$$Y_t = e_1 + e_2 + \dots + e_t \qquad e_t \overset{iid}{\sim} N(0, \sigma_e^2)$$

$$\mathbb{E}[Y_t] = \underset{0}{\mathbb{E}[e_1]} + \dots + \underset{0}{\mathbb{E}[e_t]} = 0$$

$$\mathrm{var}(Y_t) = \mathbb{E}[Y_t^2] - \underbrace{\mathbb{E}[Y_t]^2} = \mathbb{E}[(e_1 + \dots + e_t)^2] \qquad \underbrace{\mathrm{cov}(e_i, e_j)}_{=0}$$

$$= \mathbb{E}\left[\sum_{i=1}^t e_i^2\right] + \mathbb{E}\left[\sum_{i=1}^t \sum_{\substack{j=1 \\ j \neq i}}^t e_i e_j\right] = \sum_{i=1}^t \underbrace{\mathbb{E}[e_i^2]}_{\mathrm{var}(e_i) = \sigma_e^2} + \sum_{i=1}^t \sum_{j \neq i}^t \mathbb{E}[e_i e_j]$$

$$= t\sigma_e^2 \rightarrow \text{No es estacionario!}$$

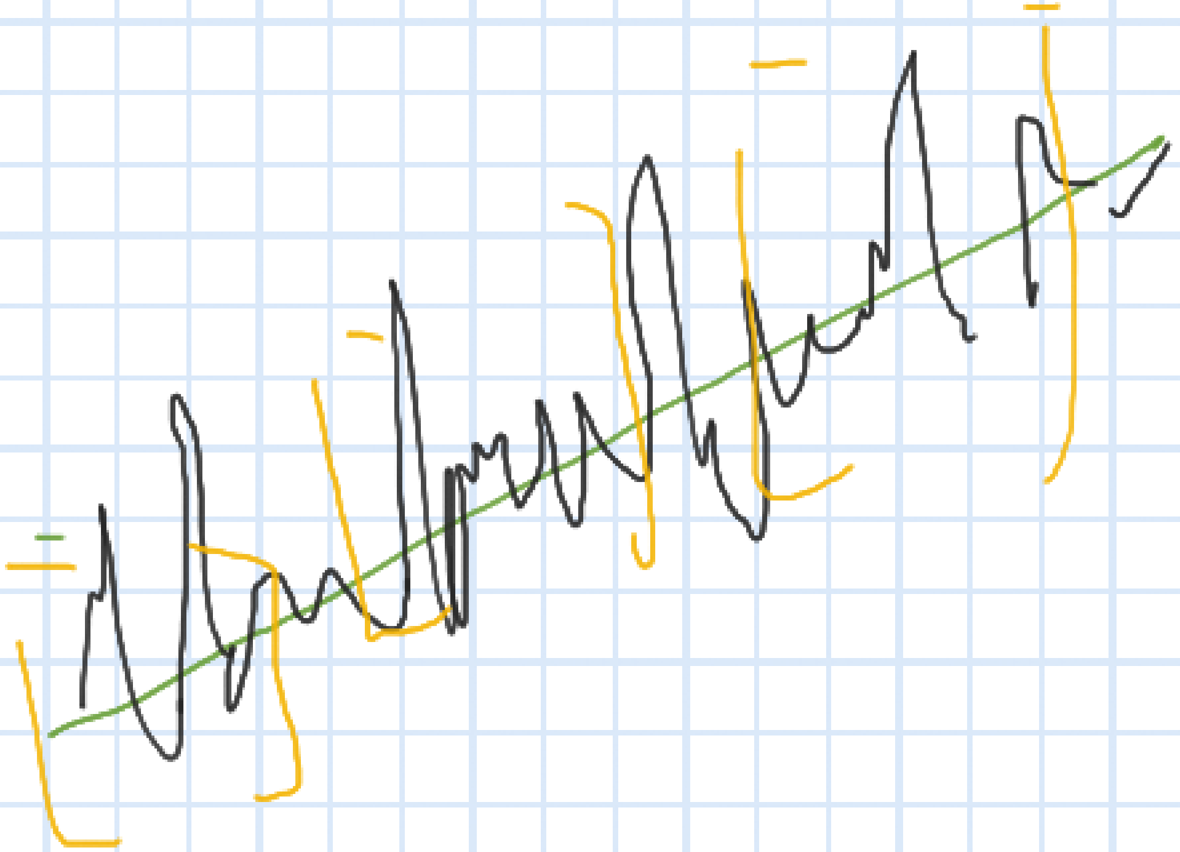
$$\mathrm{cov}(Y_t, Y_s) \overset{t \le s}{=} \mathrm{cov}(e_1 + \dots + e_t, e_1 + \dots + e_t + e_{t+1} + \dots + e_s)$$

