dealt with this issue of black money through different forms of regulations - be it random tax audits, raids by revenue department on rich business firms, huge penalties for defaulters, or some other non-conventional measures. One such non-conventional measure was taken in India on 8th November 2016 with a *sudden/ unexpected* announcement of *'Demonetisation'* of it's 2 highest denomination currency. It meant stripping the ₹500 ($\approx$ 7.5$) and ₹1000 ($\approx$ 15$) notes of their status as legal tender and replacing them with newly issued ₹500 and ₹2000 notes (on the left and right in Figure 1 respectively).

Figure 1: Old notes and New Notes in India, Demonetisation 2016

At one fell stroke, 86 percent of the cash in circulation was thereby rendered invalid (Figure 2) and gradually replaced by new notes over the next 6-10 months. The goal was to compel all individuals to reveal their money holdings in the process of having to exchange 'old' currency notes for 'new' from the government at par value. As a result, all the agents who held black money, had 3 options: (i) Declare their unaccounted wealth and pay taxes at a penalty rate, (ii) Continue to hide it, not converting their old notes and thereby suffering a tax rate of 100%, (iii) or Launder their black money by paying a cost for converting the money into 'white' (legal money). These notes were to be deposited in the banks by December 30, 2016 after which they would be rendered valueless in terms of legal tender i.e. the government would not honour them anymore. This goes back to the classic definition of unbacked fiat money - while the paper currency is not backed by actual physical goods, it is used as a money because it is backed by government/ monetary authority's promise/ fiat to honor it.

In this paper, we formalize a model to capture all these nuances when demonetisation happens and understand the implications of such a policy on the 'Black Economy' and the Monetary Equilibrium. The model uses a simplified specification of the 'Lagos and Wright (2005)' [L]

Figure 2: Currency in Circulation



Source: Economic Survey of India, 2016-17

framework with heterogenous agents (in terms of money holdings). It then adds the possibility of black money in the baseline model. Finally, we incorporate sudden demonetisation into the economy with some agents holding black money and see the implications for the Monetary Equilibrium.

One place where the black economy is predominantly effective in the Indian Economy is the informal sector - where most of the transactions are made off the books and agents have regular transactions in cash. We shall try to model the use of money for transactions between buyers and sellers, but potentially not declared for tax returns, as those, which happen in the cash intensive informal sector. The other part of the economy would be the more formal one, where agents produce and consume in a competitive market and make their inter-temporal money holding decisions. They may be required to pay taxes and they get some transfers from the government.

In Section 2, we understand the baseline model with prevalence of black money holders and random checks by the government. In Section 3, we expand the model to incorporate a one time 'Demonetisation' policy and understand how the Monetary Equilibrium changes due to the policy. Finally, we conclude with some policy implications and assess the success/ failure of the policy.

2 Baseline Model of Black Money

2.1 Primitives

We consider an infinite horizon discrete time model. Each unit of time consists of 2 sub-periods each designated to a certain type of good. The first subperiod is for the production and exchange for the 'Specialized Goods' in a 'Decentralized Market' (DM) using search and bargaining; while the second is for the production and consumption of a 'General Good' in a 'Centralized Market' (CM). Both types of goods are non-storable and perfectly divisible. Specialized goods are heterogenous and only produced by 'Specialists/ Sellers' while general goods are homogeneous and produced by all the agents. The general good is the *numeraire*.

There are two categories of agents in the economy depending on their role in the *decentralized market* - a unit mass of 'Buyers' (B) who consume specialized goods and a unit mass of 'Sellers/ Specialists' (S) who produce these specialized goods. Buyers and Sellers meet each other with probability α in this market and once they do, seller produces the amount of good the buyer needs and the buyer pays the cost to the seller using money. Money is the only non-storable perfectly-divisible medium of exchange in this economy. For simplicity, we shall assume that the buyers have all the bargaining power in this bilateral exchange.

In the *centralized market*, all the agents are endowed with 1 unit of labor. All agents produce the general good and they can consume it, or sell some of it to take money into the next period. There is *perfectly persistent* heterogenous productivity among agents i.e. they have different disutility from production. For the sellers, there is uniform disutility of 1 from putting h_t units of labor. On the other hand, buyers are randomly assigned a productivity level at the start of their life - with probability π_L, they have low productivity i.e. the disutility cost of providing h units of labor into production of general good is high (h/ϵ_L), and with probability $\pi_H = 1-\pi_L$, they have high productivity i.e. it costs less (h/ϵ_H), $\epsilon_H > \epsilon_L$.

Consequently agents maximize the following objective function,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(q_t^b) - c(q_t^s) + \log c_t - \frac{h_t}{\epsilon_t} \right] \tag{1}$$

where $\beta \in (0,1)$, q_t^b and q_t^s are the consumption and production of specialized goods in the first subperiod and c_t and h_t are consumption and hours worked in the second subperiod of time t. ϵ_t is the productivity level in the second subperiod $\in \{1, \epsilon_L, \epsilon_H\}$ for seller and buyer of each type respectively. This is how we consider a simplified model from [L] .

We shall assume that $u' > 0, u'' < 0, c' > 0, c'' \geq 0, u(0) = c(0) = 0$ and $\exists \bar{q}, q^*$ such that $u(\bar{q}) = c(\bar{q})$ and $u'(q^*) = c'(q^*)$.

All agents in this economy face an exogenous shock of death with probability δ at the beginning of the second subperiod of time t which means that they'll exit from the economy $t+1$ onwards. To ensure a stationary mass of population, a fraction ν of people will be born and exogenously assigned a type $\{B, \epsilon_L\}$, $\{B, \epsilon_H\}$ or $\{S\}$ with probability $0.5\pi_L, 0.5\pi_H, 0.5$ respectively.

There is a single financial instrument, 'Fiat Money'. The stock of money at time t is denoted by M_t with $M_0 \in \mathbb{R}_{++}$. A monetary authority (we might also refer to it as government sometimes) injects or withdraws money via lump sum transfers S_t in the second subperiod of every period in order to implement a constant growth of money supply i.e. $M_{t+1} = \mu M_t$ with $\mu > 0$. At the beginning of life, each agent is therefore endowed with their type - Buyer/ Seller (which determines their labor endowment), perfectly persistent productivity if they are buyer and a portfolio of money.

The agents' type, money holdings and bilateral transactions are common knowledge among themselves, but to the government, they are unobservable. The government has a tax-penalty policy, (lump sum for now) whereby it imposes a lump sum tax $\mathbb{T}$ on more productive agents (who turn out to be the buyers with much higher money holdings i.e. rich agents in the model) and no tax ($T = 0$) for the low productivity buyers (also the agents with lower money holdings). This tax is required to be paid at the start of the second sub-period in terms of general goods. There are no taxes on Sellers since their type is always known and they are mere facilitators in this model to support the more interesting side of heterogenous buyers. Since the government cannot observe the type of buyers, it imposes the tax based on reported types $\hat{\epsilon}$ i.e.

$$T(\hat{\epsilon}) = \begin{cases} \mathbb{T} & \text{if } \hat{\epsilon} = \epsilon_H \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

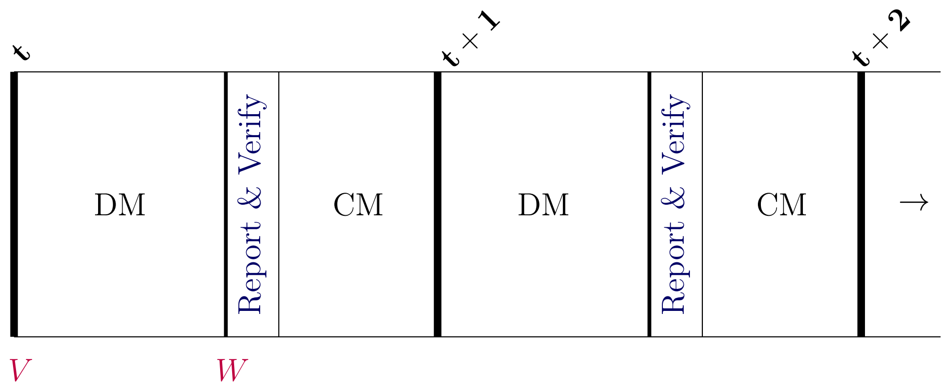
This is what leads to possible mis-reporting by the agents. In particular, rich/high productivity agents want to misreport themselves as low type in order to *evade taxes* i.e. they can potentially use money for transaction with sellers and other agents without declaring it for tax purposes. This is what leads to 'Black Money' in the economy. In order to check this misreporting behavior, the government randomly audits the buyers who report themselves to be low type, with probability p. If they are caught under-reporting, it imposes a lump sum penalty in terms of general goods $\mathbb{P}(> \mathbb{T})$ i.e.

$$P(\epsilon, \hat{\epsilon}) = \begin{cases} \mathbb{P} & \text{if } \hat{\epsilon} = \epsilon_L \neq \epsilon = \epsilon_H \\ 0 & \text{if } \hat{\epsilon} = \epsilon_L = \epsilon = \epsilon_L \end{cases} \tag{3}$$

In this simple model, the government need not audit the agents who already report to be high productivity because they are paying taxes honestly anyway. Finally, we shall assume that agents cannot make binding commitments; and trading histories are private in a way that precludes any borrowing and lending between people. So, all trade - both in the centralized market and the decentralized markets must be *quid pro quo*.

The timeline is illustrated as follows,

Figure 3: Timeline



Each time period starts with the decentralized market. We denote the value function for the agents at the start of the period with $V_t(\text{type}, m_t)$ where the state variables are the type of the agent and the amount of money with which she enters the time period. Likewise, the value function at the start of the second subperiod is denoted by $W_t(\text{type}, m_t)$. Conditional on no exit/ death shock, the second subperiod starts with the agents reporting their type, the government randomly auditing them, appropriate payment of tax/penalty/nothing, and finally consumption and saving decision in the centralized market. If the agents face the death shock, they consume whatever they have in that period and exit the economy.

Let types $\in \{S, L, C, NC\}$ where S denotes the Seller, L are the buyers with Low Productivity, C are the high productivity buyers who either pay the taxes honestly or have been 'Caught' in the past so that their perfectly persistent type is revealed forever, and NC be the high type buyers who underreport their type and have 'Not been Caught' so far. We shall see that this type space is rich enough to model black money in the economy. We shall denote money holdings of buyers at time t as m_t and that of seller as $\tilde{m}_t$. Let the distribution of money holdings across buyers be $F(m)$ and that for sellers be $G(\tilde{m})$.

2.2 Value Functions and Optimal Decisions

Given the primitives, we can now solve for the optimal decisions and values for each type of agent and characterize the Monetary Equilibrium. The details of the derivations are given in the appendix A.1. Let ϕ_t denote the price of money in terms of general goods at time t. Then, the value function at the start of the second subperiod, of each type i of agent who enters with money m_t denoted by $W_t^i(m_t)$, is as follows[1],

$$W_t^S(\tilde{m}_t) = -1 + \phi_t \tilde{m} + \phi_t S_t + (1-\delta) \max_{\tilde{m}'} \left(\beta V_{t+1}^S(\tilde{m}') - \phi_t \tilde{m}'\right) \tag{4}$$

$$W_t^L(m_t) = \log \epsilon_L - 1 + \frac{\phi_t m_t + \phi_t S_t}{\epsilon_L} + (1-\delta) \max_{m'} \left[\beta V_{t+1}^L(m') - \frac{\phi_t m'}{\epsilon_L}\right] \tag{5}$$

$$W_t^C(m_t) = \log \epsilon_H - 1 + \frac{(\phi_t m_t - \mathbb{T}) + \phi_t S_t}{\epsilon_H} + (1-\delta) \max_{m'} \left[\beta V_{t+1}^C(m') - \frac{\phi_t m'}{\epsilon_H}\right] \tag{6}$$

$$\begin{aligned} W_t^{NC}(m_t) = \log \epsilon_H - 1 + \frac{\phi_t m_t + \phi S_t}{\epsilon_H} + \mathbb{1}_{\hat{\epsilon}=\epsilon_H} \left[\frac{-\mathbb{T}}{\epsilon_H} + (1-\delta) \max_{m'} \left(\beta V_{t+1}^C(m') - \frac{\phi m'}{\epsilon_H}\right)\right] + \\ \mathbb{1}_{\hat{\epsilon}=\epsilon_L} \left[p\left(\frac{-\mathbb{P}}{\epsilon_H} + (1-\delta) \max_{m'} \left[\beta V_{t+1}^C(m') - \frac{\phi m'}{\epsilon_H}\right]\right) + (1-p)\left((1-\delta) \max_{m'} \left[\beta V_{t+1}^{NC}(m') - \frac{\phi m'}{\epsilon_H}\right]\right)\right] \end{aligned} \tag{7}$$

The sellers and low type buyers simply come in to the second subperiod and make their optimal consumption, labor input, and savings decisions. The high type 'Caught' buyers also have to pay taxes worth $\mathbb{T}$ units of general good each period. The high type 'Not Caught' agents are required to also choose a reporting strategy. If they decide to turn honest from period t onwards, they pay taxes and their value function switches to V_{t+1}^C. If on the other hand they choose to misreport, they get caught with some probability p, pay a penalty and switch to V_{t+1}^C, else with probability $(1-p)$, they continue to be 'NC' type and *'evade taxes'*. These are agents in the economy who shall be rich and potentially holding 'black money' for given value of government audit probability p. Note that the structure of the problem, induces quasi linearity in the value function W w.r.t. m i.e.

$$W_t^i(m_t) = \frac{\phi_t m_t}{\epsilon_i} + W_t^i(0) \qquad \forall i \in \{S, L, C, NC\} \tag{8}$$

which we shall use later when we solve the bargaining problem for the Decentralized market.

Having solved for the value functions at the start of second subperiod, we can move backwards to solve for the value functions $V_t^i(m_t)$ at the start of the first subperiod at time t for agent i

[1] The derivations follow very closely from [L].

who enters the time period with money holdings m_t. The buyers and sellers meet each other with probability α in the DM and enter into Nash Bargaining for exchange of the 'specialized Good'. We assume that the buyer has all the bargaining power, and she pays with money $d \leq m_t$ in exchange of the goods q_t^B she buys. This leads to the following Nash Bargaining problem for the buyer with money holdings m_t, productivity ϵ_i who meets a seller with money holdings $\tilde{m}_t$, and has an outside option of no exchange payoff,

$$\begin{aligned} &\max_{q^B,d} u_t(q_t^B(m_t,\epsilon_i,\tilde{m}_t)) + W_t^i(m_t - d(m_t,\epsilon_i,\tilde{m}_t)) - W_t^i(m_t) \\ \text{subject to} \quad &-c_t(q_t^B(m_t,\epsilon_i,\tilde{m}_t)) + W_t^S(\tilde{m}_t + d(m_t,\epsilon_i,\tilde{m}_t)) - W_t^S(\tilde{m}_t) \geq 0 \end{aligned} \tag{9}$$

The terms of trade for the bargain could in principle depend on buyers type and money holdings and on the sellers initial money holdings. It turns out that given our assumptions, the buyer exhausts all her money holdings to get as much of the 'specialized good' as she can, and she compensates the seller enough to break even. The optimal decision is solved in detail in the appendix A.2, and we get the following,

$$q_t^B(m_t,\epsilon_i) = \begin{cases} q_t^*(m_t,\epsilon_i) & \text{if } c_t(q_t^*(m_t,\epsilon_i)) \leq \phi_t m_t \\ \{\hat{q}_t | c_t(\hat{q}_t(m_t,\epsilon_i)) = \phi_t m_t\} & \text{otherwise} \end{cases} \tag{10}$$

$$d_t(m_t,\epsilon_i) = \begin{cases} \dfrac{c_t(q_t^*(m_t,\epsilon_i))}{\phi_t} & \text{if } c_t(q_t^*(m_t,\epsilon_i)) \leq \phi_t m_t \\ m_t & \text{otherwise} \end{cases} \tag{11}$$

This leads to the flow payoff in the Decentralized market for each type of agent, and combining that with the continuation values, we get their value function at the start of the first subperiod as follows:

$$V_t^S(m_t) = \phi_t m_t + W_t^S(0) \tag{12}$$

$$V_t^L(m_t) = \alpha \left[u_t(q_t^B(m_t,\epsilon_L)) - \frac{c_t(q_t^B(m_t,\epsilon_L))}{\epsilon_L} \right] + \frac{\phi_t m_t}{\epsilon_L} + W_t^L(0) \tag{13}$$

$$V_t^i(m_t) = \alpha \left[u_t(q_t^B(m_t,\epsilon_H)) - \frac{c_t(q_t^B(m_t,\epsilon_H))}{\epsilon_H} \right] + \frac{\phi_t m_t}{\epsilon_H} + W_t^i(0), \quad i \in \{C, NC\} \tag{14}$$

Using these value functions, we can go back to the equations in (4)-(7), substitute for the V_{t+1}'s, and get the value of audit probability p, such that the high type not caught agents choose to misreport their types and hold black money in a stationary monetary equilibrium.

We solve for the threshold value of p using guess and verify. The idea is to start with the guess that the value of audit probability is such that the High type NC agent chooses to misreport i.e. her value from misreporting is greater than her value from turning honest. Next, under the guess, we can solve for the optimal value of money holdings for each type of agent. Once we have the solution to all our agents' optimisation problem under the guess, we can verify that indeed the probability of audit is such that the optimal decision with subsequent optimal money holdings is for the high type agent to misreport her type. This is explained in detail in Appendix A.3 and it leads to the following threshold value of p^* and reporting strategy for the High type NC buyers.

$$\hat{\epsilon} = \begin{cases} \epsilon_L & \text{if } p \leq \frac{\mathbb{T}}{\mathbb{P}} \equiv p^* \\ \epsilon_H & \text{otherwise} \end{cases} \tag{15}$$

Proposition 1. *$\exists p^* = \frac{\mathbb{T}}{\mathbb{P}} \in (0,1)$, such that for all values of $p \leq p^*$, rich agents under-report their money holdings to evade taxes, i.e. 'Black Money' exists in the economy.*

The proof follows from the above explanation and the derivation of p^*, as derived in the appendix A.3. We assume that the government audits with a probability less than p^*, such that the high type agents misreport their type and we have agents holding 'Black Money' in this economy. Given all the value functions and the reporting strategy, we can solve for the optimal amount of money holdings for each type of agent, and we get the following stationary real value of money holdings, $z^i, i \in \{S, L, C, NC\}$, for the case when we assume $u(q) = \log q$, and $c(q) = cq$. We solve for the more general solution to the agents optimisation problem in the appendix A.3.

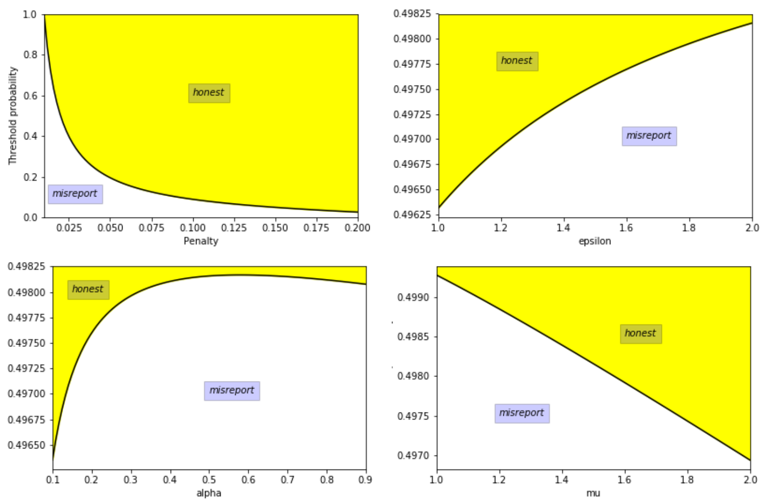
$$z^S = 0 \tag{16}$$

$$z^L = \frac{\beta\alpha\epsilon_L}{\mu - \beta(1-\alpha)} \tag{17}$$

$$z^C = z^{NC} = \frac{\beta\alpha\epsilon_H}{\mu - \beta(1-\alpha)} \tag{18}$$

I also solve for a version of the problem in which reporting and verification happens at the beginning of the first subperiod instead of the second, i.e. the agents cannot work more to pay their tax/ penalty, but have to actually save extra money to pay for it. Also, in that case the agents would get penalised more heavily and lose out the money for buying specialized goods in the DM, if they got caught at the start of the period. The value of p^*, depends on all the underlying parameters of the model in that case, $\alpha, \epsilon, \beta, \mu, \mathbb{P}, \mathbb{T}$, and the way p^* changes with changes in some of the parameter values is shown in the following figure 4. It is an example,

Figure 4: Comparative Statics for p^*



where we simply fix certain values [2] and run comparative statics for changes in one of the parameter values.

