Optimization of collective well-being in a society under a statistical physics approach

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1. Introduction

Today physics has had a dominant effect on the development of economic theory formal. The social sciences are known to be prone to borrowing ideas, or analogies, from the natural world to exemplify complex processes. However, unlike physics, economic theory lacks fundamental or universal laws; orienting itself towards choice, risk management and decision-making problems.

In 1900, Louis Bachelier, in his doctoral thesis, determined the probability of price changes. Bachelier's original proposal, a Guassian, was quickly replaced by alternative models, such as the Brownian movement, where the difference of the logarithms of the prices are distributed in a Gaussian way. Furthermore, in 1936, Majorana published his article *Il valore delle leggi statistiche nella fisica e neller scienze socialli* where he formally established analogies between statistical physics and the social sciences. Beginning in the 1970s, a growing number of physicists dedicated themselves to analyzing and modeling financial markets, and more generally, economic systems. [3]

1.1. Econophysics

Econophysics was, from the beginning, the application of the principles of physics to the study of financial markets, under the hypothesis that the economic world behaves as a set of electrons or a group of water molecules that interact with each other, and econophysicists, with the new tools of statistical physics and recent advances in understanding chaotic systems, have always been considered to be making a Controversial start by breaking down some puzzling economics and reducing them to a few elegant general principles with the help of some serious math taken from the study of messy materials. [3]

2. The Yakovenko model

In your article *Statistical mechanics of money* (2000) Yakovenko *et al* postulates that in a closed economic system money is conserved. Thus, analogous to energy, the equilibrium probability distribution of money follows the Boltzmann-Gibbs exponential law characterized by an effective temperature equal to the mean quantity of money per economic agent. Furthermore, it is demonstrated how the Boltzmann-Gibbs distribution arises in computational simulations of economic models. Likewise, a thermal machine is considered, in which the difference in temperatures allows to extract a monetary benefit. It is important to clarify that Yakovenko emphasizes that the instantaneous distribution of money among the agents of a system should not be confused with the distribution of wealth, since the latter also includes material wealth, which is not conserves, and therefore

3. Simulation

A simulation was carried out in Python, using the tool Jupyter Notebook. In which, we explore a fundamental aspect of a system of N random interacting actors: the entropic nature of the distribution of the total amount of money in society. This quantity must be kept constant, which represents a restriction for the evolution of the system over time. The initial system is totally anarchic, that is, it does not have any regulatory authority. Subsequently, the intervention of an authority through the collection of taxes and their redistribution is explored.

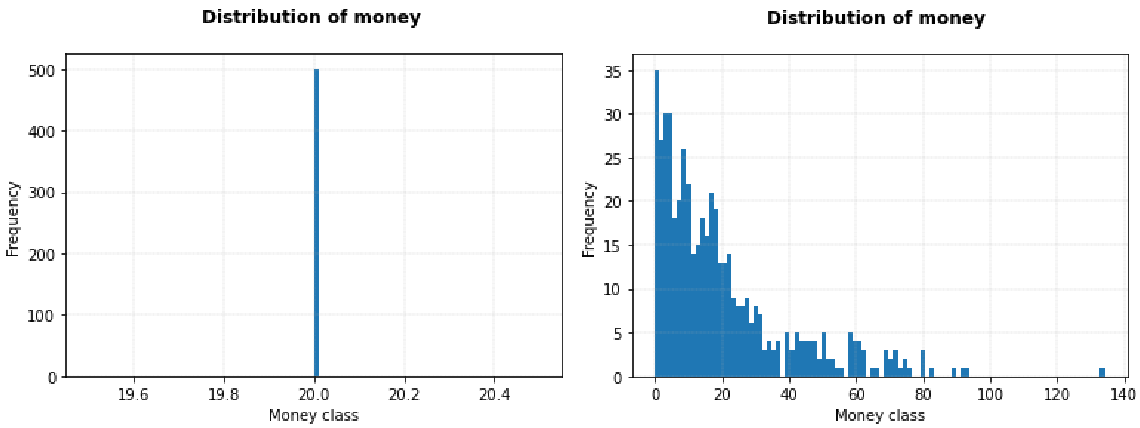
3.1. Model an´ archic

This model starts from an initial distribution Delta, given by $\delta(m_L - \frac{M}{N})$. What's more, uses a system objective function given by $OR(n_1, n_2, \ldots, n_C) = \sum_{k=1}^{C} n_k or_1(M_k)$ where M_k is the amount of money of each of the members of the class k and n_k is the number of agents in the same class, C is the number of classes and $OR_1 = 1 - e^{-aM}$. The parameters used are listed below 1.

