$$\hat{Y}_t(\ell) - \mu = \mathbb{E}\left[Y_{t+\ell} - \mu \mid Y_1, \ldots, Y_t\right] = \mathbb{E}\left[a_1(Y_{t+\ell-1} - \mu) + e_{t+\ell} \mid Y_1 \ldots Y_t\right]$$

$$Y_{t+\ell} - \mu = a_1(Y_{t+\ell-1} - \mu) + e_{t+\ell}$$

$$= a_1\Big(\underbrace{\mathbb{E}[Y_{t+\ell-1} \mid Y_1, \ldots, Y_t]}_{\hat{Y}_t(\ell-1)} - \mu\Big) + \underbrace{\mathbb{E}[e_{t+\ell} \mid Y_1 \ldots Y_t]}_{\mathbb{E}[e_{t+\ell}] = 0}$$

$$= a_1\Big(\underbrace{\hat{Y}_t(\ell-1)}_{a_1 \underbrace{\hat{Y}_t(\ell-2)}_{a_1 \hat{Y}_t(\ell-3)}} - \mu\Big) \qquad \ell \geq 1$$

$$= a_1^{\ell}(Y_t - \mu)$$

$$\sum_{n=0}^{\infty} b^n \underset{|b|<1}{=} \frac{1}{1-b}$$

$$\mathrm{var}(e_{t}) = \sum_{n=0}^{\ell-1} (a_1^2)^k = \frac{1-a_1^{2\ell}}{1-a_1^2}$$

$$1 + a_1^2 + (a_1^2)^2 + (a_1^2)^3 + \dots$$

por prop. de la esp. condic.

$$e_t \sim g(Y_1, \dots, Y_t) \Rightarrow \mathbb{E}[e_t \mid Y_1 \dots Y_t] = e_t$$

AR1

$$Y_t - \mu = a_1(Y_{t-1} - \mu) + e_t$$

$$Y_t = a_1(Y_{t-1} - \mu) + \mu + e_t.$$

RW

$$Y_{t+1} = Y_t + \theta + e_{t+1}$$

$$Y_{t+2} = Y_{t+1} + \theta + e_{t+2} = Y_t + 2\theta + e_{t+1} + e_{+2}.$$

$$\vdots$$

$$Y_{t+l} = Y_t + l\theta + \underbrace{e_{t+l}}_{} \underbrace{+ \dots + e_{t+1}}_{}$$

ARMA

$$\mathbb{E}\left[Y_{t+l} \mid Y_1 \dots Y_t\right] = \mathbb{E}\left[a_1 Y_{t+l-1} + \dots + a_p Y_{t+l-p} + e_{t+l} - b_1 e_{t+l-1} - \dots - b_q e_{t+l-q} \mid Y_1 \dots Y_t\right]$$

$$= a_1 \underbrace{\mathbb{E}\left[Y_{t+l-1} \mid Y_1 \dots Y_t\right]}_{\hat{Y}_t(l-1)} + \dots + a_p \underbrace{\mathbb{E}\left[Y_{t+l-p} \mid Y_1, \dots, Y_t\right)}_{\hat{Y}_t(l-p)}$$

$$+ b_1 \mathbb{E}\left[e_{l-1} \mid Y_1 \dots Y_t\right] - \dots - b_q \mathbb{E}\left[e_{tl-q} \mid Y_1 \dots Y_q\right)$$

Tenía $\hat{Y}_t(l+1) = \mathbb{E}\left[Y_{t+l+1} \mid Y_1 \ldots Y_t\right]$

$$\hat{Y}_{t+1}(l) = \mathbb{E}\left[Y_{t+l+1} \mid Y_1 \ldots Y_t\right]$$