Finally, let us look at the laws of motion for the distribution of agents of different types and impose stationary mass of each type of agent in our stationary monetary equilibrium. Let the

[2] $\beta = 0.95, \alpha = 0.5, c = 1, \mu = 1.5, \epsilon_H = 2, \mathbb{T} = 0.01, \mathbb{P} = 0.02$

measures be denoted by λ's. Since, agents of all types die at rate δ and are born at rate ν, we get the following laws of motion for the measure of agents, assuming that 0.5 of the agents are born as Sellers, $0.5\pi_H$ as High type Not Caught Buyers and $0.5\pi_L$ as low type buyers. Also, with probability p, some of the not caught buyers get 'Caught'. The total measure of buyers and sellers is 1 each respectively.

$$\begin{aligned}
\dot{\lambda}_S &= 0.5\nu - \delta\lambda_t^S \\
\dot{\lambda}_L &= 0.5\pi_L\nu - \delta\lambda_t^L \\
\dot{\lambda}_{NC} &= 0.5\pi_H\nu - (\delta + p)\lambda_{NC} \\
\dot{\lambda}_C &= p\lambda_{NC} - \delta\lambda_C \\
\lambda_S &= 1, \lambda_L + \lambda_C + \lambda_{NC} = 1
\end{aligned} \tag{19}$$

Together, we get

$$\boxed{\lambda_S = 1, \lambda_L = \pi_L, \lambda_{NC} = \frac{\delta}{\delta + p}\pi_H, \lambda_C = \frac{p}{\delta + p}\pi_H \text{ such that } \dot{\lambda}_i = 0, \forall i} \tag{20}$$

Also, $0.5\nu = \delta$, so for a given birth/death rate, all the variables are pinned down.

2.3 Stationary Monetary Equilibrium

Given the above model framework, we can now define the Stationary Monetary Equilibrium in this economy as follows:

Definition 1. Stationary Monetary Equilibrium *is the set of perfectly persistent types of agents $i \in \{S, L, C, NC\}$, their stationary real money holdings z^i, price of money $\phi_t > 0$, consumption, labor, savings, terms of trade, and reporting choices of the agents; government's monitoring strategy p, and transfers S_t, such that :- $\exists p^*$, and $\forall p \leq p^*$, high type agents evade taxes, all the agents satisfy their value functions (4)-(7) and (12)-(14), terms of trade in DM are as in (10)-(11), all agents choose optimal consumption $c_t^{i*} = \epsilon_i$, labor input h_t^*, and real money holdings z^i as in (16)-(18), distribution of agents satisfy (19), money market clears as follows,*

$$\phi_t \left(\lambda_L m_L + \lambda_{NC} m_{NC} + \lambda_C m_C\right) = \phi_t M_t = \phi \bar{M} \tag{21}$$

and Government budget equation holds every period, i.e.

$$\phi_t S_t = \lambda_C \mathbb{T} + \lambda_{NC} p \mathbb{P} + \phi_t (M_{t+1} - M_t) \tag{22}$$

This completes our baseline Model with Black Economy, and tax evasion. Next, we want to consider a 'surprise announcement' of Demonetisation at time τ, to an otherwise stationary equilibrium of this baseline economy, and analyze the best responses of the agents as well the implications on the monetary equilibrium due to such one time shock.

3 Model with Demonetisation

Suppose that at time period τ, the government makes a 'Sudden Announcement' that, they are demonetising all the existing money (which we shall now refer to as 'Old Money') and the only legal tender from time period $\tau + 1$ onwards, will be the new notes printed by the bank (which we shall refer to as 'New Money'). Also, the agents are given an exchange window at the end of period τ, where they can go and exchange the old notes for new notes, one for one in value. This is what we shall refer to as the *transition period*. From next period, $\tau + 1$, 'Old Money' will not longer be considered legal tender or honoured by the government.

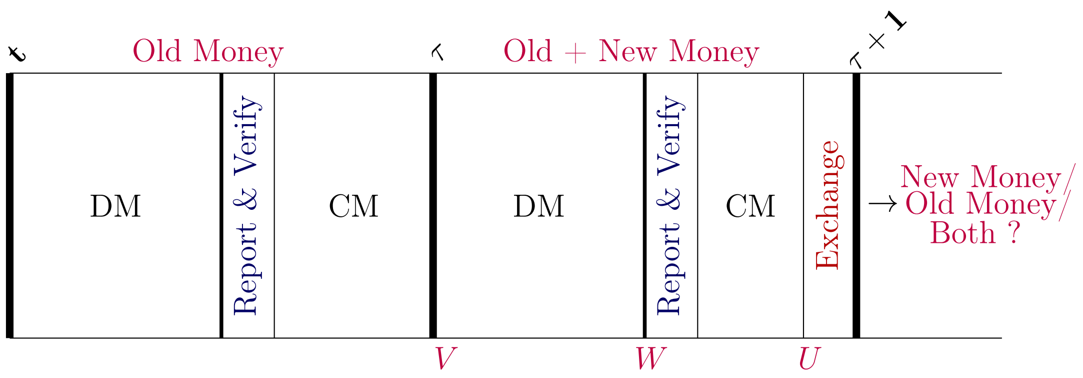
This is like the time period from the announcement of policy in India (Nov 8, 2016), to Dec 31, 2018, which was the last day the agents could go to the exchange window and government shall convert their old money deposits for new money. Thus, from time period τ, onwards we have both 'Old Money' (denoted by M^O) and, 'New Money' (denoted by M^N), and hence prices for each type of ϕ^O_τ, and ϕ^N_τ respectively.

In this model, for what we solve below, we shall assume that, going forward new money is more valuable in terms of the amount of goods it can buy as compared to old money.

Assumption. $\phi^N_{t+i} > \phi^O_{\tau+i} \quad \forall i \geq 1$

The timeline with this policy modifies as follows. We basically have the same structure as before, but now there is an additional decision to be made at the end of period τ, about how much of their old money agents want to exchange for new money at the end of the period. We shall assume that the agents can only exchange upto their own holdings of old money, i.e. we do not allow for laundering option in this simplified version. Hence, the tradeoff for the agents in this case is simple, they exchange less valuable old money for more valuable new money, and if they try to exchange more money than what their reported type entails, they could get caught in the process, heavily penalised and their type gets revealed forever.

Figure 5: Timeline with Demonetisation



The agents' problems now comprises of their decisions in the DM and CM, reporting strategy, and also 'exchange' strategy. Let $V^i_\tau(m^O_\tau, m^N_\tau)$ and $W^i_\tau(m^O_\tau, m^N_\tau)$, be the value functions at the start of the first and second subperiod respectively, same as before except that both old and new money holdings enter their state variable. Further, let us consider value function $U^i_\tau(m^O_\tau, m^N_\tau)$ at the start of the exchange window where the agents chooses how much to exchange, $x \leq m^O_\tau$ from her old money holdings. Let the government monitor them based on their reported type and the amount of money exchanged with probability $\pi^{\hat{i}}(x)$, increasing in x ($\pi'(x) > 0$). Upon monitoring the government learns the true type of the agent imposes a fine f^i, if the agent is caught laundering money (exchanging more money than what they have) or evading taxes (misreporting their type in the past). The timeline for birth and death continues to be the same as before. The value functions of the agents modify as follows. We shall again follow backward induction and start with U's. The details for each of the value functions are explained in Appendix A.4.

Suppose the agent has (m'^O, m'^N) amount of old and new money portfolio at the start of the exchange window. The agent chooses to exchange $x \leq m'^O$ amount of money to new, and gets caught with probability $\pi^i(x)$ in which case she has to pay a fine f^i. We shall consider the cost of fine in terms of loss in utility at time $\tau + 1$.

$$U_\tau^i(m'^O, m'^N) = \max_{0 \leq x \leq m'^O} \beta \left[V_{\tau+1}^i(m'^O - x, m'^N + x) - \pi^i(x) f^i\right], \quad i \in \{S, L\} \tag{23}$$

$$U_\tau^C(m'^O, m'^N) = \max_{0 \leq x \leq m'^O} \beta \left[V_{\tau+1}^C(m'^O - x, m'^N + x) - \pi^H(x) f^C\right] \tag{24}$$

$$\begin{aligned} U_\tau^{NC}(m'^O, m'^N) = \max_{0 \leq x \leq m'^O} & \pi^L(x) \left[\beta(V_{\tau+1}^C(m'^O - x, m'^N + x) - f^{NC})\right] + \\ & (1 - \pi^L(x)) \left[\beta V_{\tau+1}^{NC}(m'^O - x, m'^N + x)\right] \end{aligned} \tag{25}$$

The Value functions at the start of the second subperiod are the same as before, except the continuation value is U instead of V, and the money holdings comprise of the portfolio of old and new money.

$$\begin{aligned} W_\tau^S(m^O, m^N) = -1 + \left(\phi_\tau^O m^O + \phi_\tau^N (m^N + S_\tau)\right) + (1 - \delta) \max_{m'^O, m'^N} & U_\tau^S(m'^O, m'^N) \\ & - \left(\phi_\tau^O m'^O + \phi_\tau^N m'^N\right) \end{aligned} \tag{26}$$

$$\begin{aligned} W_\tau^L(m^O, m^N) = \log \epsilon_L - 1 + & \left(\frac{\phi_\tau^O m^O + \phi_\tau^N m^N + \phi_\tau^N S_\tau}{\epsilon_L}\right) + \\ & (1 - \delta) \max_{m'^O, m'^N} \left\{U_\tau^L(m'^O, m'^N) - \left(\frac{\phi_\tau^O m'^O + \phi_\tau^N m'^N}{\epsilon_L}\right)\right\} \end{aligned} \tag{27}$$

$$W_\tau^C(m^O, m^N) = \log \epsilon_H - 1 + \left(\frac{\phi_\tau^O m^O + \phi_\tau^N m^N - \mathbb{T} + \phi_\tau^N S_\tau}{\epsilon_H}\right) + \tag{28}$$

$$(1 - \delta) \max_{m'^O, m'^N} \left\{U_\tau^C(m'^O, m'^N) - \left(\frac{\phi_\tau^O m'^O + \phi_\tau^N m'^N}{\epsilon_H}\right)\right\} \tag{29}$$

$$\begin{aligned} W_\tau^{NC}(m^O, m^N) = \log \epsilon_H - 1 + & \left(\frac{\phi_\tau^O m^O + \phi_\tau^N m^N + \phi_\tau^N S_\tau}{\epsilon_H}\right) + \max_{\hat{\epsilon}} \\ & \left[\mathbb{1}_{\hat{\epsilon} = \epsilon_H} \left\{\frac{-\mathbb{T}}{\epsilon_H} + (1 - \delta) \max_{m'^O, m'^N} \left(U_\tau^C(m'^O, m'^N) - \frac{\phi_\tau^O m'^O + \phi_\tau^N m'^N}{\epsilon_H}\right)\right\} + \right. \\ & \mathbb{1}_{\hat{\epsilon} = \epsilon_L} \left\{p \left(\frac{-\mathbb{P}}{\epsilon_H} + (1 - \delta) \max_{m'^O, m'^N} \left(U_\tau^C(m'^O, m'^N) - \frac{\phi_\tau^O m'^O + \phi_\tau^N m'^N}{\epsilon_H}\right)\right) + \right. \\ & \left. \left. (1 - p) \left((1 - \delta) \max_{m'^O, m'^N} \left(U_\tau^{NC}(m'^O, m'^N) - \frac{\phi_\tau^O m'^O + \phi_\tau^N m'^N}{\epsilon_H}\right)\right)\right\}\right] \end{aligned} \tag{30}$$

For the start of the first subperiod, agents' value functions are,

$$V_\tau^S(m^O, m^N) = W_\tau^S(m^O, m^N) \tag{31}$$

$$V_{\tau+1}^L(m'^O, m'^N) = \alpha \Gamma_{\tau+1}^L(m'^O, m'^N) + W_{\tau+1}^L(m'^O, m'^N) \tag{32}$$

$$V_{\tau+1}^C(m'^O, m'^N) = \alpha \Gamma_{\tau+1}^H(m'^O, m'^N) + W_{\tau+1}^C(m'^O, m'^N) \tag{33}$$

$$V_{\tau+1}^{NC}(m'^O, m'^N) = \alpha \Gamma_{\tau+1}^H(m'^O, m'^N) + W_{\tau+1}^{NC}(m'^O, m'^N) \tag{34}$$

We solve this model with the above value functions for the optimal amount of money exchange and savings decisions of each type of agents. Conditional on the amount of old money saved, the agents choose the amount of exchange based on the following tradeoff - higher exchange leads to greater value for money and helps with the gains from more trade in the subsequent Decentralized market, but higher exchange also entails greater probability of getting audited in which case they might have to pay a fine in case they were holding any amount of black money. We find that for a given audit strategy of the government in the exchange window of the transition period, the agents have a threshold exchange strategy, and the threshold varies depending on agents' type and their differential fines. In particular, we get the following optimal Exchange with different, x_i, each of them derived in detail in the appendix A.4,

$$x_i^*(m'^O, m'^N) = \max\left\{0, \min\{x_i(m'^O, m'^N), m'^O\}\right\} \tag{35}$$

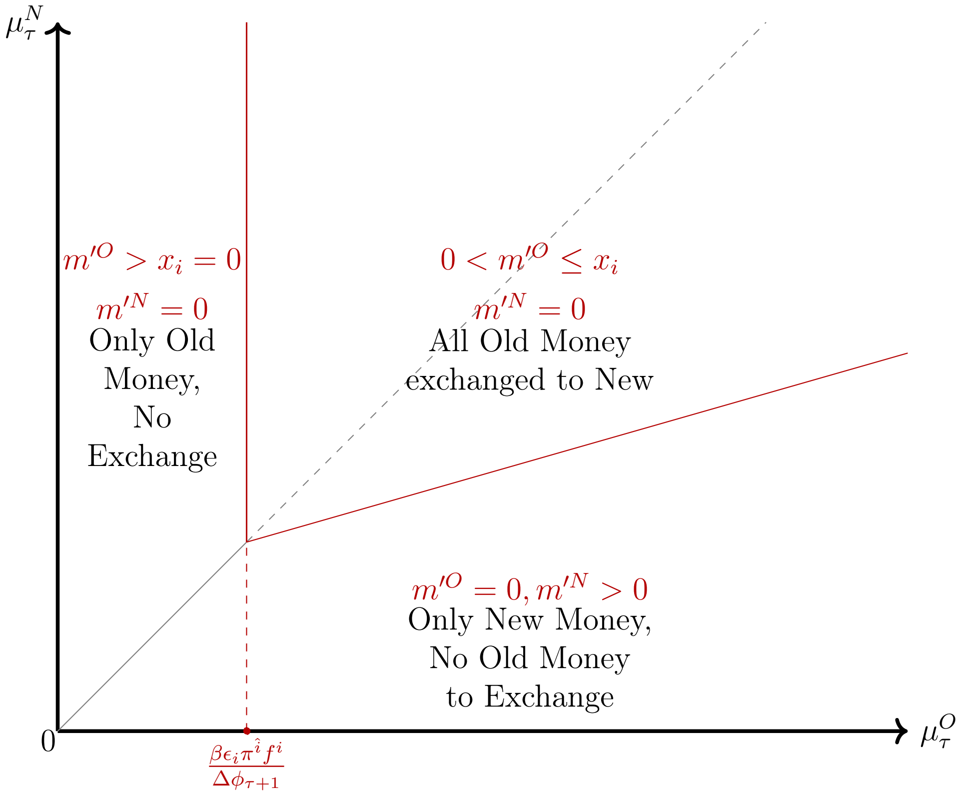
where m'^O, is the amount of old money the agent has at the start of the Exchange window. We then substitute this optimal exchange strategy into the second period value function and solve for the optimal money holding decisions (for both old and new notes) for each type of agent, and consider all the possible exchanges of old money to new for different ranges over the parameter space. The first order conditions are also given in details in the appendix A.4, but they essentially involve the key tradeoff from savings - the gains from trade in the next decentralized market (same as in the baseline model), the costs of inflation both for a given form of money and their relative inflations for old-new money portfolio composition decision, the implication in terms of how much of the old money shall be subsequently exchanged and the loss from getting audited or having to pay any fines. We consider all the possible sub cases for optimal savings of old, new notes and optimal exchange. The derivations are given in appendix A.5 for each type of agents. The final partial equilibrium from the optimisation can be summarized in the following figure 6.

In an economy with no demonetisation and 2 possible forms of money, we know that the choice of money holdings is determined by the relative inflation in the 2 forms, i.e. $\mu_\tau^N \lesseqgtr \mu_\tau^O$[3] in our model. This is indicated using the 45° line in the figure. The second dimension of decision making for the agents in our model is the tradeoff between getting caught or not, captured by the threshold strategy x_i^*. This decision crucially depends on the probability of monitoring in the exchange and the threat of fine to be paid when the agents exchange black money. In the figure, the vertical red line capture this tradeoff, to the left of the line the agent chooses to not exchange any money and too the right, they choose the optimal amount of exchange depending on their portfolio of old and new money they come into the exchange period with. Finally, the most interesting tradeoff happens to the right of this vertical line and in the case when the old notes have higher inflation than the new notes i.e. agents would prefer to hold new money if they were in the baseline economy. However, with demonetisation and exchange, the tradeoff also gets affected by the fact that the less valuable old money could be exchanged for more valuable new money in the transition period albeit with monitoring. This tilts the tradeoff between old and new money in favour of old money, which is then exchanged for new money when the fine and probability of getting caught at the exchange window is sufficiently low. The idea is that the agents can have cheaper old money converted into more valuable new money tomorrow with not much risk of getting audited.

The tradeoffs are similar for each type of buyer, but what varies is their costs from getting caught in the audit and the amount of money holdings they want to save for the next period.

[3] $\mu_\tau^i = \frac{\phi_\tau^i}{\phi_{\tau+1}^i}$

Figure 6: Best responses of Type i buyer in Partial equilibrium



Note: The axis of the figures are the inflation in each type of money

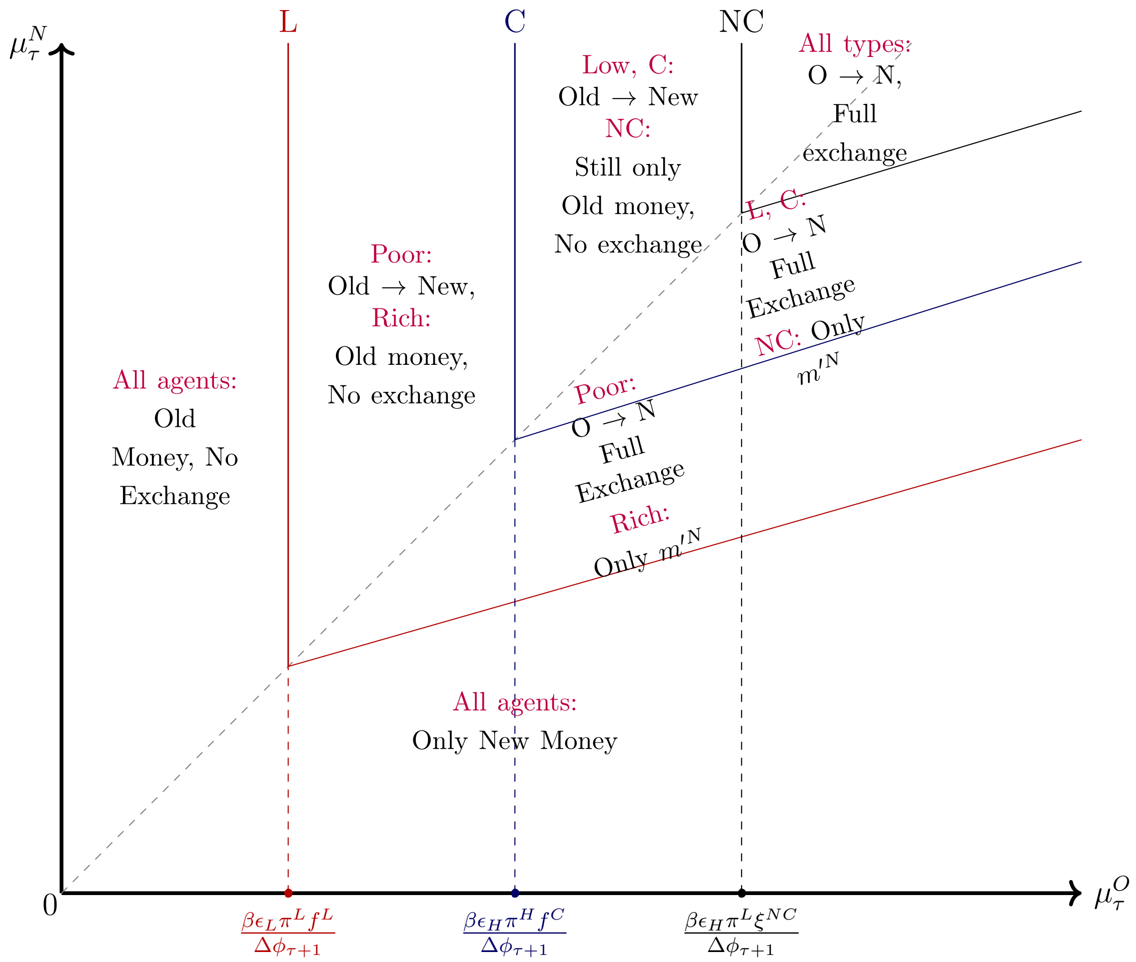
Consequently under the assumption that the agents are punished progressively more from low type to the honest high type to the dishonest high type, we get the following partial equilibrium responses for a given inflation in old and new money and given government regulations.
Assumption. $\pi^L f^L \leq \pi^H f^C \leq \pi^L \xi^{NC}$

Thus, the above analysis gives us a complete partial equilibrium of money holdings and exchange best responses to a given set or prices, and model framework (i.e. government regulations, parameters of the model, money growth etc.)