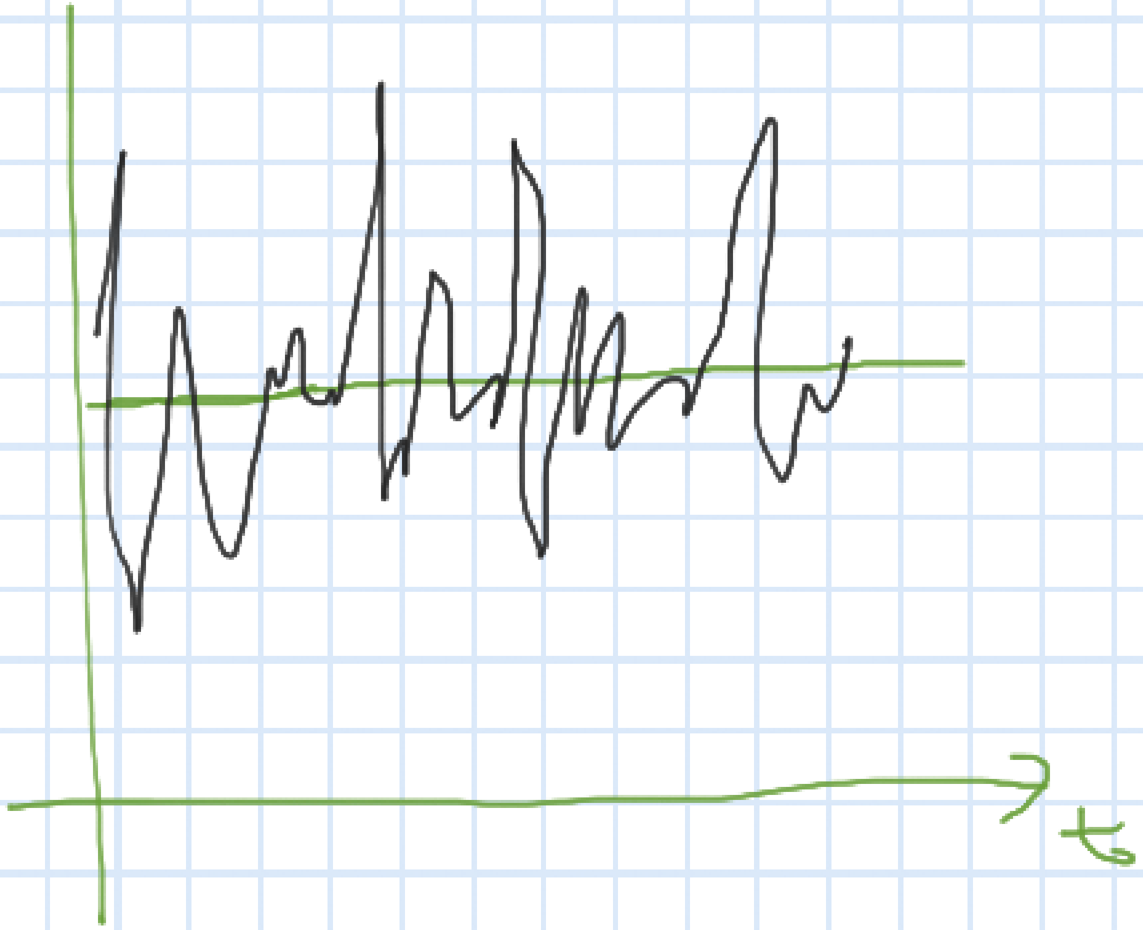
$$= \mathrm{cov}(e_1, e_1) + \dots + \mathrm{cov}(e_t, e_t)$$

