

Contrasting RW2 with ARIMA

Ziang Zhang

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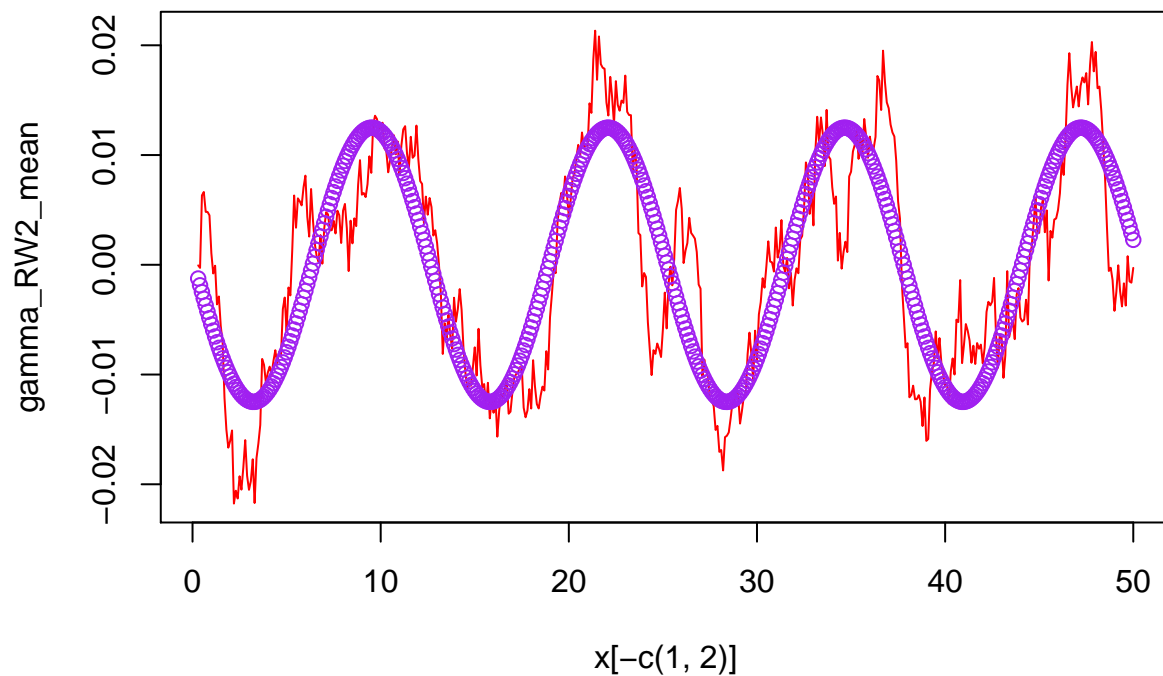
Study 1: The higher order differences

When knots are densely placed

```
### Study volatility of second order difference
true_W <- compute_g(x)
true_gamma <- diff(true_W, differences = 2)
compute_MSE <- function(vec){
  mean((vec - true_gamma)^2)
}

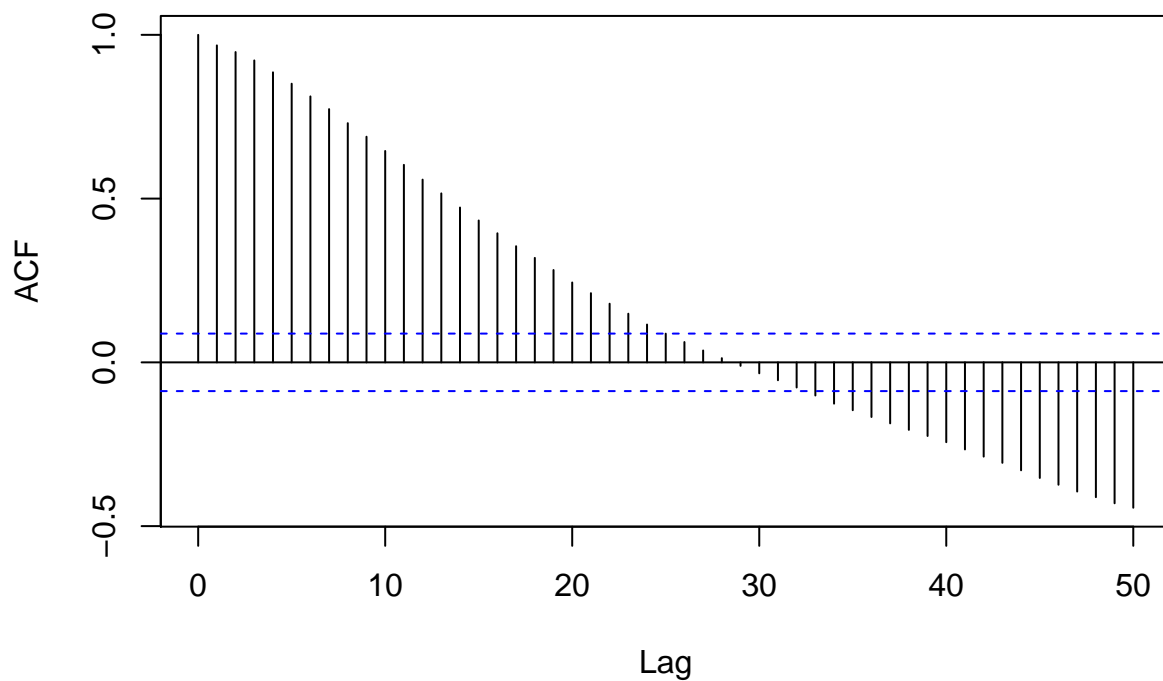
## RW2
# Mean:
gamma_RW2 <- apply(samps1$samps,2, diff, differences = 2)
gamma_RW2_mean <- apply(gamma_RW2,1, mean)
gamma_RW2_upper <- apply(gamma_RW2,1,quantile, probs = 0.975)
gamma_RW2_lower <- apply(gamma_RW2,1,quantile, probs = 0.025)

plot(gamma_RW2_mean~x[-c(1,2)], type = 'l', col = 'red')
points(true_gamma ~ x[-c(1,2)], col = 'purple')
```



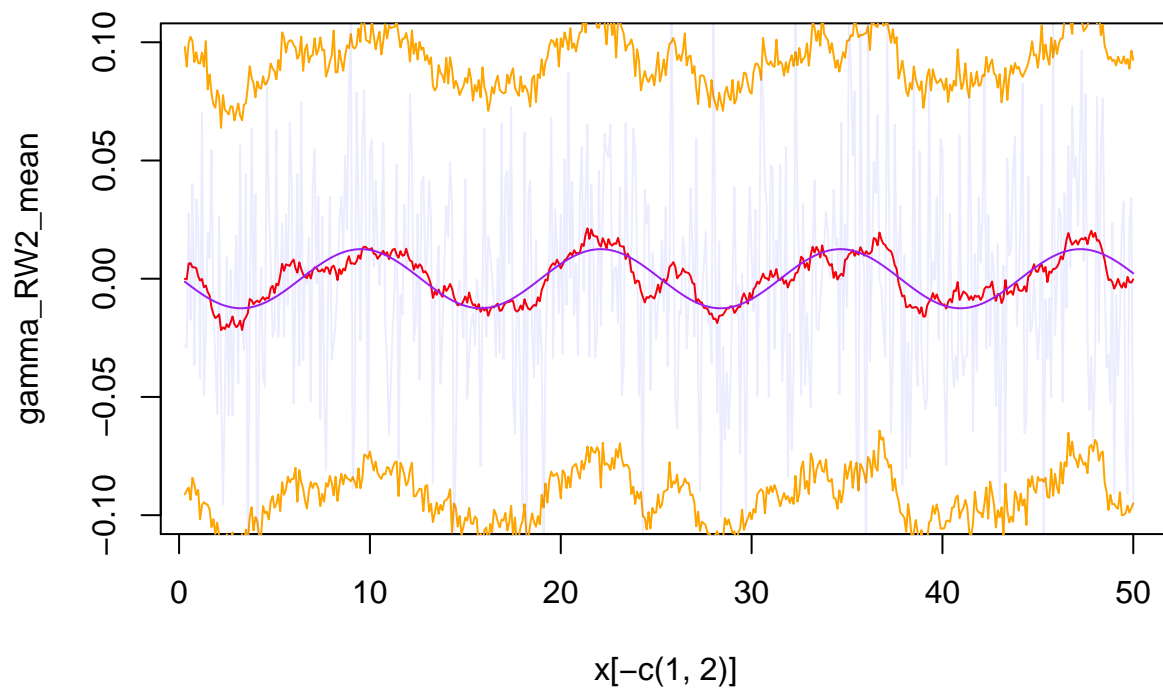
```
acf(gamma_RW2_mean, lag.max = 50)
```

