Supplementary Material B
Detailed description and illustration of the developed algorithms and their variants

This document provides a detailed and illustrated description of the 3 algorithms we implemented for the femur, tibia and patella. The 3 variants of each algorithm are also presented. The intention is to give sufficient information to fully understand the detailed of the different steps of each algorithm. We've made the commented code publicly available at: https://github.com/renaultJB/GIBOC-Knee-Coordinate-System

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I. Femoral ACS Algorithm

a. Identification of the femoral head center

1. Compute the inertia tensor of the femur. Eigen vectors of the inertia tensor :

$$\overrightarrow{V_I}, \overrightarrow{V_{II}} \ \& \ \overrightarrow{V_{III}}$$

We have :

$\overrightarrow{V_I}$: Distaloproximal

$\overrightarrow{V_{II}}$: MedioLateral

$\overrightarrow{V_{III}}$: AnteroPosterior

C_0 the centroid volumetric representation of the femur. Knowing the relative position of the proximal and distal part of the femur, we defined $\overrightarrow{DP_0}$ oriented from distal to proximal :

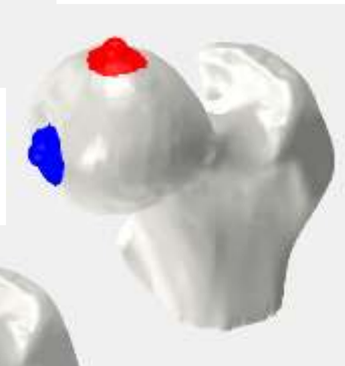
$$\overrightarrow{DP_0} = sign\left[\left(\overrightarrow{OCentroid_{ProxFemur}} - \overrightarrow{OCentroid_{DistFemur}}\right).\overrightarrow{V_I}\right].\overrightarrow{V_I}$$

And obtained a distal to proximal axis DP$_0$ = $\left(C_0, \overrightarrow{DP_0}\right)$.

2. Find the most proximal point, MPP, of the femur (red dot located on the femoral head)

3. The line connecting MPP to DP$_0$ defined a 2nd direction: $\overrightarrow{LM_0}$ lateral to medial (Yellow arrow)

4. The most medial point, MMP, was found. (Blue dot)

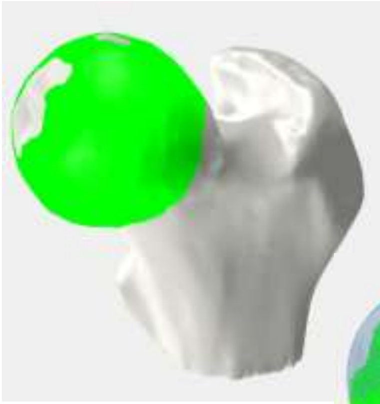


5. Region within 10 mm around the MPP and MMP were computed (blue and red surface patches)

6. A sphere was least square fitted onto the combined patches

7. Two spheres were computed from this sphere one 10% smaller the other 10% bigger.

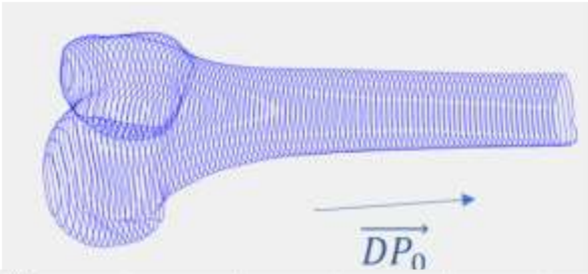
8. All vertices between those two spheres and with a surface normal direction within 10° of the corresponding spheres radius direction were identified as belonging to the femoral head AS (Green region)



9. Finally, a sphere was fitted onto the identified AS, and its center (black dot) served as the femoral head center.

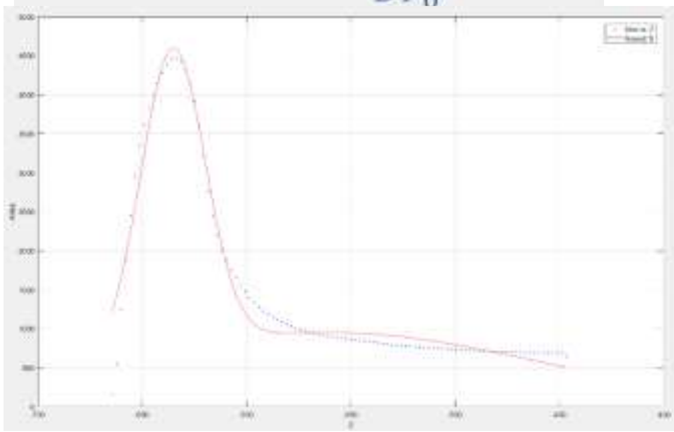
b. Identification of an initial distal condyles axis

1. The cross-section area along $\overrightarrow{DP_0}$ of the distal femur was computed



2. The evolution of the CSA along DP_0 axis, was fitted with a double gaussian function:

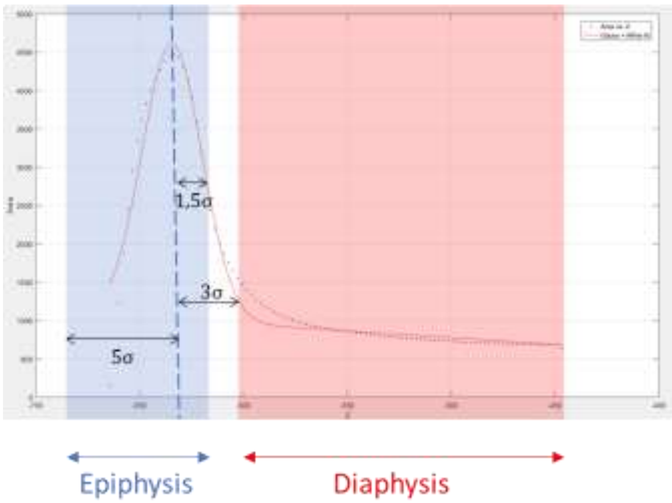
$$f_{G2} = a_1.e^{-\frac{(z-b_1)^2}{c_1}} + a_2.e^{-\frac{(z-b_2)^2}{c_2}}$$



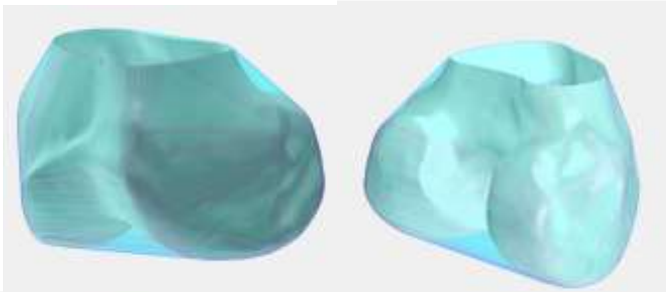
3. The identified parameters of the previous fit were used to perform a gaussian + linear fit:

$$f_{GL} = a_1.e^{-\frac{(z-z_0)^2}{\sigma}} + d.x + e$$

This allow to separate the epiphysis from the rest of the distal femur.



4. The 3D convex hull of the epiphysis was computed (cyan triangulation with light blue edges) and
    a. All the edges that have a vertex on the separation border were deleted
    b. The lengths of the remaining edges were computed

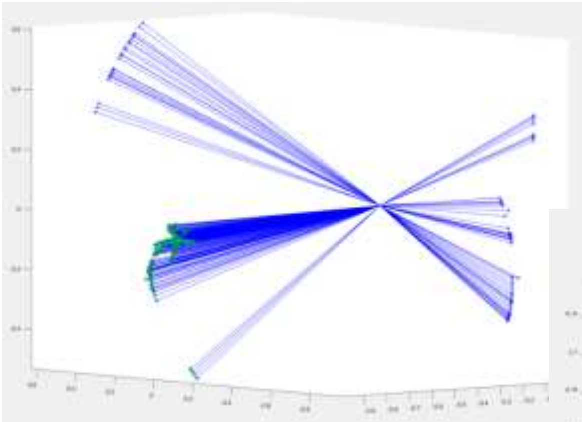


5. All the edges with a length superior to 50% of the largest convex hull edge were identified (Black lines)

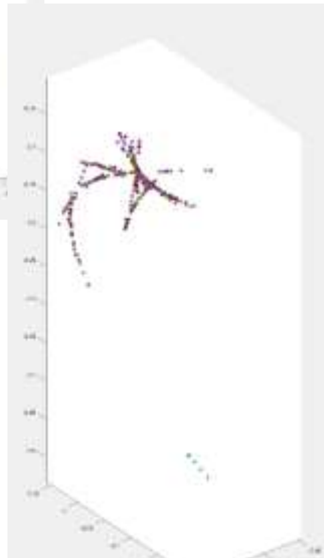
6. Eliminate the edges whose direction ($\overrightarrow{U_{edge}}$) are too far from $\overrightarrow{V_{II}}$:

$$\left|\overrightarrow{U_{edge}}.\overrightarrow{V_{II}}\right| < 0{,}75$$

The kept $\overrightarrow{U_{edge}}$ tips are in green



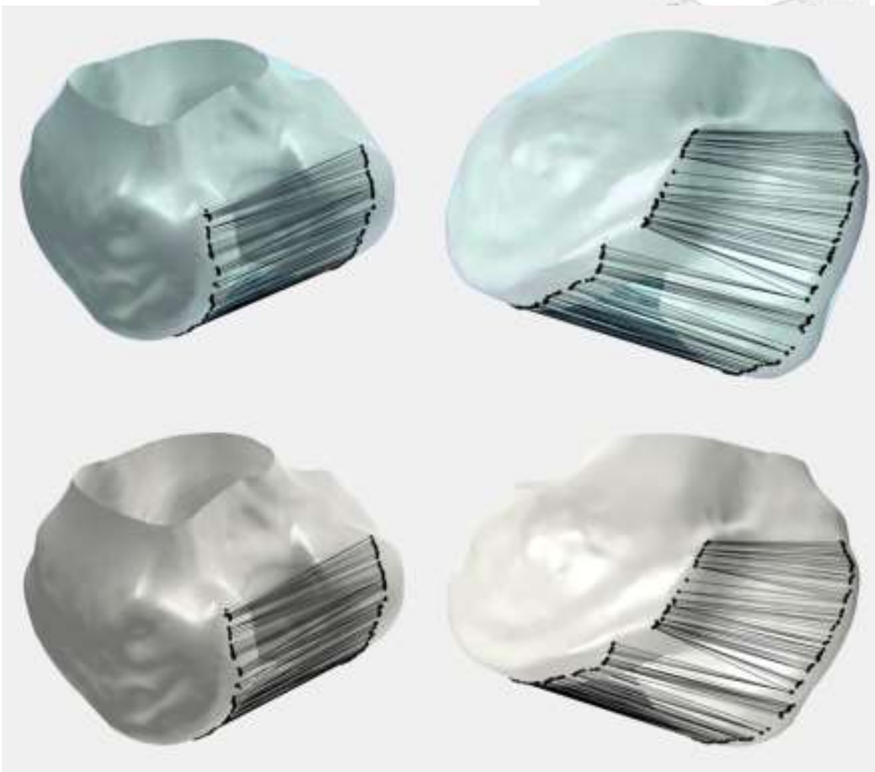
7. Keep only the largest connected patch of vector tip (max distance between vector tips of 0.1). In red all $\overrightarrow{U_{edge}}$, with between point distance <0.1 and seed point being the mean of $\overrightarrow{U_{edge}}$.



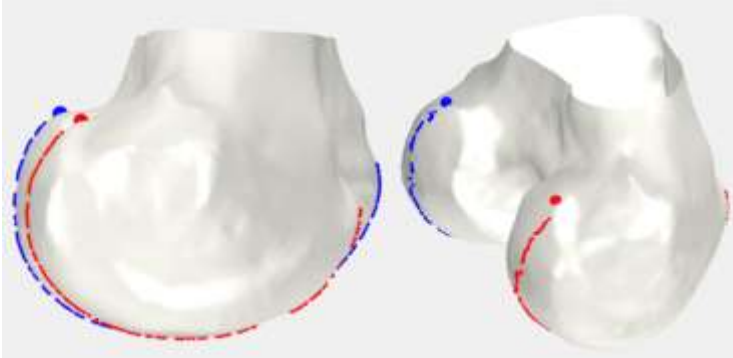
8. Calculate each edge directional unit vector $\overrightarrow{U_{edge}}$, and remove all the edges with:

$$\overrightarrow{U_{edge}}.\overrightarrow{LM_0} < 0$$

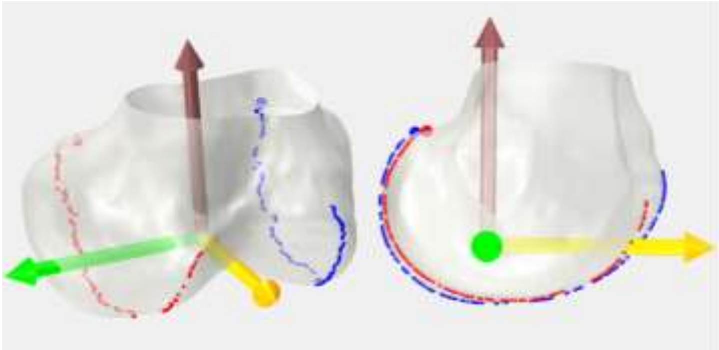
The vertices located at the origin of edges are located on the lateral condyles, and the vertices at the endings are on the medial condyles



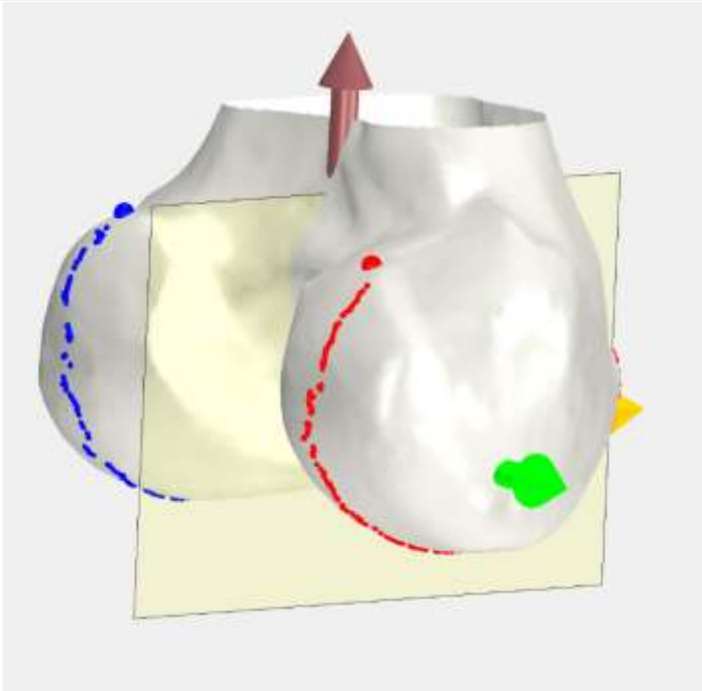
9. Lateral (blue) and medial (red) set of points correspond to the identified edges origin and ending. The two larger points are added to identify the proximal "end" of the condyle articular surfaces



10. The mean direction of all $\overrightarrow{U_{edge}}$ gives us a new temporary mediolateral vector $\overrightarrow{LM_1}$ (Green arrow), and we were able to construct a new temporary ACS.