Table 1: Anarchic parameters

Parameter	Value
C	100
M	10,000
N	500
to	0.003
t	10,000

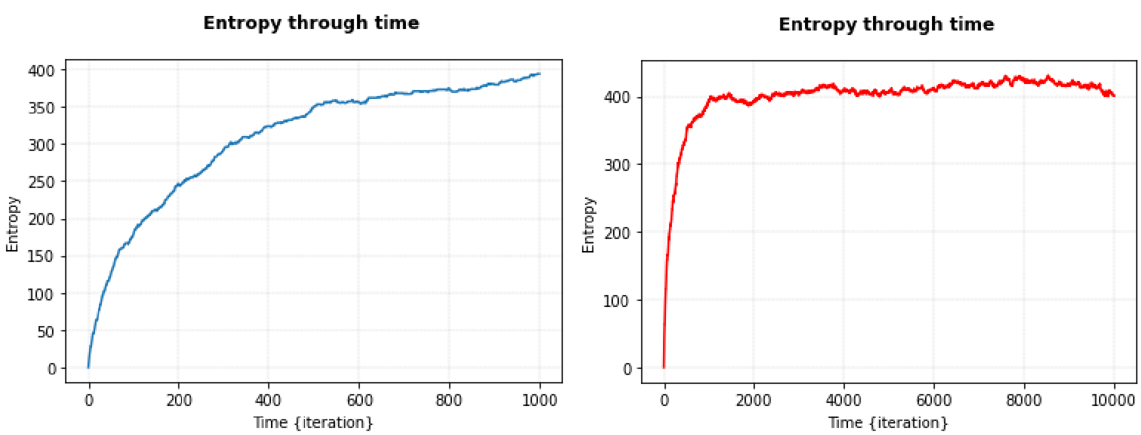
3.1.1. Results



(a) Delta distribution. (b) Final exponential distribution.

Figure 1: Final and initial distribution

We observe that the final distribution corresponds to the results predicted by Yakovenko, that is, there is a clear accumulation of agents in the lower income classes and very few in the high income classes. *A priori* it could be assumed that the system goes from a fair state to a less fair. However, in order to sustain this assertion we need to study the evolution of two key entities of the system: theentropy and the wellness.



(a) Entropy in the former for $t < 1000$. (b) Entropy through exponential time.

Figure 2: Evolution of the entropy of the system

It is interesting to note that entropy starts from 0, which represents the most 'fair' state, that is, a society where everyone has the same thing. However, we note that over time the system tended to become chaotic in the first 1000 iterations, showing less and less growth, tending to stabilize as a negative exponential. Reaching a final value of 422.029. In this way, we can affirm that the system tends to be more and more entropic and, therefore, more unfair.

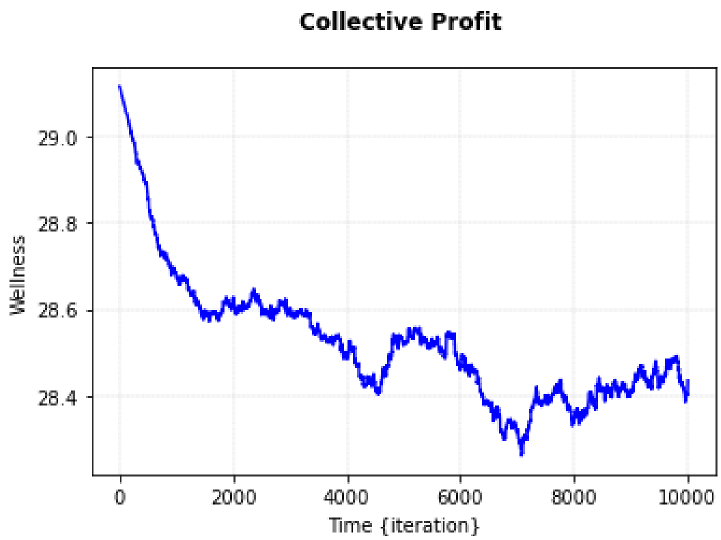


Figure 3: System Wellbeing

Similarly, it is convenient to review the evolution of the collective benefit or welfare of the system. Which is calculated using the objective function and indicates its net yield. We identify a clear downward trend, with certain outbreaks or rises, but which can be approximated by a negative logarithmic function. As expected, the initial value is maximum, since it represents the fairest way to distribute the money among the agents. Finally, it is necessary to make sure of the constancy of the money, since it is the main assumption of the model. We hope to see a straight line in*and* = 10*,* 000.