Series gamma_RW2_mean



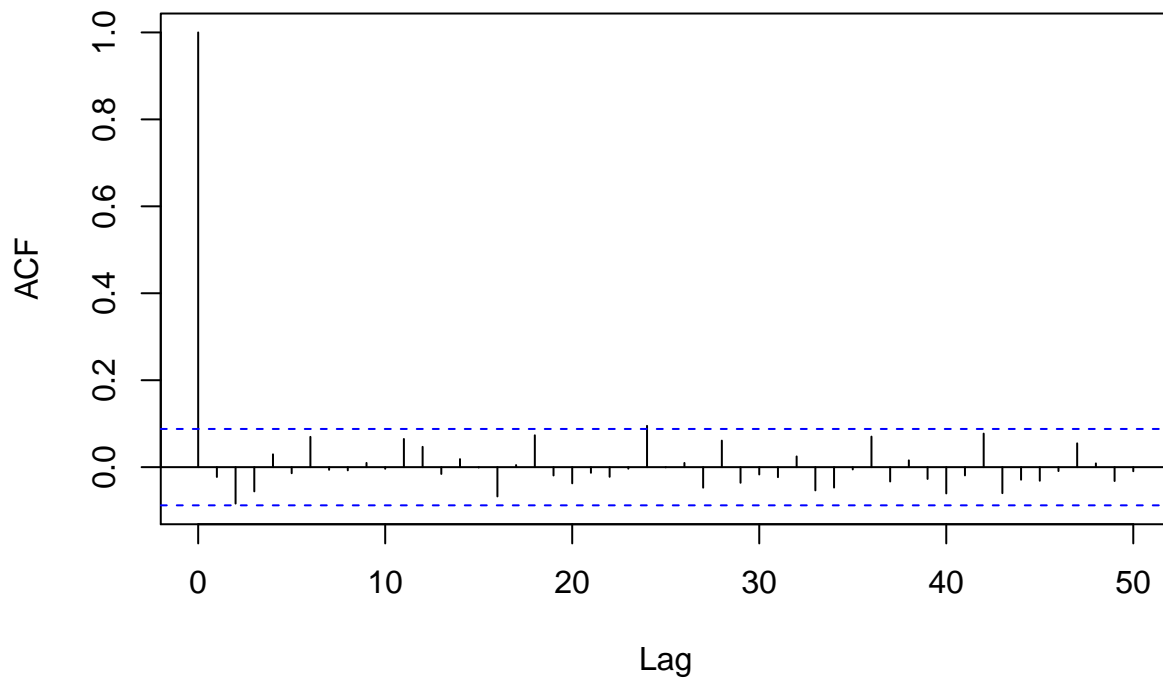
```
# Samples
plot(gamma_RW2_mean~x[-c(1,2)], type = 'l', col = 'red', ylim = c(-0.1,0.1), xlim = c(1,50))
lines(true_gamma ~ x[-c(1,2)], col = 'purple')
lines(gamma_RW2_upper ~ x[-c(1,2)], col = 'orange')
lines(gamma_RW2_lower ~ x[-c(1,2)], col = 'orange')
```

```
for (i in sample.int(1000,1)) {
  lines(gamma_RW2[,i] ~ x[-c(1,2)], col = rgb(0, 0, 255, max = 255, alpha = 20, names = "grey"))
}
```



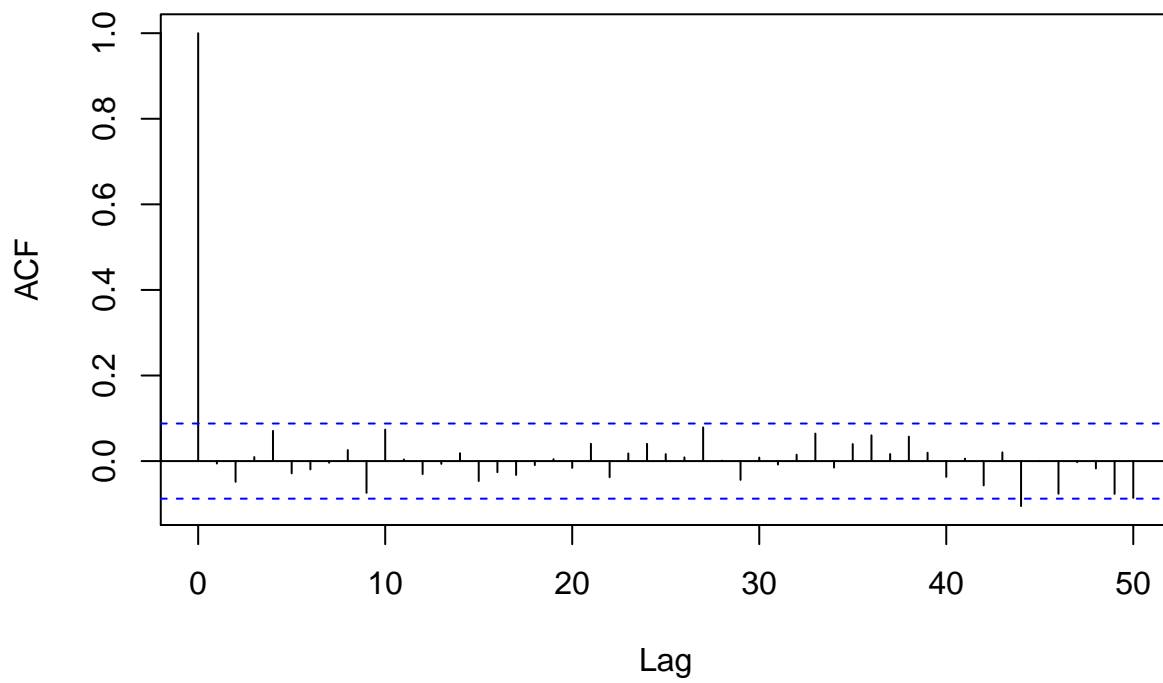
```
acf(gamma_RW2[,1], lag.max = 50)
```

Series gamma_RW2[, 1]



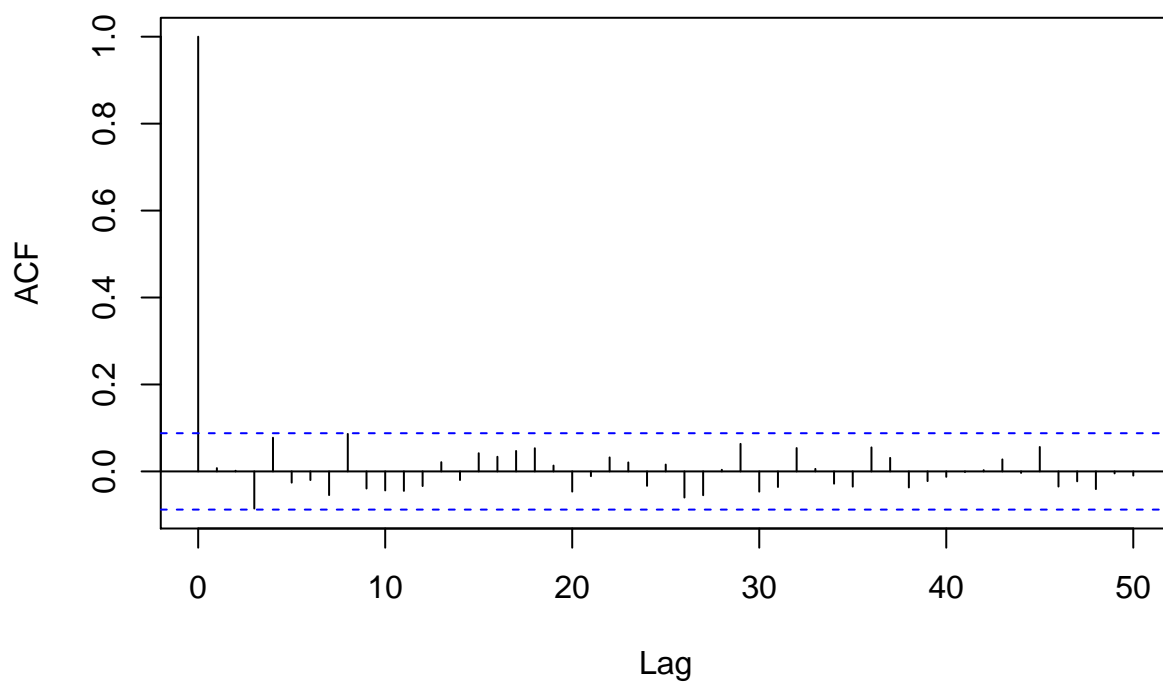
```
acf(gamma_RW2[,2], lag.max = 50)
```

Series gamma_RW2[, 2]



```
acf(gamma_RW2[,3], lag.max = 50)
```

Series gamma_RW2[, 3]



```
MSE_RW2_samples <- apply(gamma_RW2, 2, compute_MSE)
```

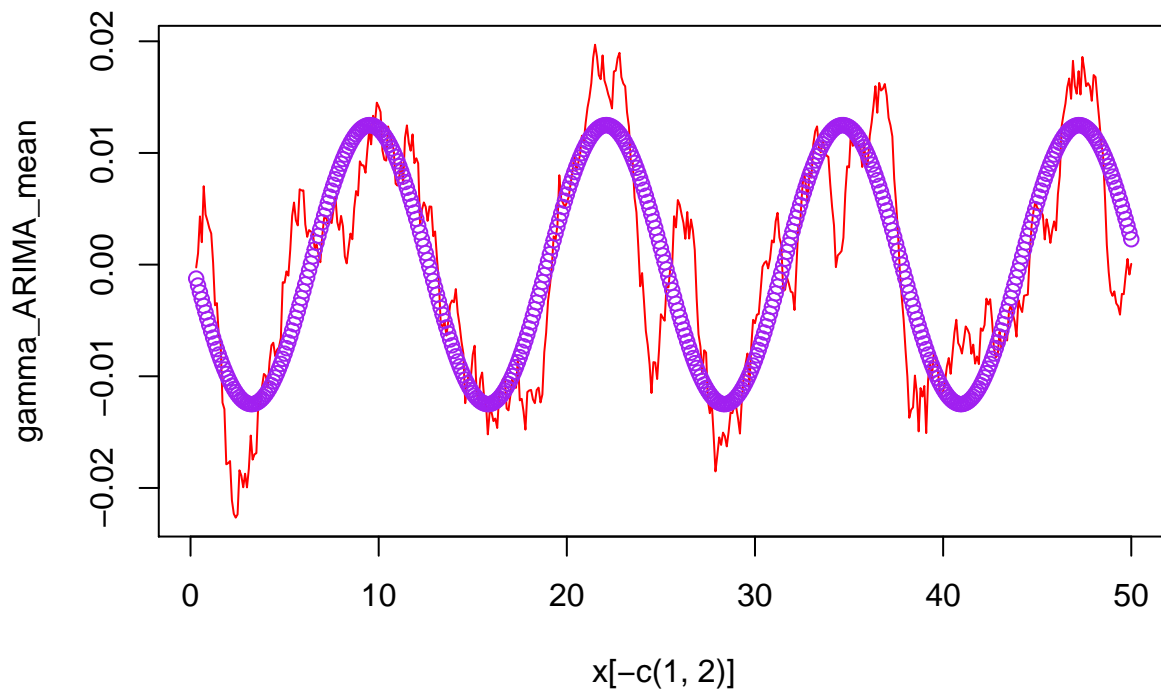
Observation: The mean is much much smoother than the sample. The credible interval is quite wide
for each sampled function only spikes at the first lag.

