$Y$, dimensionless temperature, $T/T^b$, $Y^{max} = T^{max}/T^b$, $Y_c = T_c/T^b$
$Y^{S1,S2}$, dimensionless interface temperatures
$z$, position across the catalyst, cm

*Greek Letters*

$\alpha$, normalized expansion/contraction factor, $\epsilon x_A{}^b$
$\beta$, thermicity factor, $(-\Delta H_r)C_A{}^b D_{AC}/(k_{eff}T^b)$, dimensionless
$\gamma$, dimensionless activation energy, $E/RT^b$
$\delta$, thickness of the catalyst coat, cm
$\epsilon$, expansion/contraction factor
$\eta$, effectiveness factor (see eq 11)
$\rho_c$, catalyst density, g/cm$^3$
$\phi$, dimensionless catalyst coating thickness or Thiele modulus, $\delta[\rho_c k_0 e^{-E/RT^b}[C_A{}^b]^n/C_A{}^b D_{AC}]^{1/2}$

## Literature Cited

Abramovitz, M., Stegun, I. A., Ed., "Handbook of Mathematical Functions", p 231, NBS, Applied Mathematics Service, Vol. 35, U.S. Government Printing Office, Washington, D.C., 1966.
Aris, R., "The Mathematical Theory of Diffusion and Reaction in Permeable Catalysts", Clarendon Press, Oxford, 1975.
Carberry, J. J., "Chemical and Catalytic Reaction Engineering", McGraw-Hill, New York, N.Y., 1976.
Carberry, J. J., *Ind. Eng. Chem.*, **58**(10), 40 (1966).
Forney, A. J., et al., "A Process to Make High-Btu Gas from Coal", USBM-TPR, No. 24 (1970).
Haynes, W. P., Elliott, J. J., Youngblood, A. J., Forney, A. J., "Operation of a Sprayed Raney Nickel Tube-Wall Reactor for Production of a High-Btu Gas", paper presented at Fuels Division, 160th National Meeting of the American Chemical Society, Chicago, Ill., 1970
McGreavy, C., Cresswell, D. L., *Chem. Eng. Sci.*, **24**, 608 (1969).
Senkan, S. M., Sc.D. Thesis, Massachusetts Institute of Technology, Cambridge, Mass., 1977.
Senkan, S. M., Evans, L. B., Howard, J. B., *Ind. Eng. Chem. Process Des. Dev.*, **15**, 184 (1976).
Weisz, P. B., Hicks, J. S., *Chem. Eng. Sci.*, **17**, 265 (1962).

*Received for review* November 18, 1977
*Accepted* July 14, 1978

# Momentum Transfer in Curved Pipes. 1. Newtonian Fluids

**P. Mishra* and S. N. Gupta**

*Departments of Chemical Engineering and Mechanical Engineering, Institute of Technology, Banaras Hindu University, Varanasi-221005, India*

Pressure drop data in the laminar and turbulent region for Newtonian fluids flowing through 60 helical coils are presented. Coils of wide range of diameter and pitch are investigated to fulfil the snags existing in the previous investigations and to resolve the effect of coil pitch. The following correlations are obtained for $D_t/D_c$ range 0.003 to 0.15 and $p/D_c$ range 0 to 25.4. (1) For laminar flow: $f_c/f_{SL} - 1 = 0.033(\log N_{D_m})^{4.0}$, where $1 < N_{D_m} < 3000$. (2) For critical Reynolds number: $N_{Re_c} = 2 \times 10^4(D_t/2R_c)^{0.32}$. (3) For turbulent flow: $f_c - f_{ST} = 0.0075(D_t/2R_c)^{1/2}$, where $4500 < N_{Re} < 10^5$.

## Introduction

Curved pipes in the form of helical and spiral coils are extensively used in industries for heating and cooling of Newtonian and non-Newtonian fluids. The existence of a secondary flow by the action of centrifugal force has attracted many workers, e.g., Grindley and Gybson (1908), Eustice (1910, 1911), Dean (1927, 1928), White (1929), Miyagi (1932), Tomita (1932), Adler (1934), Keulegan and Beij (1937), Hawthorne (1951), Detra (1953), Cummings (1955), Ito (1959, 1969), and Truesdell and Adler (1970), to investigate the laminar and turbulent flow behavior of Newtonian fluids flowing through the curved pipes. However, very little information is available for the case where pitch of the helical coils makes an effective contribution to the total pressure drop. It will, therefore, be an important addition to the existing literature if the effect of the coil pitch in terms of radius of curvature is experimentally investigated.

In this work (referred to as part 1), the effect of coil diameter, pitch, and coil tube diameter on the friction loss for Newtonian fluids flowing through helical coils covering a wide range of variables is studied. The investigations of this part will frame the basis for predicting the non-Newtonian flow behavior in similar coils presented in part 2.

## Experimental Setup

The schematic diagram of the experimental setup is shown in Figure 1. Thick-walled, flexible, and smooth polythene pressure tubings of uniform circular cross sections with inside diameters of 0.62, 0.78, 1.165, 1.735, and 1.905 cm were chosen for the present study. Fluid circulation was provided by a three-stage centrifugal pump suitable for handling different fluids studied in the present work. The pump used was capable of producing a 45-m head with a maximum discharge rate of 250 L/min. The fluid tank had 200 L capacity and was equipped with a propeller type of stirrer to mix and maintain a uniform temperature of the fluid throughout the tank. A cooling coil of 1.25-cm copper tubing was used to control the fluid temperature in the tank which otherwise increased rapidly because of agitation. For each experimental run the manometer readings were recorded and the corresponding fluid flow rate was measured by collecting the outgoing fluid in a measuring vessel for a known time interval.

After taking observations on straight pipe, the same was wound round a cylindrical frame of known diameter to form a helical coil. The coil diameter ($D_c$) could be varied easily by varying the diameter of the cylindrical frame, and the pitch could be adjusted to any required value by adjusting the gas between two consecutive turns. In order

* Author to whom all correspondence should be addressed at the Department of Chemical Engineering.

 

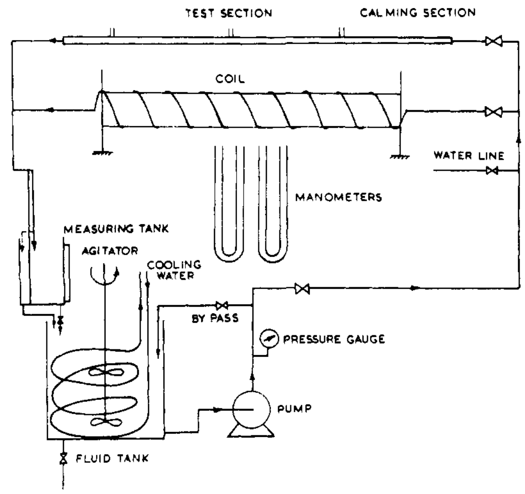


Figure 1. Schematic diagram of the straight pipe and coil systems for pressure drop measurements.

