BDA 483 (Data Science and Computational Thinking)

End Term Examination Paper

Date: 29 November 2020

Instructions

- **This question paper is open book examination. It allows you all kinds of help except help from a subject expert or your classmates.**
- **The submission date for the answers is 16 December 2020 12:00 Noon (Midday).**
- **No submission will be accepted after the mentioned time and date.**
- **In case of any doubt, instructor has a right to take viva on specific question or for all questions and then the marks will be given based on the answers during the viva.**
- **This question paper is an evaluation about the understanding of the questions as well as how to answer the questions.**
- **All questions to be submitted in one zip directory. All programming questions should have a detailed ReadMe file, with instructions about the software platform and it's version, compilation instructions and other requirements.**

---

1. Suppose that, for any patient, treatment A has $P(success) = \alpha$, and treatment B has $P(success) = \beta$, and all patients are independent. Assume that $0 < \alpha < 1$ and $0 < \beta < 1$.

   There are two different strategies for allocating patients to treatment A and B is to choose from the two treatments at random, each with probability 0.5 for each patient.

   Let p_R be the overall probability of success for each patient with the random strategy.

   Exercise: Show that $p_R = \frac{1}{2}(\alpha + \beta)$

   The two – armed bandit strategy, in which first patient, choose treatment A or B at random (probability 0.5 each). If patient n is given treatment A and it is successful, then use treatment A again for patient n+1, for all n = 1,2,3…. If A treatment if failure for patient n, switch to treatment B for patient n+1. Similar rule is applied if patient n is given treatment B; if it is successful, keep the B treatment for patient n+1; if it fails, switch to A for patient n+1.

Define the two – armed bandit process to be a Markov chain with state space {(A,S), (A,F), (B,S), (B,F)}, where (A,S) means that patient n is given treatment A and it is successful and so on.

Exercise: Draw on the arrows to show the Markov Chain based on the given state space and find their transition probabilities in terms of α and β.

Let define p_T to be the long run probability of success using the two armed bandit strategy. Find the equilibrium distribution Π for the two armed bandit process. Find the long run probability of the success for each patient under this strategy.

Exercise: Using the simulation, show that $p_T - p_R \geq 0$, regardless of the values of α and β.

2. A slide has been attached, which talks about the PageRank algorithm in it's own way. In this question, it is expected to understand the slide and do the implementation according to the details provided in slide. It's also expected to write your own report based on your understanding as well as the implementation details and results.
3. A photon counter is pointed a remote star for one minute, in order to infer the brightness, i.e., the rate of photon arriving at the counter per minute, λ. Assuming, the number of photons collected r has a Poisson distribution with mean λ,
$$P(r|\lambda) = \exp(-\lambda)\frac{\lambda^r}{r!}$$
What is the maximum likelihood estimate for λ given $r = 9$?
Also, in the same situation, now assume that the counter detects not only photon from the stars but also 'background' photons. The background rate of photons is known to be $b = 13$ photons per minute. We assume the number of photons collected, r, has a Poisson distribution with mean $\lambda + b$. Now, given $r = 9$ detected photons, what is the maximum likelihood estimate for λ?

Use the same set up and estimate the λ using Monte Carlo simulation technique and compare the results.