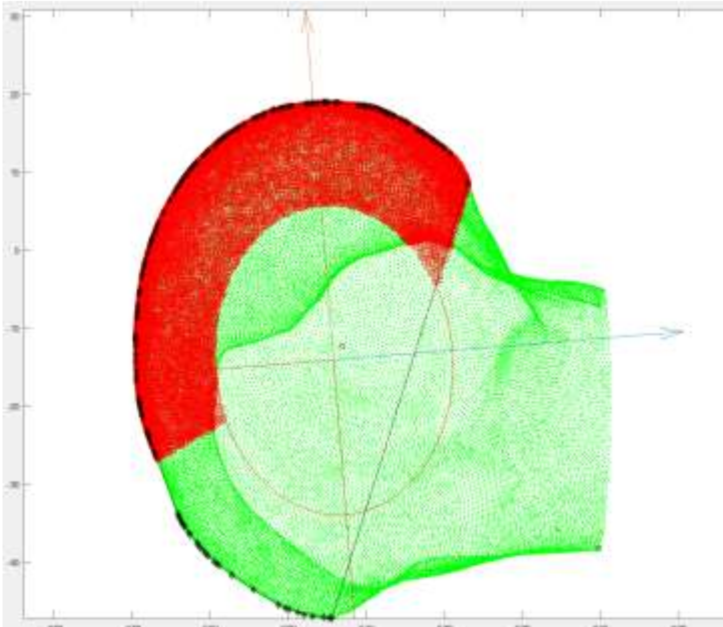
11. A plan PLM0 with normal $\overrightarrow{LM_1}$ going through the mean of the kept convex hull edges midpoints was computed (Yellow plan). This plan allows for the separation of a lateral and medial part of the epiphysis.



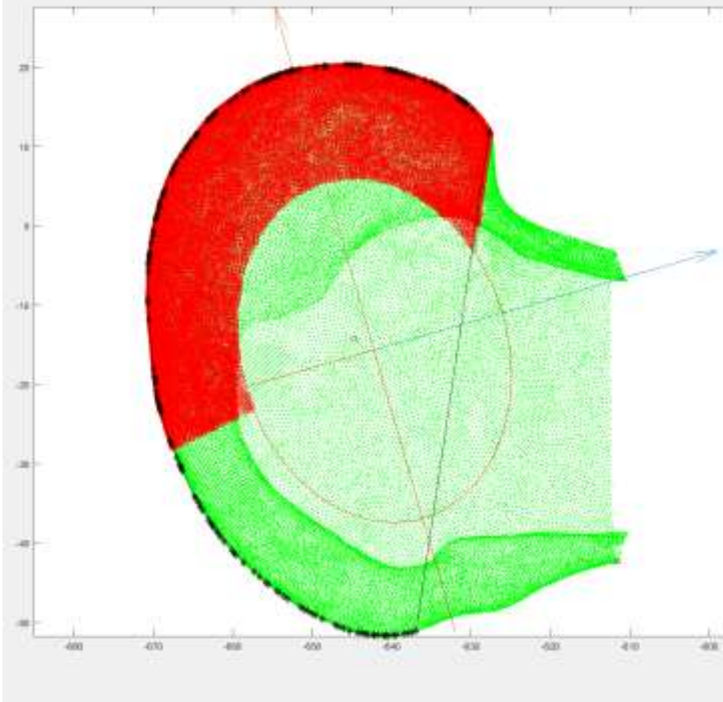
End of the common part of the femoral algorithm variants

c. PCC (Posterior Condyle Cylinder) and PCS (Posterior Condyle Spheres) Variants: Identification of the distal posterior condyles axis

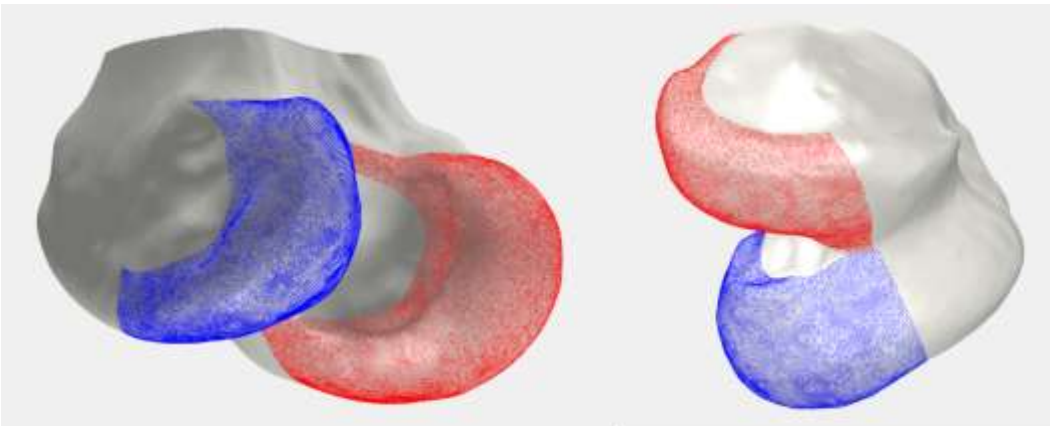
1. Identify elements that could be on the medial posterior condyle AS:
    a. Project all the medial vertices onto PLM0 (green point)
    b. Project the 3D Convex hull medial points set onto PLM0
    c. Fit an ellipse on the projected the 3D Convex hull medial points set
    d. Reduce this ellipse by **30%**
    e. Compute the 2D convex hull of the projected the 3D Convex hull medial points set
    f. Identify all the vertices ID that are outside the ellipse and inside the 2D convex hull (Red point)



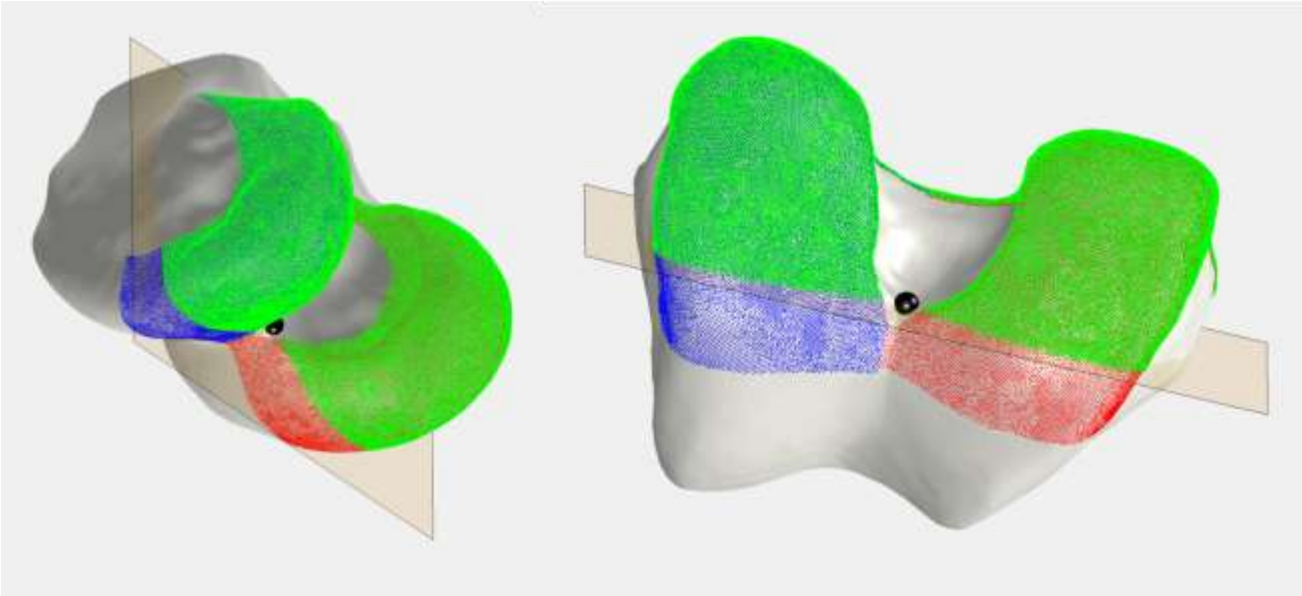
2. Do the same procedure for the lateral epiphysis part.



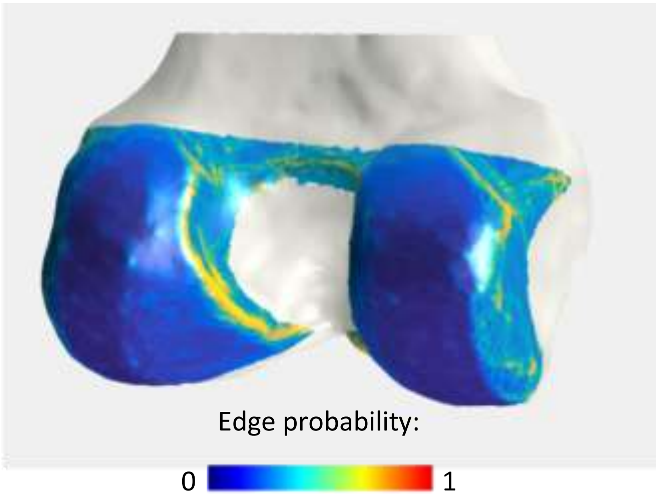
3. Use the identified ID to label the AS on the epiphysis



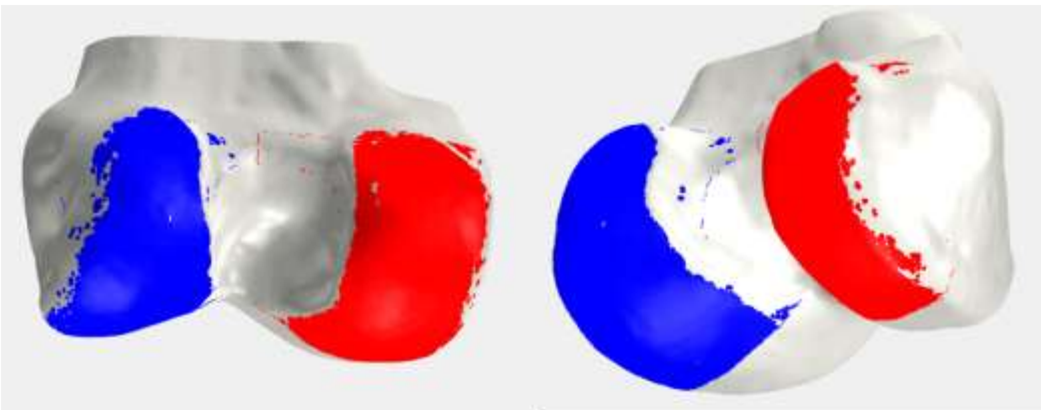
4. In green all the vertex located behind the plan (brown plan) going through the femoral notch (Black dot) with normal $\overrightarrow{AP_1}$



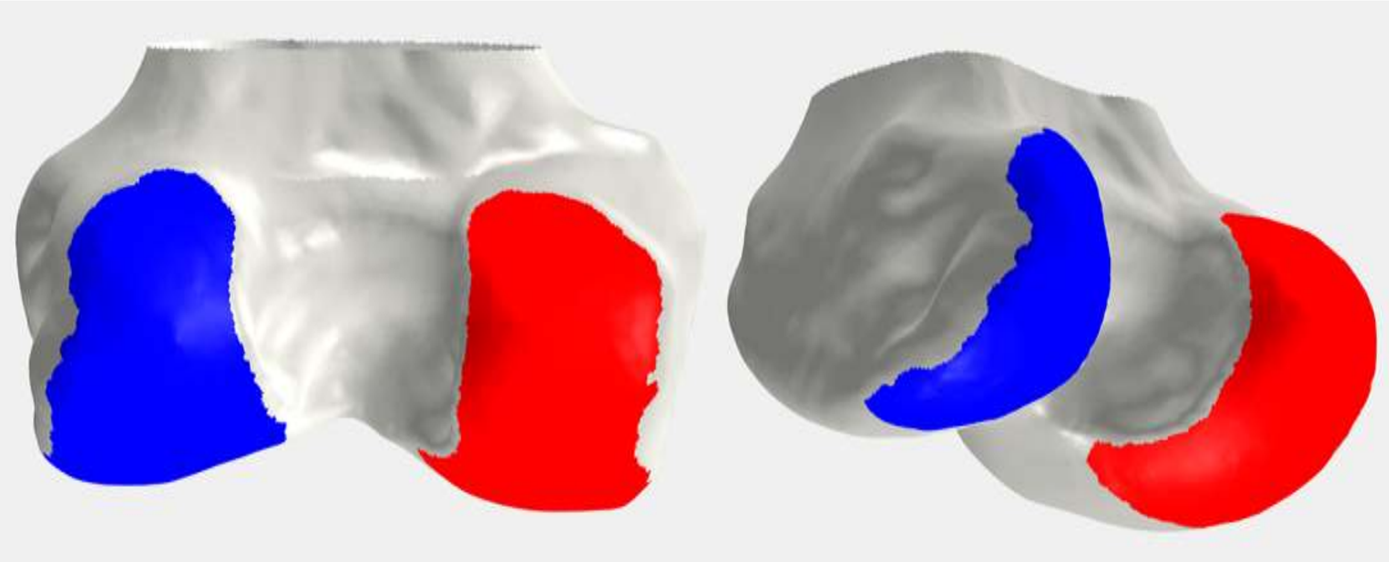
5. On the identified ASs candidates compute an articular edge probability from:
    a. Surface normal orientation relative to the $\overrightarrow{LM_1}$
    b. Surface curvature