ARIMA:

Mean:

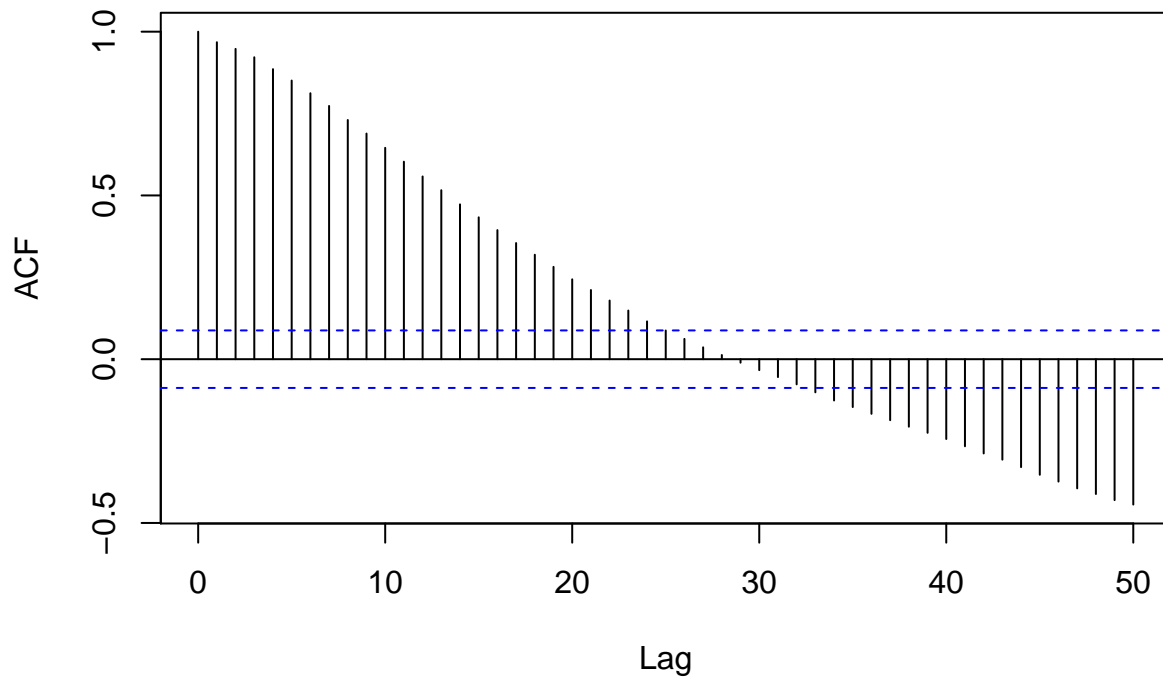
```
gamma_ARIMA <- apply(samps3$samps, 2, diff, differences = 2)  
gamma_ARIMA_mean <- apply(gamma_ARIMA, 1, mean)  
gamma_ARIMA_upper <- apply(gamma_ARIMA, 1, quantile, probs = 0.975)  
gamma_ARIMA_lower <- apply(gamma_ARIMA, 1, quantile, probs = 0.025)
```

```
plot(gamma_ARIMA_mean ~ x[-c(1, 2)], type = 'l', col = 'red')  
points(true_gamma ~ x[-c(1, 2)], col = 'purple')
```



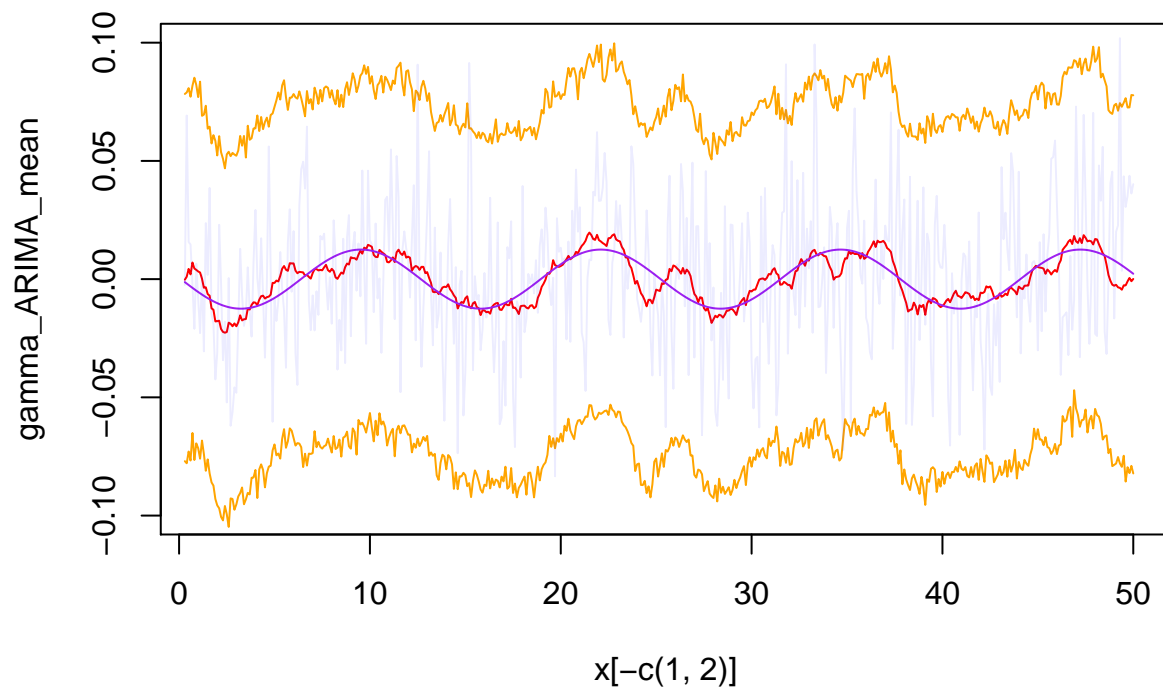
```
acf(gamma_RW2_mean, lag.max = 50)
```

Series gamma_RW2_mean



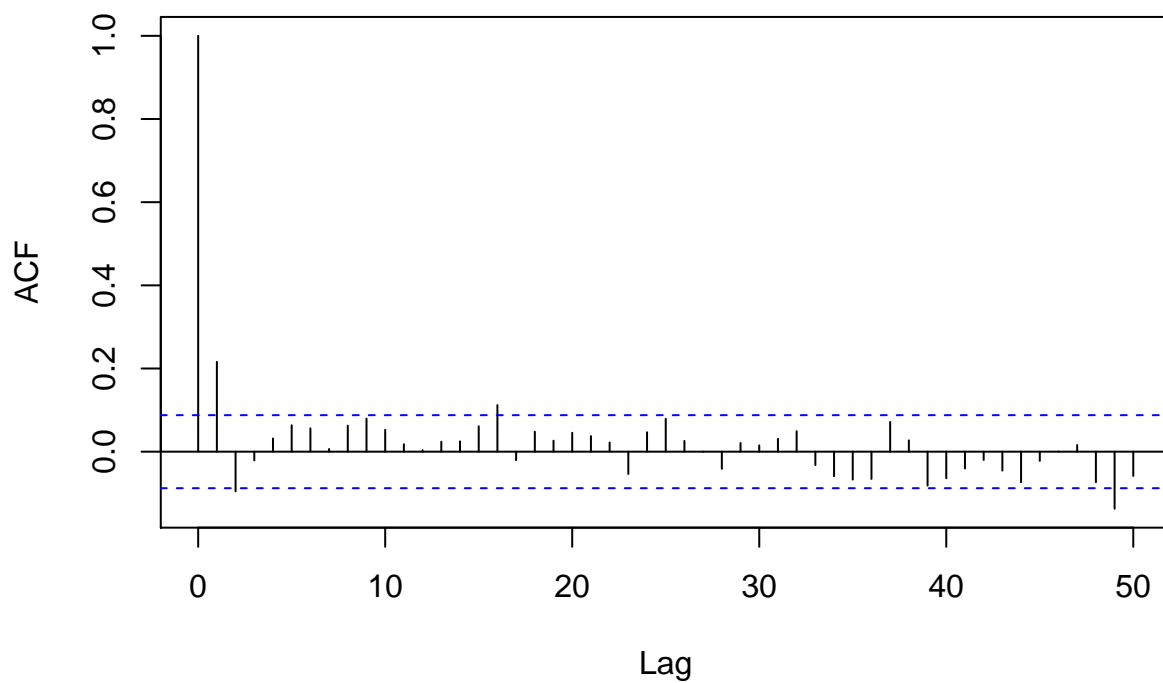
```
# Samples
plot(gamma_ARIMA_mean~x[-c(1,2)], type = 'l', col = 'red', ylim = c(-0.1,0.1), xlim = c(1,50))
lines(true_gamma ~ x[-c(1,2)], col = 'purple')
lines(gamma_ARIMA_upper ~ x[-c(1,2)], col = 'orange')
lines(gamma_ARIMA_lower ~ x[-c(1,2)], col = 'orange')

for (i in sample.int(1000,1)) {
  lines(gamma_ARIMA[,i] ~ x[-c(1,2)], col = rgb(0, 0, 255, max = 255, alpha = 20, names = "grey"))
}
```