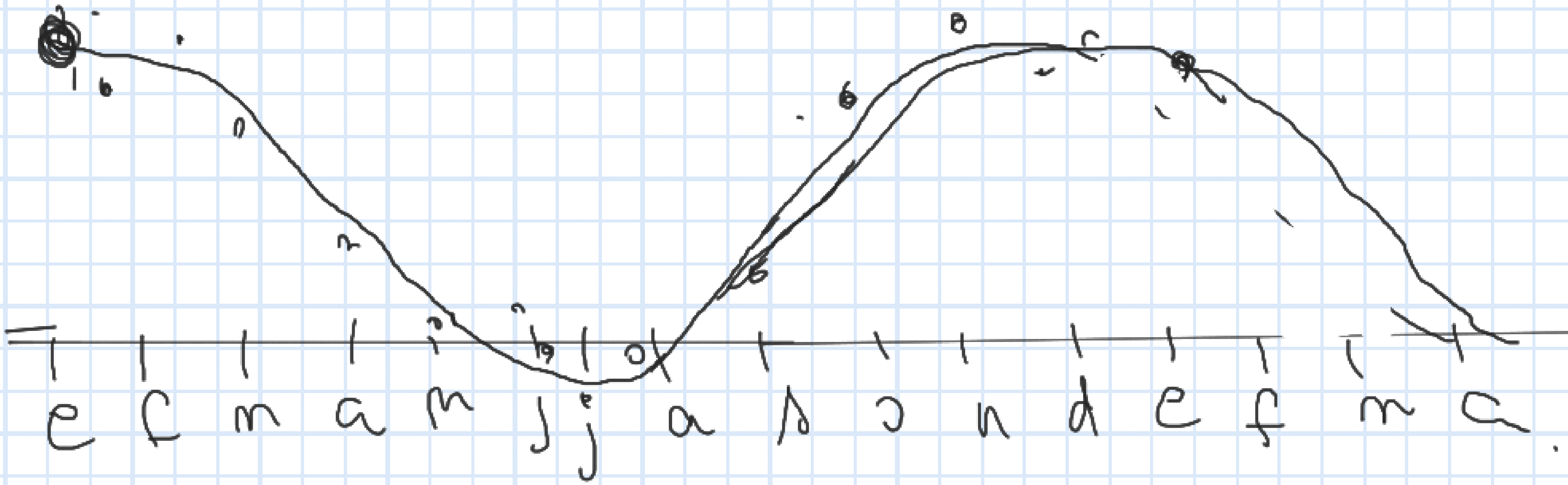
$$= t\,\mathrm{var}(e) = t\sigma_e^2, \quad t \le s \; = C_{t,s}$$

$$R_{t,s} = \frac{\operatorname{cov}(Y_t, Y_s)}{\sqrt{\operatorname{var}(Y_t)\operatorname{var}(Y_s)}} = \frac{t\,\cancel{\sigma_e^2}}{\sqrt{t\,\cancel{\sigma_e^2} \cdot s\,\cancel{\sigma_e^2}}} = \sqrt{t/s}$$

$t \leq s$



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Tendencia caso constante

$$y_1 = \mu + x_1$$

$$y_2 = \mu + x_2$$

$$\vdots$$

$$y_n = \mu + x_n$$

$$\longrightarrow \quad \mathbb{E}[y_t] = \mathbb{E}[\mu + x_t] = \mu + \underbrace{\mathbb{E}[x_t]}_{=0}$$

$$\hat{\mu} = \frac{1}{n}\sum_{t=1}^{n} y_t$$

$$y = Ax + n$$

$$y = Ax$$

$$(A^T A)^{-1} A^T y = \hat{x}$$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \mu + \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$$\underbrace{\left([1 \dots 1] \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \right)^{-1}}_{1/n} \underbrace{[1 \dots 1] \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}}_{\sum_{t=1}^{n} y_t} = \frac{1}{n}\sum_{t=1}^{n} y_t$$