4 Monetary Equilibrium

Since, we have already solved and analyzed the best responses of each type of agents, we need to next characterize the general equilibrium in this model, i.e. we need to make sure that the money market clears for both old and new notes and the price for each type of money is determined by their relative supply and demand in equilibrium. In particular, we look at each of the sub cases in the figure and consider the market clearing conditions for old and new money to get equilibrium inflation of each type of money and their exchange rates. So far, we have a preliminary analysis of the kind of equilibrium that emerge. Recall that, we have not allowed for private bilateral monetary agreements among agents, i.e. we have not allowed for laundering by construction. But since each of the agent faced a different tradeoff in the exchange window, there might be incentives to launder money during the transition whereby rich dishonest agents could bribe the poor and honest type agents to exchange some money on

Figure 7: Complete Partial Equilibrium



We interchangeably call the low type buyers as poor agents, and the high type buyers as the rich agents.

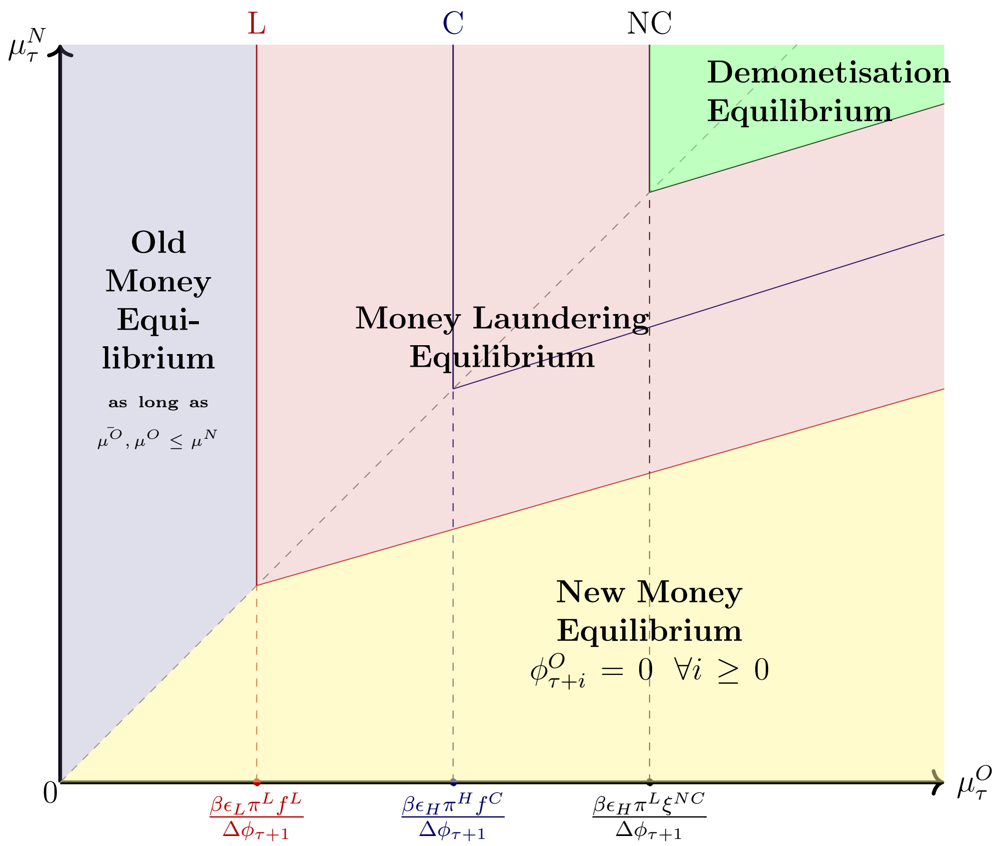
their behalf.

We analyze each of the possibilities in the above figure 7 in detail in appendix A.6, and the results on our monetary equilibrium can be summarized as follows,

Proposition 2. *The Stationary Monetary Equilibrium for a 'Black Money Economy' faced with sudden Demonetisation can be of one of the 3 types going forward:*

1. **Old Money Equilibrium** *where agents completely ignore the policy announcement and continue to hold Old money.*
2. **Money Laundering Equilibrium** *where some subset of agents exchange from old notes to new notes, while some would do so if they could launder money. The economy could continue with either both notes existing after the transition period, or only new currency.*
3. **New Money Equilibrium** *where agents discard their old money or simply exchange it all for new money. In former case, demonetisation fails to punish any of the tax evaders, while in the latter the policy is most successful it can be. We call the second case as the* **Demonetisation Equilibrium**.

Figure 8: Monetary Equilibria



Going forward from time $\tau + 1$, there is only old money in the system under the Old Money equilibrium and the new money is simply use to pay for taxes and penalty if we impose that requirement. The inflation in old money has to stay less than that of new money for the equilibrium to exist. In that sense, it is a fragile equilibrium because if, say, the Old Notes undergo some amount of wear and tear and consequently some tiny amount of money is lost which causes higher inflation in old money. Then, the economy will transition to the new economy the moment old money's inflation gets high enough to exceed new money's inflation.

In case of the New Money Equilibrium (which includes the demonetisation equilibrium) and for the area below the 45° line, we have only new notes going forward from $\tau+1$. In each of these cases $\phi_{\tau+1}=0$. This leads to the limiting case of equilibrium characterisation i.e.

Proposition 3. *When the Monetary Equilibrium is such that the economy has only New Notes after the transition period, we are in the limiting case where value of old money $\phi^O_{\tau+i}=0 \quad \forall i \geq 1$. In this case, the equilibrium money holdings of agent i is characterized by the following threshold strategy,*

$$x_i^*, m_i^{*O}, m_i^{\prime *N} \qquad s.t. \qquad \begin{cases} \textit{Exchange all Old money for New if} & \mu_\tau^N \geq \dfrac{\beta\epsilon_i\pi^{\hat{i}} f^i}{\phi^N_{\tau+1}} \\ \textit{Discard Old Money if} & \mu_\tau^N < \dfrac{\beta\epsilon_i\pi^{\hat{i}} f^i}{\phi^N_{\tau+1}} \end{cases} \tag{36}$$

According to different thresholds for different agents, we have 3 possibilities (1) When all agents discard their old money and start saving in New notes at a very low money supply, high prices equilibrium, (2) When some subset of agents exchange old notes for new, others discard old notes. In this case, agents would prefer to launder money if they could. The price of new notes will be lower than the previous case because the supply of new notes is higher after exchange by some agents (3) When all the agents exchange their old notes to new, which leads to the maximum possibility of dishonest types getting caught. In this case, the price of the new notes is even lower. In fact, in a stationary equilibrium, the new notes completely replace the new notes and the economy continues at the same inflation rate as before the transition period, with simply a change of notes.

Finally, there are the Money Laundering Equilibrium in the red region above the 45° line where both money could stay in the economy after the transition period. In these cases, some of the agents continue to hold Old Notes while some exchange them for new notes. The economy would eventually move to an equilibrium with either of the notes depending on their relative inflation in the future. This follows from the optimal money holdings decisions of the agents from $\tau+1$ onwards summarized by the following equation,

$$z^i_{\tau+t} = \begin{cases} \underbrace{\dfrac{\beta\alpha\epsilon_i}{\mu^N_{\tau+t}-\beta(1-\alpha)}}_{\text{Only New Notes}} & \text{if } \mu^N_{\tau+t} < \mu^O_{\tau+t} \\ \underbrace{\dfrac{\beta\alpha\epsilon_i}{\mu^O_{\tau+t}-\beta(1-\alpha)}}_{\text{Only Old Notes}} & \text{if } \mu^N_{\tau+t} > \mu^O_{\tau+t} \\ \underbrace{\dfrac{\beta\alpha\epsilon_i}{\mu_{\tau+t}-\beta(1-\alpha)}}_{\text{Indeterminate composition}} & \text{if } \mu^N_{\tau+t} = \mu^O_{\tau+t} = \mu_{\tau+t} \end{cases} \qquad \forall t \geq 1 \tag{37}$$

In either of the equilibria, the High type dishonest agents always exist in the economy in the stationary equilibrium, the policy does not tackle the problem of 'tax evasion' and black money completely. The laws of motion for each type fo the agents is same as before from equation 19, with p, replaced by $(p+\pi^L(x^*_{NC}))$, and we have all the type of agents still available in this economy. In this sense, we are able to explain the stylised fact observed in the data (figure

2). The government undertook the demonetisation policy hoping that out of the ₹15.4 trillion of old notes, only a fraction would be deposited by the agents for exchange and a lot of rich dishonest agents who might be evading tax will be compelled to discard some of their old notes to avoid punishments from getting caught. But we found that almost all of the New money was deposited by the agents and the New notes in circulation almost reached the same level as the Old notes. This is indicative of the Money Laundering Equilibrium with massive amounts of money laundering as in the red area below the $45°$ line in the model. The Indian Economy transitioned to only New Money equilibrium, but with black money not completely removed from the system, and we explain that through our model.

5 Policy Implications

Our model of Demonetisation helps us understand the possible equilibria that emerge in the economy when such sudden announcements are made to get rid of black money from the system. We see that almost all the equilibria entail survival of the dishonest type of rich agents who choose to launder money or incur a penalty by discarding new money in order to not reveal their true taxable money holdings. Thus, a policy like Demonetisation, despite being sudden is not a complete solution to the curb black economy. Even though in our model, the agents types got revealed forever (i.e. they could never cheat again once caught), or they were not allowed to launder; we saw that the black money still prevailed in the system. In reality, agents transition between types and launder money which makes the issue of black money even more grave instead of solving it. So, there is a need for policies which involve greater direct monitoring of the agents types instead of one time demonetisation which actually cause more inconvenience (during exchange), then the benefits from mitigating the black money from the system. Some policies which might be more effective in reducing the size of the black economy, might be increase in p, huge and prolonged penalties to agents who are caught misreporting, and the recent surge of digitisation which helps to track agent's money holdings more transparently. Digitisation enforces agents to hold more income as reported, because it reduces the cost of monitoring.

6 Conclusion

In this paper, we solved a theoretical model of Black Money and Demonetisation. We found multiplicity of Stationary Monetary Equilibrium and we could explain some of the stylised facts observed in the data from the Indian Economy's demonetisation episode of Nov, 2016. This model helps us understand the tradeoffs faced by rich agents in terms of evading taxes and getting caught in an audit by the administrative authorities. It also allows us to understand the issue of Money Laundering which is prevalent in a lot of economies across the world. The model is extremely broad in its' possibilities of any economies' responses to such policies. Demonetisation in India is not a unique episode. Economies across the world have had policies similar to or exactly like demonetisation, eg. Nigeria (1984), Pakistan (2016), North Korea (2010), Ghana (1982) and so on. Some of the economies collapsed in response to such policies, others had a phased transition and some just overruled the policy and agents never followed it. Our model, allows for all these possibilities to emerge after such a policy.

Having understood the implications on the monetary equilibrium, one should not forget the real economic implications of such policies as well. A sudden demonetisation of 2 of the highest denominations notes left the Indian Economy with 86% of its' cash in circulation invalid. It was a massive economic shock for everyone in the economy. For an economy which has a very large share of cash intensive informal sector, the policy led to a huge disruption for the real activity for the producers and consumers in these sectors. There were adverse implications on the cash intensive agriculture sector, automobiles sector catering to the middle class (eg. Scooters, Small Cars), real estate black money transactions during the transition period. The agents were also affected by a lot of difficulties in having to go the banks in order to deposit their old notes and there were limits on the amount of new notes they could withdraw in cash while the rest was credited to their bank accounts. There were long queues of depositors at the exchange window and the disruptive transition period lasted 2 months. Hence, it would be interesting to look at the effects of demonetisation in real output and welfare for our economy.

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A Appendix

A.1 2nd subperiod value Functions for the Baseline Model

Let us start by solving the value functions for each type of agent in detail. We shall solve backward and start with the value functions for each type of agent at the start of the 2nd subperiod.

First, if an agent (of type i), receives the death shock at the start of the second subperiod of time t, she does not save any money for $t+1$ and she has the following optimisation problem,

$$\begin{aligned} &\max_{c,h} u(c_t) - \frac{h_t}{\epsilon_i} \\ \text{s.t.} \quad & c_t = h_t + \phi_t m_t + \phi_t S_t \end{aligned}$$

The solution to this optimisation problem is $c^*_{i,t} = \epsilon_i$, and hence her value conditional on death shock, becomes:

$$\bar{W}_t(i, m_t) = \log \epsilon_i - 1 + \frac{\phi_t m_t + \phi_t S_t}{\epsilon_i}$$

Next, we can solve for the value functions conditional on no death sock and then combine the two to get the final expected value for type i with money holdings m_t at the start of the second subperiod $W^i_t(m_t)$ as follows. Conditional on no death shock, the agents shall optimise as follows:

Seller (S)

$$\begin{aligned} &\max_{c,h,\tilde{m}'} \log c_t - h_t + \beta V^S_{t+1}(\tilde{m}') \\ \text{s.t.} \quad & c_t + \phi_t \tilde{m}' = h_t + \phi_t \tilde{m}_t + \phi_t S_t \end{aligned}$$

where we use $\tilde{m}'$ as a short hand for $\tilde{m}_{t+1}$. Substituting out for h_t, we get,

$$\begin{aligned} &\max_{c,m'} \log c_t - [c_t + \phi_t \tilde{m}' - \phi_t \tilde{m} - \phi_t S_t] + \beta V^S_{t+1}(\tilde{m}') \\ &= -1 + [\phi_t \tilde{m} + \phi_t S_t] + \max_{\tilde{m}'} \left(\beta V^S_{t+1}(\tilde{m}') - \phi_t \tilde{m}'\right) \end{aligned}$$

The quasi linearity of preferences in consumption and labor helps to separate the problem for optimal consumption ($c^*_t = \epsilon_S = 1$) and helps to get degenerate money holdings for each agent which does not depend on the money holdings from the previous period. This greatly simplifies this setup and helps to keep track of the money holdings distribution among agents.

Combining the cases with and without the death shock, we get that

$$\begin{aligned} W^S_t(\tilde{m}) &= \delta\left[-1 + \phi_t \tilde{m}_t + \phi_t S_t\right] + (1-\delta)\left[-1 + \phi_t \tilde{m} + \phi_t S_t + \max_{\tilde{m}'} \left(\beta V^S_{t+1}(\tilde{m}') - \phi_t \tilde{m}'\right)\right] \\ &= -1 + \phi_t \tilde{m} + \phi_t S_t + (1-\delta) \max_{\tilde{m}'} \left(\beta V^S_{t+1}(\tilde{m}') - \phi_t \tilde{m}'\right) \end{aligned}$$

Low Type Buyer (L)

Likewise, we can solve the problem for each of the buyers. The low type buyers simply consume and produce in the second sub-period and do not have to pay taxes, or participate in misreporting i.e. they have the following maximisation problem conditional on death shock.

$$\begin{aligned} &\max_{c,h,m'} \log c_t - \frac{h_t}{\epsilon_L} + \beta V^L_{t+1}(m') \\ \text{s.t.} \quad & c_t + \phi_t m' = h_t + \phi_t m_t + \phi_t S_t \end{aligned}$$

$$\begin{aligned} \implies &\max_{c,m'} \log c_t - \frac{1}{\epsilon_L}\left[c_t + \phi_t m' - \phi_t m_t - \phi_t S_t\right] + \beta V^L_{t+1}(m') \\ &= \log \epsilon_L - 1 + \frac{\phi_t m_t + \phi_t S_t}{\epsilon_L} + \max_{m'}\left[\beta V^L_{t+1}(m') - \frac{\phi_t m'}{\epsilon_L}\right] \end{aligned}$$

Combining the cases with and without death shock, we get that

$$W^L_t(m_t) = \log \epsilon_L - 1 + \frac{\phi_t m_t + \phi_t S_t}{\epsilon_L} + (1-\delta)\max_{m'}\left[\beta V^L_{t+1}(m') - \frac{\phi_t m'}{\epsilon_L}\right]$$

High type Caught Buyer (C)

The high type 'Caught' buyer has a similar problem as that of the low type with no misreporting incentive, except she has to pay taxes (either because she chose to be honest in reporting her type or she was caught in some previous time period and her true type is revealed forever). So, she solves the following,

$$\begin{aligned} &\max_{c,h,m'} \log c_t - \frac{h_t}{\epsilon_H} + \beta V^C_{t+1}(m') \\ \text{s.t.} \quad & c_t + \phi_t m' = h_t + \phi_t m_t - \mathbb{T} + \phi_t S_t \end{aligned}$$

$$\begin{aligned} \implies &\max_{c,m'} \log c_t - \frac{1}{\epsilon_H}\left[c_t + \phi_t m' - (\phi_t m_t - \mathbb{T}) - \phi_t S_t\right] + \beta V^C_{t+1}(m') \\ &= \log \epsilon_H - 1 + \frac{(\phi_t m_t - \mathbb{T}) + \phi_t S_t}{\epsilon_H} + \max_{m'}\left[\beta V^C_{t+1}(m') - \frac{\phi_t m'}{\epsilon_H}\right] \end{aligned}$$

Combining the cases with and without death shock, we get that

$$W^C_t(m_t) = \log \epsilon_H - 1 + \frac{(\phi_t m_t - \mathbb{T}) + \phi_t S_t}{\epsilon_H} + (1-\delta)\max_{m'}\left[\beta V^C_{t+1}(m') - \frac{\phi_t m'}{\epsilon_H}\right]$$

High type Not Caught Buyer (NC)

Finally, the high type 'Not Caught' buyer has an additional decision to make: whether to report her true type and pay taxes, or to continue to misreport and take the chances of getting caught with some probability (p) of government audit. Hence, her maximisation problem also involves reporting strategy, and using the same steps as above, simplifies to the following:

$$\begin{aligned} \implies W^{NC}_t(m_t) =& \log \epsilon_H - 1 + \frac{\phi m + \phi S_t}{\epsilon_H} + \mathbb{1}_{\hat{\epsilon}=\epsilon_H}\left[\frac{-\mathbb{T}}{\epsilon_H} + (1-\delta)\max_{m'}\left(\beta V^C_{t+1}(m') - \frac{\phi m'}{\epsilon_H}\right)\right] + \\ &\mathbb{1}_{\hat{\epsilon}=\epsilon_L}\left[p\left(\frac{-\mathbb{P}}{\epsilon_H} + (1-\delta)\max_{m'}\left[\beta V^C_{t+1}(m') - \frac{\phi m'}{\epsilon_H}\right]\right) + (1-p)\left((1-\delta)\max_{m'}\left[\beta V^{NC}_{t+1}(m') - \frac{\phi m'}{\epsilon_H}\right]\right)\right] \end{aligned}$$

A.2 Nash Bargaining Problem for the Baseline Model

We know that the buyer maximises as follows:

$$\begin{aligned} &\max_{q^B, d\leq m} u_t(q_t^B(m_t,\epsilon_i,\tilde{m}_t)) + W_t^i(m_t - d(m_t,\epsilon_i,\tilde{m}_t)) - W_t^i(m_t) \\ \text{subject to} \quad &- c_t(q_t^B(m_t,\epsilon_i,\tilde{m}_t)) + W_t^S(\tilde{m}_t + d(m_t,\epsilon_i,\tilde{m}_t)) - W_t^S(\tilde{m}_t) \geq 0 \end{aligned}$$

Using the quasi linearity result in 8, we can simplify this as follows,

$$\begin{aligned} &\max_{q^B, d\leq m} u_t(q_t^B(m_t,\epsilon_i,\tilde{m}_t)) - \frac{\phi_t d(m_t,\epsilon_i,\tilde{m}_t)}{\epsilon_i} \\ \text{subject to} \quad &- c_t(q_t^B(m_t,\epsilon_i,\tilde{m}_t)) + \phi_t d(m_t,\epsilon_i,\tilde{m}_t) \geq 0 \end{aligned}$$