6. Keep all the elements with a probability above 20%

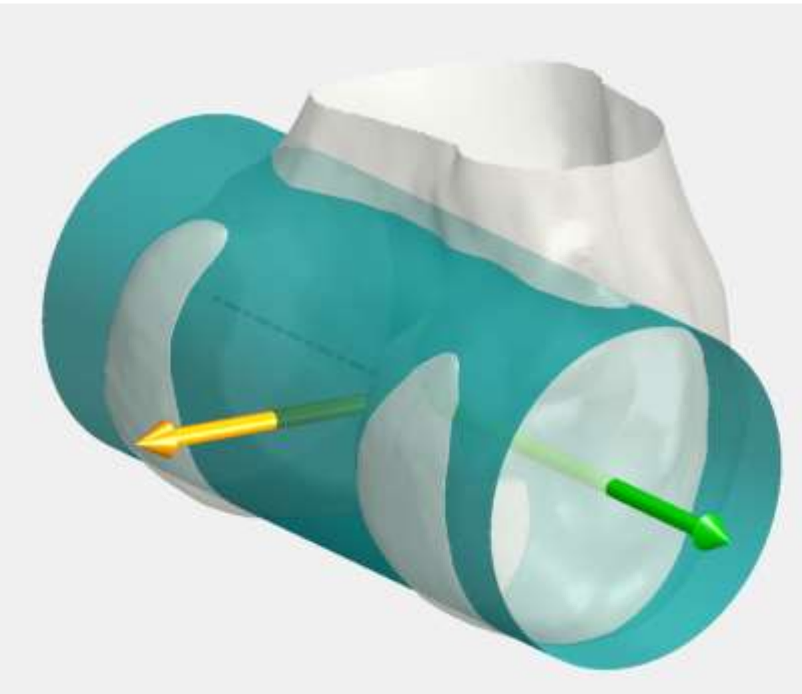


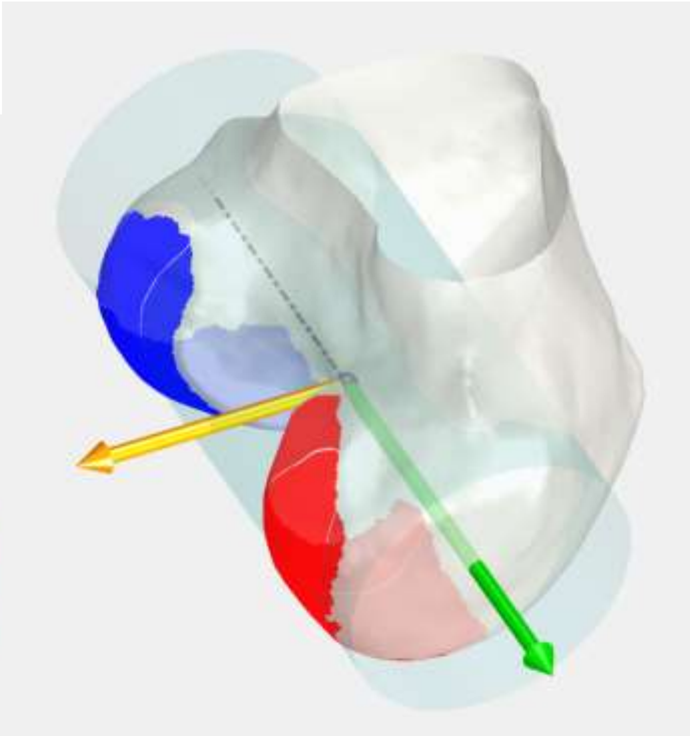
7. Filter small part and smooth the results, keep only the patches that contain an element form the medial or lateral 3D Convex hull points set.

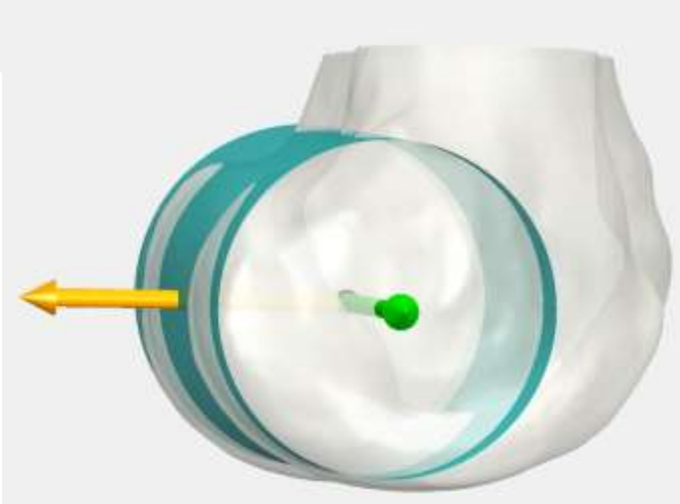


d. Ending of the PCC variant

1. Fit a cylinder (least square method) on the combined identified AS

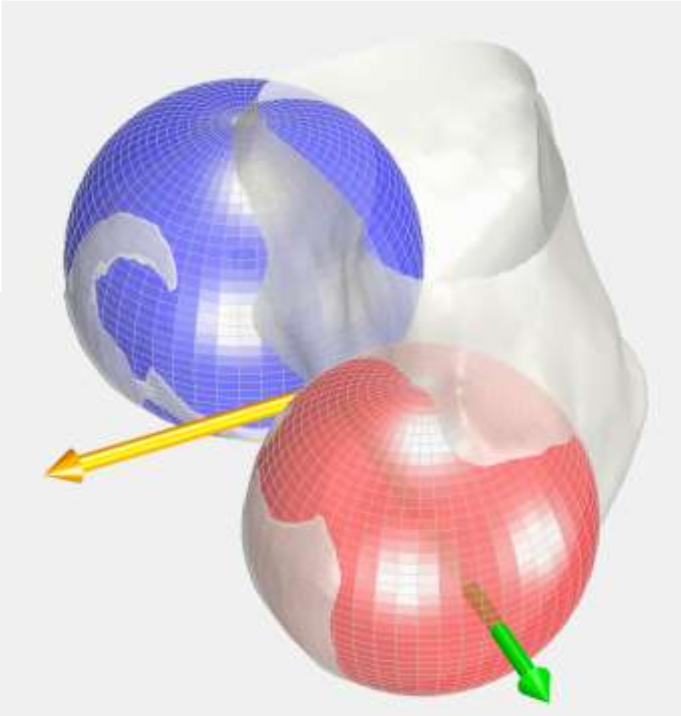


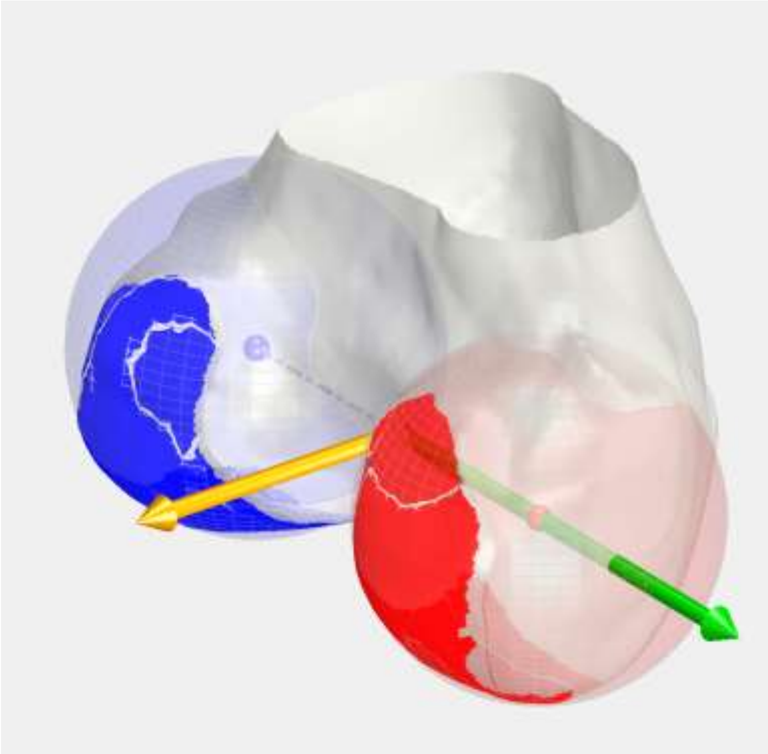


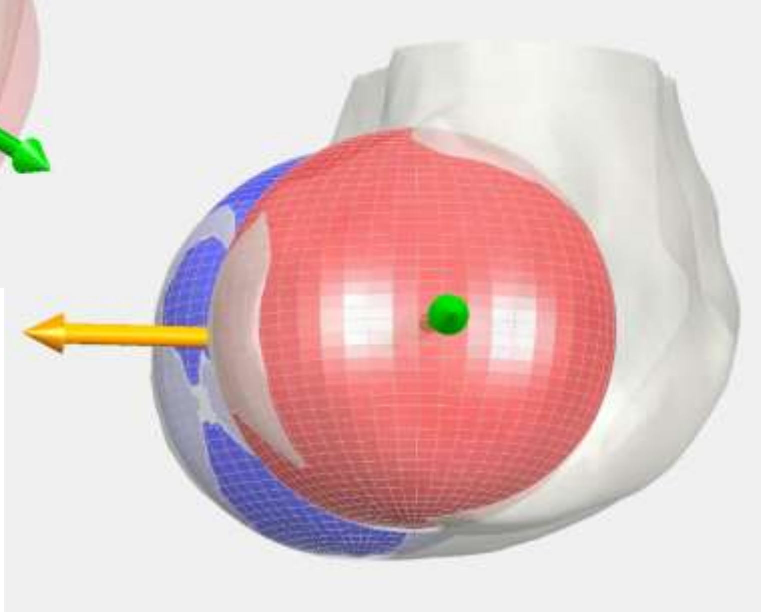


e. Ending of the PCS variant

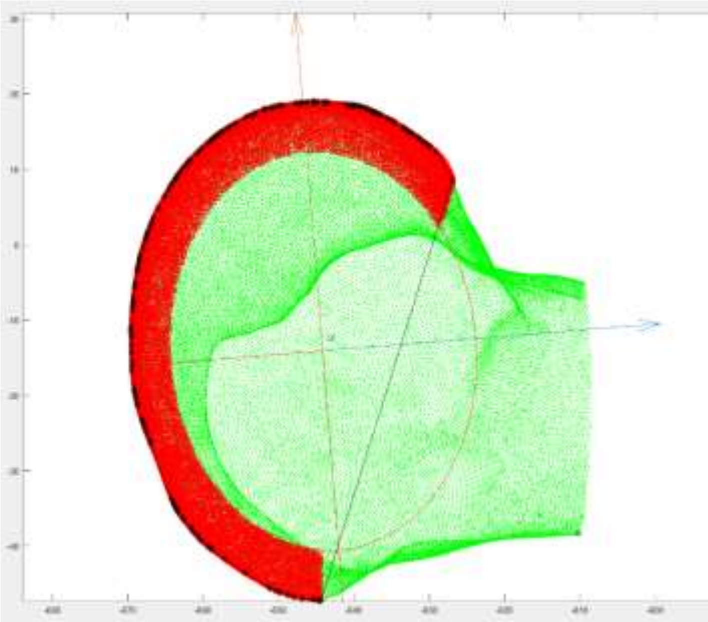
1. Fit two spheres one on the lateral posterior condyles ASs and another on the medial posterior ASs



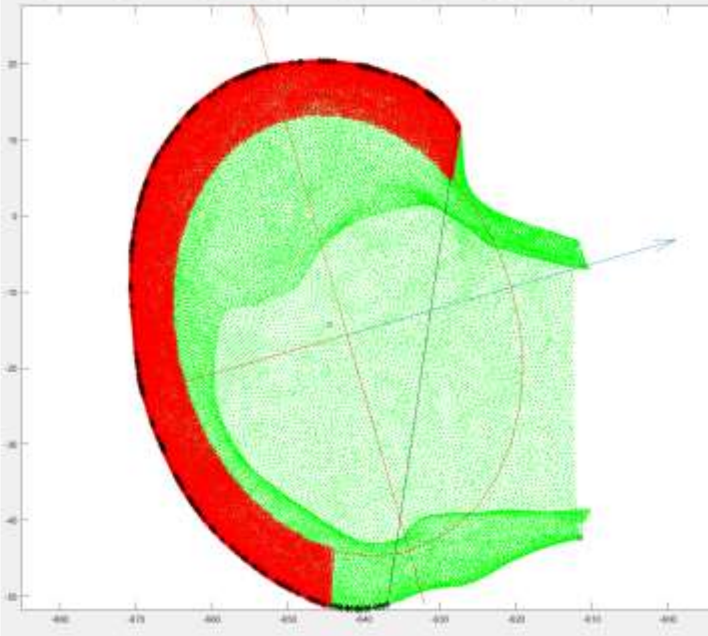




f. CE (Condyle Ellipsoids) Variant: Identification of the distal posterior condyles axis