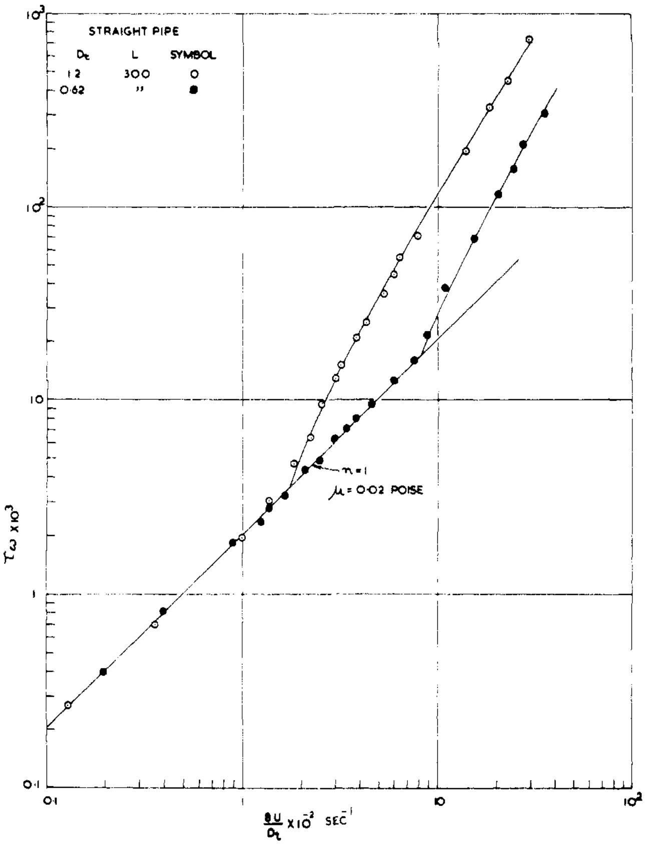


Figure 2. Flow diagram for glycerine-water solution.

to examine the effect of all the major variables, experimental measurements were made under widely varying geometric and flow conditions. Polythene pressure tubes had sufficient strength and wall thickness and by using these tubes it was possible to obtain large variations in $D_t/D_c$ and $D_c/p$ or $D_t/2R_c$ ratios without affecting their original circular cross sections. Sixty coils of varying radii of curvature were made from these polythene tubings. Three copper coils of different coil diameters were also used and measurements were made to see if there is any difference in the friction loss for copper and polythene coils under identical conditions of experimentation.

## Results

In order to test the accuracy of the experimental setup, pressure drop data were taken on straight tubes of all the

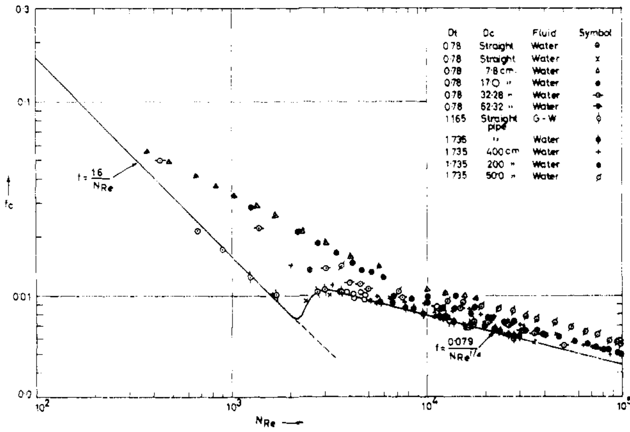


Figure 3. Friction factor for straight tube and helical coils showing the effect of coil diameter (Newtonian fluid).

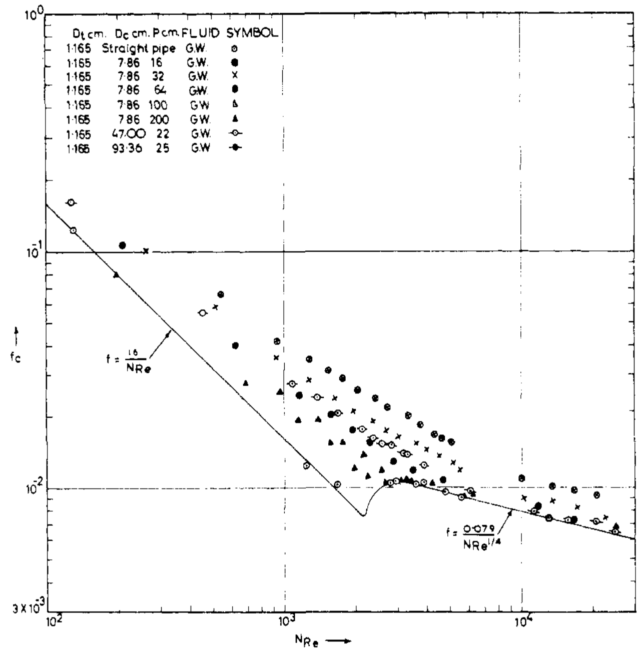


Figure 4. Friction factor for straight tube and helical coils showing pitch effect (Newtonian fluid).

diameters used for making the test coils. The results were found to be in close agreement with the following resistance formulas for smooth straight pipes: (1) laminar flow

$$f_{SL} = 16/N_{Re} \tag{1}$$

(2) turbulent flow

$$f_{ST} = 0.079/(N_{Re})^{0.25} \qquad 4000 < N_{Re} < 10^5 \tag{2}$$

Figure 2 shows the flow curve for 40% aqueous glycerine flowing through straight tubes. Laminar flow data for 0.62- and 1.165-cm diameter tubes fall on the same straight line, whereas turbulent flow data follow different curves. The critical Reynolds number evaluated from points where laminar and turbulent flow curves cut each other are found to be in close agreement with the known value of critical Reynolds number 2100 in smooth straight pipes.