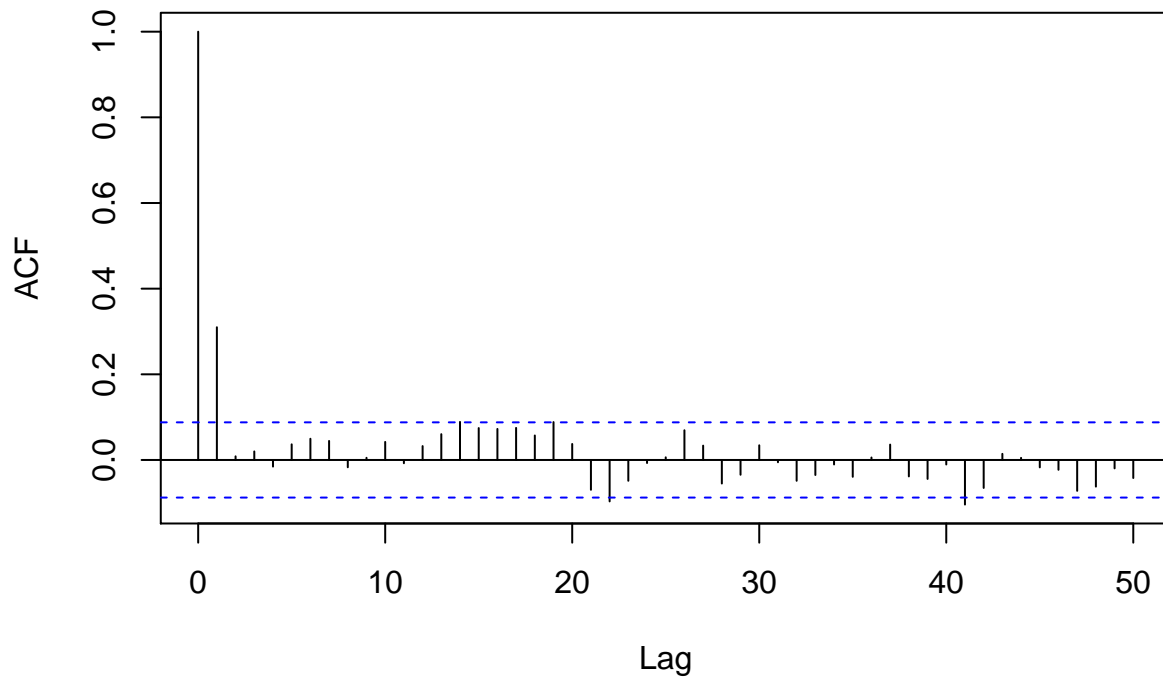
```
acf(gamma_ARIMA[,1], lag.max = 50)
```

Series gamma_ARIMA[, 1]



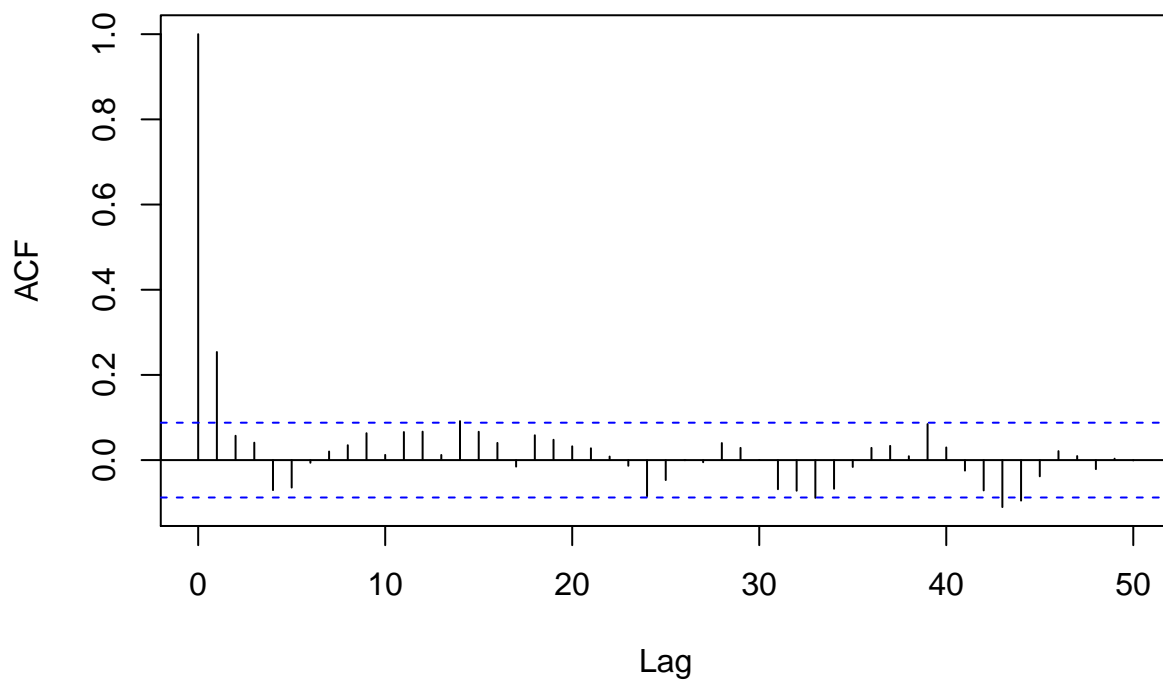
```
acf(gamma_ARIMA[,2], lag.max = 50)
```

Series gamma_ARIMA[, 2]



```
acf(gamma_ARIMA[,3], lag.max = 50)
```

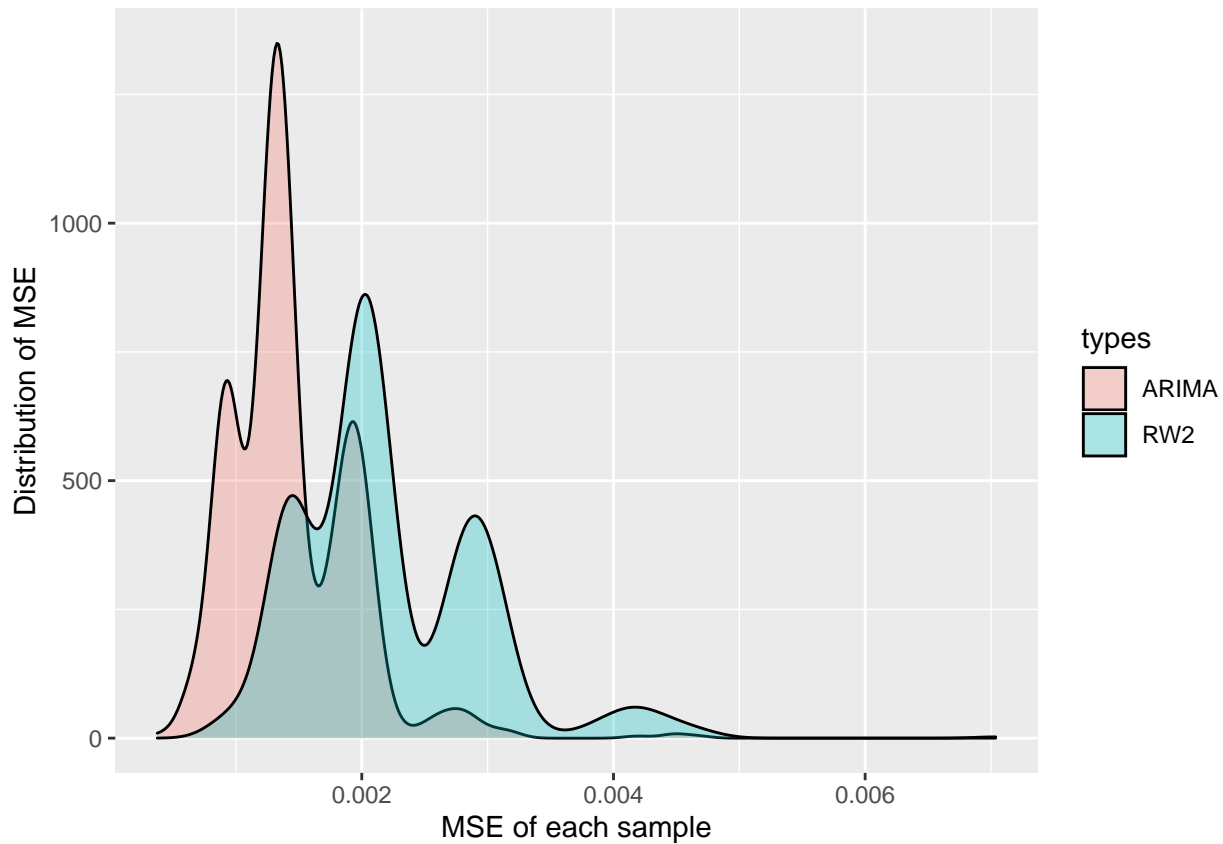
Series gamma_ARIMA[, 3]



```
MSE_ARIMA_samples <- apply(gamma_ARIMA, 2, compute_MSE)
```



```
hist_data <- data.frame(MSE = c(MSE_RW2_samples,MSE_ARIMA_samples), types = rep(c("RW2", "ARIMA"), each
ggplot(data = hist_data) + geom_density(aes(x=MSE, fill = types), alpha = 0.3) + ylab("Distribution of MSE of each sample")
```



```
### Mean credible width
```

```
credible_width_rw2 <- mean(gamma_RW2_upper - gamma_RW2_lower)
credible_width_rw2
```

```
## [1] 0.1842096
```

```
credible_width_ARIMA <- mean(gamma_ARIMA_upper - gamma_ARIMA_lower)
credible_width_ARIMA
```

```
## [1] 0.1481282
```