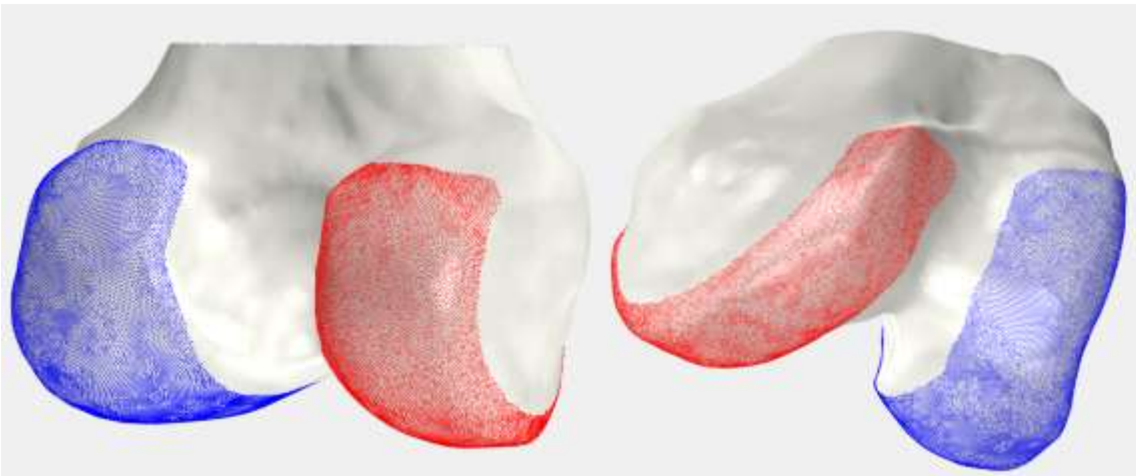


1. Identify elements that could be on the medial condyle AS:
   a. Project all the medial vertices onto PLM0 (green point)
   b. Project the 3D Convex hull medial points set onto PLM0
   c. Fit an ellipse on the projected the 3D Convex hull medial points set
   d. Reduce this ellipse by **20%**
   e. Compute the 2D convex hull of the projected the 3D Convex hull medial points set
   f. Identify all the vertices ID that are outside the ellipse and inside the 2D convex hull (Red point)



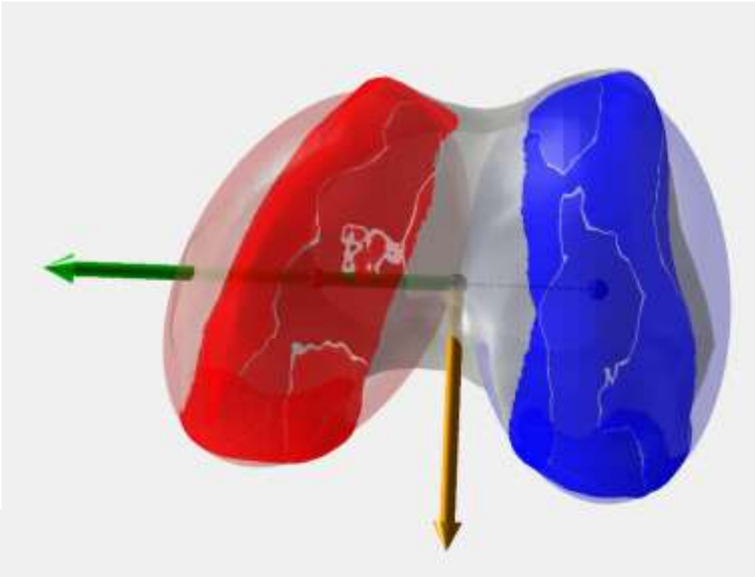
2. Do the same procedure for the lateral epiphysis part.

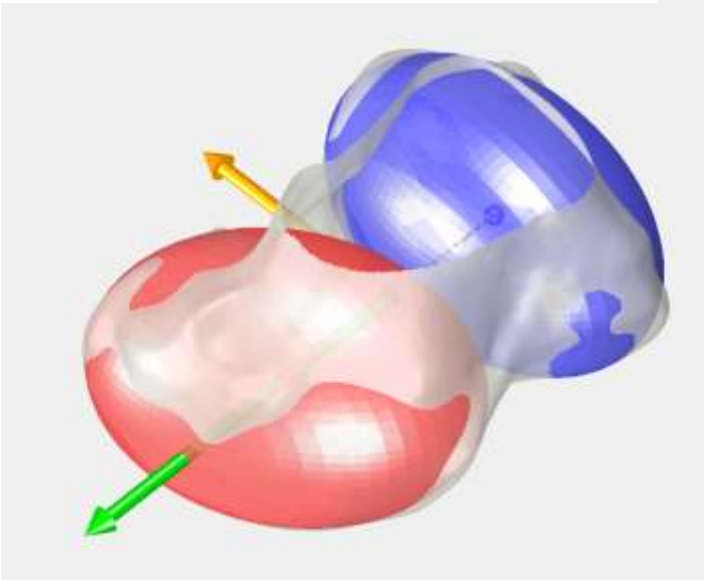
3. Use the identified ID to label the AS on the epiphysis



g. Ending of the CE Variant

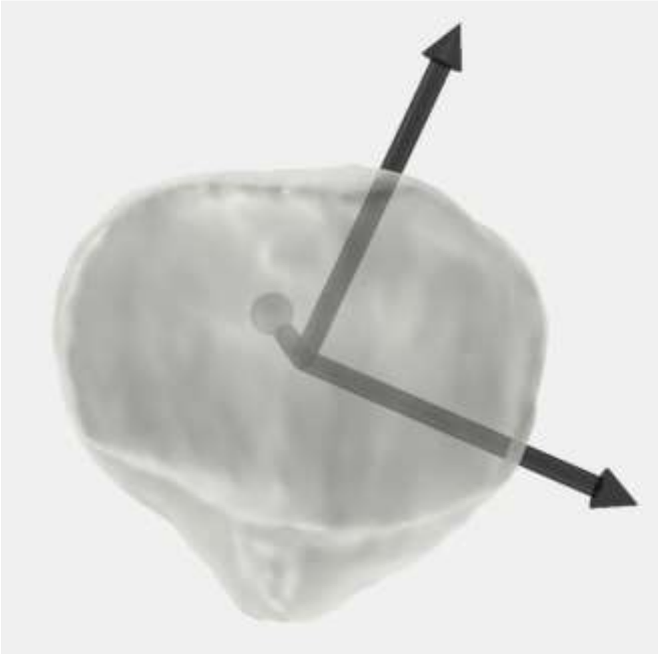
1. Fit two ellipsoids one on the lateral posterior condyles ASs and another on the medial posterior ASs





II. Patella ACS Algorithm

a. Identification of the articular surface ridge



1. Compute the inertia tensor of the patella. Eigen vectors of the inertia tensor :

$$\overrightarrow{V_I}, \overrightarrow{V_{II}} \,\&\, \overrightarrow{V_{III}}$$

We have :

$\overrightarrow{V_I}$: Undifined

$\overrightarrow{V_{II}}$: Undifined

$\overrightarrow{V_{III}}$: Anterior to Posterior or Posterior to Anterior

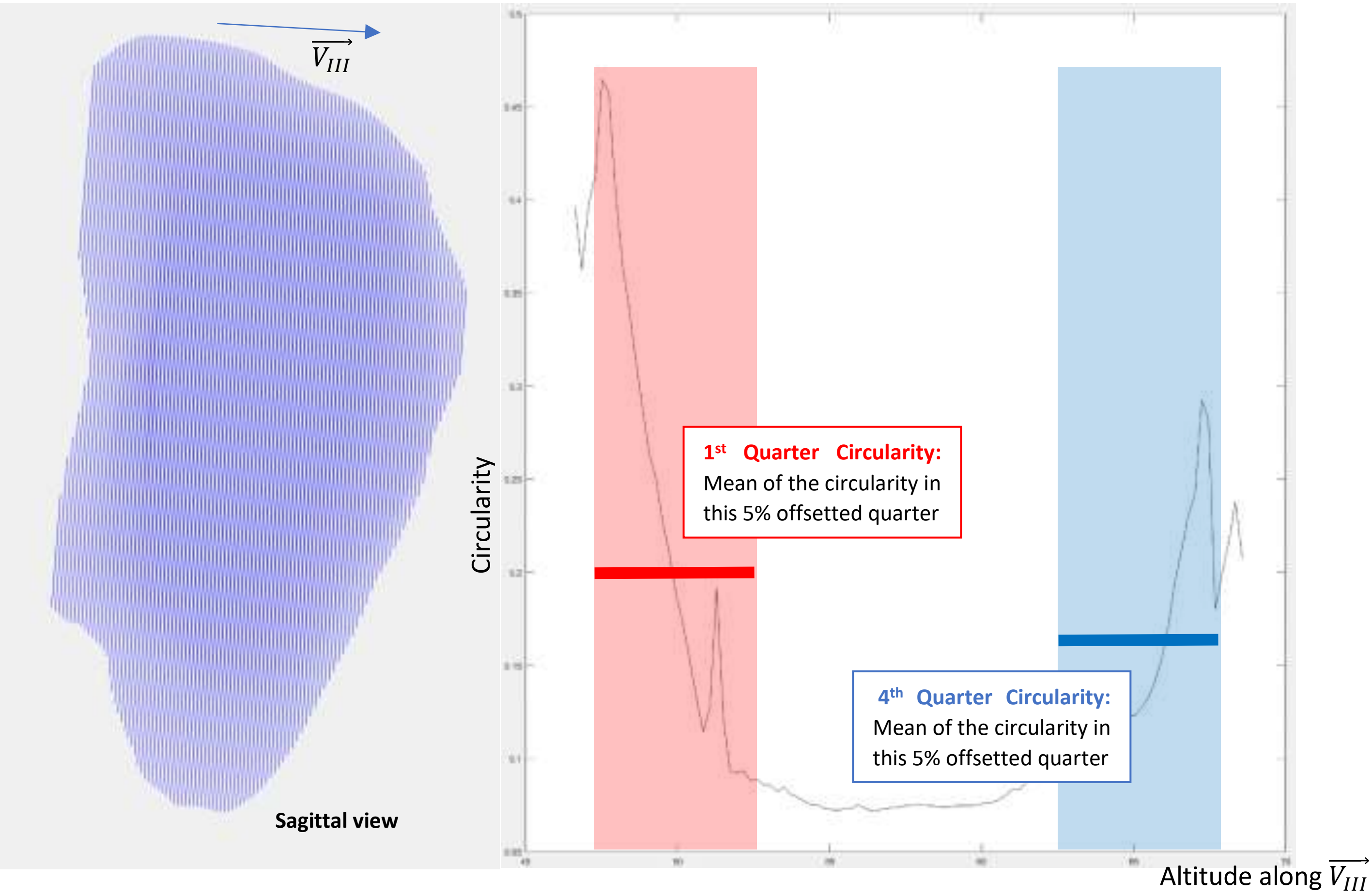
C_0 the centroid of the volumetric representation.

2. To make sure the 3rd PIA ($\overrightarrow{V_{III}}$) points from Anterior to Posterior, the circularity of the cross sections along $\overrightarrow{V_{III}}$ are computed.

$$Circularity = \frac{std(DistanceToSectionCenter)}{mean(DistanceToSectionCenter)}$$

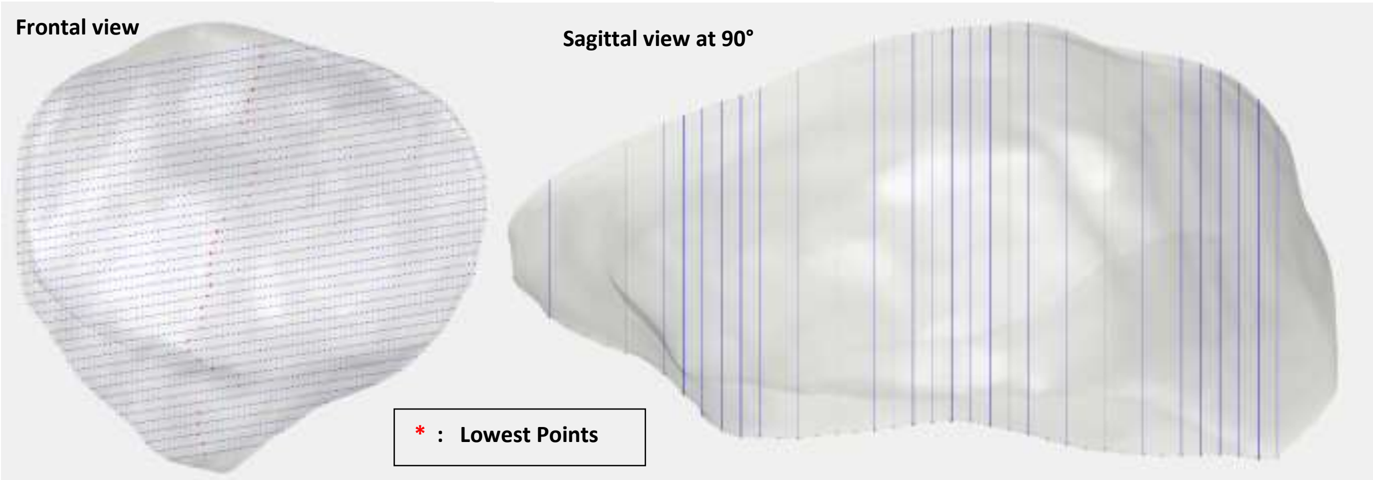
If 1stQuarter_Mean_Centrality < 4thQuarter_Mean_Centrality:

$$\overrightarrow{V_{III}} := -\overrightarrow{V_{III}} \quad and \quad \overrightarrow{V_{II}} := -\overrightarrow{V_{II}}$$