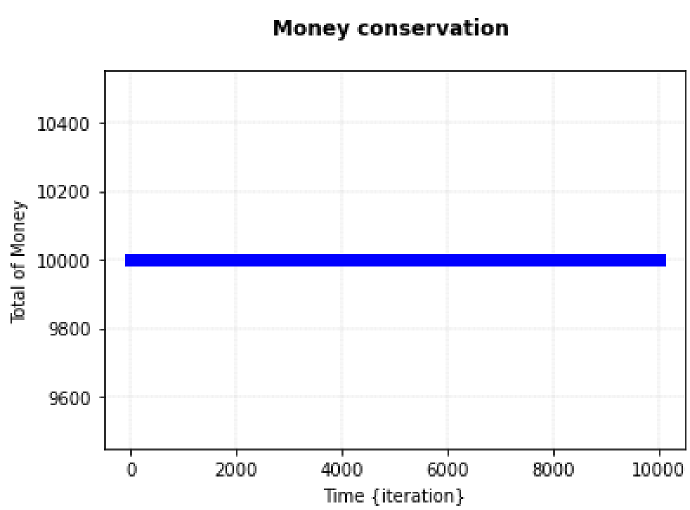


Figure 4: Total system money over time

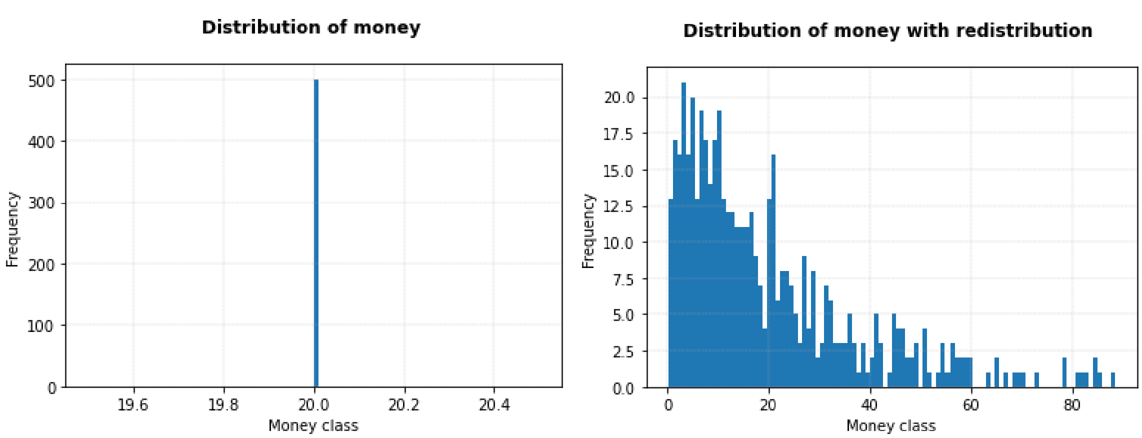
3.2. Model with redistribution on

A case was simulated with a special agent, simulating the effect of a government, collect a fraction λ_t in each transaction and the amount collected is redistributed in each weather τ_s. Similarly, random exchanges of a fraction of the average of the two agents are made. The parameters used are listed in table 3.

Table 2: Parameters with redistribution

Parameter	Value
C	100
M	10,000
N	500
to	0.003
t	10,000
tax	0.16
timer	10

3.2.1. Results

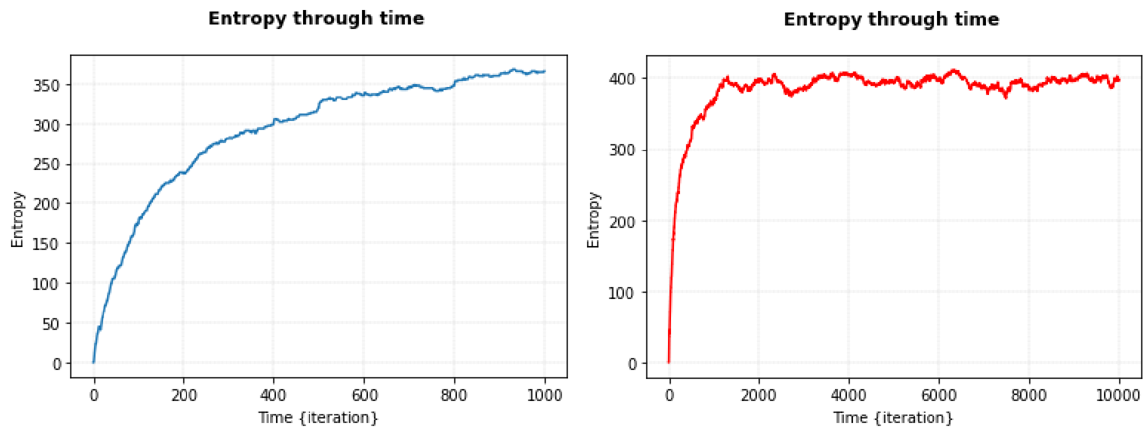


(a) Delta distribution. (b) Final exponential distribution.