When knots are sparsely placed

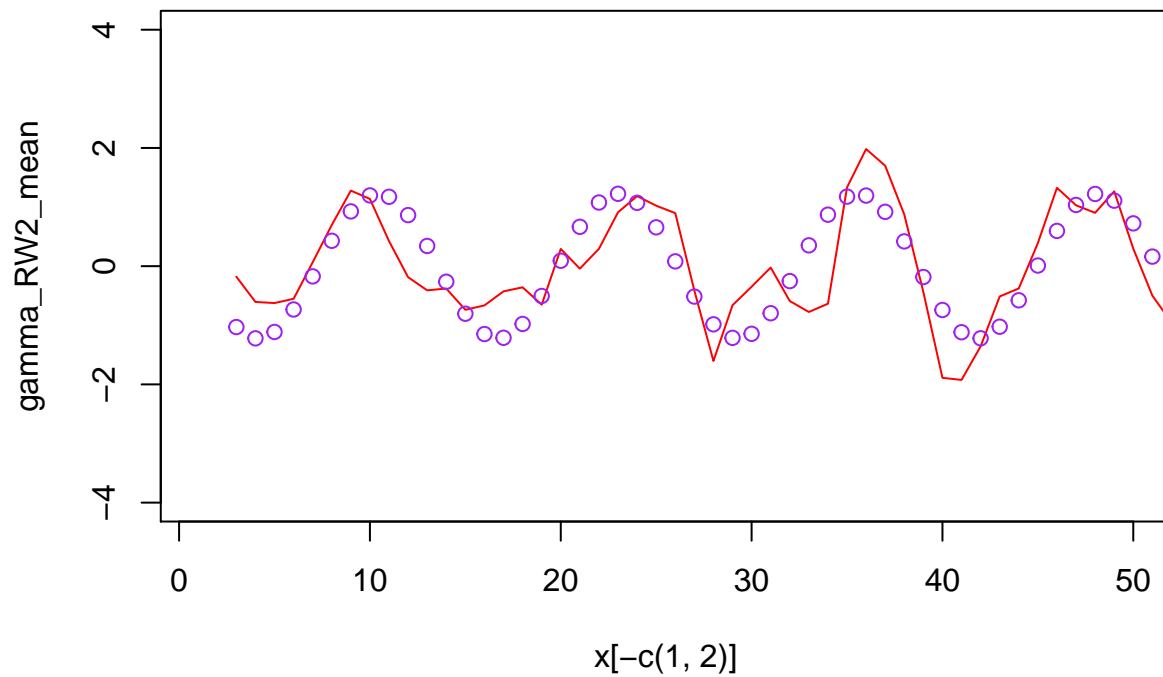
```
## Warning: 4 external pointers will be removed
```

```
### Study volatility of second order difference
```

```
true_W <- compute_g(x)
true_gamma <- diff(true_W, differences = 2)
compute_MSE <- function(vec){
  mean((vec - true_gamma)^2)
}
```

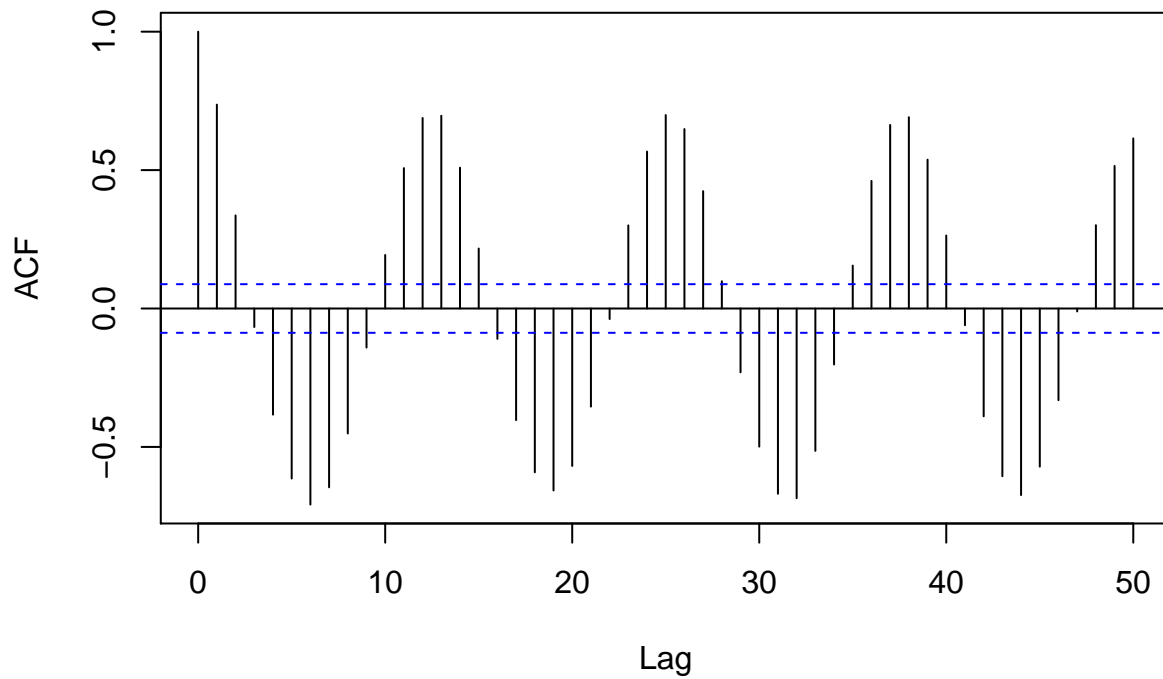
```
## RW2
# Mean:
gamma_RW2 <- apply(samps1$samps,2, diff, differences = 2)
gamma_RW2_mean <- apply(gamma_RW2,1, mean)
gamma_RW2_upper <- apply(gamma_RW2,1,quantile, probs = 0.975)
gamma_RW2_lower <- apply(gamma_RW2,1,quantile, probs = 0.025)

plot(gamma_RW2_mean~x[-c(1,2)], type = 'l', col = 'red', ylim = c(-4,4), xlim = c(1,50))
points(true_gamma ~ x[-c(1,2)], col = 'purple')
```



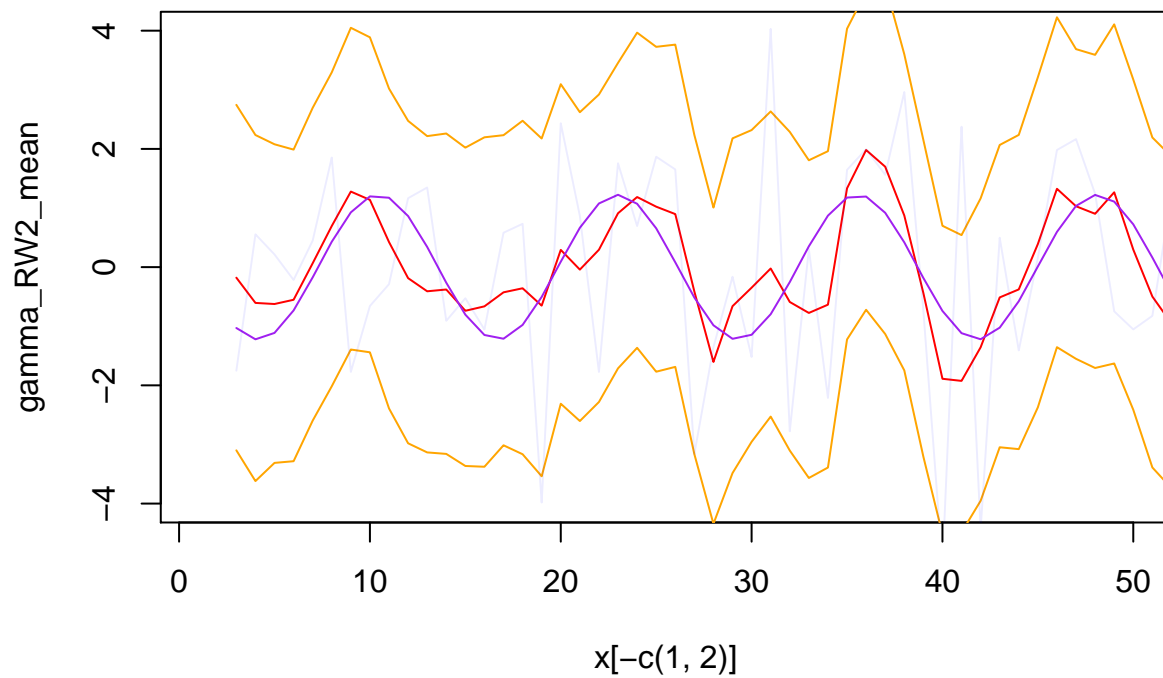
```
acf(gamma_RW2_mean, lag.max = 50)
```