The friction factor $f_c$ for a helical coil is calculated by the well known Fanning friction factor equation

$$f_c = \frac{D_t \Delta P}{2L\rho U^2} \tag{3}$$

Figure 3 gives the variation of friction factor $f_c$ with Reynolds number for water, where the effect of coil diameter is shown for 0.78- and 1.135-cm diameter coil

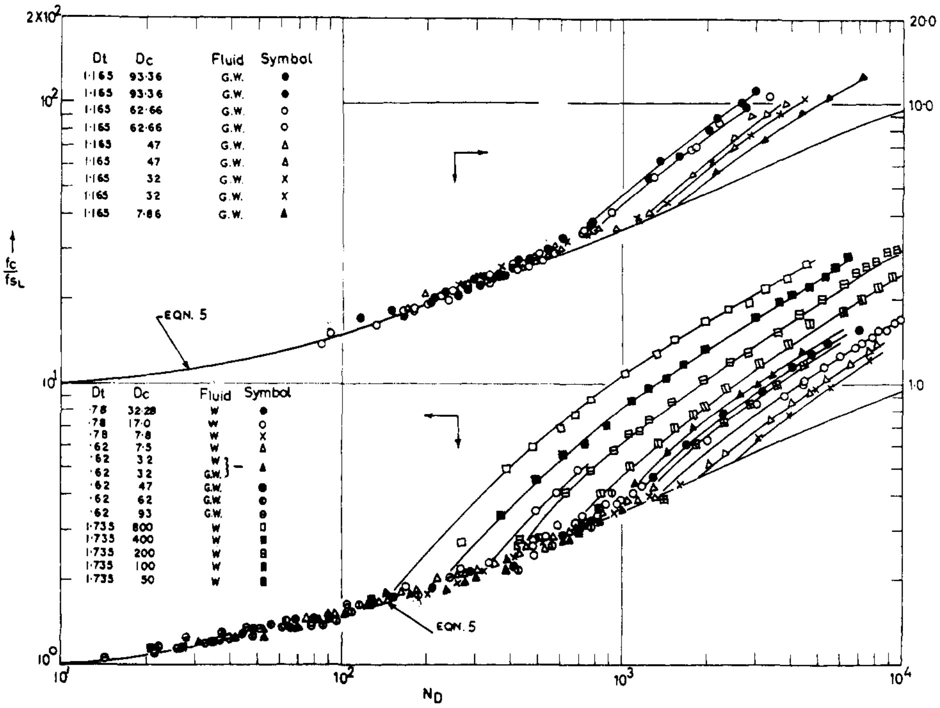


**Figure 5.** Laminar flow friction factor in helical coils as a factor of Dean number (Newtonian fluids).

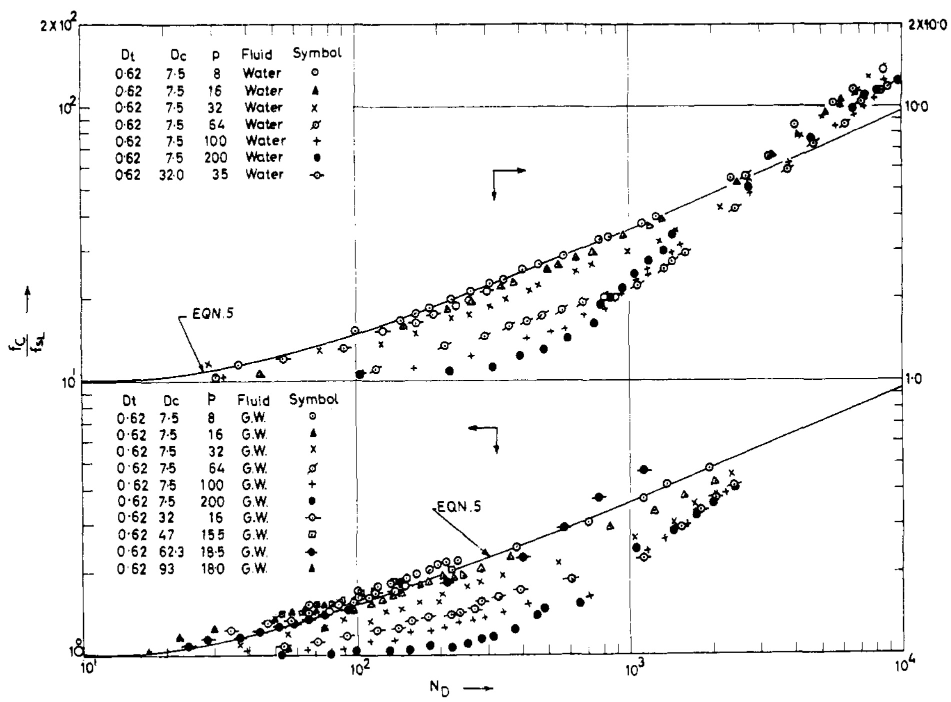


**Figure 6.** Laminar flow friction factor in helical coils as a function of Dean number showing pitch effect (Newtonian fluid).

tubing. The friction factor for turbulent flow through the largest coil diameter falls very near to the Blassius equation for straight pipes while a considerable increase in resistance with the decrease in coil diameter is observed. Similar behavior is noticed for other coils also (Gupta, 1974).

Let us now examine Figure 4 where the results of 40% aqueous glycerine flowing through coils of tube diameter 1.165 cm and coil diameter 7.86 cm are plotted. The coil pitch was varied from 16 to 200 cm. A remarkable effect of coil pitch on the resistance is observed. The friction factor decreased with the increase in coil pitch and in the turbulent flow region it almost coincides with the Blassius equation for larger pitches. From similar results of many coils of various coils diameters and pitches it is concluded that the radius of curvature is the only responsible factor for increased pressure drop. By changing the coil diameter or coil pitch, the radius of curvature is changed, thereby altering the intensity of secondary flow. At very low Reynolds number, velocity being small, fluid particles remain blind to the curvature and the viscous forces dominate the centrifugal forces. As the flow rate increases, centrifugal force becomes of considerable magnitude and thus inertia force in the fluid has to overcome the effect of both the viscous and centrifugal forces. The difference between friction coefficients for the coil and that for the straight pipe decreases for a maxium value in the laminar flow to a minimum in turbulent flow. In turbulent flow the action of centrifugal force is dampened by the dom-