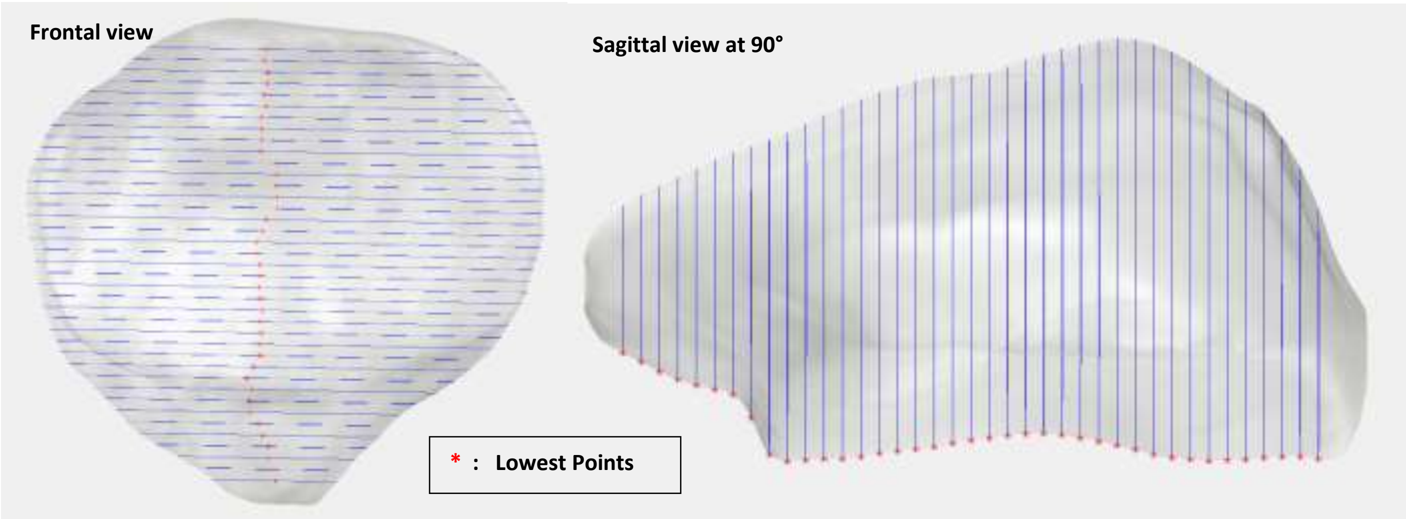


3. Then the patella is moved to the PIA coordinate system, and the Rainbow et Al. 2013 ridge identification is perfomed :

1st Iteration

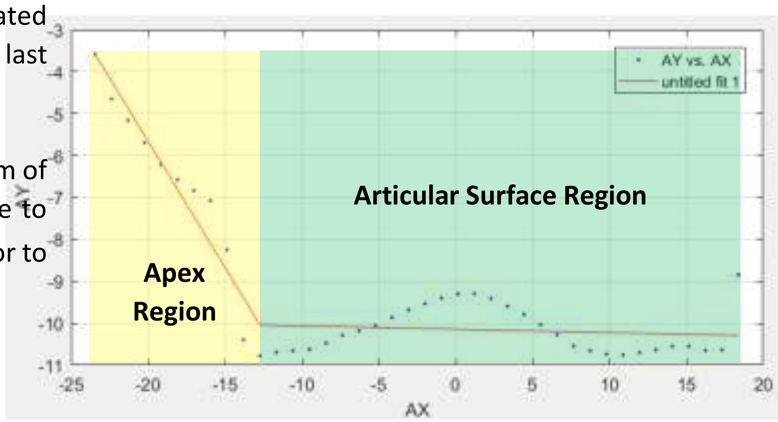


Last iteration



4. The patellar apex and articular surfaces are separated thanks to a non-linear fit of lowest points (at the last iteration) projected on a temporary sagital plan.

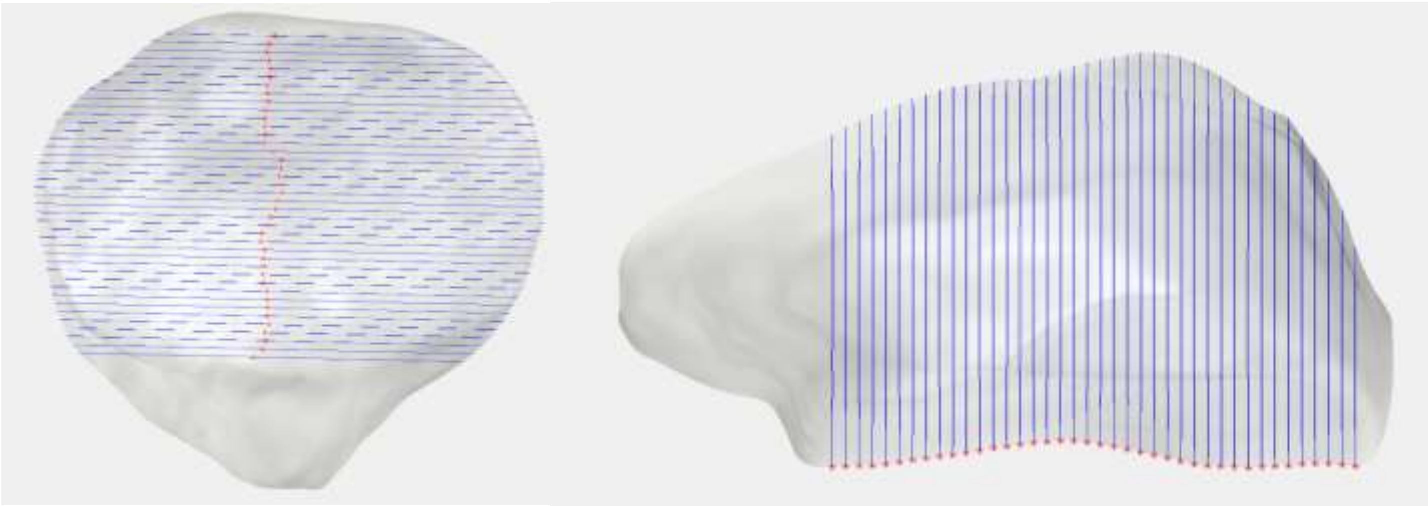
   This separation allow to know where the top and bottom of the patella are located. Thanks to that, we will be able to ensure that the patella $\overrightarrow{DP}$ axis is oriented from inferirior to superior.



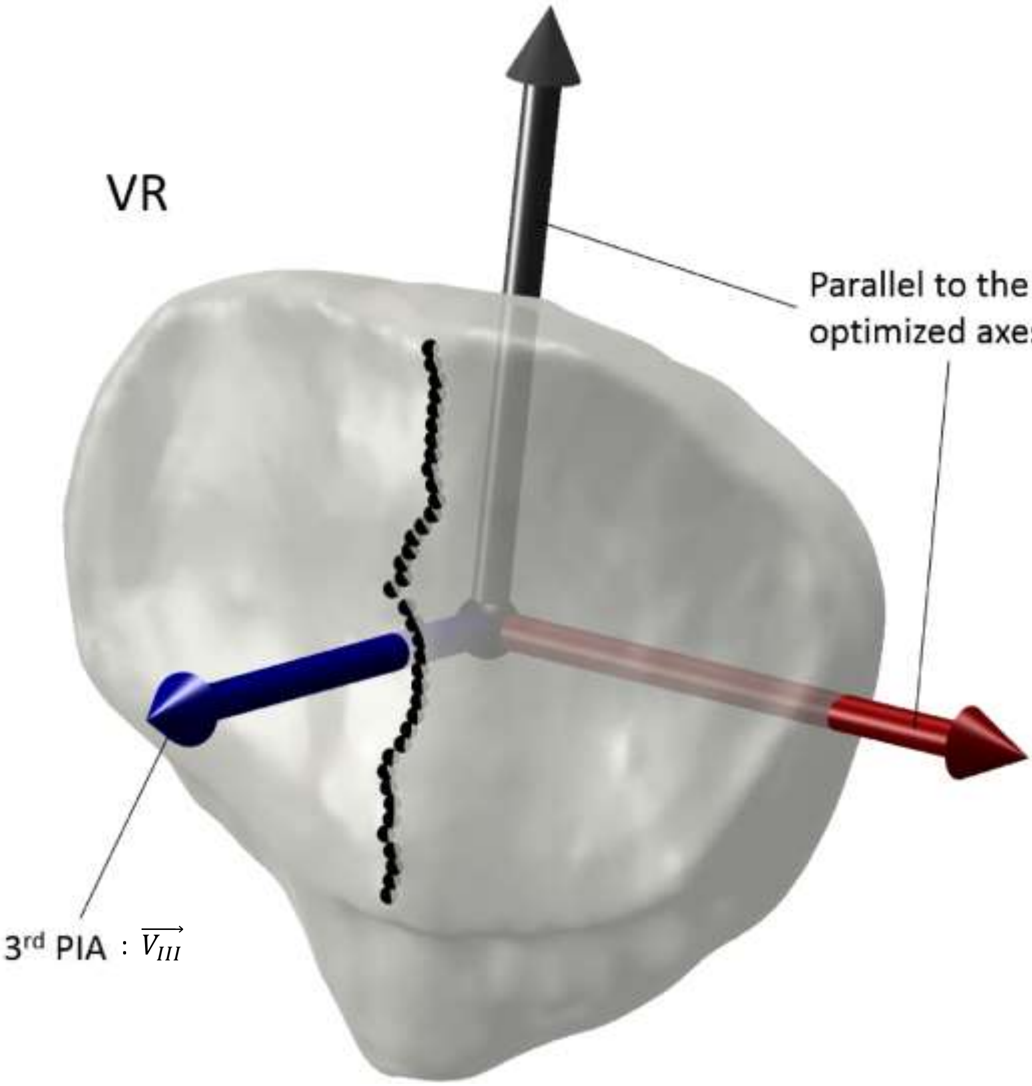
End of the common part of the Patellar algorithm variants

b. VR (Volume and Ridge) variant

1. Re-perform the ridge identification on only the articular surface part, using the previous result as initialization.

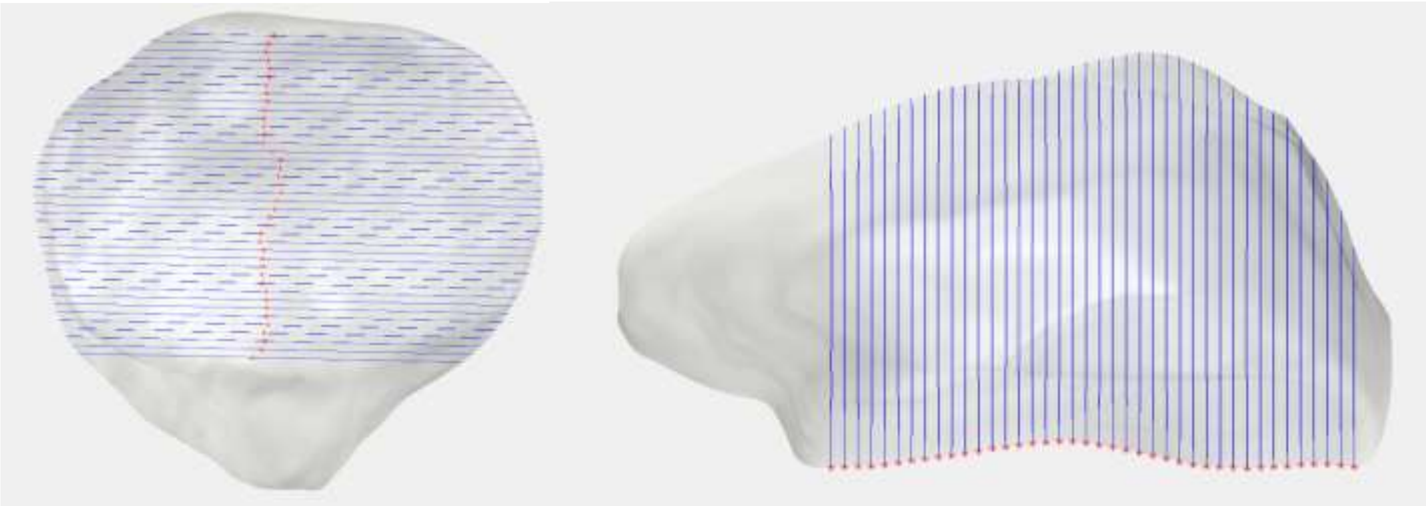


2. After the ridge identification is performed all the identified axis are transposed to the original coordinate system, to get the final ACS :



c. RL (Ridge Line) variant

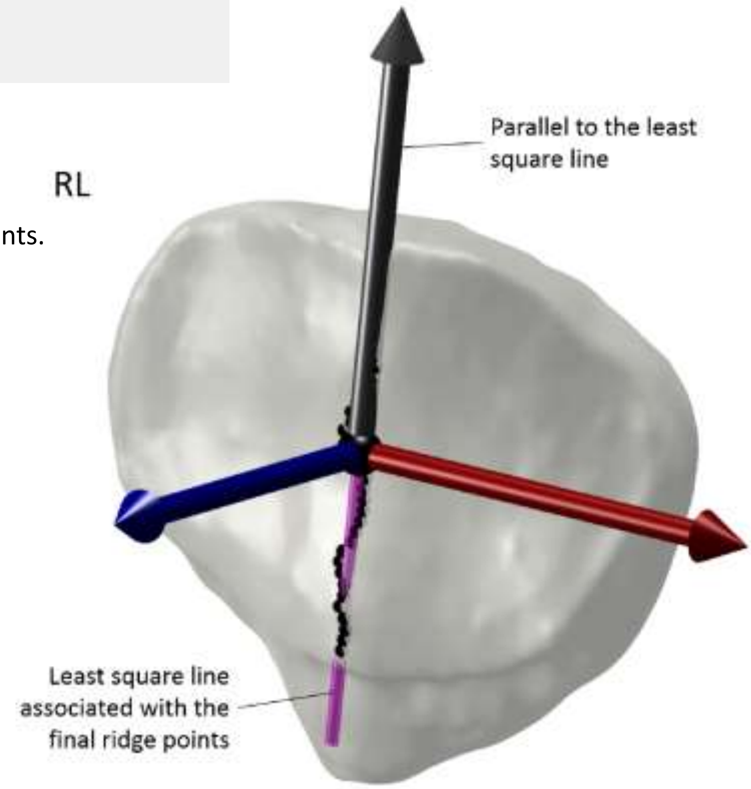
1. Re-perform the ridge identification on only the articular surface part, using the previous result as initialization.