Series gamma_RW2_mean



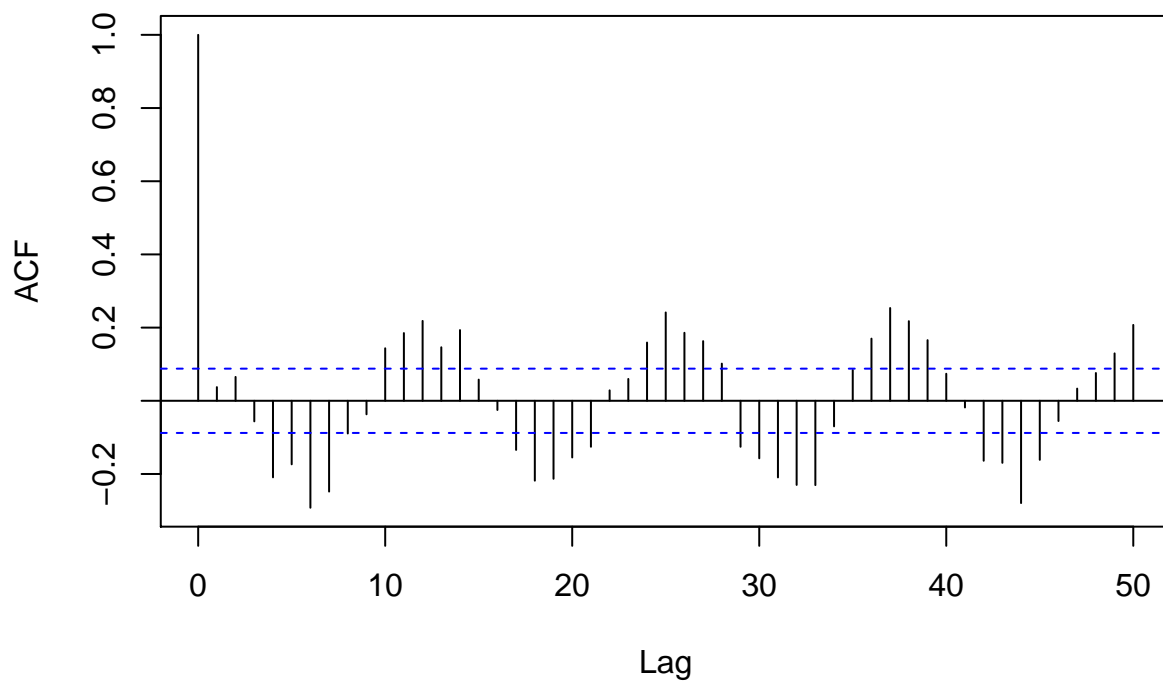
```
# Samples
plot(gamma_RW2_mean~x[-c(1,2)], type = 'l', col = 'red', ylim = c(-4,4), xlim = c(1,50))
lines(true_gamma ~ x[-c(1,2)], col = 'purple')
lines(gamma_RW2_upper ~ x[-c(1,2)], col = 'orange')
lines(gamma_RW2_lower ~ x[-c(1,2)], col = 'orange')

for (i in sample.int(1000,1)) {
  lines(gamma_RW2[,i] ~ x[-c(1,2)], col = rgb(0, 0, 255, max = 255, alpha = 20, names = "grey"))
}
```



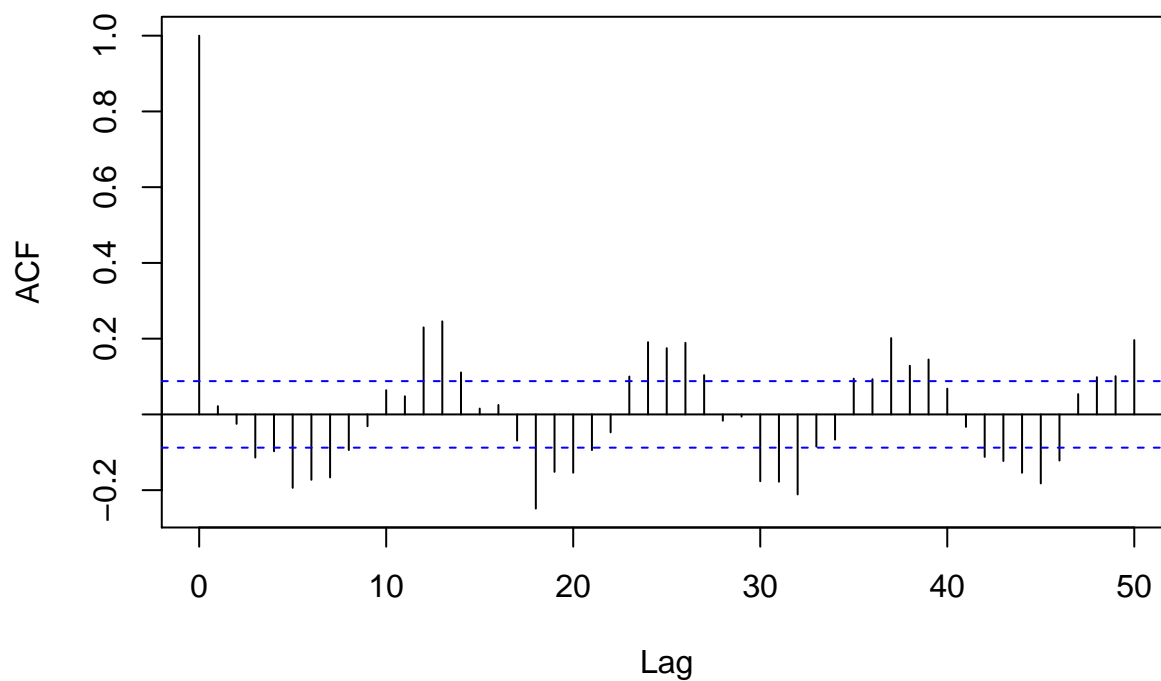
```
acf(gamma_RW2[,1], lag.max = 50)
```

Series gamma_RW2[, 1]



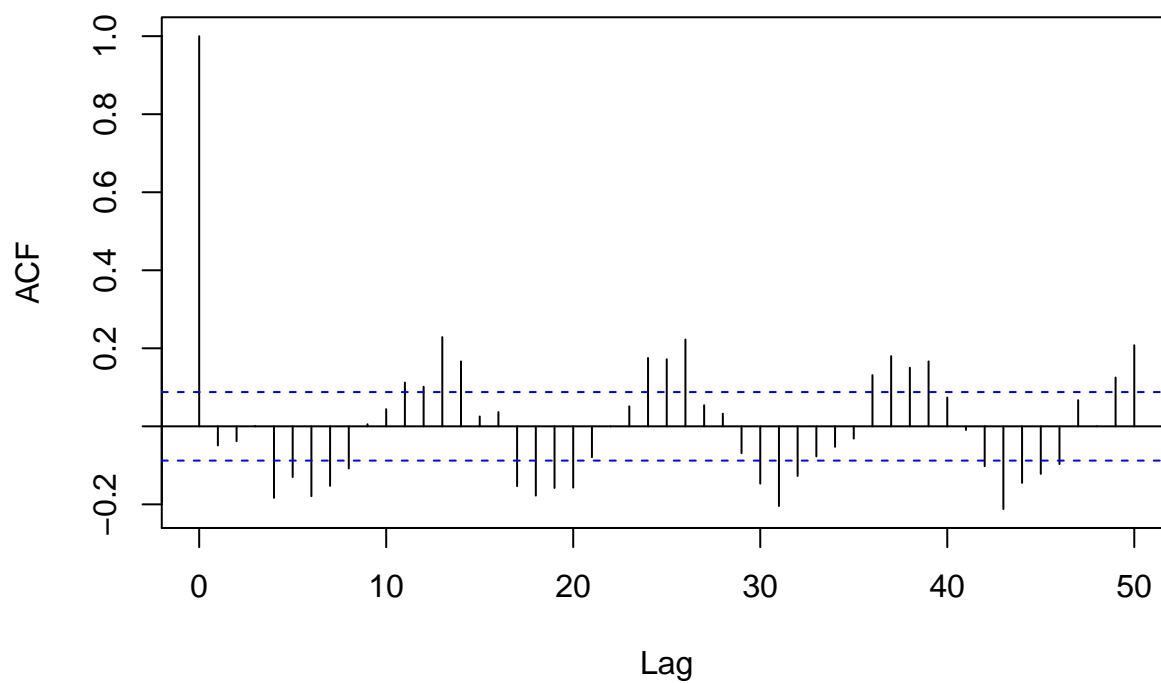
```
acf(gamma_RW2[,2], lag.max = 50)
```

Series gamma_RW2[, 2]



```
acf(gamma_RW2[,3], lag.max = 50)
```

Series gamma_RW2[, 3]



```
MSE_RW2_samples <- apply(gamma_RW2, 2, compute_MSE)
```

```
##### Observation: The mean is much much smoother than the sample. The credible interval is quite wide
##### for each sampled function only spikes at the first lag.
```

```
## ARIMA:
```

```
# Mean:
```

```
gamma_ARIMA <- apply(samps3$samps,2, diff, differences = 2)
```

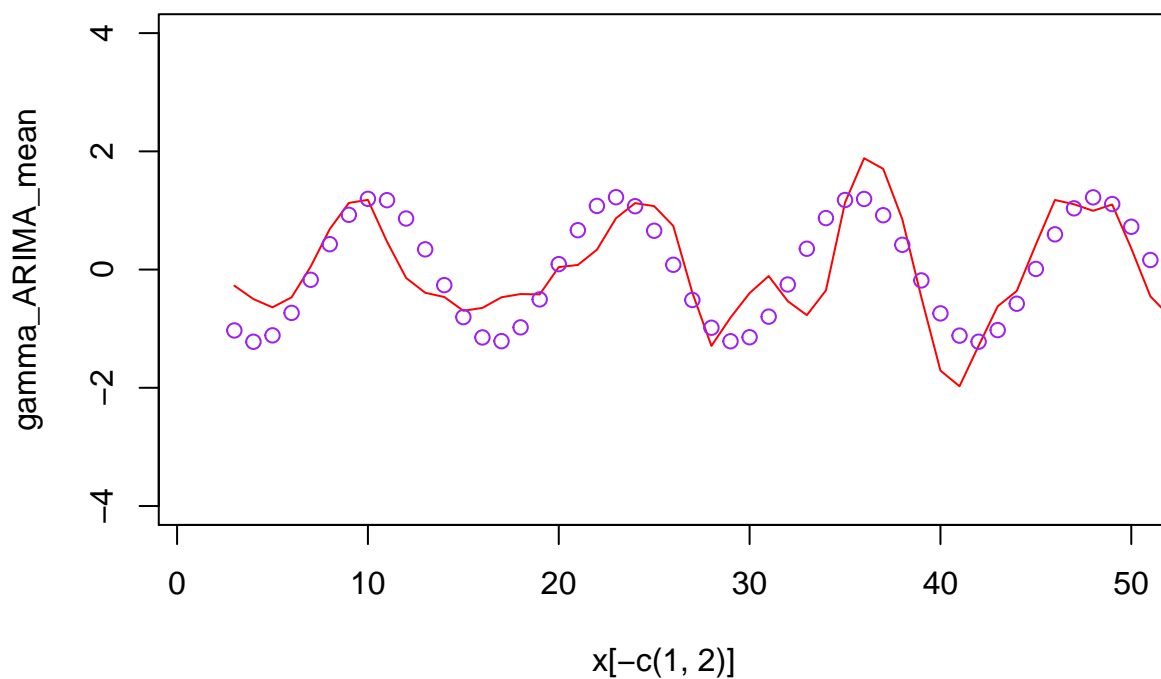
```
gamma_ARIMA_mean <- apply(gamma_ARIMA,1, mean)
```

```
gamma_ARIMA_upper <- apply(gamma_ARIMA,1,quantile, probs = 0.975)
```

```
gamma_ARIMA_lower <- apply(gamma_ARIMA,1,quantile, probs = 0.025)
```