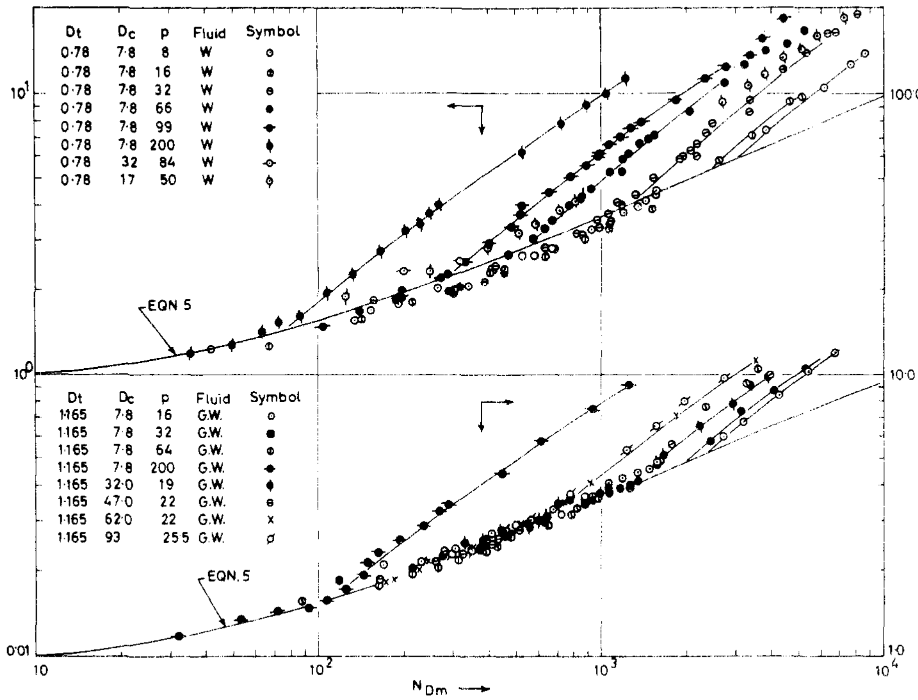

Figure 7. Laminar flow friction factor in helical coils as a function of modified Dean number (Newtonian fluids).

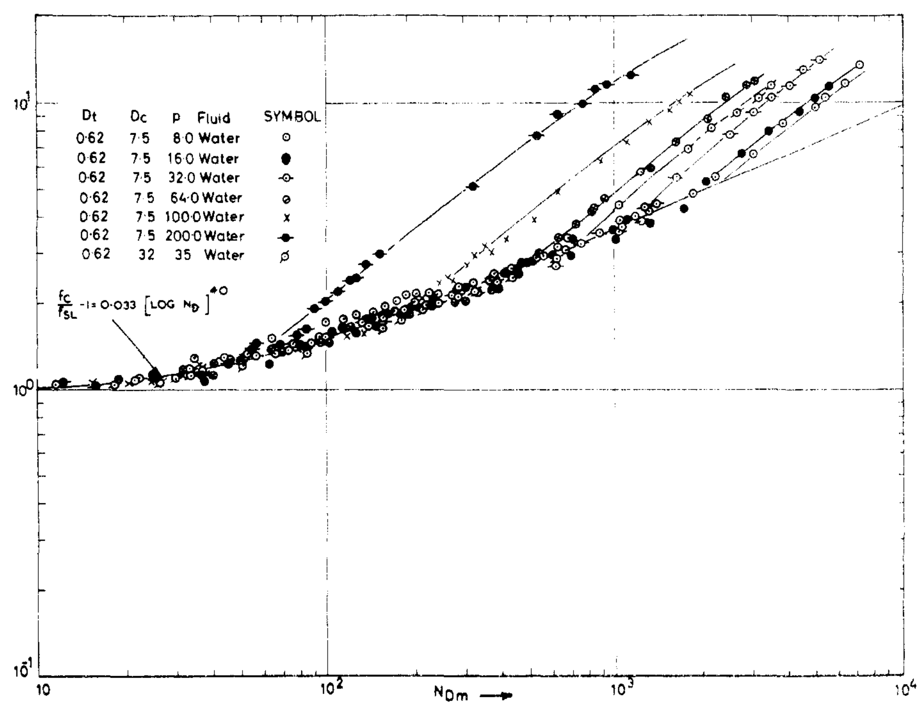

Figure 8. Laminar flow friction factor in helical coils as a function of modified Dean number (Newtonian fluids).

inating inertia force where excessive mixing of the fluid particles reduces the effect of the secondary flow. Thus, the difference between $f_c$ and $f_s$ in turbulent flow is much less compared to that in laminar flow.

## Correlation of Data

**(1) Laminar Flow.** The ratio of the friction factors for helical coils $f_c$ to that for straight pipe $f_{SL}$ is shown in Figures 5 to 8 as a function of Dean number. In Figure 5, the ratio of $f_c/f_{SL}$ is plotted against Dean number, $N_D$, for the coils having the least possible pitch. It is seen that the ratio $f_c/f_{SL}$ for laminar flow is a function of $N_D$ only. The influence of coil pitch in Figure 6 is not unexpected as $f_c/f_{SL}$ is plotted against Dean number $N_D$ defined as $N_{Re}(D_t/D_c)^{1/2}$ which contains the coil diameter, $D_c$, not the true radius of curvature. Further, it is also seen that the pitch has a negligible effect on pressure drop if it is less than the diameter of the coil. Thus, the correlation for coils having appreciable pitch would be inappropriate unless the diameter of the coil in the Dean number is replaced by the radius of curvature of the coil. The derivation of the expression for the radius of curvature of a helix is presented by Thomas (1964). Truesdell and Adler (1970) suggested that as long as the pitch is small with respect to helix circumference, the increase in radius of curvature due to finite pitch may be compensated to a high degree of approximation by replacing the radius of the coil $r_c$ by $R_c$, where