Figure 5: Final and initial distribution

With the redistribution model, we start from an initial delta distribution and arrive at to a final non-Boltzmann-Gibbs distribution, which is characterized by a displacement in the equilibrium of the distribution, which leaves a gap in $m = 0$, shifting the distribution to the right. In this way, the low-income population is suppressed, through what could be described as a government 'push'.

We found a similar behavior obtained in the case without redistribution. In which, the entropy starts at its minimum value $t(0) = 0$ and tends towards a quasi-static or equilibrium behavior when $t >> 1$, reaching a final value of 398.198. It is important to emphasize that the entropy value, in the steady state, of the model with redistribution is less than



(a) Entropy for $t < 1000$ (b) Entropy through time.

Figure 6: Evolution of the entropy of the system

obtained in the model without external regulatory agents. Therefore, it could be concluded that this consideration leads to a fairer distribution.

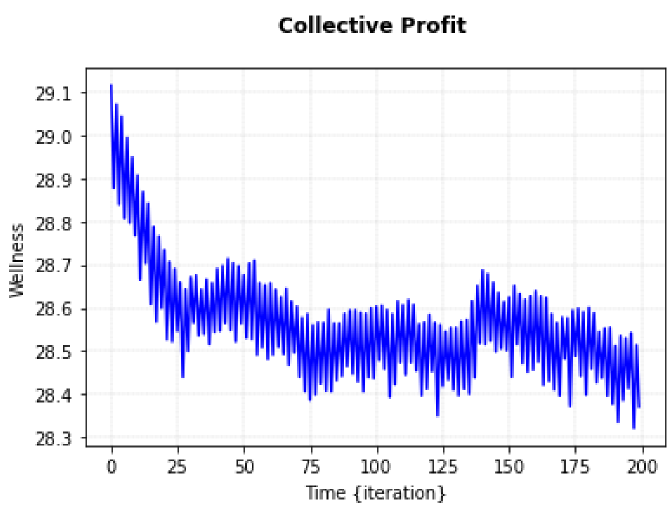
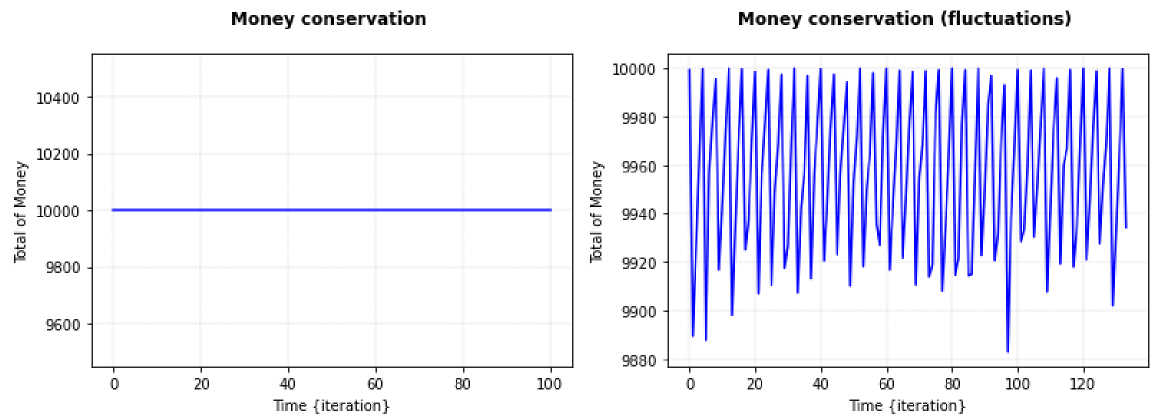


Figure 7: System welfare with redistribution

Analyzing the graph of the collective benefit for the case with redistribution, we note a greater oscillation of the values over time. This behavior has its origin in the amount of money subtracted from the system, that is, there are periods where the amount of money is greater or less (but always less than 10,000), which directly impacts the calculation of welfare, since that, the objective function of the system depends directly on money that resides in each class.