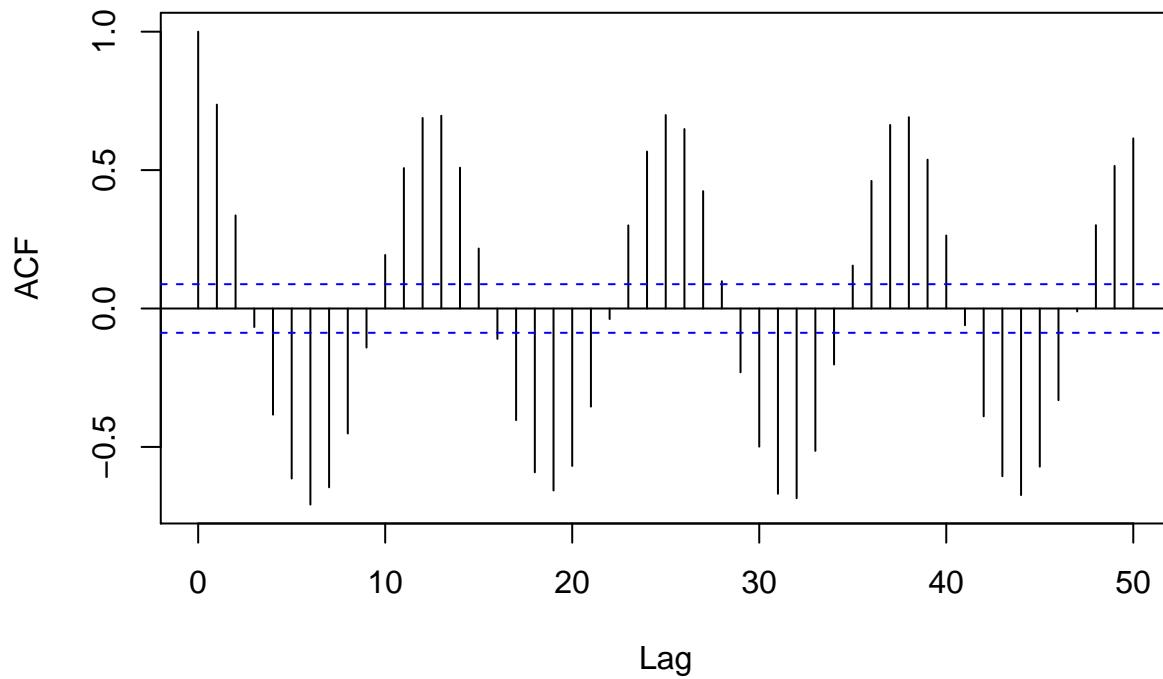
```
plot(gamma_ARIMA_mean~x[-c(1,2)], type = 'l', col = 'red', ylim = c(-4,4), xlim = c(1,50))
```

```
points(true_gamma ~ x[-c(1,2)], col = 'purple')
```



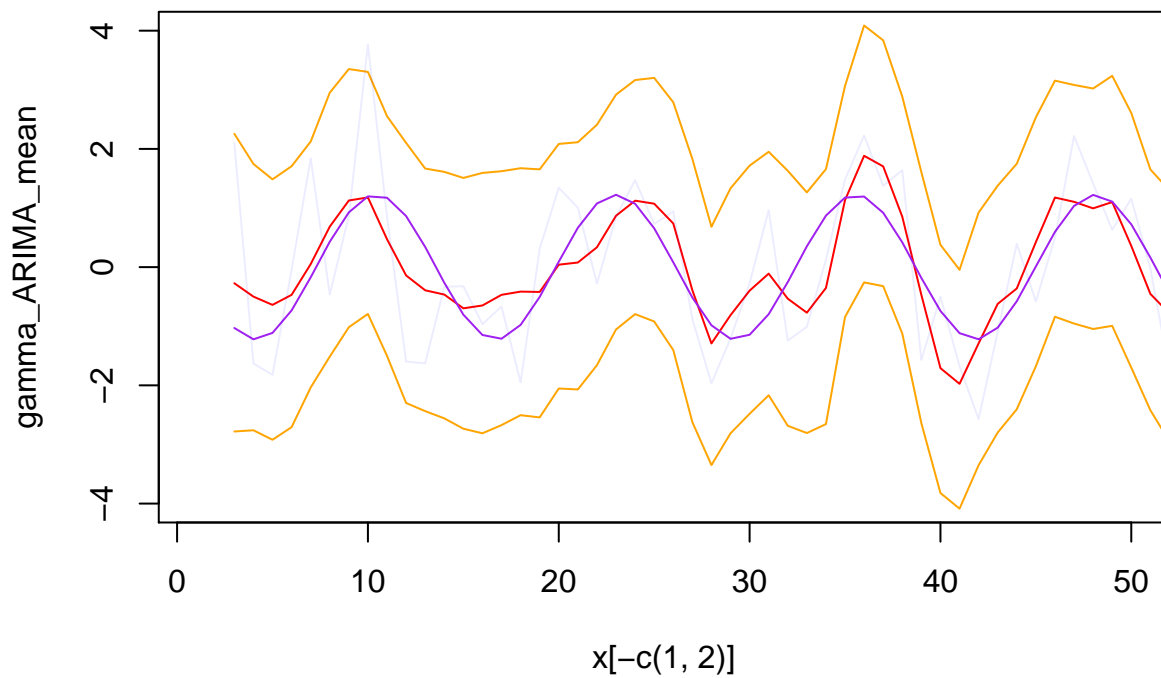
```
acf(gamma_RW2_mean, lag.max = 50)
```

Series gamma_RW2_mean



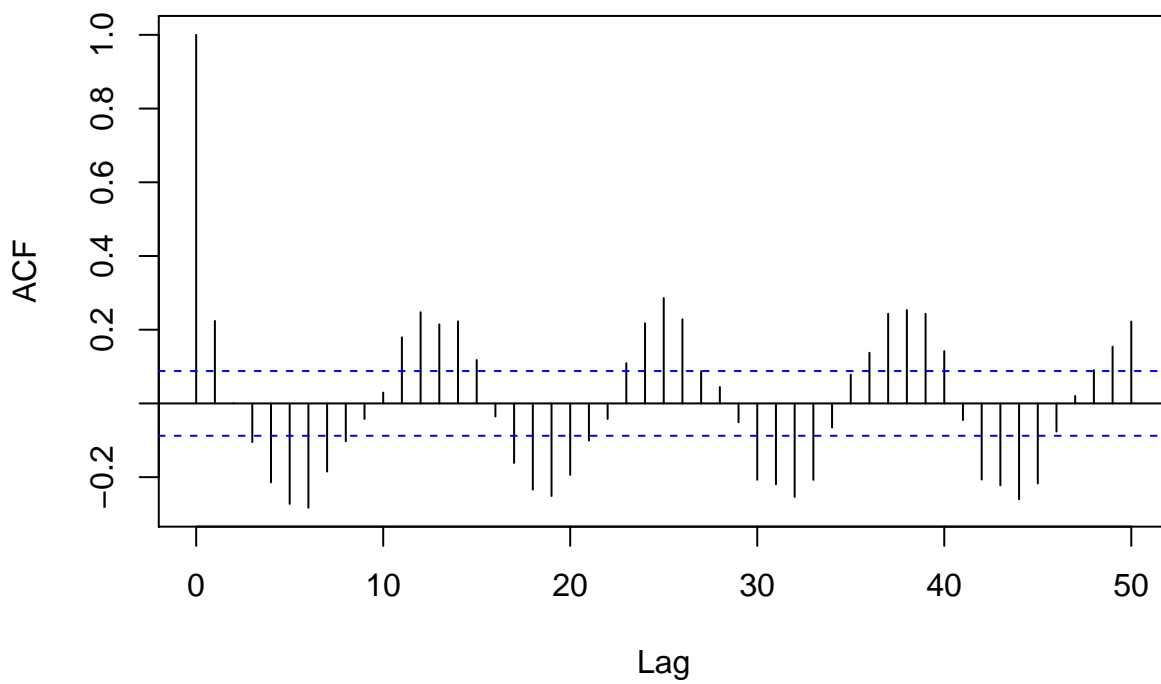
```
# Samples
plot(gamma_ARIMA_mean~x[-c(1,2)], type = 'l', col = 'red', ylim = c(-4,4), xlim = c(1,50))
lines(true_gamma ~ x[-c(1,2)], col = 'purple')
lines(gamma_ARIMA_upper ~ x[-c(1,2)], col = 'orange')
lines(gamma_ARIMA_lower ~ x[-c(1,2)], col = 'orange')

for (i in sample.int(1000,1)) {
  lines(gamma_ARIMA[,i] ~ x[-c(1,2)], col = rgb(0, 0, 255, max = 255, alpha = 20, names = "grey"))
}
```