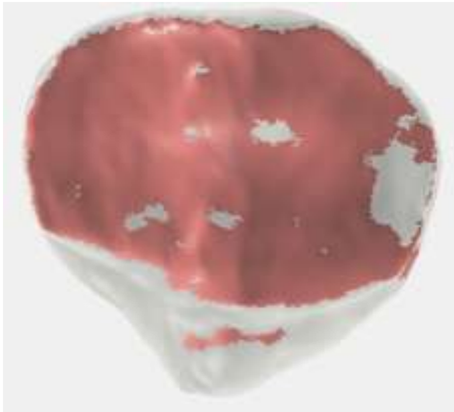
2. Fit a line on the lowest points, this line direction give $\overrightarrow{DP}$

3. Define $\overrightarrow{ML}$ as $\overrightarrow{ML} = \frac{\overrightarrow{DP} \wedge \overrightarrow{V_{III}}}{\|\overrightarrow{DP} \wedge \overrightarrow{V_{III}}\|}$ and $\overrightarrow{AP} = \overrightarrow{ML} \wedge \overrightarrow{DP}$

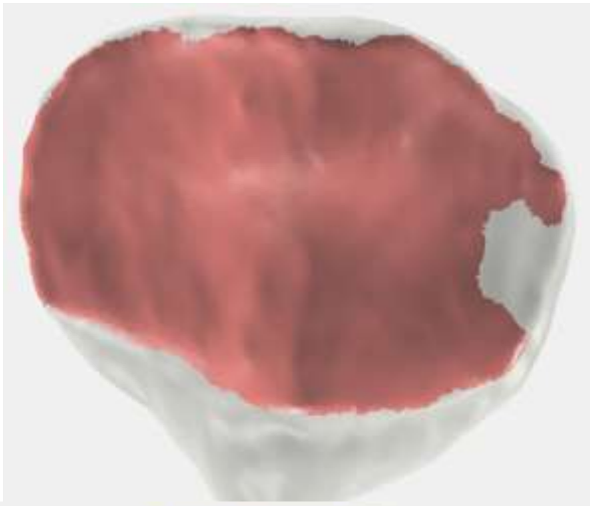
   The origin of the ACS is the barycenter of the lowest points.



d. Principal Inertia Axis of the Articular Surface (PIAAS) variant

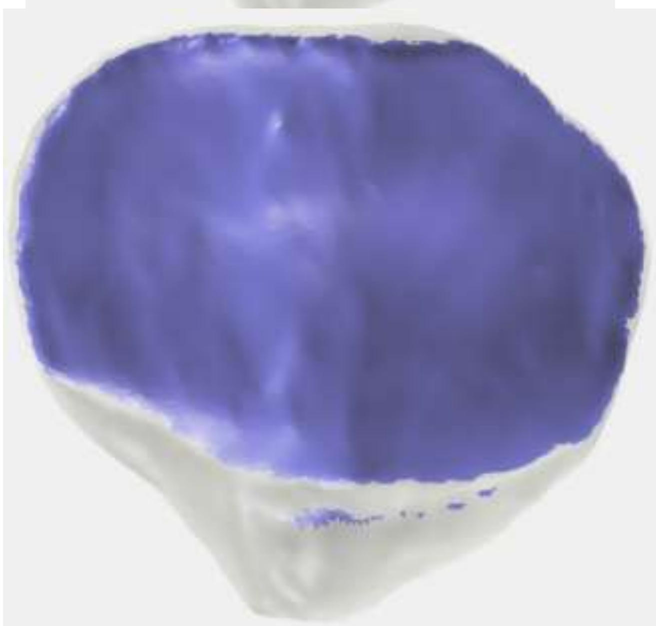


1. From RL algorithm ACS, an initial set of articular surface elements is identified with 3 conditions:
   a. Elment normals should make an angle with $\overrightarrow{DP}$ superior to 15°
   b. Elment normals should make an angle with $\overrightarrow{AP}$ inferior to 60°
   c. Element centers should be above (in the $\overrightarrow{AP}$ direction) a points located a quarter of the ridge length under the lowest ridge point.



2. Filter the isolated elemnts with an "open" morphology operation (2 elements), then fill the holes with a "close" one (4 elements). And keep only the regions that contain at least one of the ridge point. This gets us $ArtSurf_0$

3. Compute the least square plane of the $ArtSurf_0$. Normal of the plan: $\vec{n}$.



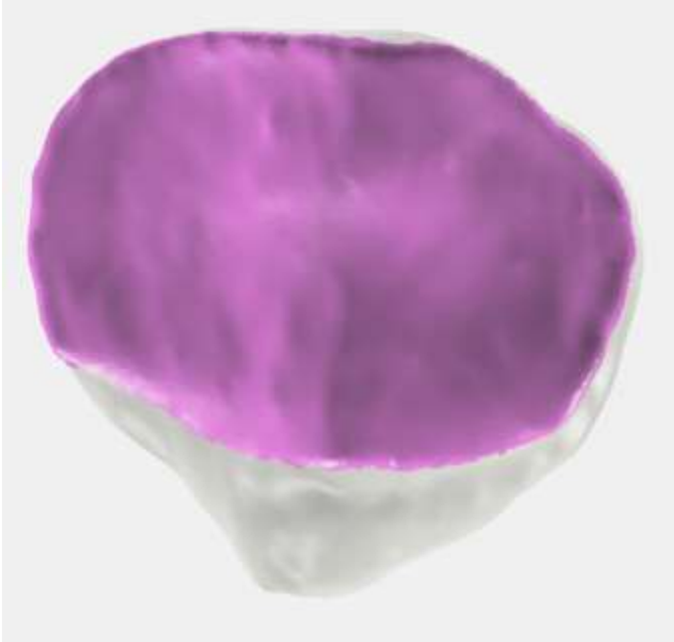
4. Dilate $ArtSurf_0$ by 10 elements and find the elements respecting this two conditions:
   a. Elment normals should make an angle with $\overrightarrow{DP}$ superior to 22°
   b. Elment normals should make an angle with $\vec{n}$ inferior to 54°

   Or this one :

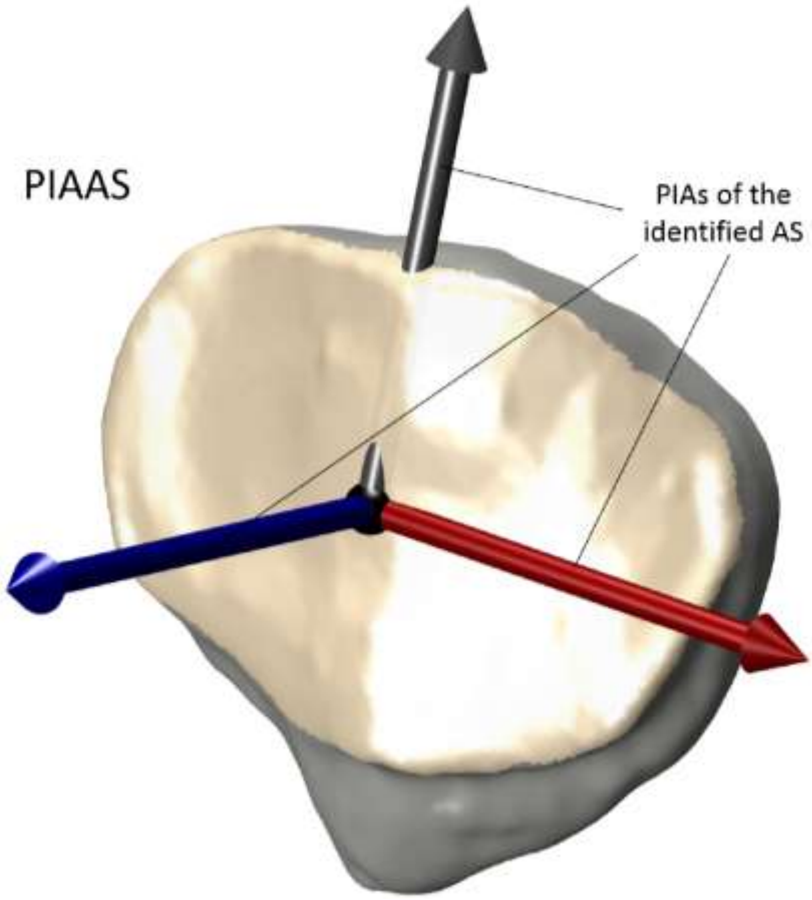
   c. Element centers should be above (in the $\overrightarrow{AP}$ direction) a points located 15% of the ridge length above the lowest ridge point. And the element normal sould make an angle inferior to 30° with $\vec{n}$.

   This generates $ArtSurf_1$

5. $ArtSurf_1$ and $ArtSurf_0$ are united and only the region that contains at ridge points are kept. Then the region is iteratively 'closed', 'opened' and 'eroded' to generate the final $ArtSurf_{End}$.



6. Then the principal ineria axis of $ArtSurf_{End}$ point cloud is computed and they serve as basis vector for the ACS, after being properly oriented from inferior to superior, from anterior to posterior. $ArtSurf_{End}$ barycenter serves as the origin of the ACS.



III. Tibia ACS Algorithm

a. Indentification of an initial distal-to-proximal axis

1. Compute the inertia tensor of the tibia. Eigen vectors of the inertia tensor are :

$$\overrightarrow{V_I}, \overrightarrow{V_{II}} \ \& \ \overrightarrow{V_{III}}$$

   We have :

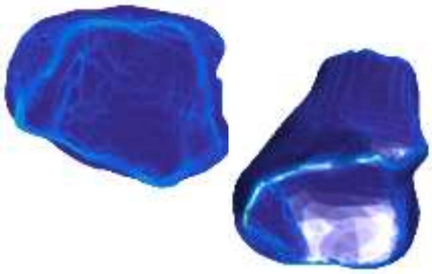
   $\overrightarrow{V_I}$: Distaloproximal or proximodistal
   $\overrightarrow{V_{II}}$: unconsistent direction
   $\overrightarrow{V_{III}}$: unconsistent direction

   C_0 the centroid volumetric representation of the tibia. Knowing the relative position of the proximal and distal part of the tibia, we defined $\overrightarrow{DP_0}$ oriented from distal to proximal :

$$\overrightarrow{DP_0} = sign\left[\left(\overrightarrow{OCentroid_{DistTibia}} - \overrightarrow{OCentroid_{ProxTibia}}\right).\overrightarrow{V_I}\right].\overrightarrow{V_I}$$

   And obtained a distal to proximal axis DP$_0$ = $\left(C_0, \overrightarrow{DP_0}\right)$.

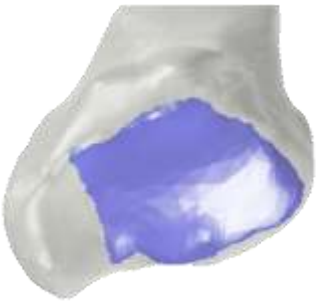
b. Identification of the ankle center



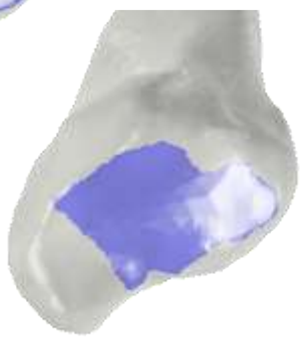
1. Compute the curvature of the distal tibia and identify elements that are in the highest third of the mean curvature and whose normal vectors make an angle inferior to 30° with $-\overrightarrow{DP_0}$. Close the region by 6 elements and open it by 4, then keep the region closest to the identified elements barycenter.
   We define our curvature C:

   $C = \sqrt{{k_1}^2 + {k_2}^2}$ with k_1 and k_2 the principal curvatures.

2. Fit a 2nd order polynomial surface (poly22 in Matlab) to it and exclude points that are two far (1mm) from the fitted surface, then smooth the results with open & close morphology operations

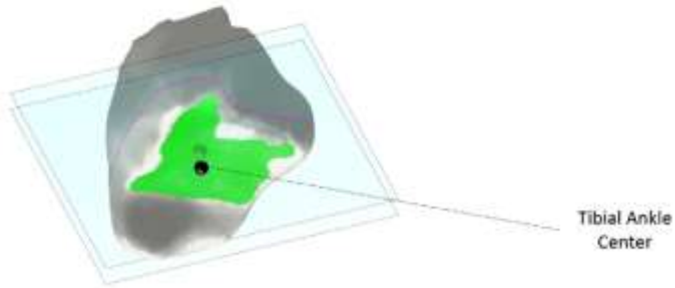


3. Delete elements whose normal doesn't go towards the Articular surface centroid.



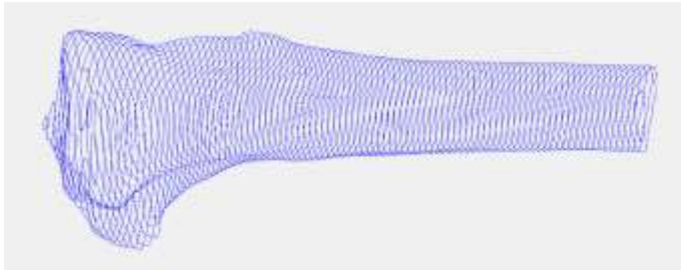
4. Open articular surface by 4 elements and close it by 10, to smooth the results.