Caso lineal $\qquad y_t = \underbrace{\beta_0 + \beta_1 t}_{\mu} + x_t$

↳ serie estacionaria de media cero.

$$\underbrace{\begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ \vdots & \vdots \\ 1 & m \end{bmatrix}}_{A} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} + \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & m \end{bmatrix} \begin{bmatrix} 1 & 1 \\ \vdots & \vdots \\ 1 & m \end{bmatrix}$$

$$\begin{bmatrix} \hat\beta_0 \\ \hat\beta_1 \end{bmatrix} = \left[A^T A\right]^{-1} A^T y = \begin{bmatrix} m & \sum_{i=1}^{m} i \\ \sum_{i=1}^{m} i & \sum_{i=1}^{m} i^2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & \cdots & 1 \\ 1 & \cdots & m \end{bmatrix} y$$

$$= \frac{1}{m\sum i^2 - \left(\sum i\right)^2} \begin{bmatrix} \sum i^2 & -\sum i \\ -\sum i & m \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{m} y_i \\ \sum_{i=1}^{m} i\, y_i \end{bmatrix}$$

Caso cuadrático $y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + x_t$

$$\begin{bmatrix} y_n \\ \vdots \\ y_1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & n & n^2 \\ 1 & (n-1) & (n-1)^2 \\ \vdots & \vdots & \vdots \\ 1 & 1 & 1 \end{bmatrix}}_{A} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} x_n \\ \vdots \\ x_1 \end{bmatrix}$$

$$\begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix} = \left(A^T A\right)^{-1} A^T \begin{bmatrix} y_n \\ \vdots \\ y_1 \end{bmatrix}$$

Caso cíclico

$$y_t = x_t + \begin{cases} \beta_1 & t = 1, T+1, 2T+1, \dots \\ \beta_2 & t = 2, T+2, 2T+2, \\ \vdots & \\ \beta_T & t = T, t = 2T, \dots \end{cases}$$

$$\begin{bmatrix} y_n \\ \vdots \\ y_{T+1} \\ y_T \\ \vdots \\ y_2 \\ y_1 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \dots & 0 & 1 & 0 \\ & & & & \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 1 \\ & & \vdots & & \\ 0 & 1 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \end{bmatrix}}_{A} \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_T \end{bmatrix} + \begin{bmatrix} x_n \\ \vdots \\ x_1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \dots & 1 & 0 & \dots & 1 \\ 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ 1 & & 0 & 0 & & \\ 0 & & 0 & 0 & & \\ & & 0 & 1 & & 0 \end{bmatrix} \begin{bmatrix} y_n \\ \vdots \\ y_1 \end{bmatrix} = \begin{bmatrix} \sum_{k=0} y_{kT+1} \\ \sum_{k} y_{kT+2} \\ \vdots \end{bmatrix}$$

$$\hat{\beta}_k = \frac{1}{m/\!/T} \sum_{j=0}^{m/\!/T} y_{jT+k}$$

caso coseno $y_t = \beta \cos(2\pi f \cdot t + \varphi) \; \beta_0 + \cancel{\beta_1} t$

$= \beta_1 \cos(2\pi f t) + \beta_2 \sin(2\pi f t)$

$\beta_1 = \beta \cos(\phi)$, $\beta_2 = \beta \sin(\phi)$

$\cos(a+b) = \cos(a)\cos(b) + \sin(a)\sin(b)$

$$\begin{bmatrix} y_n \\ \vdots \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 & \cos(2\pi f n) & \sin(2\pi f n) \\ \vdots & \vdots & \\ 1 & \cos(2\pi f 1) & \sin(2\pi f 1) \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} x_n \\ \vdots \\ x_1 \end{bmatrix}$$

Test Dickey-Fuller

$$y_t = a_1 y_{t-1} + e_t$$

$$y_t - y_{t-1} = a_1 y_{t-1} - y_{t-1} + e_t$$

$$\rightarrow \Delta y_t = (a_1 - 1) y_{t-1} + e_t$$