Since the buyer has all the bargaining power and her utility is increasing in the quantity of specialized good consumed, the seller's participation constraint binds, i.e. the buyer only pays enough money to the seller to compensate her for the cost of producing the good i.e.

$$\begin{aligned} &\phi_t d(m_t,\epsilon_i,\tilde{m}_t) = c_t(q_t^B(m_t,\epsilon_i,\tilde{m}_t)) \\ \implies &\max_{q^B, d\leq m} u_t(q_t^B(m_t,\epsilon_i,\tilde{m}_t)) - \frac{c_t(q_t^B(m_t,\epsilon_i,\tilde{m}_t))}{\epsilon_i} \end{aligned}$$

Define $q_t^*(m,\epsilon_i) = \left\{ q | u_t'(q) = \frac{c_t'(q)}{\epsilon_i} \right\}$, then the optimal decision of the Nash Bargaining is,

$$q_t^B(m_t,\epsilon_i) = \begin{cases} q_t^*(m_t,\epsilon_i) & \text{if } c_t(q_t^*(m_t,\epsilon_i)) \leq \phi_t m_t \\ \{\hat{q}_t | c_t(\hat{q}_t(m_t,\epsilon_i)) = \phi_t m_t\} & \text{otherwise} \end{cases}$$

and for the payment,

$$d_t(m_t,\epsilon_i) = \begin{cases} \dfrac{c_t(q_t^*(m_t,\epsilon_i))}{\phi_t} & \text{if } c_t(q_t^*(m_t,\epsilon_i)) \leq \phi_t m_t \\ m_t & \text{otherwise} \end{cases}$$

Value functions at the start of the 1st subperiod

With the solution to the Nash Bargaining, we now know the flow payoff during the first subperiod and we can get the value functions $V_t^i(m_t)$ for each type i of the agent as follows:

Seller (S)

With probability $\alpha\pi_L$, the seller meets the low type buyer, produces the good as demanded, and gets the continuation payoff after receiving the payment, with $\alpha\pi_H$, she meets the high type buyer and with $1-\alpha$, she does not meet a counter-party to produce and exchange the specialized good.

$$\begin{aligned} V_t^S(\tilde{m}_t) = \alpha\pi_L &\left[-c_t(q_t^B(m_t,\epsilon_L)) + W_t^S(\tilde{m}_t + d(m_t,\epsilon_L)) \right] + \\ &\alpha\pi_H \left[-c_t(q_t^B(m_t,\epsilon_H)) + W_t^S(\tilde{m}_t + d(\tilde{m}_t,\epsilon_H)) \right] + (1-\alpha) W_t^S(\tilde{m}_t) \end{aligned}$$

Since, the seller gets no gain from trade in the DM, from the above bargaining solution where $\phi_t d_t = c_t(q_t^B)$ always, we get that her value at the start of the period, is just equal to her payoff at the start of the second subperiod,

$$V_t^S(m_t) = W_t^S(m_t) = \phi_t m_t + W_t^S(0)$$

In this sense, the seller in this model is more like a facilitating agent that supports the more interesting side of heterogenous buyers.

Low Type Buyers (L)

For the low type buyers, they only get flow payoff in the DM, if they meet a seller with probability α,

$$\begin{aligned}
V_t^L(m_t) &= \alpha \left[u_t(q_t^B(m_t, \epsilon_L)) + W_t^L(m_t - d(m_t, \epsilon_L))\right] + (1-\alpha)W_t^L(m_t) \\
&= \alpha \left[u_t(q_t^B(m_t, \epsilon_L)) - \frac{\phi_t d(m_t, \epsilon_L))}{\epsilon_L}\right] + \frac{\phi_t m_t}{\epsilon_L} + W_t^L(0) \\
&= \alpha \left[u_t(q_t^B(m_t, \epsilon_L)) - \frac{c_t(q_t^B(m_t, \epsilon_L))}{\epsilon_L}\right] + \frac{\phi_t m_t}{\epsilon_L} + W_t^L(0)
\end{aligned}$$

High Type Buyers $i \in \{C, NC\}$

Likewise for the high type buyer whether of not she has been caught in the past, we get the following value functions at the start of the first subperiod,

$$V_t^i(m_t) = = \alpha \left[u_t(q_t^B(m_t, \epsilon_H)) - \frac{c_t(q_t^B(m_t, \epsilon_H))}{\epsilon_H}\right] + \frac{\phi_t m_t}{\epsilon_H} + W_t^i(0)$$

A.3 Guess and Verify solution for threshold value of audit probability

We know that the value function at the start of the second subperiod, for the high type NC buyer, when she decides on her reporting strategy is as follows:

$$\begin{aligned}
W_t^{NC}(m_t) =& \log \epsilon_H - 1 + \frac{\phi_t m_t + \phi S_t}{\epsilon_H} + \mathbb{1}_{\hat{\epsilon}=\epsilon_H}\left[\frac{-\mathbb{T}}{\epsilon_H} + (1-\delta)\max_{m'}\left(\beta V_{t+1}^C(m') - \frac{\phi m'}{\epsilon_H}\right)\right] + \\
&\mathbb{1}_{\hat{\epsilon}=\epsilon_L}\left[p\left(\frac{-\mathbb{P}}{\epsilon_H} + (1-\delta)\max_{m'}\beta V_{t+1}^C(m') - \frac{\phi m'}{\epsilon_H}\right) + (1-p)\left((1-\delta)\max_{m'}\beta V_{t+1}^{NC}(m') - \frac{\phi m'}{\epsilon_H}\right)\right]
\end{aligned}$$

We guess that the value of p is such that the agent prefers to misreport than to turn honest i.e.

$$p\left(\frac{-\mathbb{P}}{\epsilon_H}+(1-\delta)\max_{m'}\beta V^C_{t+1}(m')-\frac{\phi m'}{\epsilon_H}\right)+(1-p)\left((1-\delta)\max_{m'}\beta V^{NC}_{t+1}(m')-\frac{\phi m'}{\epsilon_H}\right)$$
$$\geq\frac{-\mathbb{T}}{\epsilon_H}+(1-\delta)\max_{m'}\left(\beta V^C_{t+1}(m')-\frac{\phi m'}{\epsilon_H}\right)\quad(38)$$

Under the guess, we know that her value function is as follows,

$$W^{NC}_t(m_t)=\log\epsilon_H-1+\frac{\phi_t m_t+\phi S_t}{\epsilon_H}+\left[p\left(\frac{-\mathbb{P}}{\epsilon_H}+(1-\delta)\max_{m'}\beta V^C_{t+1}(m')-\frac{\phi m'}{\epsilon_H}\right)+\right.$$
$$\left.(1-p)\left((1-\delta)\max_{m'}\beta V^{NC}_{t+1}(m')-\frac{\phi m'}{\epsilon_H}\right)\right]$$

Next, we solve for the **optimal value of money holdings for each type of agent under the guess**. Let us define

$$\Gamma^i_t(m_t)\equiv u_t(q^B_t(m_t,\epsilon_i))-\frac{c_t(q^B_t(m_t,\epsilon_i))}{\epsilon_i}$$

Also, let us solve for the derivative of $\Gamma^i_t(m_t)$ w.r.t. m_t as follows:

$$\frac{\partial\Gamma^i_t(m_t)}{\partial m_t}=\left(\frac{\partial\Gamma^i_t(m_t)}{\partial q^B_t}\right)\frac{\partial q^B_t}{\partial m_t}=\left[u'_t(q^B_t)-\frac{c'_t(q^B_t)}{\epsilon_i}\right]\frac{\partial q^B_t}{\partial m_t}$$

From, 10, we get that

$$\frac{\partial\Gamma^i_t(m_t)}{\partial m_t}=\begin{cases}0 & \text{if } c_t(q^*_t(m_t,\epsilon_i))\leq\phi_t m_t\\ \left[u'_t(\hat{q})-\frac{c'_t(\hat{q})}{\epsilon_i}\right]\frac{\partial\hat{q}}{\partial m_t} & \text{otherwise}\end{cases}$$

using the definition of q^* s.t. $u'(q^*)=\frac{c'(q^*)}{\epsilon_i}$. Also, using the definition of $\hat{q}$, we get that $\frac{\partial\hat{q}}{\partial m_t}=\frac{\phi_t}{c'_t(\hat{q}_t(m_t,\epsilon_i))}$. Substituting this in the second case above, we get that

$$\frac{\partial\Gamma^i_t(m_t)}{\partial m_t}=\begin{cases}0 & \text{if } c_t(q^*_t(m_t,\epsilon_i))\leq\phi_t m_t\\ \left[\frac{u'_t(\hat{q}_t(m_t,\epsilon_i))}{c'_t(\hat{q}_t(m_t,\epsilon_i))}-\frac{1}{\epsilon_i}\right]\phi_t & \text{otherwise}\end{cases}$$

With this notation in hand, let us solve for the optimal money holding decision for each type of agent.

Seller (S)

First for the seller, she faces the following optimisation problem once we substitute for V^S_{t+1} into W^S_t,

$$W_t^S(\tilde{m}_t) = -1 + \phi_t \tilde{m} + \phi_t S_t + (1-\delta) \max_{\tilde{m}'} \left(\beta \left(\phi_{t+1} m' + W_{t+1}^S(0)\right) - \phi_t \tilde{m}'\right)$$

This leads to the following first order condition for money holdings,

$$\{m'^S\}: \qquad (1-\delta)(\beta\phi_{t+1} - \phi_t)$$

We shall look at the stationary equilibrium for this economy i.e. $\phi_t M_t = \phi_{t+1} M_{t+1}$ in equilibrium. This implies $\frac{\phi_t}{\phi_{t+1}} = \frac{M_{t+1}}{M_t} = \mu$. Moreover, we shall only consider the case when $\beta \leq \mu$, else all agents would want to hold infinite amount of money and that would not be supported by market clearing in equilibrium. Further, we shall look at a stationary equilibrium in which all agents hold a constant amount of real money holdings $\phi_t m_t^i = z^i$ over time. Considering all this, we get from the above FOC for seller, that

$$\beta - \mu < 0 \implies m'^S = 0 \implies \boxed{z^S = 0}$$

Low Type Buyer (L)

The low type buyer has the following optimisation problem once we substitute for the value function $V_{t+1}^L(m_{t+1})$ from next period into $W_t^L(m_t)$ and use the definition of $\Gamma_{t+1}^L(m_{t+1})$ as follows,

$$W_t^L(m_t) = \log \epsilon_L - 1 + \frac{\phi_t m_t + \phi_t S_t}{\epsilon_L} + (1-\delta) \max_{m'} \left[\beta \left(\alpha \Gamma_{t+1}^L(m') + \frac{\phi_{t+1} m'}{\epsilon_L} + W_{t+1}^L(0)\right) - \frac{\phi_t m'}{\epsilon_L}\right]$$

This implies the following FOC for money holdings. We rule out the case in which $\dfrac{\partial \Gamma_{t+1}^L(m')}{\partial m'} = 0$, i.e. $q_t^B = q^*$, because then we get that the agents choice of money holdings depend on $\dfrac{\beta - \mu}{\epsilon_L} < 0$, i.e. she holds 0 money but consumes optimal amount of specialized good next period (contradiction). This shows that the buyer is only able to consume less than her optimal level of specialized good, and she exhausts all her money holdings in doing so, i.e. each buyer shall only be able to demand $\hat{q}$ units of specialized good in the DM.

$$\begin{aligned}
\{m'^L\}: \qquad & (1-\delta)\left[\beta\left(\alpha \frac{\partial \Gamma_{t+1}^L(m')}{\partial m'} + \frac{\phi_{t+1}}{\epsilon_L}\right) - \frac{\phi_t}{\epsilon_L}\right] \leq 0 \\
& \implies (1-\delta)\left[\beta\left(\alpha\left[\frac{u'_{t+1}(\hat{q}_{t+1}(m',\epsilon_L))}{c'_{t+1}(\hat{q}_{t+1}(m',\epsilon_L))} - \frac{1}{\epsilon_L}\right]\phi_{t+1} + \frac{\phi_{t+1}}{\epsilon_L}\right) - \frac{\phi_t}{\epsilon_L}\right] \leq 0 \\
& \implies \beta\alpha\left[\left(\frac{u'}{c'}\right)_{t+1}(\hat{q}_{t+1}(m',\epsilon_L))\right]\phi_{t+1} + \frac{\beta(1-\alpha)\phi_{t+1}}{\epsilon_L} - \frac{\phi_t}{\epsilon_L} \leq 0
\end{aligned}$$

Assuming that we have an interior solution for money holdings, and using the definition of $\hat{q}$, we get the FOC, which leads to some degenerate value of money holdings $m'^L(\phi_{t+1}, \beta, \alpha, \epsilon_L, \mu)$

depending on the functional form of u, and c, such that $z^L = \phi_{t+1}m'$ is constant and solves the following FOC:

$$\{z^L\}: \qquad \left[\left\{ \frac{u'}{c'} \right\}_{t+1} (c_{t+1}^{-1}(\underbrace{\phi_{t+1}m'^L}_{z^L}) \right] = \frac{\mu - \beta(1-\alpha)}{\beta\alpha\epsilon_L}$$
$$\implies z^L = c_{t+1}\left(\left\{ \frac{u'}{c'} \right\}_{t+1}^{-1} \left[\frac{\mu - \beta(1-\alpha)}{\beta\alpha\epsilon_L} \right] \right)$$

If for example, we have $u_t(q_t^B) = \log q_t^B$, and $c_t(q_t^B) = cq_t^B$, we get that

$$\boxed{z^L = \frac{\beta\alpha\epsilon_L}{\mu - \beta(1-\alpha)}}$$

High Type Caught Buyer (C)

Using the exact same steps as for the low type, it is easy to show that the money holdings for the high type Caught buyer can be solved from the following FOC, which for the case of log utility and linear cost is as follows:

$$z^C = c_{t+1}\left(\left\{ \frac{u'}{c'} \right\}_{t+1}^{-1} \left[\frac{\mu - \beta(1-\alpha)}{\beta\alpha\epsilon_H} \right] \right) \implies \boxed{z^C = \frac{\beta\alpha\epsilon_H}{\mu - \beta(1-\alpha)}}$$

High Type Not Caught Buyer (NC)

For this type, we look at the value function at the start of the second subperiod, and substitute for the optimal value function after getting caught, and the value function for the NC type V_{t+1}^{NC} as follows,

$$W_t^{NC}(m_t) = \log\epsilon_H - 1 + \frac{\phi_t m_t - p\mathbb{P} + \phi S_t}{\epsilon_H} + (1-\delta)\left[p\left(\beta V_{t+1}^C(m'^C) - \frac{\phi_t m'^C}{\epsilon_H} \right) + \right.$$
$$\left. (1-p)\left(\max_{m'} \beta \left[\alpha\Gamma_{t+1}^{NC}(m') + \frac{\phi_{t+1}m'}{\epsilon_H} + W_{t+1}^{NC}(0) \right] - \frac{\phi_t m'}{\epsilon_H} \right) \right]$$

Getting the FOC for money holdings,

$$\{m'^{NC}\}: \qquad (1-\delta)(1-p)\left(\beta\left[\alpha\frac{\partial\Gamma_{t+1}^{NC}(m')}{\partial m'} + \frac{\phi_{t+1}}{\epsilon_H} \right] - \frac{\phi_t}{\epsilon_H} \right)$$
$$= (1-\delta)(1-p)\left(\beta\left[\alpha\frac{\partial\Gamma_{t+1}^{NC}(m')}{\partial m'} + \frac{\phi_{t+1}}{\epsilon_H} \right] - \frac{\phi_t}{\epsilon_H} \right)$$

Using the same steps as before, we get that the high type not caught holds the same amount of money holdings as the high type caught buyer but works less since she does not have to pay taxes.

$$z^{NC} = c_{t+1}\left(\left\{\frac{u'}{c'}\right\}_{t+1}^{-1}\left[\frac{\mu-\beta(1-\alpha)}{\beta\alpha\epsilon_H}\right]\right) \implies \boxed{z^{NC} = \frac{\beta\alpha\epsilon_H}{\mu-\beta(1-\alpha)}}$$

Having solved for the optimal values of real money holdings under the guess, we can go back to the condition in 38 and verify when the guess is satisfied in equilibrium. From 38, and our solution,

$$\begin{aligned}
p\left(\frac{-\mathbb{P}}{\epsilon_H}+(1-\delta)\left[\beta V^{C}_{t+1}(m'^{C})-\frac{\phi_t m'^{C}}{\epsilon_H}\right]\right)+(1-p)\left((1-\delta)\left[\beta V^{NC}_{t+1}(m'^{NC})-\frac{\phi_t m'^{NC}}{\epsilon_H}\right]\right)&\\
\geq \frac{-\mathbb{T}}{\epsilon_H}+(1-\delta)\left[\beta V^{C}_{t+1}(m'^{C})-\frac{\phi_t m'^{C}}{\epsilon_H}\right]&\\
\implies p\left[\frac{\mathbb{P}}{\epsilon_H}+(1-\delta)\left[\left(\beta V^{NC}_{t+1}(m'^{NC})-\frac{\phi_t m'^{NC}}{\epsilon_H}\right)-\left(\beta V^{C}_{t+1}(m'^{C})-\frac{\phi_t m'^{C}}{\epsilon_H}\right)\right]\right]&\\
\leq \frac{\mathbb{T}}{\epsilon_H}+(1-\delta)\left[\left(\beta V^{NC}_{t+1}(m'^{NC})-\frac{\phi_t m'^{NC}}{\epsilon_H}\right)-\left(\beta V^{C}_{t+1}(m'^{C})-\frac{\phi_t m'^{C}}{\epsilon_H}\right)\right]&
\end{aligned}$$

Since, $z^C = z^{NC}$, we can simplify this to,

$$p\left[\frac{\mathbb{P}}{\epsilon_H}+(1-\delta)\beta\left(V^{NC}_{t+1}(m'^{NC})-V^{C}_{t+1}(m'^{C})\right)\right] \leq \frac{\mathbb{T}}{\epsilon_H}+(1-\delta)\beta\left(V^{NC}_{t+1}(m'^{NC})-V^{C}_{t+1}(m'^{C})\right)$$

Next, note that $V^{NC}_{t+1}(m'^{NC})-V^{C}_{t+1}(m'^{C}) = W^{NC}_{t+1}(0)-W^{C}_{t+1}(0)$, since they have the same amount of optimal money holdings, and from (6) - (7), we can substitute that out to get the following within the framework of a stationary equilibrium,

$$\begin{aligned}
W^{NC}_{t+1}(0)-W^{C}_{t+1}(0) &= \frac{-p\mathbb{P}+\mathbb{T}}{\epsilon_H}+(1-\delta)(1-p)\beta(W^{NC}_{t+2}(0)-W^{C}_{t+2}(0))\\
\implies W^{NC}-W^{C} &= \frac{-p\mathbb{P}+\mathbb{T}}{[1-(1-\delta)(1-p)\beta]\epsilon_H}
\end{aligned}$$

Going back, to the inequality, we can substitute this out to get,

$$\begin{aligned}
p\left[\frac{\mathbb{P}}{\epsilon_H}+(1-\delta)\beta\left(\frac{-p\mathbb{P}+\mathbb{T}}{[1-(1-\delta)(1-p)\beta]\epsilon_H}\right)\right] &\leq \frac{\mathbb{T}}{\epsilon_H}+(1-\delta)\beta\left(\frac{-p\mathbb{P}+\mathbb{T}}{[1-(1-\delta)(1-p)\beta]\epsilon_H}\right)\\
\implies \frac{p\mathbb{P}-\mathbb{T}}{\epsilon_H} &\leq (1-p)(1-\delta)\beta\left(\frac{-p\mathbb{P}+\mathbb{T}}{[1-(1-\delta)(1-p)\beta]\epsilon_H}\right)\\
\implies 0 &\leq \frac{\mathbb{T}-p\mathbb{P}}{\epsilon_H}\left[1+\frac{(1-p)(1-\delta)\beta}{[1-(1-\delta)(1-p)\beta]}\right]\\
\implies 0 &\leq \frac{\mathbb{T}-p\mathbb{P}}{\epsilon_H}\left[\underbrace{\frac{1}{[1-(1-\delta)(1-p)\beta]}}_{+ve}\right]\\
\implies p &\leq \frac{\mathbb{T}}{\mathbb{P}} \equiv p^*
\end{aligned}$$

A.4 Value functions for the Model with Demonetisation

First let us simplify the generic second subperiod value function incorporating for the death shock. The agents again have the same optimisation problem upon death, i.e.

$$\begin{aligned}
&\max_{c,h} \log c_\tau - \frac{h_\tau}{\epsilon_i} \\
\text{s.t.} \quad & c_\tau = h_\tau + \phi_\tau^O m_\tau^O + \phi_\tau^N m_\tau^N + \phi_\tau^N S_\tau \\
&\implies \bar{W}_\tau^i(m^O, m^N) = \log \epsilon_i - 1 + \frac{\phi_\tau^O m_\tau^O + \phi_\tau^N m_\tau^N + \phi_\tau^N S_\tau}{\epsilon_i}
\end{aligned}$$

In case of no death shock, the agents also have same problem as before with the continuation values being U'S instead of V's now.

$$\begin{aligned}
&\max_{c,h,m'^O,m'^N} \log c_\tau - \frac{h_\tau}{\epsilon_i} + U_\tau^i(m'^O, m'^N) \\
\text{s.t.} \quad & c_\tau + \phi_\tau^O m_\tau'^O + \phi_\tau^N m_\tau'^N = h_\tau + \phi_\tau^O m_\tau^O + \phi_\tau^N m_\tau^N + \phi_\tau^N S_\tau \\
&= \log \epsilon_i - 1 + \frac{\phi_\tau^O m_\tau^O + \phi_\tau^N m_\tau^N + \phi_\tau^N S_\tau}{\epsilon_i} + \max_{m'^O,m'^N} U_\tau^i(m'^O, m'^N) - \frac{\phi_\tau^O m_\tau'^O + \phi_\tau^N m_\tau'^N}{\epsilon_i}
\end{aligned}$$

Combining the 2, we get $W_\tau^i(m^O, m^N) = \delta(\bar{W}_\tau^i(m^O, m^N)) + (1-\delta)(\text{above value})$, which we shall now specify in detail for each type of agents.