5. Then a plan is fitted and offset upward by 5 mm, and the centroid of bone section at the offset plan, projected back on the original plan serves as an ankle center.

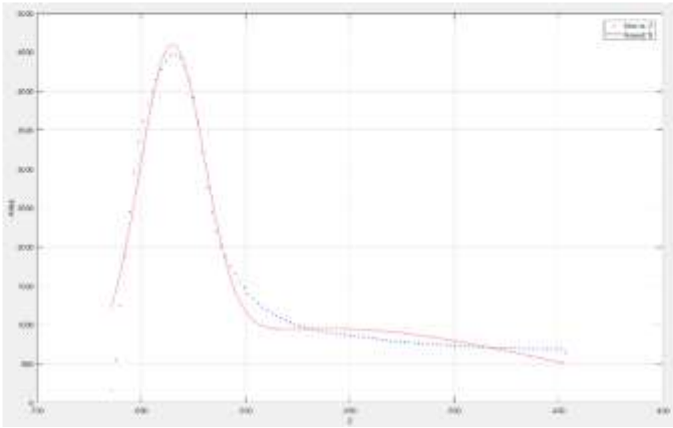


6. Next the vector between the most distal point (tip of the medial malleolus) and the tibial ankle center, is made perpendicular to $\overrightarrow{DP_0}$ to get a first lateral to medial vector $\overrightarrow{LM_0}$

c. Identification of the proximal condyles axis

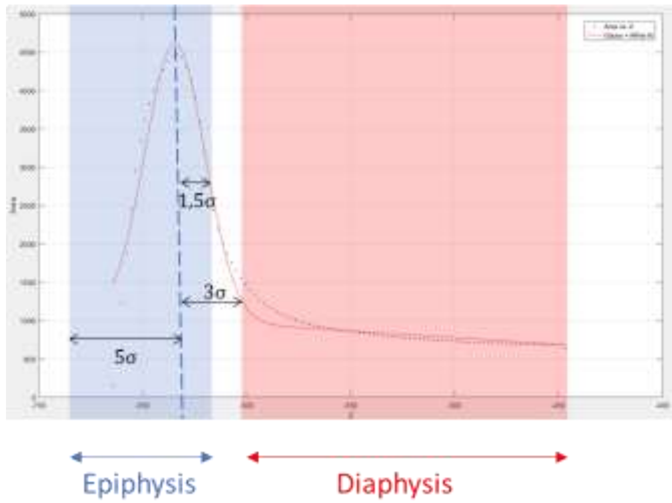


7. The cross-section area along $\overrightarrow{DP_0}$ of the proximal tibia was computed



8. The evolution of the CSA along DP_0 axis, was fitted with a double gaussian function:

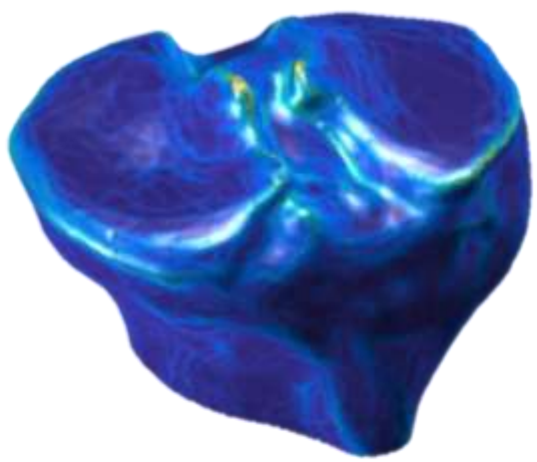
$$f_{G2} = a_1 . e^{-\frac{(z-b_1)^2}{c_1}} + a_2 . e^{-\frac{(z-b_2)^2}{c_2}}$$



9. The identified parameters (a_1, b_1, c_1) of the previous fit were used as initialization values to perform a gaussian + linear fit:

$$f_{GL} = a_3 . e^{-\frac{(z-b_3)^2}{c_3}} + d.x + e$$

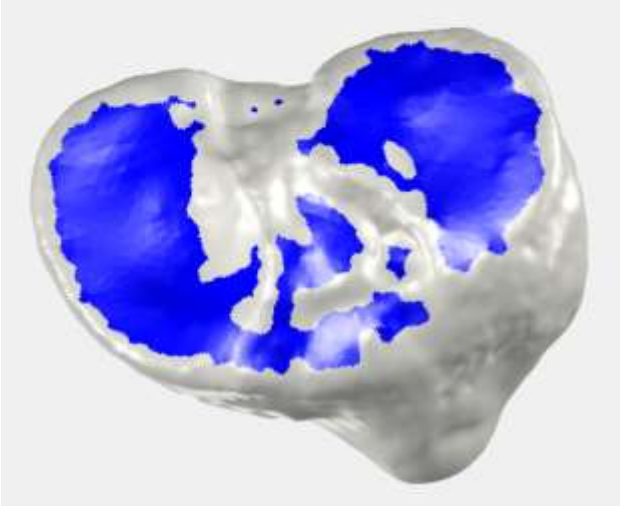
This allow to separate the epiphysis from the rest of the proximal tibia.



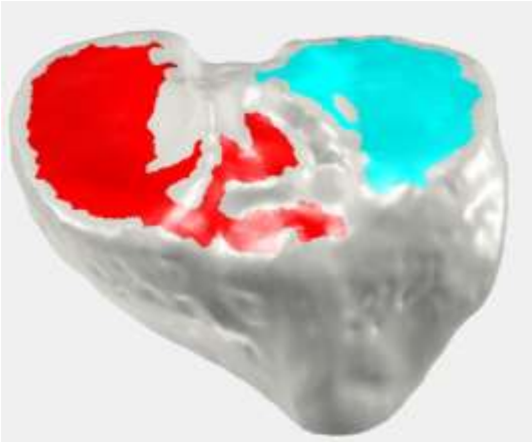
10. The curvature of the epiphysis is computed, since we are interested on AS elements that lie on flat region of the bone. We define our curvature C:

$C = \sqrt{{k_1}^2 + {k_2}^2}$ with $k_1$and k_2 the principal curvatures.

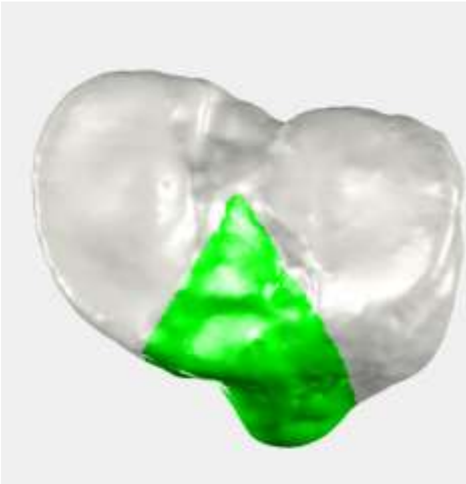
11. Elements that are above the first quartile of C, and whose normal make an angle with $\overrightarrow{DP_0}$ superior to 35° [To account for extreme Varus or Valgus], are removed.



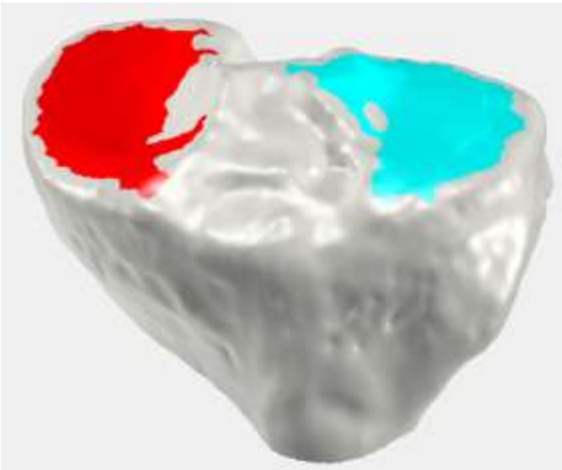
12. By fitting an ellipse on the exterior edges of the identified elements projected onto their least square plan, a medial and lateral region were obtained (thanks to $\overrightarrow{LM_0}$).



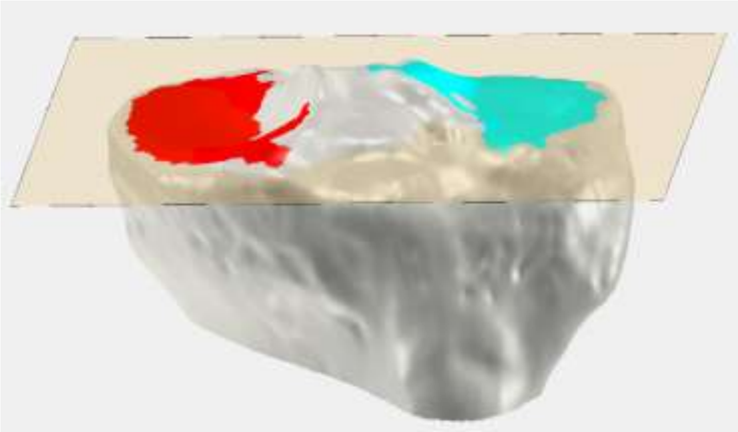
13. Thanks to the fitted ellipse orientation, a zone between the spines is identified as a non-articular surface area.



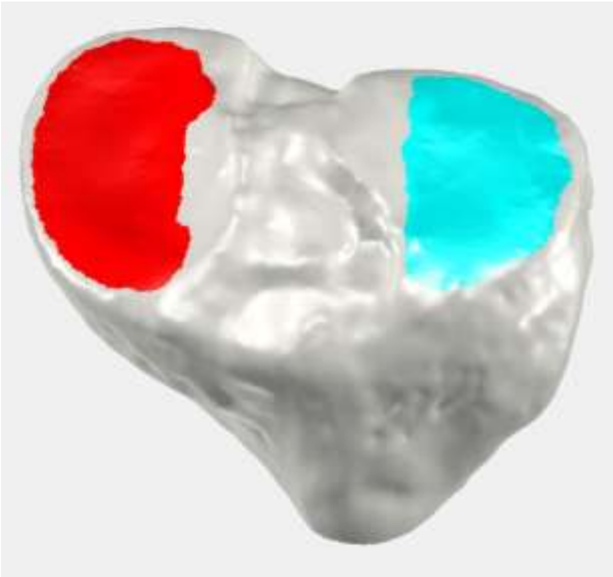
14. The articular surfaces element previously identified that are in the between spines zone (green) are deleted.



15. A least square plan is associated to the articular surfaces



16. All the elements that are within 5mm of the plane and normal that makes an angle with the plane normal inferior to 25°, are kept and the region are opened with 2 elements closed with 10 elements, to smooth the AS regions.



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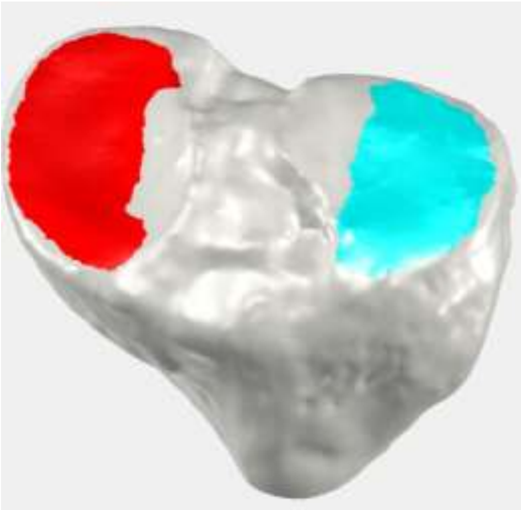
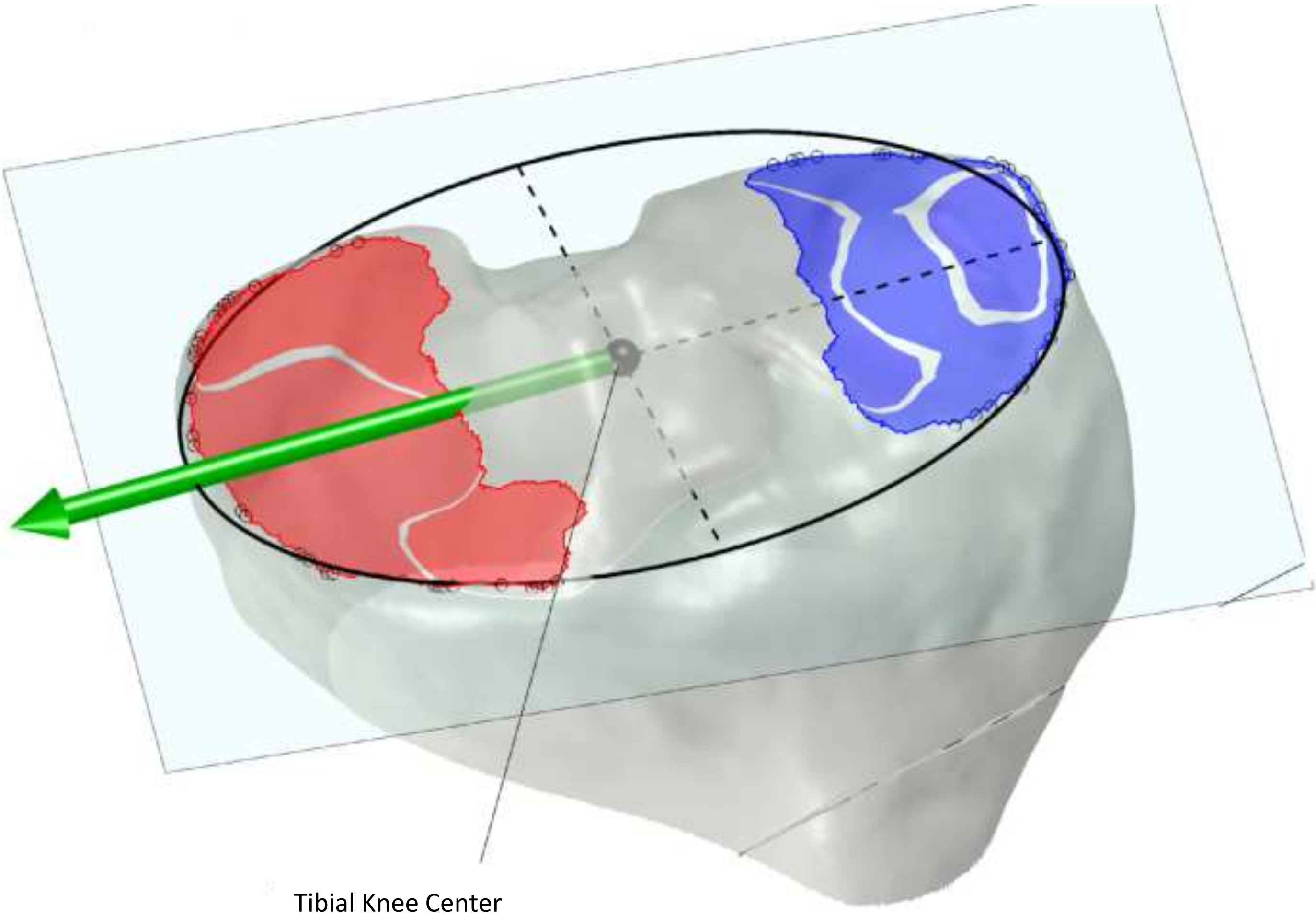
17. The previous step is repeated with an angles inferior to 18°