$$R_c = r_c\left[1 + \left(\frac{p}{2\pi r_c}\right)^2\right] \tag{4}$$

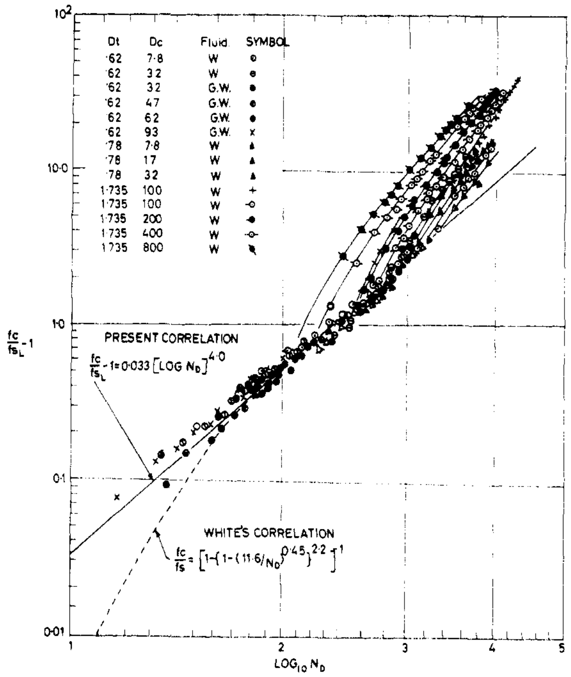


**Figure 9.** Friction factor correlation for laminar flow through helical coils (Newtonian fluid).

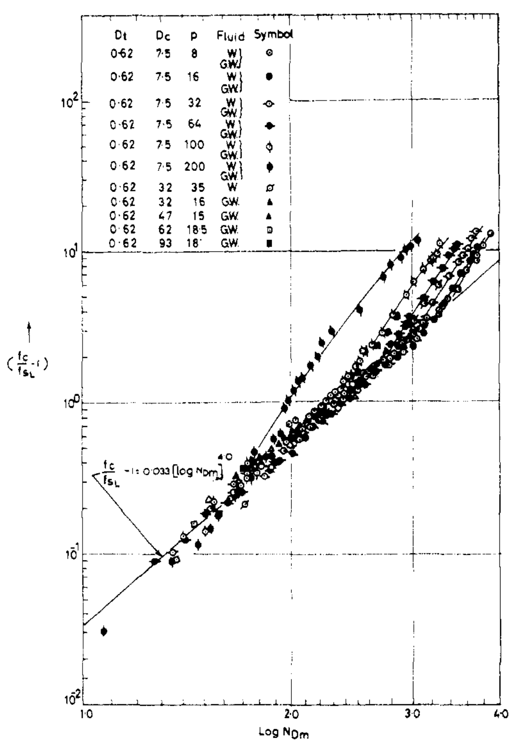


**Figure 10.** Friction factor correlation for laminar flow through helical coils (Newtonian fluids).

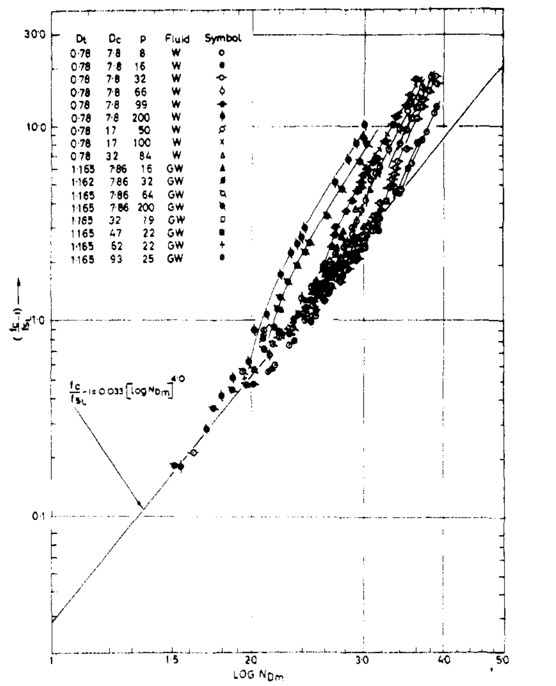


**Figure 11.** Friction factor correlation for laminar flow through helical coils (Newtonian fluids).

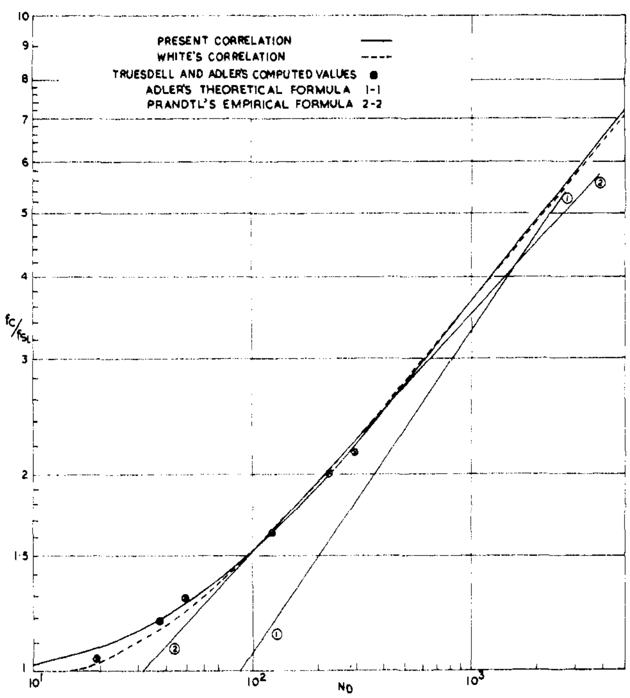


**Figure 12.** Comparison of present correlation with that of Truesdell et al. (1970), computed values, and White's (1929) correlation.

Therefore the ratio $f_c/f_{SL}$ is plotted against the modified Dean number $N_{D_m}$ defined as $N_{Re}(D_t/2R_c)^{1/2}$ and is shown in Figures 7 and 8. All the data for coils of different tubes, coil diameters, and pitches are seen to give a regular correlation without any wide scattering when presented as a $f_c/f_{SL}$ vs. $N_{D_m}$ plot.

Figures 3 and 4 further reveal that there is gradual departure of laminar friction factor data of curved pipes from that of straight pipes and at a very low Reynolds number $f_c$ almost coincides with $f_{SL}$. Thus, is is advisable to correlate $(f_c/f_{SL} - 1)$ as a function of $N_D$ or $N_{D_m}$. In an attempt to obtain such a desirable correlation, $(f_c/f_{SL} - 1)$ was plotted against log $N_D$ on a log–log scale. Laminar flow data of 14 coils having negligible pitch are shown in Figure 9 and those of 27 coils having a wide range of coil pitch are shown in Figures 10 and 11. A least-square analysis of all the laminar flow data points gave the following correlation

$$(f_c/f_{SL}) - 1 = 0.033(\log N_{D_m})^{4.0} \qquad (5)$$

for the entire range of $N_D$ from 1 to 3000, $N_{D_m}$ from 1 to 3000, $D_t/D_c$ from $0.289 \times 10^{-2}$ to 0.155, and $p/D_c$ from 0 to 25.4 with standard deviation of 5%.

In Figure 12, the present correlation is compared with White's (1929) and Adler's (1934) theoretical formula,

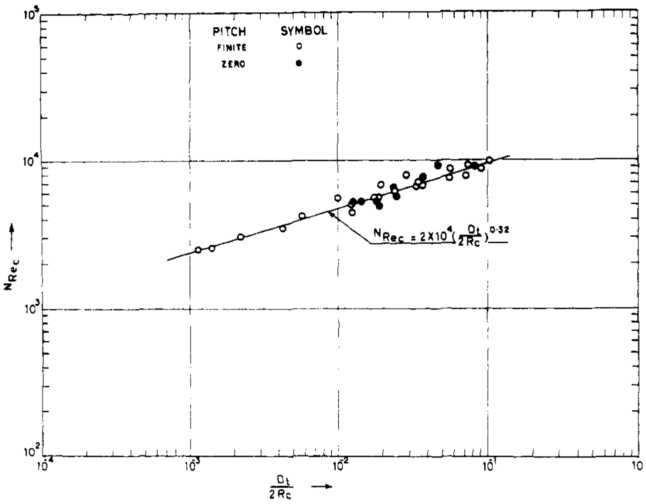

Figure 13. Critical Reynolds number of helical coils as a function of curvature.