<u>**Seller (S)**</u>

At the start of the exchange period, agents come with certain holdings of new and old money. They choose how much of the old money ($0 \leq x \leq m'^O$), should they convert into new money, given they can gain for the higher value of new money but also get caught and penalised if they try to launder money.

$$U_\tau^S(m'^O, m'^N) = \max_{0 \leq x \leq m'^O} \beta \left[V_{\tau+1}^S(m'^O - x, m'^N + x) - \pi^S(x) f^S \right]$$

Using the above explanation,

$$\begin{aligned}
W_\tau^S(m^O, m^N) &= -1 + \left(\phi_\tau^O m^O + \phi_\tau^N m^N + \phi_\tau^N S_\tau\right) + (1-\delta) \max_{m'^O,m'^N} U_\tau^S(m'^O, m'^N) - \left(\phi_\tau^O m'^O + \phi_\tau^N m'^N\right) \\
&= \left(\phi_\tau^O m^O + \phi_\tau^N m^N\right) + W_\tau^S(0,0)
\end{aligned}$$

Note that the quasi linearity of W still holds and we use it in exactly the same way in solving for the terms of trade in DM. Also, since the bargaining problem is essentially the same as before with $\phi_t m_t$ replaced by $\phi_t^O m_t^O + \phi_t^N m_t^N$, and the seller got no flow payoff from Nash Bargaining, we get

$$V_\tau^S(m^O, m^N) = W_\tau^S(m^O, m^N)$$

Solving for the optimal amount of money holdings for the seller, using the above value functions, we get

$$U^S_\tau(m'^O, m'^N) = \max_{0\le x\le m'^O} \beta\left[W^S_{\tau+1}(m'^O - x, m'^N + x) - \pi^S(x) f^S\right]$$

FOC for $\{x\}$ is decreasing in x using the assumption that $\pi' > 0$, we get the following threshold strategy,

$$\begin{aligned} \text{Let } x_S &: \beta\left[(\phi^N_{\tau+1} - \phi^O_{\tau+1}) - \pi'^S(x) f^S\right] = 0 \\ &\implies \boxed{x^*(m'^O) = \max\{0, \min\{m'^O, x_S\}\}} \end{aligned}$$

i.e. the Seller shall exchange all her money holdings upto x_S. Next, we substitute this back into the period before i.e. W, and solve for the optimal amount of money holdings of Old and New Money as follows:

$$\begin{aligned} W^S_\tau(m^O, m^N) = -1 + \left(\phi^O_\tau m^O + \phi^N_\tau m^N + \phi^N_\tau S_\tau\right) + (1-\delta) \max_{m'^O, m'^N} \\ \left[\beta\left[W^S_{\tau+1}(m'^O - x^*(m'^O), m'^N + x^*(m'^O)) - \pi^S(x^*(m'^O)) f^S\right]\right] - \left(\phi^O_\tau m'^O + \phi^N_\tau m'^N\right) \end{aligned}$$

FOC for $\{m'^O\}, \{m'^N\}$:

$$\begin{aligned} \{m'^O\}: \quad & \beta\left[\phi^O_{\tau+1}\left(1 - \frac{\partial x^*(m'^O)}{\partial m'^O}\right) + \phi^N_{\tau+1}\left(\frac{\partial x^*(m'^O)}{\partial m'^O}\right) - \pi'^S(x^*(m'^O))\left(\frac{\partial x^*(m'^O)}{\partial m'^O}\right) f^S\right] - \phi^O_\tau \\ &= \begin{cases} \beta\left[\phi^N_{\tau+1} - \pi'^S(m'^O) f^S\right] - \phi^O_\tau & \text{when } m'^O \le x_S \\ \beta\phi^O_{\tau+1} - \phi^O_\tau & \text{when } x_S \le m'^O \end{cases} \\ \{m'^N\}: \quad & \beta\phi^N_{\tau+1} - \phi^N_\tau \end{aligned}$$

Assume that $\beta \le \mu^N_\tau, \beta \le \mu^O_\tau \implies$

$$\begin{aligned} & m'^O_S \in [0, x_S] : \beta\left[\phi^N_{\tau+1} - \pi'^S(m'^O_S) f^S\right] - \phi^O_\tau = 0 \\ & m'^N_S = 0 \end{aligned}$$

This gives us the optimal money holdings decision for the sellers.

<u>Low type buyer (L)</u>

We follow the same steps as for the Seller, to get the value functions for the low type buyer. First, she chooses the exchange amount at time τ, she reports $\hat{i} = L$, i.e. she is monitored with probability π^L, and if she deposits more money than what she holds, i.e. in order to launder, the punishment kicks in and stops her. Further, her value functions at the start of the first and second subperiod are also derived as follows:

$$
\begin{aligned}
U^L_\tau(m'^O, m'^N) &= \max_{0\le x\le m'^O} \beta\left[V^L_{\tau+1}(m'^O - x, m'^N + x) - \pi^L(x) f^L\right] \\
W^L_\tau(m^O, m^N) &= \log\epsilon_L - 1 + \left(\frac{\phi^O_\tau m^O + \phi^N_\tau m^N + \phi^N_\tau S_\tau}{\epsilon_L}\right) + \\
&\qquad (1-\delta)\max_{m'^O, m'^N}\left\{U^L_\tau(m'^O, m'^N) - \left(\frac{\phi^O_\tau m'^O + \phi^N_\tau m'^N}{\epsilon_L}\right)\right\} \\
&= \left(\frac{\phi^O_\tau m^O + \phi^N_\tau m^N}{\epsilon_L}\right) + W^L_\tau(0,0) \\
V^L_{\tau+1}(m'^O, m'^N) &= \alpha\left[\Gamma^L_{\tau+1}(m'^O, m'^N)\right] + W^L_{\tau+1}(m'^O, m'^N)
\end{aligned}
$$

where $\Gamma^L_{\tau+1}(m'^O, m'^N) = \left[u_{\tau+1}(q^b_{\tau+1}(m'^O, m'^N, \epsilon_L)) - \dfrac{c_{\tau+1}(q^b_{\tau+1}(m'^O, m'^N, \epsilon_L))}{\epsilon_L}\right]$ is the modified version of Γ with a portfolio of old and new notes.

We start with solving for the optimal amount of exchange for a given value of money holdings of old notes and new notes by substituting out for the $V_{\tau+1}$

$$
\implies U^L_\tau(m'^O, m'^N) = \max_{0\le x\le m'^O} \beta\left[\alpha\Gamma^L_{\tau+1}(m'^O - x, m'^N + x) + W^L_{\tau+1}(m'^O - x, m'^N + x) - \pi^L(x) f^L\right]
$$

$$
\text{FOC for } \{x\}: \qquad \beta\left[\alpha\frac{\partial\Gamma^L_{\tau+1}(m'^O - x, m'^N + x)}{\partial x} + \left(\frac{\phi^N_{\tau+1} - \phi^O_{\tau+1}}{\epsilon_L}\right) - \pi'^L(x) f^L\right]
$$

We know that, under our utility and cost functions, where in we assumes $u(q) = \log q, c(q) = cq$,

$$
\frac{\partial\Gamma^L_{\tau+1}(m'^O - x, m'^N + x)}{\partial m'^O} = \left[\frac{1}{\boldsymbol{\phi}.\mathbf{m}} - \frac{1}{\epsilon_L}\right]\phi^O_{\tau+1}, \quad \frac{\partial\Gamma^L_{\tau+1}(m'^O - x, m'^N + x)}{\partial m'^N} = \left[\frac{1}{\boldsymbol{\phi}.\mathbf{m}} - \frac{1}{\epsilon_L}\right]\phi^N_{\tau+1}
$$

with $\boldsymbol{\phi}.\mathbf{m} = \phi^O_{\tau+1}(m'^O - x) + \phi^N_{\tau+1}(m'^N + x)$, $\implies$ the following FOC decreasing in x. Consequently, we shall get a threshold strategy also for the poor buyer. Let $x_L(m'^O, m'^N)$ be such that,

$$
\beta\left[\alpha\left(\frac{(\phi^N_{\tau+1} - \phi^O_{\tau+1})}{\phi^O_{\tau+1} m'^O + \phi^N_{\tau+1} m'^N + (\phi^N_{\tau+1} - \phi^O_{\tau+1}) x_L}\right) + (1-\alpha)\left(\frac{\phi^N_{\tau+1} - \phi^O_{\tau+1}}{\epsilon_L}\right) - \pi'^L(x_L) f^L\right] = 0
$$

$$
\implies x^*(m'^O, m'^N) = \max\{0, \min\{m'^O, x_L(m'^O, m'^N)\}\}
$$

Again, once we solve for the optimal value of exchange as a function of money holdings, we can move back to the value function W at the start of the second subperiod as follows,

$$
\begin{aligned}
\implies W^L_\tau(m^O, m^N) = \log\epsilon_L - 1 + \left(\frac{\phi^O_\tau m^O + \phi^N_\tau m^N + \phi^N_\tau S_\tau}{\epsilon_L}\right) + (1-\delta)\max_{m'^O, m'^N}\Big\{\beta\big[\alpha\Gamma^L_{\tau+1}(m'^O - x^*_L, m'^N \\
+ W^L_{\tau+1}(m'^O - x^*_L, m'^N + x^*_L) - \pi^L(x^*) f^L\big] - \left(\frac{\phi^O_\tau m'^O + \phi^N_\tau m'^N}{\epsilon_L}\right)\Big\}
\end{aligned}
$$

Getting our first order conditions for money holdings,

$$\{m'^O\}: \qquad \beta\left[\alpha\left(\frac{\partial\Gamma^L_{\tau+1}(\cdot)}{\partial m'^O}\left\{1-\frac{\partial x^*(\cdot)}{\partial m'^O}\right\}+\frac{\partial\Gamma^L_{\tau+1}(\cdot)}{\partial m'^N}\left\{\frac{\partial x^*(\cdot)}{\partial m'^O}\right\}\right)+\frac{\phi^O_{\tau+1}}{\epsilon_L}\left\{1-\frac{\partial x^*(\cdot)}{\partial m'^O}\right\}+\frac{\phi^N_{\tau+1}}{\epsilon_L}\left\{\frac{\partial x^*(\cdot)}{\partial m'^O}\right\}\right.$$
$$\left.-\pi'^L(x^*)\left\{\frac{\partial x^*(\cdot)}{\partial m'^O}\right\}f^L\right]-\frac{\phi^O_\tau}{\epsilon_L}$$

$$\{m'^N\}: \qquad \beta\left[\alpha\left(\frac{\partial\Gamma^L_{\tau+1}(\cdot)}{\partial m'^O}\left\{-\frac{\partial x^*(\cdot)}{\partial m'^N}\right\}+\frac{\partial\Gamma^L_{\tau+1}(\cdot)}{\partial m'^N}\left\{1+\frac{\partial x^*(\cdot)}{\partial m'^N}\right\}\right)+\frac{\phi^O_{\tau+1}}{\epsilon_L}\left\{-\frac{\partial x^*(\cdot)}{\partial m'^N}\right\}+\frac{\phi^N_{\tau+1}}{\epsilon_L}\left\{1+\frac{\partial x^*(\cdot)}{\partial m'^N}\right\}\right.$$
$$\left.-\pi'^L(x^*)\left\{\frac{\partial x^*(\cdot)}{\partial m'^N}\right\}f^L\right]-\frac{\phi^N_\tau}{\epsilon_L}$$

Simplifying and substituting for the partial derivatives under our example with log utility and constant marginal cost of production, we get the following,

$$\{m'^O\}: \qquad \begin{cases}\beta\left[\frac{\alpha\phi^O_{\tau+1}}{\phi^O_{\tau+1}m'^O+\phi^N_{\tau+1}m'^N+(\phi^N_{\tau+1}-\phi^O_{\tau+1})x_L}+\frac{(1-\alpha)\phi^O_{\tau+1}}{\epsilon_L}\right]-\frac{\phi^O_\tau}{\epsilon_L} & \text{if } m'^O > x_L \geq 0\\ \beta\left[\frac{\alpha}{(m'^N+m'^O)}+\frac{(1-\alpha)\phi^N_{\tau+1}}{\epsilon_L}-\pi^L f^L\right]-\frac{\phi^O_\tau}{\epsilon_L} & \text{if } m'^O \leq x_L\end{cases}$$

$$\{m'^N\}: \qquad \begin{cases}\beta\left[\frac{\alpha}{(m'^N+m'^O)}+\frac{(1-\alpha)\phi^N_{\tau+1}}{\epsilon_L}\right]-\frac{\phi^N_\tau}{\epsilon_L} & \text{if } x_L \geq m'^O\\ \beta\left[\frac{\alpha\phi^N_{\tau+1}}{\phi^O_{\tau+1}m'^O+\phi^N_{\tau+1}m'^N+(\phi^N_{\tau+1}-\phi^O_{\tau+1})x_L}+\frac{(1-\alpha)\phi^N_{\tau+1}}{\epsilon_L}\right]-\frac{\phi^N_\tau}{\epsilon_L} & \text{if } x_L \leq m'^O\end{cases}$$

High type Caught Buyer (C)

The value functions for the high type buyer whose type is already revealed is as follows. Her reposted type is $\hat{i}=H$, and the corresponding fine f^C.

$$U^C_\tau(m'^O,m'^N) = \max_{0\leq x\leq m'^O}\beta\left[V^C_{\tau+1}(m'^O-x,m'^N+x)-\pi^H(x)f^C\right]$$

$$W^C_\tau(m^O,m^N) = \log\epsilon_H - 1 + \left(\frac{\phi^O_\tau m^O+\phi^N_\tau m^N-\mathbb{T}+\phi^N_\tau S_\tau}{\epsilon_H}\right)+$$
$$(1-\delta)\max_{m'^O,m'^N}\left\{U^C_\tau(m'^O,m'^N)-\left(\frac{\phi^O_\tau m'^O+\phi^N_\tau m'^N}{\epsilon_H}\right)\right\}$$
$$=\left(\frac{\phi^O_\tau m^O+\phi^N_\tau m^N}{\epsilon_H}\right)+W^C_\tau(0,0)$$

$$V^C_{\tau+1}(m'^O,m'^N) = \alpha\Gamma^H_{\tau+1}(m'^O,m'^N)+W^C_{\tau+1}(m'^O,m'^N)$$

where $\Gamma^H_{\tau+1}(m'^O,m'^N) = \left[u_{\tau+1}(q^B_{\tau+1}(m'^O,m'^N,\epsilon_H))-\frac{c_{\tau+1}(q^B_{\tau+1}(m'^O,m'^N,\epsilon_H))}{\epsilon_H}\right]$

Substituting for $V_{\tau+1}$ in U_τ, we get

$$\implies U^C_\tau(m'^O,m'^N) = \max_{0\leq x\leq m'^O}\beta\left[\alpha\Gamma^H_{\tau+1}(m'^O-x,m'^N+x)+W^C_{\tau+1}(m'^O-x,m'^N+x)-\pi^H(x)f^C\right]$$

This leads to the following First order condition for exchange amount,

$$\{x\}: \qquad \beta\left[\alpha\frac{\partial\Gamma^H_{\tau+1}(m'^O-x,m'^N+x)}{\partial x}+\left(\frac{\phi^N_{\tau+1}-\phi^O_{\tau+1}}{\epsilon_H}\right)-\pi'^H(x)f^C\right]$$

For out choice of utility and cost functions,

$$\frac{\partial \Gamma^H_{\tau+1}(m'^O - x, m'^N + x)}{\partial m'^O} = \left[\frac{1}{\boldsymbol{\phi}.\mathbf{m}} - \frac{1}{\epsilon_H}\right]\phi^O_{\tau+1}, \quad \frac{\partial \Gamma^L_{\tau+1}(m'^O - x, m'^N + x)}{\partial m'^N} = \left[\frac{1}{\boldsymbol{\phi}.\mathbf{m}} - \frac{1}{\epsilon_H}\right]\phi^N_{\tau+1}$$

with $\boldsymbol{\phi}.\mathbf{m} = \phi^O_{\tau+1}(m'^O - x) + \phi^N_{\tau+1}(m'^N + x)$. $\implies$ the following FOC decreasing in x. Consequently, we shall get a threshold strategy also for the high type caught buyer. Let $x_C(m'^O, m'^N)$ be such that,

$$\beta\left[\alpha\left(\frac{(\phi^N_{\tau+1} - \phi^O_{\tau+1})}{\phi^O_{\tau+1}m'^O + \phi^N_{\tau+1}m'^N + (\phi^N_{\tau+1} - \phi^O_{\tau+1})x_C}\right) + (1-\alpha)\left(\frac{\phi^N_{\tau+1} - \phi^O_{\tau+1}}{\epsilon_H}\right) - \pi'^H(x_C)f^C\right] = 0$$

$$\implies x^*_C(m'^O, m'^N) = \max\{0, \min\{m'^O, x_C(m'^O, m'^N)\}\}$$

Substituting this back into the value function at the start of the second subperiod,

$$W^C_\tau(m^O, m^N) = \log\epsilon_H - 1 + \left(\frac{\phi^O_\tau m^O + \phi^N_\tau m^N - \mathbb{T} + \phi^N_\tau S_\tau}{\epsilon_H}\right) + (1-\delta)\max_{m'^O, m'^N}$$
$$\left\{\beta\left[\alpha\Gamma^H_{\tau+1}(m'^O - x^*, m'^N + x^*) + W^C_{\tau+1}(m'^O - x^*, m'^N + x^*) - \pi^H(x^*)f^C\right] - \left(\frac{\phi^O_\tau m'^O + \phi^N_\tau m'^N}{\epsilon_H}\right)\right\}$$