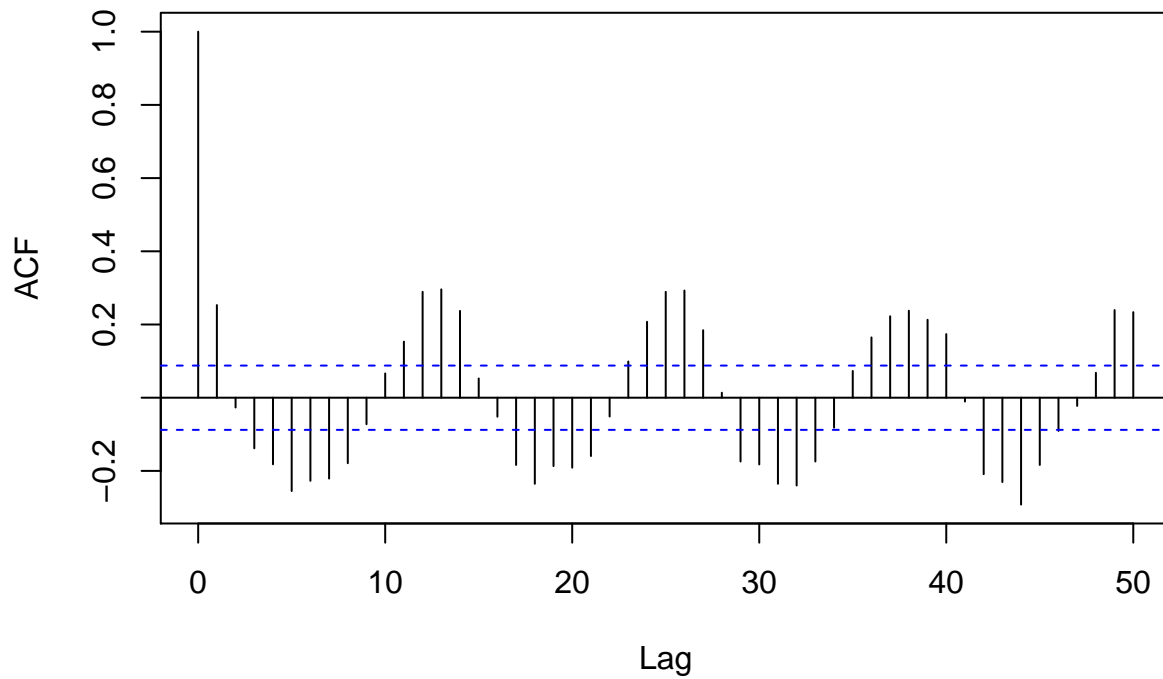
```
acf(gamma_ARIMA[,1], lag.max = 50)
```

Series $\gamma_{ARIMA}[, 1]$



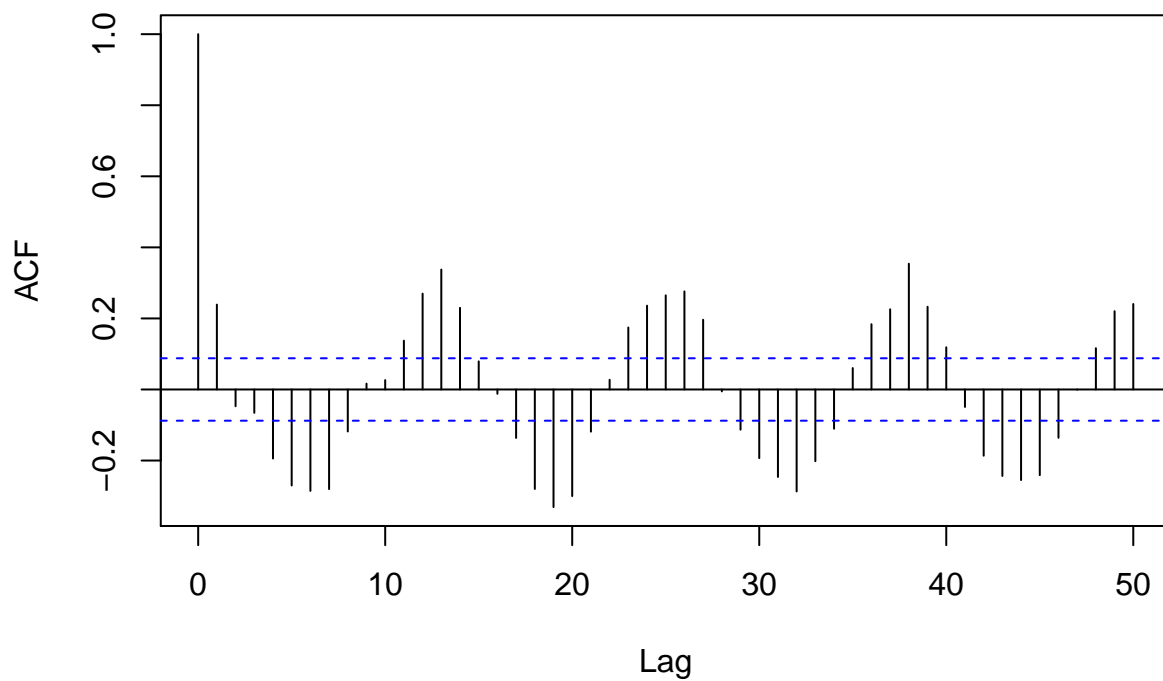
```
acf(gamma_ARIMA[,2], lag.max = 50)
```


Series gamma_ARIMA[, 2]



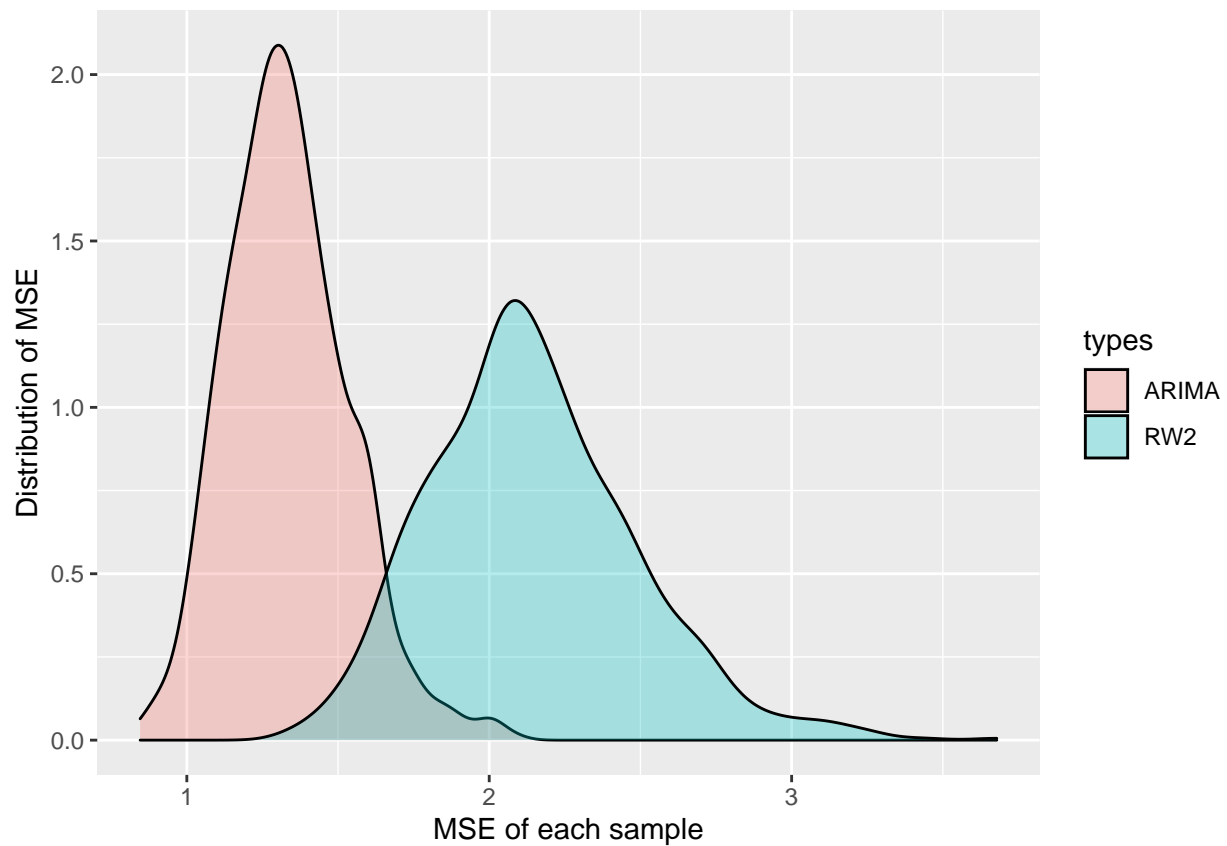
```
acf(gamma_ARIMA[,3], lag.max = 50)
```

Series gamma_ARIMA[, 3]



```
MSE_ARIMA_samples <- apply(gamma_ARIMA, 2, compute_MSE)
```

```
hist_data <- data.frame(MSE = c(MSE_RW2_samples,MSE_ARIMA_samples), types = rep(c("RW2", "ARIMA"), each
ggplot(data = hist_data) + geom_density(aes(x=MSE, fill = types), alpha = 0.3) + ylab("Distribution of MSE of each sample")
```



```
### Mean credible width
```

```
credible_width_rw2 <- mean(gamma_RW2_upper - gamma_RW2_lower)
credible_width_rw2
```

```
## [1] 5.432599
```

```
credible_width_ARIMA <- mean(gamma_ARIMA_upper - gamma_ARIMA_lower)
credible_width_ARIMA
```

```
## [1] 4.216738
```