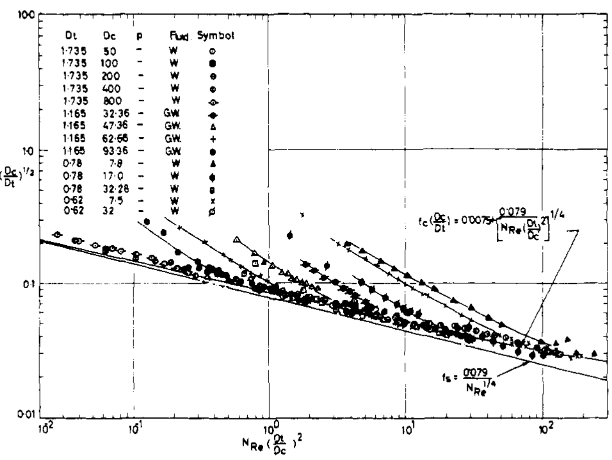

Figure 14. Friction factor correlation for turbulent flow in helical coils (Newtonian fluids).

Prandtl's empirical formula, and Truesdell and Adler's (1970) computed values for laminar flow of Newtonian fluids flowing through helical coils. White's correlation

$$f_c/f_{SL} = \left[1 - \left\{1 - \left(\frac{11.6}{N_D}\right)^{0.45}\right\}^{2.2}\right]^{-1} \qquad (6)$$

is found to be in excellent agreement with the present correlation (eq 5). A very small difference between the two equations is seen only in the very low Dean number range.

Figures 3 and 4 show that the transition from laminar to turbulent flow is delayed in the curved pipes as compared to that in straight ones. The usual two- and three-dimensional propagation of turbulent eddies is suppressed by secondary flow caused by the centrifugal forces. The critical Reynolds number is found to increase with a decrease in coil diameter. An increase in the pitch of the coil decreases the critical Reynolds number. The breakdown of laminar flow is more clear from Figures 4 and 7 to 11. The critical Reynolds number evaluated from these curves is shown in Figure 13 as a function of curvature ratio $D_t/2R_c$. The empirical equation

$$N_{Re_c} = 2 \times 10^4 (D_t/2R_c)^{0.32} \qquad (7)$$

for critical Reynolds number presented by Ito (1959) is also shown in Figure 13. This equation is in good agreement with the experimental results in the range of $10^{-13} < (D_t/2R_c) < 10^{-1}$.

**(ii) Turbulent Flow.** With the view to interpret the present experimental results of turbulent flow in terms of the resistance formula deduced from the $^1/_7$th power law by Ito (1959), $f_c(D_c/D_t)^{1/2}$ is plotted against $N_{Re}(D_t/D_c)^2$ in Figure 14 for various coils having the least possible pitch. Laminar flow data fall on distinct separate curves for different coils, whereas turbulent flow data fall on the same curve showing that $f_c(D_c/D_t)^{1/2}$ is a function of $N_{Re}(D_t/D_c)^2$. Exactly similar behavior is also observed for the coils having small and large pitches in Figure 15 where $f_c$-

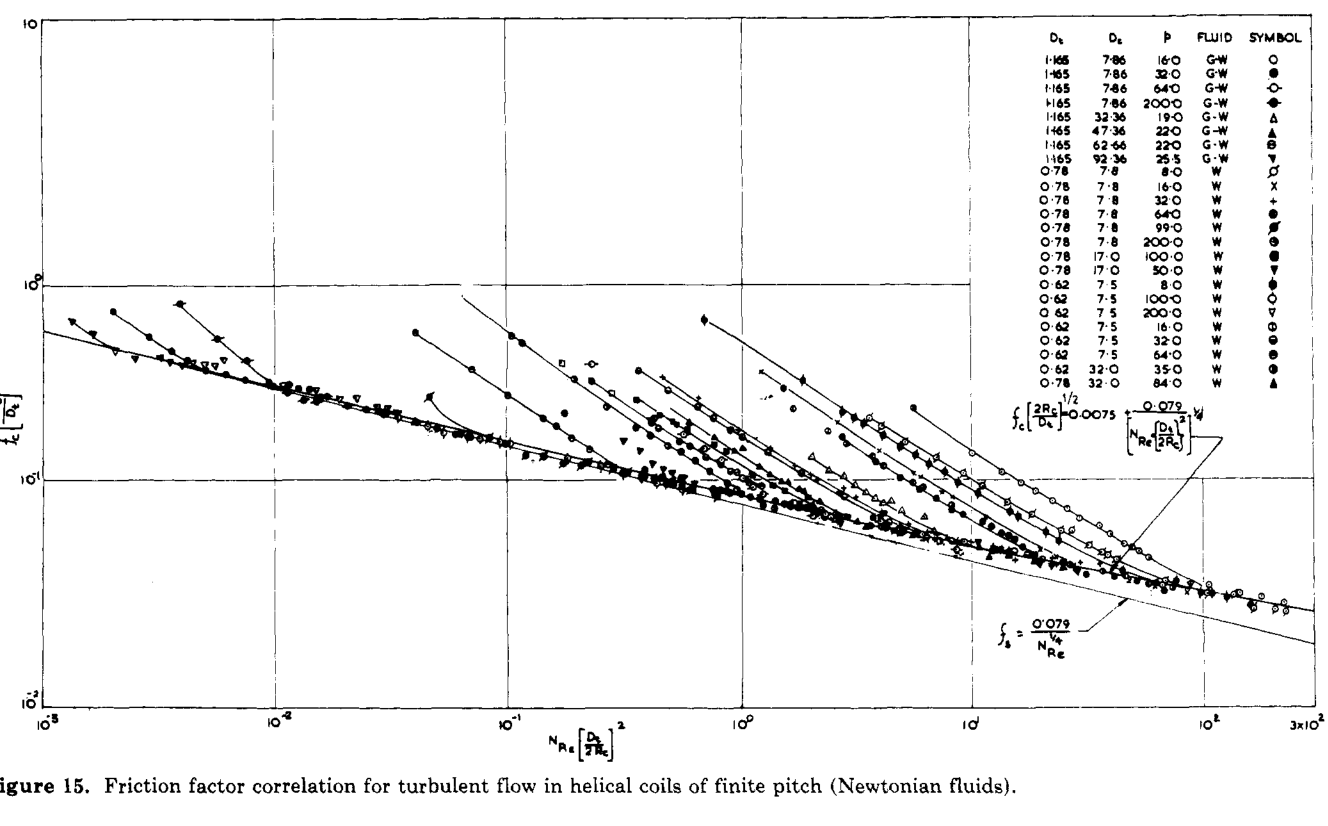

Figure 15. Friction factor correlation for turbulent flow in helical coils of finite pitch (Newtonian fluids).