This leads to the following first order conditions for money holdings,

$$\{m'^O\}: \quad \beta\left[\alpha\left(\frac{\partial\Gamma^H_{\tau+1}(\cdot)}{\partial m'^O}\left\{1 - \frac{\partial x^*(\cdot)}{\partial m'^O}\right\} + \frac{\partial\Gamma^H_{\tau+1}(\cdot)}{\partial m'^N}\left\{\frac{\partial x^*(\cdot)}{\partial m'^O}\right\}\right) + \frac{\phi^O_{\tau+1}}{\epsilon_H}\left\{1 - \frac{\partial x^*(\cdot)}{\partial m'^O}\right\} + \frac{\phi^N_{\tau+1}}{\epsilon_H}\left\{\frac{\partial x^*(\cdot)}{\partial m'^O}\right\}\right.$$
$$\left. -\pi'^H(x^*)\left\{\frac{\partial x^*(\cdot)}{\partial m'^O}\right\}f^C\right] - \frac{\phi^O_\tau}{\epsilon_H}$$

$$\{m'^N\}: \quad \beta\left[\alpha\left(\frac{\partial\Gamma^H_{\tau+1}(\cdot)}{\partial m'^O}\left\{-\frac{\partial x^*(\cdot)}{\partial m'^N}\right\} + \frac{\partial\Gamma^H_{\tau+1}(\cdot)}{\partial m'^N}\left\{1 + \frac{\partial x^*(\cdot)}{\partial m'^N}\right\}\right) + \frac{\phi^O_{\tau+1}}{\epsilon_H}\left\{-\frac{\partial x^*(\cdot)}{\partial m'^N}\right\} + \frac{\phi^N_{\tau+1}}{\epsilon_H}\left\{1 + \frac{\partial x^*(\cdot)}{\partial m'^N}\right\}\right.$$
$$\left. -\pi'^H(x^*)\left\{\frac{\partial x^*(\cdot)}{\partial m'^N}\right\}f^C\right] - \frac{\phi^N_\tau}{\epsilon_H}$$

which can be simplified as follows,

$$\{m'^O\}: \quad = \begin{cases} \beta\left[\frac{\alpha}{(m'^N + m'^O)} + \frac{(1-\alpha)\phi^N_{\tau+1}}{\epsilon_H} - \pi'^H(m'^O)f^C\right] - \frac{\phi^O_\tau}{\epsilon_H} & \text{if } 0 \le m'^O \le x_C \\ \beta\left[\frac{\alpha\phi^O_{\tau+1}}{\phi^O_{\tau+1}m'^O + \phi^N_{\tau+1}m'^N + (\phi^N_{\tau+1} - \phi^O_{\tau+1})x_C} + \frac{(1-\alpha)\phi^O_{\tau+1}}{\epsilon_H}\right] - \frac{\phi^O_\tau}{\epsilon_H} & \text{if } x_C < m'^O \end{cases}$$

$$\{m'^N\}: \quad = \begin{cases} \beta\left[\frac{\alpha}{(m'^N + m'^O)} + \frac{(1-\alpha)\phi^N_{\tau+1}}{\epsilon_H}\right] - \frac{\phi^N_\tau}{\epsilon_H} & \text{if } 0 \le m'^O \le x_C \\ \beta\left[\frac{\alpha\phi^N_{\tau+1}}{\phi^O_{\tau+1}m'^O + \phi^N_{\tau+1}m'^N + (\phi^N_{\tau+1} - \phi^O_{\tau+1})x_C} + \frac{(1-\alpha)\phi^N_{\tau+1}}{\epsilon_H}\right] - \frac{\phi^N_\tau}{\epsilon_H} & \text{if } x_C < m'^O \end{cases}$$

High type Not Caught Buyer (NC)

The high type not caught buyer is the most interesting one, since she faces the tradeoff of getting her type revealed in case she is caught exchanging more money than her reported type could have. The in the event that she gets audited for exchange, her penalty is much higher, and her value function is as follows.

$$
\begin{aligned}
U_\tau^{NC}(m'^O, m'^N) &= \max_{0\le x\le m'^O} \pi^L(x)\left[\beta(V_{\tau+1}^C(m'^O - x, m'^N + x) - f^{NC})\right] + \\
&\qquad\qquad\qquad\qquad (1-\pi^L(x))\left[\beta V_{\tau+1}^{NC}(m'^O - x, m'^N + x)\right] \\
W_\tau^{NC}(m^O, m^N) &= \log\epsilon_H - 1 + \left(\frac{\phi_\tau^O m^O + \phi_\tau^N m^N + \phi_\tau^N S_\tau}{\epsilon_H}\right) + \max_{\hat\epsilon} \\
&\Bigg[\mathbb{1}_{\hat\epsilon=\epsilon_H}\left\{\frac{-\mathbb{T}}{\epsilon_H} + (1-\delta)\max_{m'^O, m'^N}\left(U_\tau^C(m'^O, m'^N) - \frac{\phi_\tau^O m'^O + \phi_\tau^N m'^N}{\epsilon_H}\right)\right\} + \\
&\mathbb{1}_{\hat\epsilon=\epsilon_L}\Bigg\{p\left(\frac{-\mathbb{P}}{\epsilon_H} + (1-\delta)\max_{m'^O, m'^N}\left(U_\tau^C(m'^O, m'^N) - \frac{\phi_\tau^O m'^O + \phi_\tau^N m'^N}{\epsilon_H}\right)\right) + \\
&\qquad (1-p)\left((1-\delta)\max_{m'^O, m'^N}\left(U_\tau^{NC}(m'^O, m'^N) - \frac{\phi_\tau^O m'^O + \phi_\tau^N m'^N}{\epsilon_H}\right)\right)\Bigg\}\Bigg] \\
V_{\tau+1}^{NC}(m'^O, m'^N) &= \alpha\Gamma_{\tau+1}^H(m'^O, m'^N) + W_{\tau+1}^{NC}(m'^O, m'^N)
\end{aligned}
$$

where $\Gamma_{\tau+1}^H(m'^O, m'^N) = \left[u_{\tau+1}(q_{\tau+1}^b(m'^O, m'^N, \epsilon_H)) - \frac{c_{\tau+1}(q_{\tau+1}^b(m'^O, m'^N, \epsilon_H))}{\epsilon_H}\right]$

Substituting, for tomorrow's value function $V_{\tau+1}$ into U, and assuming that the agent prefers to still misreport her type when she is not caught, we get the following,

$$
\begin{aligned}
\implies U_\tau^{NC}(m'^O, m'^N) &= \max_{0\le x\le m'^O} \pi^L(x)\beta\left[\alpha\Gamma_{\tau+1}^H(m'^O - x, m'^N + x) + W_{\tau+1}^C(m'^O - x, m'^N + x) - f^{NC}\right] \\
&\qquad + (1-\pi^L(x))\beta\left[\alpha\Gamma_{\tau+1}^H(m'^O - x, m'^N + x) + W_{\tau+1}^{NC}(m'^O - x, m'^N + x)\right]
\end{aligned}
$$

This leads to the following first order condition for $\{x\}$:

$$
\begin{aligned}
&\beta\left[\alpha\frac{\partial\Gamma_{\tau+1}^H(m'^O - x, m'^N + x)}{\partial x} + \frac{\phi_{\tau+1}^N - \phi_{\tau+1}^O}{\epsilon_H}\right] + \\
&\qquad\qquad \pi'^L(x)\beta\left[W_{\tau+1}^C(m'^O - x, m'^N + x) - f^{NC} - W_{\tau+1}^{NC}(m'^O - x, m'^N + x)\right]
\end{aligned}
$$

I solve the baseline model for the case with both old and new money and it is easy to show that the high type caught and not caught agents hold the same amount of money holdings going forward from $\tau + 1$ onwards, so we can easily subtract the 2 values as follows:

$$
\begin{aligned}
W_{\tau+1}^C(m'^O, m'^N) - W_{\tau+1}^{NC}(m'^O, m'^N) &= \frac{-\mathbb{T}}{\epsilon_H} - p\frac{-\mathbb{P}}{\epsilon_H} + (1-p)(1-\delta)\beta[W_{\tau+2}^C - W_{\tau+2}^{NC}] \\
&= \frac{1}{1-(1-p)(1-\delta)\beta}\frac{p\mathbb{P} - \mathbb{T}}{\epsilon_H}
\end{aligned}
$$

Substituting this back in the above FOC, we get:

$$\{x\}: \qquad \beta\left[\alpha\left(\frac{(\phi^N_{\tau+1}-\phi^O_{\tau+1})}{\phi^O_{\tau+1}m'^O+\phi^N_{\tau+1}m'^N+(\phi^N_{\tau+1}-\phi^O_{\tau+1})x}\right)+\frac{(1-\alpha)(\phi^N_{\tau+1}-\phi^O_{\tau+1})}{\epsilon_H}\right] \\ -\pi'^L(x)\beta\left[\frac{1}{1-(1-p)\beta}\frac{\mathbb{T}-p\mathbb{P}}{\epsilon_H}+f^{NC}\right]$$

Again, this is decreasing in x, which implies a unique threshold value $x_{NC}(m'^O, m'^N)$ such that

$$\beta\left[\alpha\left(\frac{(\phi^N_{\tau+1}-\phi^O_{\tau+1})}{\phi^O_{\tau+1}m'^O+\phi^N_{\tau+1}m'^N+(\phi^N_{\tau+1}-\phi^O_{\tau+1})x_{NC}}\right)+\frac{(1-\alpha)(\phi^N_{\tau+1}-\phi^O_{\tau+1})}{\epsilon_H}\right] \\ -\pi'^L(x_{NC})\beta\left[\frac{1}{1-(1-p)\beta}\frac{\mathbb{T}-p\mathbb{P}}{\epsilon_H}+f^{NC}\right]=0$$

$$\implies x^* = \max\{0, \min\{m'^O, x_{NC}(m'^O, m'^N)\}\}$$

Substituting this back to get the optimal value at the start of the exchange window which will also be the continuation value to the second subperiod value function.

$$\implies U^{NC}_\tau(m'^O, m'^N) = \pi^L(x^*)\beta\left[\alpha\Gamma^H_{\tau+1}(m'^O-x^*, m'^N+x^*)+W^C_{\tau+1}(m'^O-x^*, m'^N+x^*)-f^{NC}\right] \\ +(1-\pi^L(x^*))\beta\left[\alpha\Gamma^H_{\tau+1}(m'^O-x^*, m'^N+x^*)+W^{NC}_{\tau+1}(m'^O-x^*, m'^N+x^*)\right]$$

Next, we want the threshold value of p_τ at time τ to be also such that the agent does not want to reveal her type in the Report and Verify window, i.e. we solve for the threshold value of p^* also for time τ,

$$\left\{\frac{-\mathbb{T}}{\epsilon_H}+\max_{m'^O,m'^N}\left(U^C_\tau(m'^O,m'^N)-\frac{\phi^O_\tau m'^O+\phi^N_\tau m'^N}{\epsilon_H}\right)\right\}\leq \\ \left\{p\left(\frac{-\mathbb{P}}{\epsilon_H}+\max_{m'^O,m'^N}\left(U^C_\tau(m'^O,m'^N)-\frac{\phi^O_\tau m'^O+\phi^N_\tau m'^N}{\epsilon_H}\right)\right)+\right. \\ \left.(1-p)\left(\max_{m'^O,m'^N}\left(U^{NC}_\tau(m'^O,m'^N)-\frac{\phi^O_\tau m'^O+\phi^N_\tau m'^N}{\epsilon_H}\right)\right)\right\}$$

$$\implies \frac{p\mathbb{P}-\mathbb{T}}{\epsilon_H}\leq(1-p)\left\{\max_{m'^O,m'^N}\left(U^{NC}_\tau(m'^O,m'^N)-\frac{\phi^O_\tau m'^O+\phi^N_\tau m'^N}{\epsilon_H}\right)-\right. \\ \left.\max_{m'^O,m'^N}\left(U^C_\tau(m'^O,m'^N)-\frac{\phi^O_\tau m'^O+\phi^N_\tau m'^N}{\epsilon_H}\right)\right\}$$

Now, we know the solution to get U^C_τ, but we should solve for U^{NC}_τ under such guess for probability of monitoring. This implies, the agent chooses to never reveal her type.

Under the guess, we get the following value function at the start of the second subperiod,

$$
\begin{aligned}
W_\tau^{NC}(m^O, m^N) &= \log \epsilon_H - 1 + \left(\frac{\phi_\tau^O m^O + \phi_\tau^N m^N + \phi_\tau^N S_\tau}{\epsilon_H} \right) + \\
&\qquad \left\{ p \left(\frac{-\mathbb{P}}{\epsilon_H} + (1-\delta) \max_{m'^O, m'^N} \left(U_\tau^C(m'^O, m'^N) - \frac{\phi_\tau^O m'^O + \phi_\tau^N m'^N}{\epsilon_H} \right) \right) + \right. \\
&\qquad \left. (1-p) \left((1-\delta) \max_{m'^O, m'^N} \left(U_\tau^{NC}(m'^O, m'^N) - \frac{\phi_\tau^O m'^O + \phi_\tau^N m'^N}{\epsilon_H} \right) \right) \right\} \\
&= \log \epsilon_H - 1 + \left(\frac{\phi_\tau^O m^O + \phi_\tau^N m^N + \phi_\tau^N S_\tau}{\epsilon_H} \right) + \\
&\qquad \left\{ p \left(\frac{-\mathbb{P}}{\epsilon_H} + (1-\delta) \left(\beta [V_{\tau+1}^C(m_C'^O - x_C^*, m_C'^N + x_C^*) - \pi^H(x_C^*) f^C] - \frac{\phi_\tau^O m_C'^O + \phi_\tau^N m_C'^N}{\epsilon_H} \right) \right) \right. \\
&\qquad + (1-p)(1-\delta) \max_{m'^O, m'^N} \left(\left(\pi^L(x_{NC}^*) \left[\beta (V_{\tau+1}^C(m'^O - x_{NC}^*, m'^N + x_{NC}^*) - f^{NC}) \right] + \right. \right. \\
&\qquad \left. \left. \left. (1 - \pi^L(x_{NC}^*)) \left[\beta V_{\tau+1}^{NC}(m'^O - x_{NC}^*, m'^N + x_{NC}^*) \right] - \frac{\phi_\tau^O m'^O + \phi_\tau^N m'^N}{\epsilon_H} \right) \right) \right\}
\end{aligned}
$$

So, we can get the FOC for the money holdings of the high type not caught buyer in the last period when exchange is possible, by taking the first order conditions for m'^O and m'^N in the above value function.

$$
\begin{aligned}
\{m'^O\}: \qquad & \left(\pi'^L(x_{NC}^*) \frac{\partial x^*(\cdot)}{\partial m'^O} \right) \beta \left[V_{\tau+1}^C(\cdot) - f^{NC} - V_{\tau+1}^{NC}(\cdot) \right] \\
& + \pi^L(x_{NC}^*) \beta \left[\frac{\partial V_{\tau+1}^C(\cdot)}{\partial m'^O} \left\{ 1 - \frac{\partial x_{NC}^*(\cdot)}{\partial m'^O} \right\} + \frac{\partial V_{\tau+1}^C(\cdot)}{\partial m'^N} \left\{ \frac{\partial x_{NC}^*(\cdot)}{\partial m'^O} \right\} \right] \\
& + (1 - \pi^L(x_{NC}^*)) \beta \left[\frac{\partial V_{\tau+1}^{NC}(\cdot)}{\partial m'^O} \left\{ 1 - \frac{\partial x_{NC}^*(\cdot)}{\partial m'^O} \right\} + \frac{\partial V_{\tau+1}^{NC}(\cdot)}{\partial m'^N} \left\{ \frac{\partial x_{NC}^*(\cdot)}{\partial m'^O} \right\} \right] - \frac{\phi_\tau^O}{\epsilon_H} \\
\{m'^N\}: \qquad & \left(\pi'^L(x_{NC}^*) \frac{\partial x^*(\cdot)}{\partial m'^N} \right) \beta \left[V_{\tau+1}^C(\cdot) - f^{NC} - V_{\tau+1}^{NC}(\cdot) \right] \\
& + \pi^L(x_{NC}^*) \beta \left[\frac{\partial V_{\tau+1}^C(\cdot)}{\partial m'^O} \left\{ - \frac{\partial x_{NC}^*(\cdot)}{\partial m'^N} \right\} + \frac{\partial V_{\tau+1}^C(\cdot)}{\partial m'^N} \left\{ 1 + \frac{\partial x_{NC}^*(\cdot)}{\partial m'^N} \right\} \right] \\
& + (1 - \pi^L(x_{NC}^*)) \beta \left[\frac{\partial V_{\tau+1}^{NC}(\cdot)}{\partial m'^O} \left\{ - \frac{\partial x_{NC}^*(\cdot)}{\partial m'^N} \right\} + \frac{\partial V_{\tau+1}^{NC}(\cdot)}{\partial m'^N} \left\{ 1 + \frac{\partial x_{NC}^*(\cdot)}{\partial m'^N} \right\} \right] - \frac{\phi_\tau^N}{\epsilon_H}
\end{aligned}
$$

Again, we can solve for each of these things and we get the following final FOC for money holdings for the not caught high type agent.

$$
\{m'^O\}: \quad \begin{cases} \beta \left[\frac{\alpha}{m'^O + m'^N} + \frac{(1-\alpha)\phi_{\tau+1}^N}{\epsilon_H} - \pi'^L(m'^O) \left(\frac{1}{1-(1-p)(1-\delta)\beta} \frac{\mathbb{T} - p\mathbb{P}}{\epsilon_H} + f^{NC} \right) \right] - \frac{\phi_\tau^O}{\epsilon_H} & \text{if } m'^O \leq x_{NC} \\ \beta \left[\frac{\alpha \phi_{\tau+1}^O}{\phi_{\tau+1}^O m'^O + \phi_{\tau+1}^N m'^N + (\phi_{\tau+1}^N - \phi_{\tau+1}^O) x_{NC}} + \frac{(1-\alpha)\phi_{\tau+1}^O}{\epsilon_H} \right] - \frac{\phi_\tau^O}{\epsilon_H} & \text{if } m'^O \geq x_{NC} \end{cases}
$$

$$
\{m'^N\}: \quad \begin{cases} \beta \left[\frac{\alpha}{m'^O + m'^N} + \frac{(1-\alpha)\phi_{\tau+1}^N}{\epsilon_H} \right] - \frac{\phi_\tau^N}{\epsilon_H} & \text{if } m'^O \leq x_{NC} \\ \beta \left[\frac{\alpha \phi_{\tau+1}^N}{\phi_{\tau+1}^O m'^O + \phi_{\tau+1}^N m'^N + (\phi_{\tau+1}^N - \phi_{\tau+1}^O) x_{NC}} + \frac{(1-\alpha)\phi_{\tau+1}^N}{\epsilon_H} \right] - \frac{\phi_\tau^N}{\epsilon_H} & \text{if } m'^O \geq x_{NC} \end{cases}
$$

Let $\xi^{NC} = \left(\frac{1}{1-(1-p)(1-\delta)\beta} \frac{\mathbb{T} - p\mathbb{P}}{\epsilon_H} + f^{NC} \right)$

$\implies$ the following FOC's for the high type not caught buyer in the transition period,

$$\{m'^O\}: \quad \begin{cases} \beta\left[\frac{\alpha}{m'^O+m'^N}+\frac{(1-\alpha)\phi^N_{\tau+1}}{\epsilon_H}-\pi'^L(m'^O)\xi^{NC}\right]-\frac{\phi^O_\tau}{\epsilon_H} & \text{if } m'^O\leq x_{NC}\\ \beta\left[\frac{\alpha\phi^O_{\tau+1}}{\phi^O_{\tau+1}m'^O+\phi^N_{\tau+1}m'^N+(\phi^N_{\tau+1}-\phi^O_{\tau+1})x_{NC}}+\frac{(1-\alpha)\phi^O_{\tau+1}}{\epsilon_H}\right]-\frac{\phi^O_\tau}{\epsilon_H} & \text{if } m'^O\geq x_{NC}\end{cases}$$

$$\{m'^N\}: \quad \begin{cases} \beta\left[\frac{\alpha}{m'^O+m'^N}+\frac{(1-\alpha)\phi^N_{\tau+1}}{\epsilon_H}\right]-\frac{\phi^N_\tau}{\epsilon_H} & \text{if } m'^O\leq x_{NC}\\ \beta\left[\frac{\alpha\phi^N_{\tau+1}}{\phi^O_{\tau+1}m'^O+\phi^N_{\tau+1}m'^N+(\phi^N_{\tau+1}-\phi^O_{\tau+1})x_{NC}}+\frac{(1-\alpha)\phi^N_{\tau+1}}{\epsilon_H}\right]-\frac{\phi^N_\tau}{\epsilon_H} & \text{if } m'^O\geq x_{NC}\end{cases}$$

A.5 Analysing the first order conditions for each type of agent over the parameter space

We can divide the first order conditions for exchange and money holdings into sub cases, the basics are the same of reach type of buyer, so i shall go over the low type buyer in detail and it should be easy to see that we get similar results for each of the high type buyers.

Low type Buyer (L)

To solve the complete system, let us look at all the possible cases and the corresponding restrictions on the parameter space.