End of the common part of the tibial algorithm variants

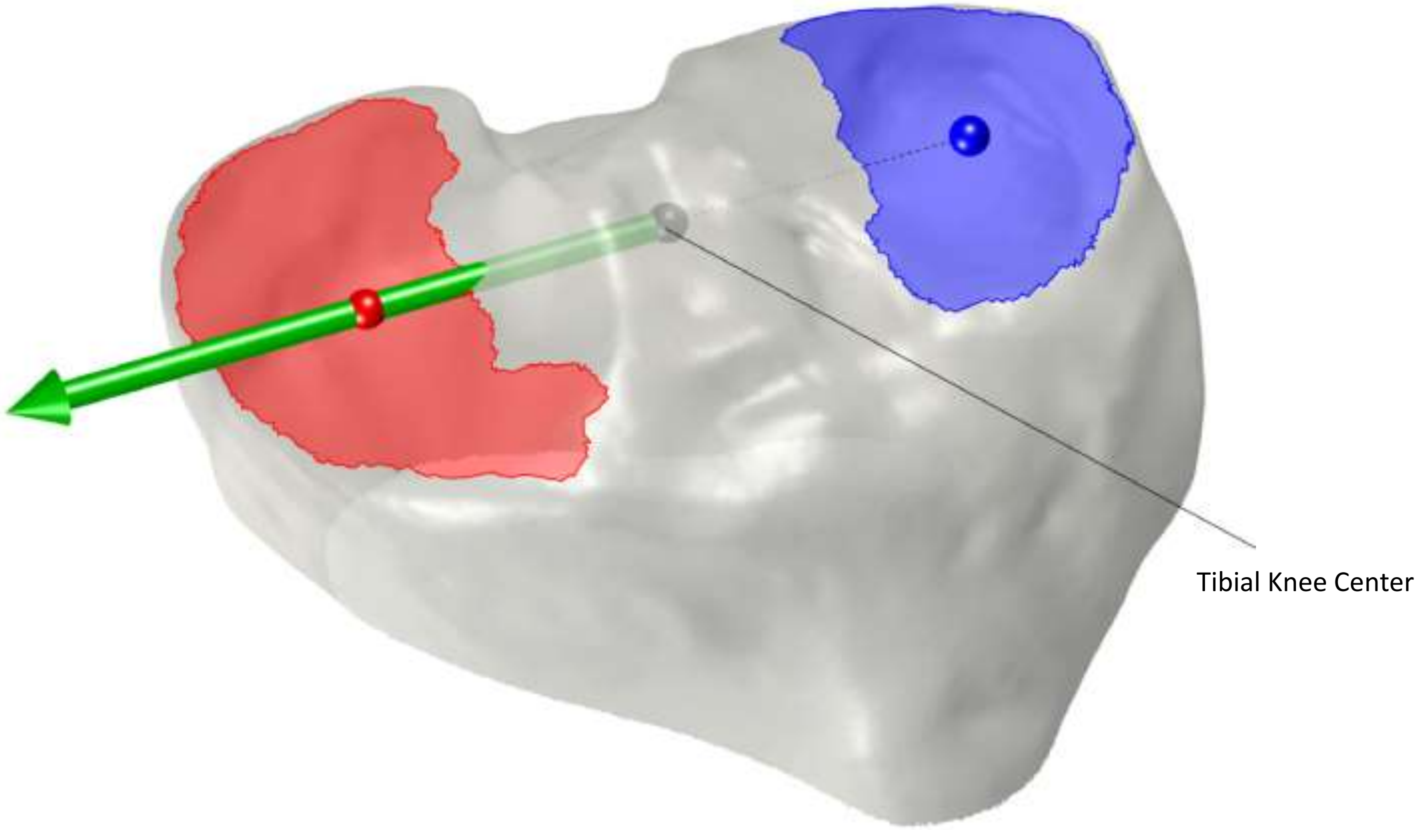
d. ECASE (Ellipse on Condyle Articular Surface Edges) variant

1. The identified Articular Surface Elements are projected on their associated least square plane and a 2D convex Hull is computed. Then an ellipse is fitted on the convex hull vertices. The ellipse major axis served as the condyle axis and its center as the tibial knee center.



e. CASC (Condyle Articular Surface Centroids) variant

1. The centroids of the lateral and medial Articular Surfaces are identified. They are connected in a segment whose midpoint define the tibial knee center and direction the condyle axis.

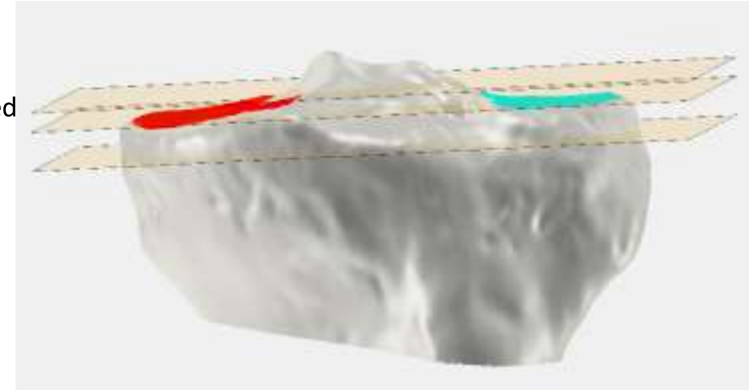


f. PIAASL (Principal Inertia Axis of the Articular Surface Layer) variant

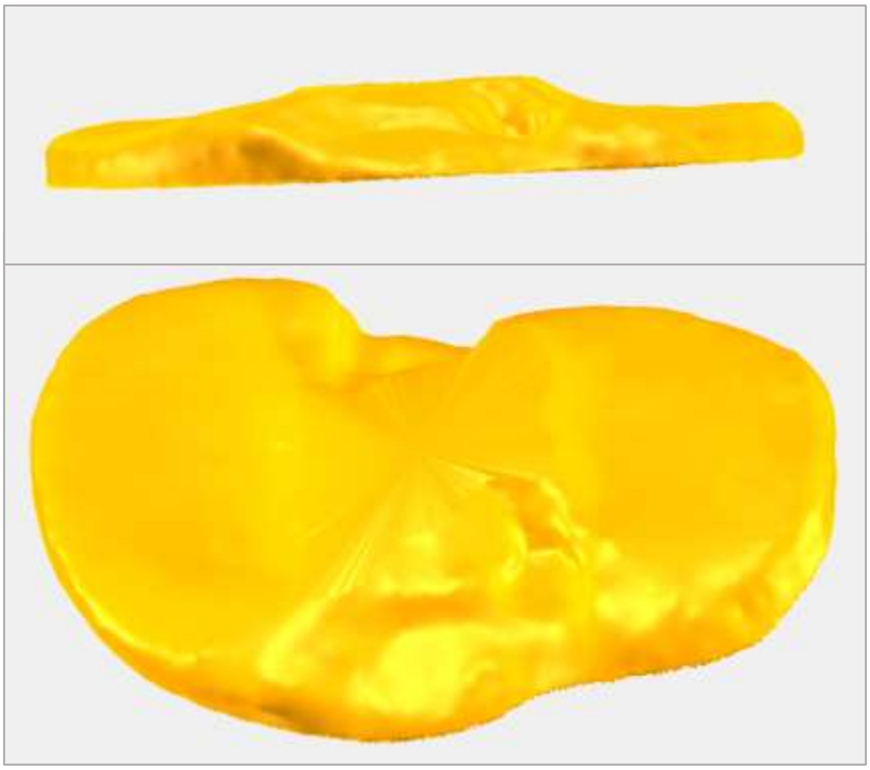
1. From the max cross section area a distance H is computed:

$$H = \sqrt{\frac{3}{\pi} Area_{Max}}$$

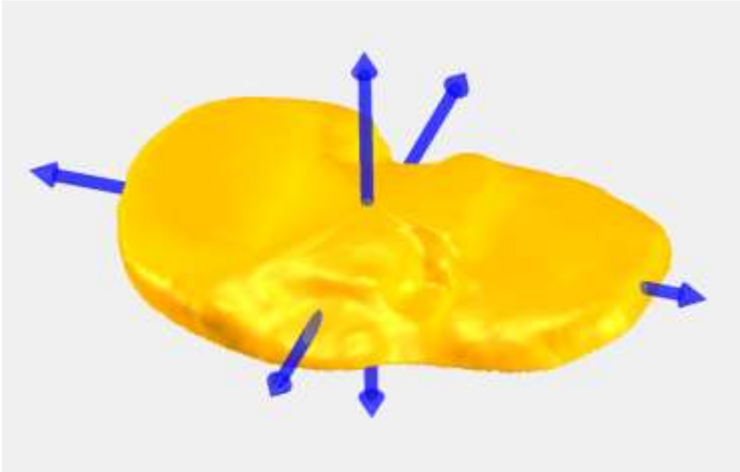
2. Then the articular surface least square plan is offsetted upward by 0.5*H and downward by H



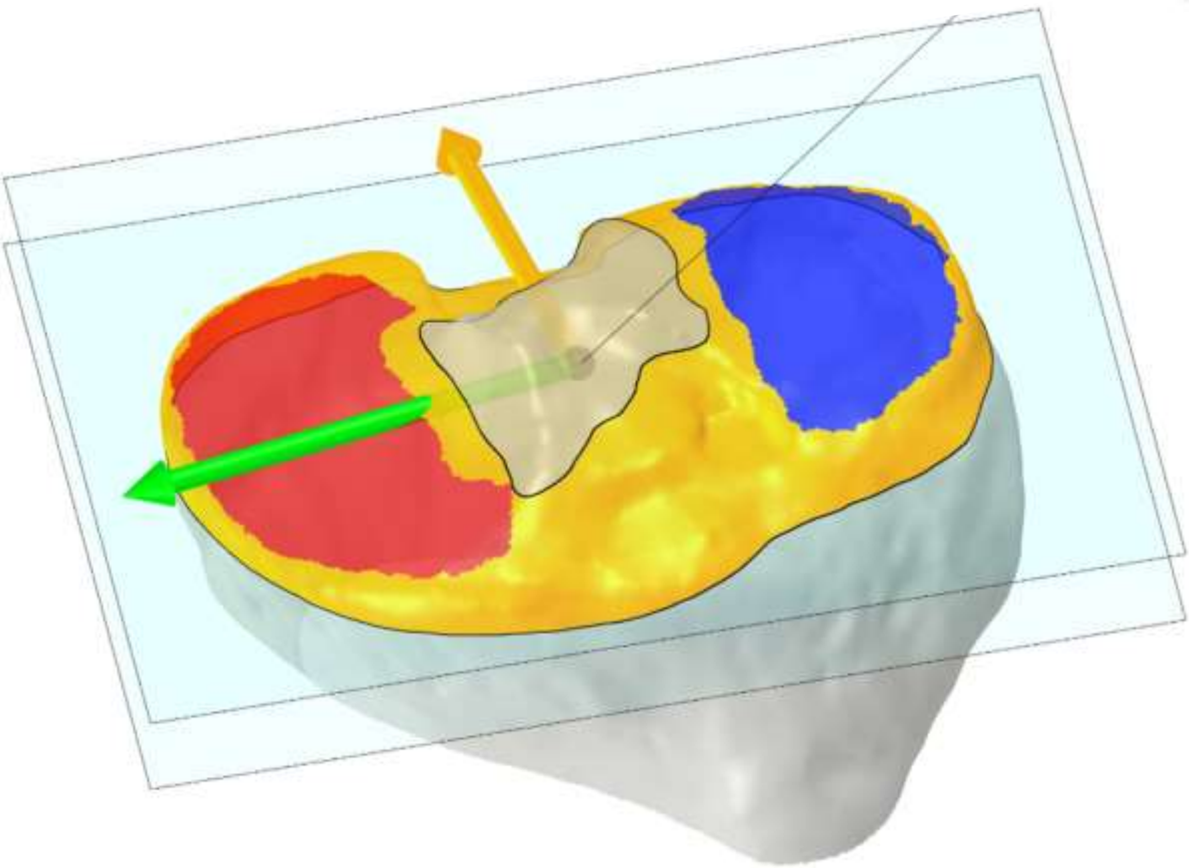
3. Then the tibia layer between those two planes is extracted.



4. The principal inertia axis of this layer are computed



5. The centroid of this layer is projected on the articular surface plane to define the knee center. And the first principal inertia axis serves as the condyle axis.



End of Supplementary Material-B