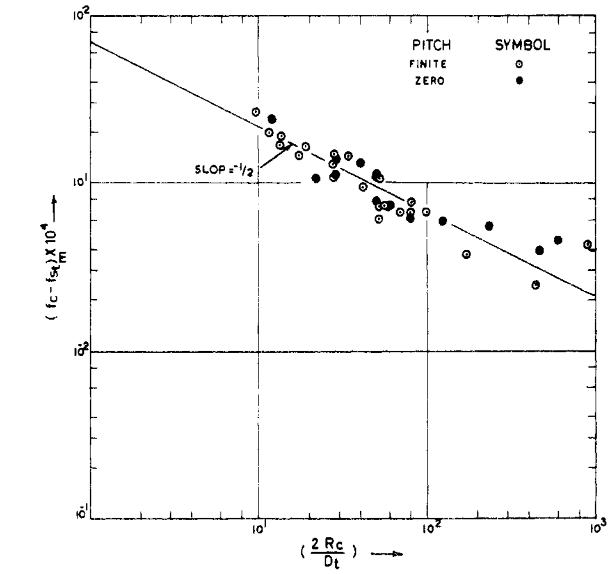


**Figure 16.** Variation of $(f_c - f_{st})_m$ with $2R_c/D_t$ for turbulent flow through helical coils (Newtonian fluids).

$(2R_c/D_t)^{1/2}$ is plotted against $N_{Re}(D_t/2R_c)^2$. For comparison of turbulent friction factor of curved pipes with that of straight pipes, the Blassius eq 2, which may readily be written in the following alternative form

$$f_{ST}(D_c/D_t)^{1/2} = 0.079\ [N_{Re}(D_t/D_c)^2]^{-1/4} \qquad (8)$$

is also shown in these figures. On comparing the turbulent flow data of curved pipes and the Blassius equation it is seen that the difference between $f_c(D_c/D_t)^{1/2}$ or $f_c(2R_c/D_t)^{1/2}$ and $f_{ST}(D_c/D_t)^{1/2}$ or $f_{ST}(2R_c/D_t)^{1/2}$ is almost constant. This information may be expressed as

$$f_c - f_{ST} = A(2R_c/D_t)^{-1/2} \qquad (9)$$

where $A$ is a constant.

The values of $(f_c - f_{ST})$ obtained from experimental $f_c$ and $f_{ST}$ calculated from eq 2 at the Reynolds number prevailing in the coil for various curvature ratios $(2R_c/D_t)$ are shown in Figure 16. The mean line drawn through the data points has a slope of $-^1/_2$. In order to obtain the value of $A$ and to verify eq 9, $f_c(D_c/D_t)^{1/2}$ is plotted against $f_{ST}(D_c/D_t)^{1/2}$ in Figure 17 for coils having negligible pitch and $f_c(2R_c/D_t)^{1/2}$ is plotted against $f_{ST}(2R_c/D_t)^{1/2}$ in Figure 18 for small and large coil piitches. At least-square analysis of all the data resulted in the following correlations

$$f_c - f_{ST} = 0.0075(D_t/D_c)^{1/2} \qquad (10)$$

$$f_c - f_{ST} = 0.0075(D_t/2R_c)^{1/2} \qquad (11)$$

for the Reynolds number range of 4500 to $10^5$, $D_t/D_c$ range of $0.289 \times 10^{-2}$ to 0.15, and $p/D_c$ range of 0 to 25.4 with a standard deviation of 6.8%.

## Conclusion

Equation 5 represents about 900 data points of laminar flow through helical coils covering a wide range of coil pitch and diameters. The radius of curvature calculated from eq 4 gives a satisfactory value for practical purposes even for large coil pitches. Equation 9 shows that the increase in turbulent drag depends upon the ratio of the coil tube diameter and its radius of curvature only.

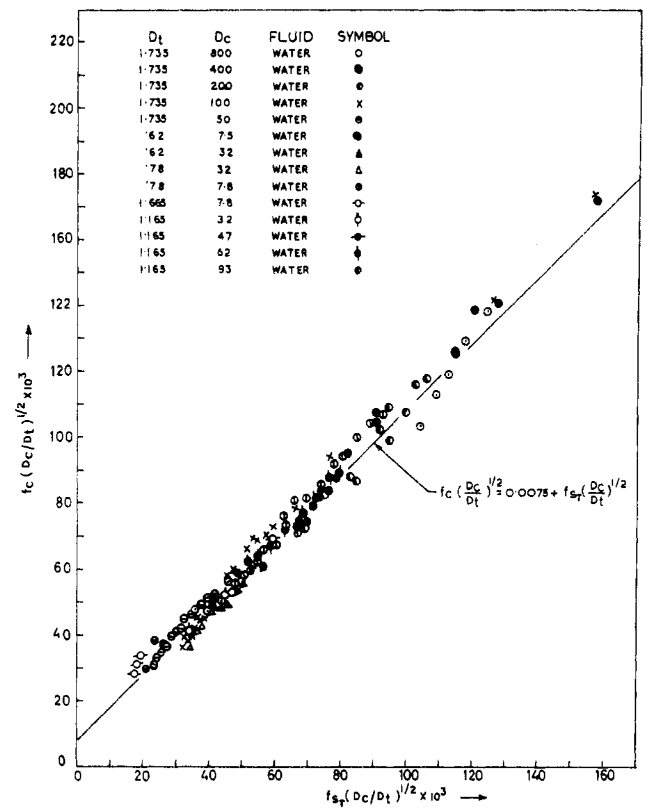


**Figure 17.** Turbulent flow friction factor correlation for helical coils (Newtonian fluids).

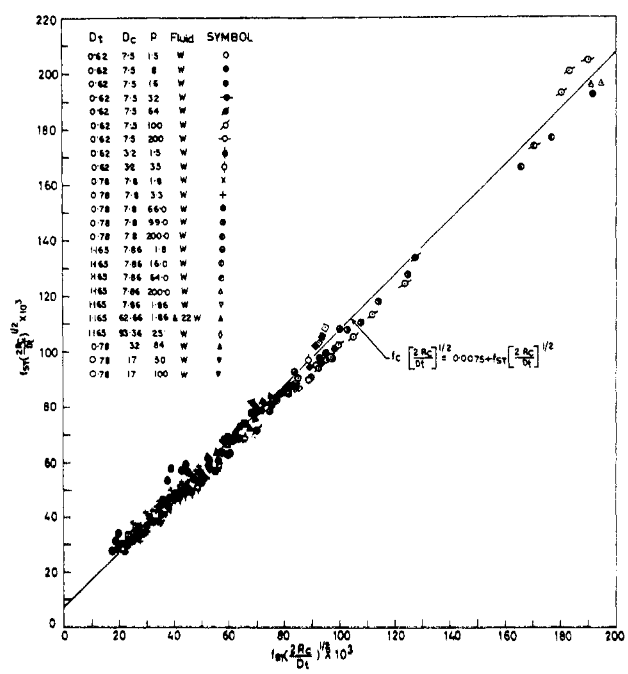


**Figure 18.** Friction factor correlation for turbulent flow in helical coils (Newtonian fluids).

## Acknowledgment

The authors wish to thank the Banaras Hindu University for providing the experimental facilities.