- **[Case 1]: No exchange:** $x_L \leq 0, m'^O > x_L = 0 \implies$

$$\{x\}: \quad \beta\Delta\phi_{\tau+1}\left[\frac{\alpha}{\phi^O_{\tau+1}m'^O+\phi^N_{\tau+1}m'^N}+\frac{(1-\alpha)}{\epsilon_L}-\frac{\pi^Lf^L}{\Delta\phi_{\tau+1}}\right]<0$$

$$\{m'^O\}: \quad \beta\phi^O_{\tau+1}\left[\frac{\alpha}{\phi^O_{\tau+1}m'^O+\phi^N_{\tau+1}m'^N}+\frac{(1-\alpha)}{\epsilon_L}-\frac{\mu^O_\tau}{\beta\epsilon_L}\right]\leq 0$$

$$\{m'^N\}: \quad \beta\phi^N_{\tau+1}\left[\frac{\alpha}{\phi^O_{\tau+1}m'^O+\phi^N_{\tau+1}m'^N}+\frac{(1-\alpha)}{\epsilon_L}-\frac{\mu^N_\tau}{\beta\epsilon_L}\right]\leq 0$$

[SubCase 1]: $\mu^O_\tau > \mu^N_\tau \implies m'^O = 0, m'^N \geq 0, z'^N = \phi^N_{\tau+1}m'^N = \frac{\alpha\beta\epsilon_L}{\mu^N_\tau-\beta(1-\alpha)}$, $\frac{\Delta\phi_{\tau+1}\mu^N_\tau}{\epsilon_L} < \beta\pi^Lf^L$
[SubCase 2]: $\mu^O_\tau < \mu^N_\tau \implies m'^N = 0, m'^O \geq 0, z'^O = \phi^O_{\tau+1}m'^O = \frac{\alpha\beta\epsilon_L}{\mu^O_\tau-\beta(1-\alpha)}$, $\frac{\Delta\phi_{\tau+1}\mu^O_\tau}{\epsilon_L} < \beta\pi^Lf^L$
[SubCase 3]: $\mu^O_\tau = \mu^N_\tau \implies \phi^N_{\tau+1}m'^N+\phi^O_{\tau+1}m'^O \geq 0, z'^O+z'^N = \frac{\alpha\beta\epsilon_L}{\mu^N_\tau-\beta(1-\alpha)}$, $\frac{\Delta\phi_{\tau+1}\mu^N_\tau}{\epsilon_L} < \beta\pi^Lf^L$

- **[Case 2]: Exchange<Old Money Holdings:** $x_L > 0, m'^O > x_L > 0$ **i.e.** $x^*_L =$

$x_L \implies$

$$\{x\}: \qquad \beta\Delta\phi_{\tau+1}\left[\frac{\alpha}{\phi^O_{\tau+1}(m'^O - x_L) + \phi^N_{\tau+1}(m'^N + x_L)} + \frac{(1-\alpha)}{\epsilon_L} - \frac{\pi^L f^L}{\Delta\phi_{\tau+1}}\right] = 0$$

$$\{m'^O\}: \qquad \beta\phi^O_{\tau+1}\left[\frac{\alpha}{\phi^O_{\tau+1}(m'^O - x_L) + \phi^N_{\tau+1}(m'^N + x_L)} + \frac{(1-\alpha)}{\epsilon_L} - \frac{\mu^O_\tau}{\beta\epsilon_L}\right] = \beta\phi^O_{\tau+1}\left[\frac{\pi^L f^L}{\Delta\phi_{\tau+1}} - \frac{\mu^O_\tau}{\beta\epsilon_L}\right]$$

$$\{m'^N\}: \qquad \beta\phi^N_{\tau+1}\left[\frac{\alpha}{\phi^O_{\tau+1}(m'^O - x_L) + \phi^N_{\tau+1}(m'^N + x_L)} + \frac{(1-\alpha)}{\epsilon_L} - \frac{\mu^N_\tau}{\beta\epsilon_L}\right] = \beta\phi^N_{\tau+1}\left[\frac{\pi^L f^L}{\Delta\phi_{\tau+1}} - \frac{\mu^N_\tau}{\beta\epsilon_L}\right]$$

[SubCase 1]: $\mu^O_\tau > \mu^N_\tau \implies m'^O = 0$ Contradiction
[SubCase 2]: $\mu^O_\tau < \mu^N_\tau \implies m'^N = 0, \beta\left[\frac{\pi^L f^L}{\Delta\phi_{\tau+1}}\right] = \frac{\mu^O_\tau}{\epsilon_L}$
[SubCase 3]: $\mu^O_\tau = \mu^N_\tau, \beta\left[\frac{\pi^L f^L}{\Delta\phi_{\tau+1}}\right] = \frac{\mu^O_\tau}{\epsilon_L}$ Indeterminate $\phi^O_{\tau+1}m'^O + \phi^N_{\tau+1}m'^N$ but x_L and total money pinned down if $\pi'^L(x)$ function.

- **[Case 3]: Exchange Threshold > Old Money Holdings:** $x_L > 0, 0 \le m'^O \le x_L \implies x^*_L = m'^O \implies$

$$\{x\}@x_L: \qquad \beta\Delta\phi_{\tau+1}\left[\frac{\alpha}{\phi^O_{\tau+1}(m'^O - x_L) + \phi^N_{\tau+1}(m'^N + x_L)} + \frac{(1-\alpha)}{\epsilon_L} - \frac{\pi^L f^L}{\Delta\phi_{\tau+1}}\right] = 0$$

$$\{x\}@m'^O: \qquad \beta\Delta\phi_{\tau+1}\left[\frac{\alpha}{\phi^N_{\tau+1}(m'^N + m'^O)} + \frac{(1-\alpha)}{\epsilon_L} - \frac{\pi^L f^L}{\Delta\phi_{\tau+1}}\right] \ge 0$$

$$\{m'^O\}: \qquad \beta\phi^N_{\tau+1}\left[\frac{\alpha}{\phi^N_{\tau+1}(m'^N + m'^O)} + \frac{(1-\alpha)}{\epsilon_L} - \frac{\pi^L f^L}{\phi^N_{\tau+1}} - \frac{\phi^O_\tau}{\beta\phi^N_{\tau+1}\epsilon_L}\right] \le 0$$

$$\{m'^N\}: \qquad \beta\phi^N_{\tau+1}\left[\frac{\alpha}{\phi^N_{\tau+1}(m'^N + m'^O)} + \frac{(1-\alpha)}{\epsilon_L} - \frac{\mu^N_\tau}{\beta\epsilon_L}\right] \le 0$$

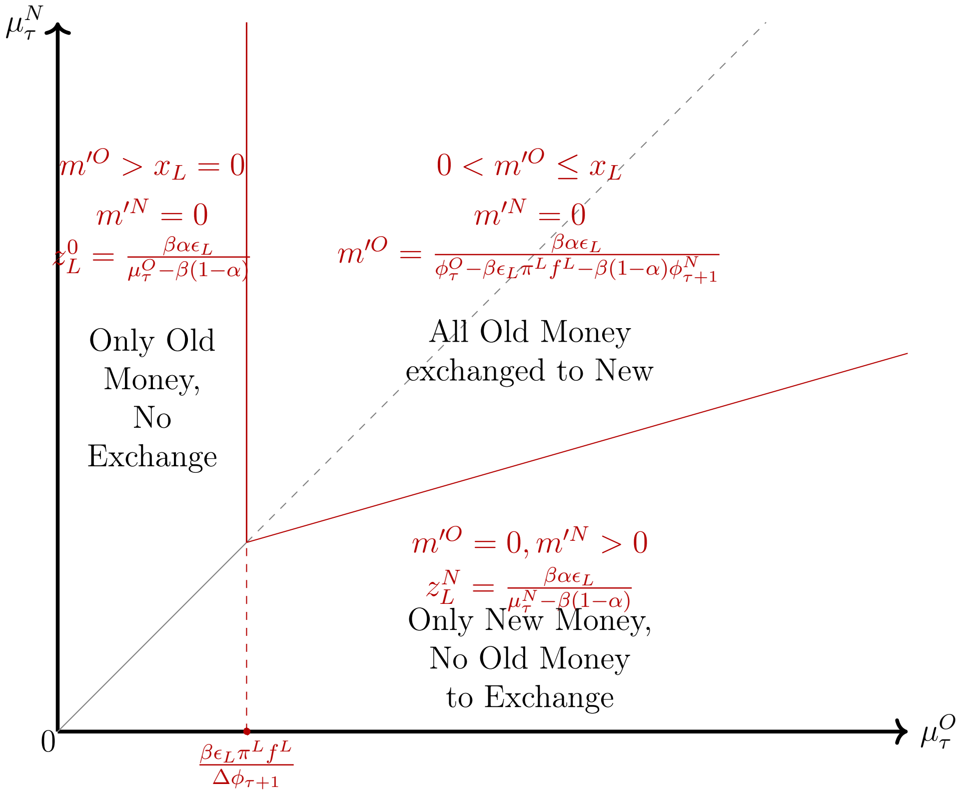
[SubCase 1]: $\phi^N_\tau - \phi^O_\tau > \beta\epsilon_L\pi^L f^L \implies m'^N = 0, m'^O > 0, \pi^L f^L\beta\epsilon_L \le \mu^O_\tau\Delta\phi_{\tau+1}$,
$m'^O = \dfrac{\beta\alpha\epsilon_L}{\phi^O_\tau + \beta\epsilon_L\pi^L f^L - (1-\alpha)\beta\phi^N_{\tau+1}}, z'^N_L = \phi^N_{\tau+1}m'^O$
[SubCase 2]: $\phi^N_\tau - \phi^O_\tau < \beta\epsilon_L\pi^L f^L \implies m'^O = 0, m'^N > 0, \pi^L f^L\beta\epsilon_L \le \mu^N_\tau\Delta\phi_{\tau+1}$,
$m'^N = \dfrac{\beta\alpha\epsilon_L}{\phi^N_\tau - (1-\alpha)\beta\phi^N_{\tau+1}}, z'^N_L = \phi^N_{\tau+1}m'^N$
[SubCase 3]: $\phi^N_\tau - \phi^O_\tau = \beta\epsilon_L\pi^L f^L \implies m'^O + m'^N > 0, \pi^L f^L\beta\epsilon_L \le \mu^N_\tau\Delta\phi_{\tau+1}$,
$m'^O + m'^N = \dfrac{\beta\alpha\epsilon_L}{\phi^N_\tau - (1-\alpha)\beta\phi^N_{\tau+1}}, z'^N_L = \phi^N_{\tau+1}(m'^O + m'^N)$

We summarize the results of the above cases in the following figure 9.

High type Caught Buyer (C)

We can similarly solve for the optimal money holding decisions for the high type caught and the high type not caught buyers respectively. They have similar sub cases, except the relevant threshold for the no exchange to optimal exchange on the x-axis changes.

Figure 9: Best responses of Low type buyer in Partial equilibrium



- **[Case1]: No exchange i.e.** $x_C = 0, m'^O \geq 0 \implies$ **the following FOC's**

$$\{x\}: \qquad \beta\Delta\phi_{\tau+1}\left[\frac{\alpha}{\phi_{\tau+1}^O m'^O + \phi_{\tau+1}^N m'^N} + \frac{(1-\alpha)}{\epsilon_H} - \frac{\pi^H f^C}{\Delta\phi_{\tau+1}}\right] < 0$$

$$\{m'^O\}: \qquad \beta\phi_{\tau+1}^O\left[\frac{\alpha}{\phi_{\tau+1}^O m'^O + \phi_{\tau+1}^N m'^N} + \frac{(1-\alpha)}{\epsilon_H} - \frac{\mu_\tau^O}{\beta\epsilon_H}\right] \leq 0$$

$$\{m'^N\}: \qquad \beta\phi_{\tau+1}^N\left[\frac{\alpha}{\phi_{\tau+1}^O m'^O + \phi_{\tau+1}^N m'^N} + \frac{(1-\alpha)}{\epsilon_H} - \frac{\mu_\tau^N}{\beta\epsilon_H}\right] \leq 0$$

[SubCase 1]: $\mu_\tau^O > \mu_\tau^N \implies m'^O = 0, m'^N \geq 0, z'^N = \phi_{\tau+1}^N m'^N = \frac{\alpha\beta\epsilon_H}{\mu_\tau^N - \beta(1-\alpha)}$, $\frac{\Delta\phi_{\tau+1}\mu_\tau^N}{\epsilon_H} < \beta\pi^H f^C$
[SubCase 2]: $\mu_\tau^O < \mu_\tau^N \implies m'^N = 0, m'^O \geq 0, z'^O = \phi_{\tau+1}^O m'^O = \frac{\alpha\beta\epsilon_H}{\mu_\tau^O - \beta(1-\alpha)}$, $\frac{\Delta\phi_{\tau+1}\mu_\tau^O}{\epsilon_H} < \beta\pi^H f^C$
[SubCase 3]: $\mu_\tau^O = \mu_\tau^N \implies \phi_{\tau+1}^N m'^N + \phi_{\tau+1}^O m'^O \geq 0, z'^O + z'^N = \frac{\alpha\beta\epsilon_H}{\mu_\tau^N - \beta(1-\alpha)}$, $\frac{\Delta\phi_{\tau+1}\mu_\tau^N}{\epsilon_H} < \beta\pi^H f^C$

- **[Case 2]: Exchange** $<$ **Old Money Holdings:** $x_C > 0, m'^O > x_C > 0 \implies x_C^* =$

$x_C \implies$

$$\{x_C\}: \qquad \beta\Delta\phi_{\tau+1}\left[\frac{\alpha}{\phi^O_{\tau+1}(m'^O - x_C) + \phi^N_{\tau+1}(m'^N + x_C)} + \frac{(1-\alpha)}{\epsilon_H} - \frac{\pi^H f^C}{\Delta\phi_{\tau+1}}\right] = 0$$

$$\{m'^O\}: \qquad \beta\phi^O_{\tau+1}\left[\frac{\alpha}{\phi^O_{\tau+1}(m'^O - x_C) + \phi^N_{\tau+1}(m'^N + x_C)} + \frac{(1-\alpha)}{\epsilon_H} - \frac{\mu^O_\tau}{\beta\epsilon_H}\right] = \beta\phi^O_{\tau+1}\left[\frac{\pi^H f^C}{\Delta\phi_{\tau+1}} - \frac{\mu^O_\tau}{\beta\epsilon_H}\right]$$

$$\{m'^N\}: \qquad \beta\phi^N_{\tau+1}\left[\frac{\alpha}{\phi^O_{\tau+1}(m'^O - x_C) + \phi^N_{\tau+1}(m'^N + x_C)} + \frac{(1-\alpha)}{\epsilon_H} - \frac{\mu^N_\tau}{\beta\epsilon_H}\right] = \beta\phi^N_{\tau+1}\left[\frac{\pi^H f^C}{\Delta\phi_{\tau+1}} - \frac{\mu^N_\tau}{\beta\epsilon_H}\right]$$

[SubCase 1]: $\mu^O_\tau > \mu^N_\tau \implies m'^O = 0$ Contradiction
[SubCase 2]: $\mu^O_\tau < \mu^N_\tau \implies m'^N = 0, \beta\left[\frac{\pi^H f^C}{\Delta\phi_{\tau+1}}\right] = \frac{\mu^O_\tau}{\epsilon_L}$
[SubCase 3]: $\mu^O_\tau = \mu^N_\tau, \beta\left[\frac{\pi^H f^C}{\Delta\phi_{\tau+1}}\right] = \frac{\mu^O_\tau}{\epsilon_L}$ Indeterminate $\phi^O_{\tau+1}m'^O + \phi^N_{\tau+1}m'^N$ but x_L and total money pinned down if $\pi'^H(x)$ function.

- **[Case 3]: Exchange Threshold $>$ Old Money Holdings:** $x_C > 0, 0 \leq m'^O \leq x_C \implies x^*_C = m'^O \implies$

$$\{x\}@x_C: \qquad \beta\Delta\phi_{\tau+1}\left[\frac{\alpha}{\phi^O_{\tau+1}(m'^O - x_C) + \phi^N_{\tau+1}(m'^N + x_C)} + \frac{(1-\alpha)}{\epsilon_H} - \frac{\pi^H f^C}{\Delta\phi_{\tau+1}}\right] = 0$$

$$\{x\}@m'^O: \qquad \beta\Delta\phi_{\tau+1}\left[\frac{\alpha}{\phi^N_{\tau+1}(m'^N + m'^O)} + \frac{(1-\alpha)}{\epsilon_H} - \frac{\pi^H f^C}{\Delta\phi_{\tau+1}}\right] \geq 0$$

$$\{m'^O\}: \qquad \beta\phi^N_{\tau+1}\left[\frac{\alpha}{\phi^N_{\tau+1}(m'^N + m'^O)} + \frac{1-\alpha}{\epsilon_H} - \pi^H f^C - \frac{\phi^O_\tau}{\beta\phi^N_{\tau+1}\epsilon_H}\right] \leq 0$$

$$\{m'^N\}: \qquad \beta\phi^N_{\tau+1}\left[\frac{\alpha}{\phi^N_{\tau+1}(m'^N + m'^O)} + \frac{1-\alpha}{\epsilon_H} - \frac{\mu^N_\tau}{\beta\epsilon_H}\right] \leq 0$$

[SubCase 1]: $\phi^N_\tau - \phi^O_\tau > \beta\epsilon_H\pi^H f^C \implies m'^N = 0, m'^O > 0, \beta\epsilon_H\pi^H f^C \leq \mu^O_\tau\Delta\phi_{\tau+1}$,
$m'^O = \dfrac{\beta\alpha\epsilon_H}{\phi^O_\tau + \beta\epsilon_H\pi^H f^C - (1-\alpha)\beta\phi^N_{\tau+1}}, z'^N_L = \phi^N_{\tau+1}m'^O$
[SubCase 2]: $\phi^N_\tau - \phi^O_\tau < \beta\epsilon_H\pi^H f^C \implies m'^O = 0, m'^N > 0, \beta\epsilon_H\pi^H f^C \leq \mu^N_\tau\Delta\phi_{\tau+1}$,
$m'^N = \dfrac{\beta\alpha\epsilon_H}{\phi^N_\tau - (1-\alpha)\beta\phi^N_{\tau+1}}, z'^N_L = \phi^N_{\tau+1}m'^N$
[SubCase 3]: $\phi^N_\tau - \phi^O_\tau = \beta\epsilon_H\pi^H f^C \implies m'^O + m'^N > 0, \beta\epsilon_H\pi^H f^C \leq \mu^N_\tau\Delta\phi_{\tau+1}$,
$m'^O + m'^N = \dfrac{\beta\alpha\epsilon_H}{\phi^N_\tau - (1-\alpha)\beta\phi^N_{\tau+1}}, z'^N_L = \phi^N_{\tau+1}(m'^O + m'^N)$

High Type Not Caught Buyer (NC)

Define, $\xi^{NC} = \left(\dfrac{1}{1-(1-p)(1-\delta)\beta}\dfrac{\mathbb{T} - p\mathbb{P}}{\epsilon_H} + f^{NC}\right)$

- **[Case 1]: No exchange:** $x_{NC} \leq 0, m'^{O} > x_{NC} = 0 \implies$

$$\begin{aligned}
\{x\}: &\qquad \beta\Delta\phi_{t+1}\left[\frac{\alpha}{\phi^{O}_{\tau+1}m'^{O}+\phi^{N}_{\tau+1}m'^{N}}+\frac{(1-\alpha)}{\epsilon_H}-\frac{\pi^{L}\xi^{NC}}{\Delta\phi_{t+1}}\right]<0\\
\{m'^{O}\}: &\qquad \beta\phi^{O}_{\tau+1}\left[\frac{\alpha}{\phi^{O}_{\tau+1}m'^{O}+\phi^{N}_{\tau+1}m'^{N}}+\frac{(1-\alpha)}{\epsilon_H}-\frac{\mu^{O}_{\tau}}{\beta\epsilon_H}\right]\leq 0\\
\{m'^{N}\}: &\qquad \beta\phi^{N}_{\tau+1}\left[\frac{\alpha}{\phi^{O}_{\tau+1}m'^{O}+\phi^{N}_{\tau+1}m'^{N}}+\frac{(1-\alpha)}{\epsilon_H}-\frac{\mu^{N}_{\tau}}{\beta\epsilon_H}\right]\leq 0
\end{aligned}$$

[SubCase 1]: $\mu^{O}_{\tau} > \mu^{N}_{\tau} \implies m'^{O} = 0, m'^{N} \geq 0, z'^{N} = \phi^{N}_{\tau+1}m'^{N} = \frac{\alpha\beta\epsilon_H}{\mu^{N}_{\tau}-\beta(1-\alpha)}$, $\frac{\Delta\phi_{\tau+1}\mu^{N}_{\tau}}{\epsilon_H} < \beta\pi^{L}\xi^{NC}$
[SubCase 2]: $\mu^{O}_{\tau} < \mu^{N}_{\tau} \implies m'^{N} = 0, m'^{O} \geq 0, z'^{O} = \phi^{O}_{\tau+1}m'^{O} = \frac{\alpha\beta\epsilon_H}{\mu^{O}_{\tau}-\beta(1-\alpha)}$, $\frac{\Delta\phi_{\tau+1}\mu^{O}_{\tau}}{\epsilon_H} < \beta\pi^{L}\xi^{NC}$
[SubCase 3]: $\mu^{O}_{\tau} = \mu^{N}_{\tau} \implies \phi^{N}_{\tau+1}m'^{N}+\phi^{O}_{\tau+1}m'^{O} \geq 0, z'^{O}+z'^{N} = \frac{\alpha\beta\epsilon_H}{\mu^{N}_{\tau}-\beta(1-\alpha)}$, $\frac{\Delta\phi_{\tau+1}\mu^{N}_{\tau}}{\epsilon_H} < \beta\pi^{L}\xi^{NC}$

- **[Case 2]: Exchange<Old Money Holdings:** $x_{NC} > 0, m'^{O} > x_{NC} > 0$ **i.e.** $x^{*}_{NC} = x_{NC} \implies$

$$\begin{aligned}
\{x\}: &\qquad \beta\Delta\phi_{t+1}\left[\frac{\alpha}{\phi^{O}_{\tau+1}(m'^{O}-x^{NC})+\phi^{N}_{\tau+1}(m'^{N}+x^{NC})}+\frac{(1-\alpha)}{\epsilon_H}-\frac{\pi^{L}\xi^{NC}}{\Delta\phi_{t+1}}\right]=0\\
\{m'^{O}\}: &\qquad \beta\phi^{O}_{\tau+1}\left[\frac{\alpha}{\phi^{O}_{\tau+1}(m'^{O}-x_{NC})+\phi^{N}_{\tau+1}(m'^{N}+x_{NC})}+\frac{(1-\alpha)}{\epsilon_H}-\frac{\mu^{O}_{\tau}}{\beta\epsilon_H}\right]=\beta\phi^{O}_{\tau+1}\left[\frac{\pi^{L}\xi^{NC}}{\Delta\phi_{t+1}}-\frac{\mu^{O}_{\tau}}{\beta\epsilon_H}\right.\\
\{m'^{N}\}: &\qquad \beta\phi^{N}_{\tau+1}\left[\frac{\alpha}{\phi^{O}_{\tau+1}(m'^{O}-x_{NC})+\phi^{N}_{\tau+1}(m'^{N}+x_{NC})}+\frac{(1-\alpha)}{\epsilon_H}-\frac{\mu^{N}_{\tau}}{\beta\epsilon_H}\right]=\beta\phi^{N}_{\tau+1}\left[\frac{\pi^{L}\xi^{NC}}{\Delta\phi_{t+1}}-\frac{\mu^{N}_{\tau}}{\beta\epsilon_H}\right.
\end{aligned}$$

[SubCase 1]: $\mu^{O}_{\tau} > \mu^{N}_{\tau} \implies m'^{O} = 0$ Contradiction
[SubCase 2]: $\mu^{O}_{\tau} < \mu^{N}_{\tau} \implies m'^{N} = 0, \beta\left[\frac{\pi^{L}\xi^{NC}}{\Delta\phi_{\tau+1}}\right] = \frac{\mu^{O}_{\tau}}{\epsilon_H}$
[SubCase 3]: $\mu^{O}_{\tau} = \mu^{N}_{\tau}, \beta\left[\frac{\pi^{L}\xi^{NC}}{\Delta\phi_{\tau+1}}\right] = \frac{\mu^{O}_{\tau}}{\epsilon_H}$ Indeterminate $\phi^{O}_{\tau+1}m'^{O}+\phi^{N}_{\tau+1}m'^{N}$ but x_{NC} and total money pinned down if $\pi'^{L}(x)$ function.

- **[Case 3]: Exchange Threshold > Old Money Holdings:** $x_{NC} > 0, 0 \leq m'^{O} \leq x_{NC} \implies x^{*}_{NC} = m'^{O} \implies$

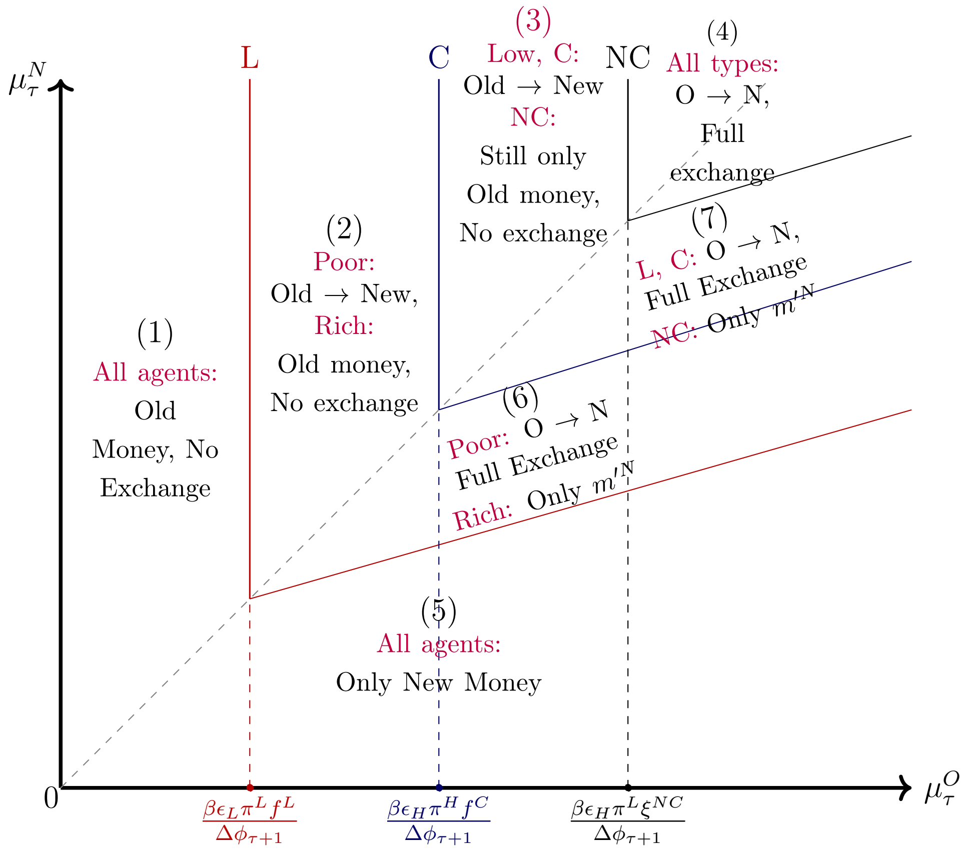
$$\begin{aligned}
\{x\}@x_{NC}: &\qquad \beta\Delta\phi_{t+1}\left[\frac{\alpha}{\phi^{O}_{\tau+1}(m'^{O}-x^{NC})+\phi^{N}_{\tau+1}(m'^{N}+x^{NC})}+\frac{(1-\alpha)}{\epsilon_H}-\frac{\pi^{L}\xi^{NC}}{\Delta\phi_{t+1}}\right]=0\\
\{x\}@m'^{O}: &\qquad \beta\Delta\phi_{t+1}\left[\frac{\alpha}{\phi^{N}_{\tau+1}(m'^{N}+m'^{O})}+\frac{(1-\alpha)}{\epsilon_H}-\frac{\pi^{L}\xi^{NC}}{\Delta\phi_{t+1}}\right]\geq 0\\
\{m'^{O}\}: &\qquad \beta\phi^{N}_{\tau+1}\left[\frac{\alpha}{\phi^{N}_{\tau+1}(m'^{O}+m'^{N})}+\frac{(1-\alpha)}{\epsilon_H}-\pi^{L}\xi^{NC}-\frac{\phi^{O}_{\tau}}{\beta\phi^{N}_{\tau+1}\epsilon_H}\right]\leq 0\\
\{m'^{N}\}: &\qquad \beta\phi^{N}_{\tau+1}\left[\frac{\alpha}{\phi^{N}_{\tau+1}(m'^{O}+m'^{N})}+\frac{(1-\alpha)}{\epsilon_H}-\frac{\mu^{N}_{\tau}}{\beta\epsilon_H}\right]\leq 0
\end{aligned}$$

[SubCase 1]: $\phi^N_\tau - \phi^O_\tau > \beta\epsilon_H\pi^H\xi^{NC} \implies m'^N = 0, m'^O > 0, \beta\epsilon_H\pi^H\xi^{NC} \leq \mu^O_\tau\Delta\phi_{\tau+1}$,
$m'^O = \dfrac{\beta\alpha\epsilon_H}{\phi^O_\tau + \beta\epsilon_H\pi^H\xi^{NC} - (1-\alpha)\beta\phi^N_{\tau+1}}, z'^N_L = \phi^N_{\tau+1}m'^O$
[SubCase 2]: $\phi^N_\tau - \phi^O_\tau < \beta\epsilon_H\pi^H\xi^{NC} \implies m'^O = 0, m'^N > 0, \beta\epsilon_H\pi^H\xi^{NC} \leq \mu^N_\tau\Delta\phi_{\tau+1}$,
$m'^N = \dfrac{\beta\alpha\epsilon_H}{\phi^N_\tau - (1-\alpha)\beta\phi^N_{\tau+1}}, z'^N_L = \phi^N_{\tau+1}m'^N$
[SubCase 3]: $\phi^N_\tau - \phi^O_\tau = \beta\epsilon_H\pi^H\xi^{NC} \implies m'^O + m'^N > 0, \beta\epsilon_H\pi^H\xi^{NC} \leq \mu^N_\tau\Delta\phi_{\tau+1}$,
$m'^O + m'^N = \dfrac{\beta\alpha\epsilon_H}{\phi^N_\tau - (1-\alpha)\beta\phi^N_{\tau+1}}, z'^N_L = \phi^N_{\tau+1}(m'^O + m'^N)$

A.6 Deriving the Monetary Equilibria

We shall consider each of the sub cases, for each of the agents and analyze each of the 7 possible regions in the complete picture (figure 7), and see which one of the them satisfies market clearing for the equilibrium to exist. Let us number them as follows and solve for the equilibrium prices in each case,

Figure 10: Complete Partial Equilibrium



We interchangeably call the low type buyers as poor agents, and the high type buyers as the rich agents.

We know that in any of of the cases the supply of Old and New Money comes either from the previous periods savings or from government seignorage. For the old money, the real value of money supply from the previous period is as follows,

$$\phi_\tau^O\left(\lambda_L\left(\frac{\beta\alpha\epsilon_L}{\phi_{\tau-1}^O-\beta(1-\alpha)\phi_\tau^O}\right)+\lambda_H\left(\frac{\beta\alpha\epsilon_H}{\phi_{\tau-1}^O-\beta(1-\alpha)\phi_\tau^O}\right)\right)=\phi_\tau^O\left(\frac{\beta\alpha\bar{\epsilon}}{\phi_{\tau-1}^O-\beta(1-\alpha)\phi_\tau^O}\right)\equiv MS_\tau^O$$

where $\bar{\epsilon}=\pi_L\epsilon_L+\pi_H\epsilon_H$, and the $\lambda_L=\pi_L,\lambda_H=\pi_H$. Likewise, the money supply of new money cones either from exchange at the window or from government transfers, we shall call it MS_τ^N. We calculate the price for old money ϕ_τ^O before the exchange and the price of new money ϕ_τ^N, after the exchange. For all cases, we still have our assumption that $\phi_{\tau+1}^N>\phi_{\tau+1}^O$,

(1): All agents saving in only Old Money

In this case, all the agents have the following demand for real money,

$$\frac{\alpha\beta\bar{\epsilon}}{\mu_\tau^O-\beta(1-\alpha)}=MD_\tau^O$$

Equating this with the money supply, we get that the given equilibrium exists whenever, $\mu_\tau^O<\mu_\tau^N$, and $\mu_t^O=\bar{\mu},\forall t\geq\tau$. For new notes, the only source of supply is government transfers S_τ, and if we impose that the agents pay their taxes and penalty in terms of new money, we can pin down the new money prices as follows,

$$\phi_\tau^N=\frac{\lambda_C\mathbb{T}+\lambda_{NC}p\mathbb{P}}{S_\tau}$$

The prices would be determined for a given level of government transfers. In particular, the government could manipulate and enforce the agents from not staying in this equilibrium by dictating a certain level of inflation using S_t, but right now, this is an equilibrium. We call it the *'Old Money Equilibrium'*.

(2): Only poor agents are willing to exchange Old Notes for New

In this case, the agents have the following demand for old money in nominal terms - used for exchange by the poor agents (and hence real value is $\phi_{\tau+1}^N*m'^O$, and for savings to the next period by the rich agents with real value $\phi_{\tau+1}^O*m'^O$, on the LHS and the real money supply from the previous period is as follows

$$\lambda_L\frac{\beta\alpha\epsilon_L}{\mu_\tau^O\frac{\phi_{\tau+1}^O}{\phi_{\tau+1}^N}-\frac{\beta\epsilon_L\pi^Lf^L}{\phi_{\tau+1}^N}-\beta(1-\alpha)}+\lambda_H\frac{\beta\alpha\epsilon_H}{\mu_\tau^O-\beta(1-\alpha)}=\frac{\beta\alpha\bar{\epsilon}}{\mu_{\tau-1}^O-\beta(1-\alpha)}$$

$$\implies\mu_\tau^O>\mu_{\tau-1}^O$$

Further, we consider the possible cases next period,

1. $\mu_{\tau+1}^N>\mu_{\tau+1}^O$ - in this case, we get that $\mu_{\tau+1+i}=\mu_{\tau+1}^O>\mu_\tau^O>\mu_{\tau-1}^O$, i.e. the inflation in old money increases as it is scarce in supply when the poor agents exchange it for new, but then everyone demands only old money next period. In this case the equilibrium breaks down if the increased inflation in old money surpasses the inflation in new money at any time.

2. $\mu^N_{\tau+1} < \mu^O_{\tau+1}$ - when old money has higher inflation tomorrow, it is not demanded by anyone, it's excess supply makes it value less and inflation μ^O_τ actually goes to infinity. This contradicts that the economy is in area 2.

New money prices are determined as in case 1 above. So, the equilibrium either does not exist in region 2, or is extremely fragile to parameters, as any increase in the excess supply of old money between τ, and $\tau+1$ can break down the equilibrium. However, this is an interesting equilibrium region because it allows for money laundering and consequent possibility of new monwy equilibrium as in case (4) below.

(3): Only the Poor agents and Rich honest agents exchange Old Notes for New

This case is the same as the case 2, except even more fragile. The only possibility of an equilibrium is when the agents continue to hold old money eventually from period $\tau+1$ onwards, enough such that the inflation in old money is $\leq$ inflation in new money forever. However if anything like say wear and tear of old notes decreases its money supply, then it's inflation will increase and the old money will vanish the moment it's inflation is greater than that of new money. Again, there could be a quick transition to New Money Equilibrium in case the agents could launder money in period τ.

(4): All agents exchange old money for new - Full Demonetisation

In this case all the agents exchange old money for new, the money market claring condition in old money is as follows,

$$\lambda_L \frac{\beta\alpha\epsilon_L}{\mu^O_\tau \frac{\phi^O_{\tau+1}}{\phi^N_{\tau+1}} - \frac{\beta\epsilon_L\pi^L f^L}{\phi^N_{\tau+1}} - \beta(1-\alpha)} + \lambda_C \frac{\beta\alpha\epsilon_H}{\mu^O_\tau \frac{\phi^O_{\tau+1}}{\phi^N_{\tau+1}} - \frac{\beta\epsilon_H\pi^H f^C}{\phi^N_{\tau+1}} - \beta(1-\alpha)} +$$
$$\lambda_{NC} \frac{\beta\alpha\epsilon_H}{\mu^O_\tau \frac{\phi^O_{\tau+1}}{\phi^N_{\tau+1}} - \frac{\beta\epsilon_H\pi^L \xi^{NC}}{\phi^N_{\tau+1}} - \beta(1-\alpha)} = \frac{\beta\alpha\bar{\epsilon}}{\mu^O_{\tau-1} - \beta(1-\alpha)} \equiv MS^O_\tau$$

and all the new money after exchange which is the new money supply for the next period, is equal to the new money demanded in next period,

$$\lambda_L \frac{\beta\alpha\epsilon_L}{\mu^O_\tau \frac{\phi^O_{\tau+1}}{\phi^N_{\tau+1}} - \frac{\beta\epsilon_L\pi^L f^L}{\phi^N_{\tau+1}} - \beta(1-\alpha)} + \lambda_C \frac{\beta\alpha\epsilon_H}{\mu^O_\tau \frac{\phi^O_{\tau+1}}{\phi^N_{\tau+1}} - \frac{\beta\epsilon_H\pi^H f^C}{\phi^N_{\tau+1}} - \beta(1-\alpha)} +$$
$$\lambda_{NC} \frac{\beta\alpha\epsilon_H}{\mu^O_\tau \frac{\phi^O_{\tau+1}}{\phi^N_{\tau+1}} - \frac{\beta\epsilon_H\pi^L \xi^{NC}}{\phi^N_{\tau+1}} - \beta(1-\alpha)} = \frac{\beta\alpha\bar{\epsilon}}{\mu^N_{\tau+1} - \beta(1-\alpha)} = MD^N_{\tau+1}$$

This puts restrictions on the inflation in new money after transition and inflation in old money before the announcement. Since we are only looking at the stationary equilibrium, we need that the new money replaces old money and follows the same path as old money would otherwise have. The old money completely vanishes from this economy and it is like the $\phi^O_{\tau+1} = 0$. I solved for the equilibrium when old money disappears separately, and it appears like the infinite limit of the x-axis in the complete picture above. i.e. the agents either hold old money and completely exchange it for new, when inflation in new money is sufficiently higher than the fine threshold, or they just directly transition to new money and discard all the old money they have (in the absence of laundering) when μ_τ^N, is below the fine threshold. The tilted 45°, in the partial equilibrium figures, becomes completely horizontal at the threshold values marked on the x- axis.

(5): All agents only hold new money

This is the simplest case of an equilibrium. There is excess supply of Old money, since all the agents got old money from $\tau - 1$ equal to MS_τ^O, and no body wants to save in old money anymore. This implies that the price of old money goes to 0 immediately and the old money seizes to exist. While it may seem that then the equilibrium cannot exist in this area because $\mu_\tau^O = \frac{\phi_\tau^O}{\phi_{\tau+1}^O}$, and $\phi_\tau^O \downarrow 0$, we cannot directly infer this result, because even $\phi_{\tau+1}^O = 0$. So we move to the limiting case explained above and all the agents simply hold whatever new money there is in the system with very high price of buying it at τ, and a correspondingly high price of of new money tomorrow. So, they are in a high prices high inflation equilibrium for new money, forever (due to stationarity) but the fine in the exchange is sufficiently high for them to want to be in this equilibrium.

$$MD_\tau^N = \frac{\beta\alpha\bar{\epsilon}}{\mu_\tau^N - \beta(1-\alpha)} + \lambda_C \mathbb{T} + \lambda_{NC} p \mathbb{P} = \phi_\tau^N S_\tau = MS_\tau^N$$

assuming taxes and penalties are to be paid in terms of new money. This is what we shall call the new money equilibrium. The equilibrium could be made better once all the agents take the one time loss of all their old money in the transition period, if the government decides to give lots of transfers in new money to the agents in the future. But, then the problem of black money does not get mitigated from the system, and none of the agents ever get caught in the exchange window imposed by the Demonetisation policy. This equilibrium is the one in which the use of demonetisation to curb black money fails miserably.

(6): Honest agents exchange old for new, dishonest agents discard old money

In this equilibrium, all the old money is exchange for new money by the Honest agents (Low types and the High type Caught agents), but the dishonest agents have a huge penalty cost of keeping or exchanging old money, so they simply transition to holding new money and discard whatever old money holdings they had. Again, there is scope for money laundering in this case i.e. the dishonest agents could use their old money and launder if for exchanging to new money at a lower cost by giving bribes to the honest agents. In this case, the supply and demand of new money is as follows,

$$MD_\tau^N = \lambda_{NC} \frac{\beta\alpha\epsilon_H}{\mu_\tau^N - \beta(1-\alpha)} + \lambda_C \mathbb{T} + \lambda_{NC} p \mathbb{P} = \phi_\tau^N S_\tau = MS_\tau^N$$

$$\begin{aligned} MS_{\tau+1}^N = \lambda_L & \frac{\beta\alpha\epsilon_L}{\mu_\tau^O \frac{\phi_{\tau+1}^O}{\phi_{\tau+1}^N} - \frac{\beta\epsilon_L \pi^L f^L}{\phi_{\tau+1}^N} - \beta(1-\alpha)} + \lambda_C \frac{\beta\alpha\epsilon_H}{\mu_\tau^O \frac{\phi_{\tau+1}^O}{\phi_{\tau+1}^N} - \frac{\beta\epsilon_H \pi^H f^C}{\phi_{\tau+1}^N} - \beta(1-\alpha)} \\ & + \lambda_{NC} \frac{\beta\alpha\epsilon_H}{\mu_\tau^N - \beta(1-\alpha)} + S_\tau \phi_{\tau+1}^N = \frac{\beta\alpha\bar{\epsilon}}{\mu_{\tau+1}^N - \beta(1-\alpha)} = MD_{\tau+1}^N \end{aligned}$$

(7): Poor agents exchange old for new, Rich agents discard old money and save in new

This is the same as the previous case except that the honest agents also choose to take 100% penalty on their old money and directly save in more expensive new money. Again laundering possibilities will be used by the agents in this case.