## Nomenclature

$A$, constant in eq 9
$D_t$, inner diameter of the straight or coiled pipe
$D_c$, diameter of the coil helix
$f_c$, friction factor in the coil
$f_{SL}$, laminar flow friction factor in the straight pipe

$f_{ST}$, turbulent flow friction factor in the straight pipe
$L$, length of the straight pipe or the coil
$N_{Re}$, Reynolds number, $D_t \rho U/\mu$
$N_D$, Dean number, $N_{Re}(D_t/D_c)^{1/2}$
$N_{Dm}$, modified Dean number, $N_{Re}(D_t/2R_c)^{1/2}$
$p$, pitch of the helical coil, distance between center lines of two adjacent wraps
$\Delta P$, pressure difference
$r_c$, radius of the coil
$R_c$, radius of curvature of the coil, see eq 4
$U$, average fluid velocity
$\mu$, viscosity
$\rho$, density

## Literature Cited


Adler, *ZAMM*, **14**, 257 (1934).
Cummings, G. H., Aeronautical Research Council Reports and Memoranda No. 2880, 1955.
Dean, W. R., *Phil. Mag. S.T.*, **4**, 208 (1927).
Dean, W. R., *Phil. Mag. S.T.*, **5**, 673 (1928).
Detra, R. W., Mitteilungen aus dem Institut fuer Aerodynamik Herausgegeben Var, Prof. J. Ackert, No. 20, 1953.
Eustice, J., *Proc. R. Soc.*, **A84**, 107 (1910).
Grindley, J. H., Gibson, A. H., *Proc. R. Soc.*, **A80**, 114 (1908).
Gupta, S. N., Ph.D. Thesis, Banaras Hindu University, 1974.
Hawthorne, W. R., *Proc. R. Soc.*, 374 (1951).
Ito, H., *J. Basic Eng., Trans. A.S.M.E.*, **81D**, 123 (1959).
Ito, H., *ZAMM*, **49**, 653 (1969).
Keulegan, G. H., Beij, K. H., *J. Res. Nat. Bur. Stand.*, **18**, 89 (1937).
Miyagi, C., *J. Jpn. Soc. Mech. Eng.*, **35**, 1121 (1932).
Thomas, G. B., Jr., "Calculus", Addison-Wesley Publishing Co., Inc. (London), 2nd ed, p 464, 1964.
Tomita, K., *J. Jpn. Soc. Mech. Eng.*, **35**, 293 (1932).
Truesdell, L. C., Jr., Adler, R. J., *AIChE J.*, **16**, 1010 (1970).
White, C. M., *Proc. R. Soc.*, **123A**, 645 (1929).


*Received for review* December 1, 1977
*Accepted* June 29, 1978

# Momentum Transfer in Curved Pipes. 2. Non-Newtonian Fluids

**P. Mishra* and S. N. Gupta**

*Departments of Chemical Engineering and Mechanical Engineering, Institute of Technology, Banaras Hindu University, Varanasi-221005, India*

Experimental data for non-Newtonian fluids flowing through helical coils are presented. An appropriate viscosity term prevailing at the wall shear stress is defined and is used in the Reynolds number to obtain a correlation between friction factor and Dean number for helical coils. The use of differential viscosity in turbulent flow Reynolds number is found to give a single valued correlation for both the Newtonian and non-Newtonian fluids.

## Introduction

In spite of the increasing importance of non-Newtonian fluids in industries, little effort has been made to study the behavior of such fluids flowing through curved pipes. Rajshekharan et al. (1964, 1966) obtained diametrical pressure drop data for helical coils of different geometries for non-Newtonian fluids and correlated the results in terms of flow rate and pressure drop. Later (1970) they presented the data on non-Newtonian fluid flow through helical and spiral coils in the form of a plot of friction factor vs. Reynolds number. No correlation is available so far in the literature for laminar and turbulent flow of non-Newtonian fluids through curved pipes. With little data available it is not possible to present any correlation in terms of friction coefficient as a function of Dean number, the well known dimensionless variable. It is, therefore, felt necessary to obtain more experimental data for non-Newtonian fluids flowing through helical coils.

## Theoretical Background

**(i) Laminar Flow.** The shear stress–shear rate relationship, velocity profile equation, and friction factor–Reynolds number relationships for the laminar flow of non-Newtonian fluids (Ostwald power law type) flowing through straight tubes of round cross section as given by Skelland (1967) are summarized below: (a) the constitutive equation of a power law fluid

$$\tau = -K\left(\frac{du}{dr}\right)^n \tag{1}$$

(b) the generalized power law

$$\tau_w = K'\left(\frac{8U}{d_t}\right)^{n'} \tag{2}$$

where

$$K' = K\left(\frac{3n+1}{4n}\right)^n; \quad n = n' \tag{3}$$

(c) the velocity profile

$$\frac{u}{U} = \frac{3n+1}{n+1}\left[1 - \left(\frac{r}{r_w}\right)^{(n+1)/n}\right]$$

and the velocity gradient at the wall

$$\left(-\frac{du}{dr}\right)_w = \frac{3n+1}{4n}(8U/D_t) \tag{4}$$

(d) the laminar friction factor

$$f_{SL} = 16/N'_{Re} \tag{5}$$

where

$$N_{Re}' = \frac{D_t^n U^{(2-n)}\rho}{K'8^{n-1}} \tag{6}$$

and

$$f = \frac{2\tau_w}{\rho U^2} = \frac{D_t \Delta P}{2\rho U^2 L} \tag{7}$$

The prediction of the pressure drop for laminar flow of non-Newtonian fluids through straight pipe systems poses

* Author to whom all correspondence should be addressed at the Department of Chemical Engineering.

 