In graphs 8 we can observe the variations over time of money in the system. In figure 8a we observe the sum of money withdrawn (reservoir) and in 8b we observe only the amount in the system for a given time. So, the variations in the amount of money in the system for a time t it causes a greater variation in the collective benefit. Using this model, it follows that the Boltzmann-Gibbs distribution is not completely



(a) Sum of money in the system and in the reservoir

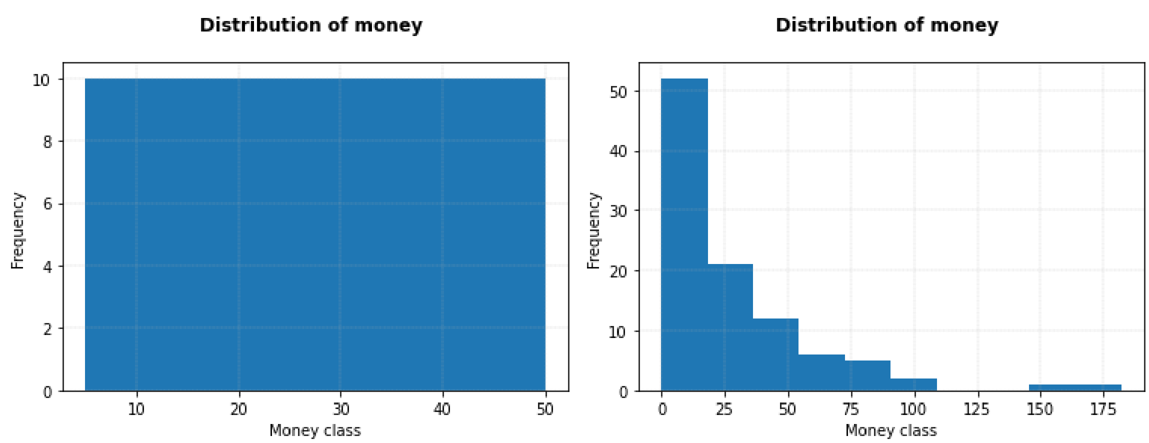
(b) Money in the system over time.

Figure 8: Evolution of total money over time

universal, that is, it does not conform to any exchange model that conserves a fixed amount of money. However, it can be said that it is universal for models with time symmetry.

3.3. Model an´ archic with uniform distribution

For this case, a uniform initial distribution given by $F(m_i) = cte$, on the one that each class has the same frequency as can be seen in the figure.

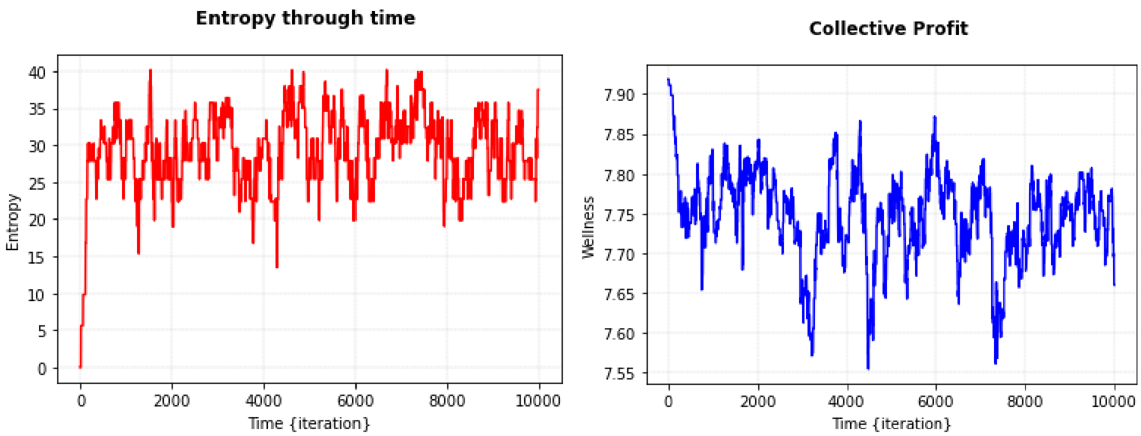


(a) Uniform initial distribution

(b) Final Boltzmann-Gibbs distribution.

Figure 9: Initial and final distribution

For this case, we again observe the emergence of an exponential Boltzmann-Gibbs distribution, in which there is a greater concentration of agents in the low-income classes and few in the high-income class. Furthermore, in figure 10a we can observe a more oscillatory behavior than that obtained with the delta distribution, having as the final value of the entropy 37.512



(a) Temporal evolution of entropy (b) Collective Welfare

Figure 10: System characteristics

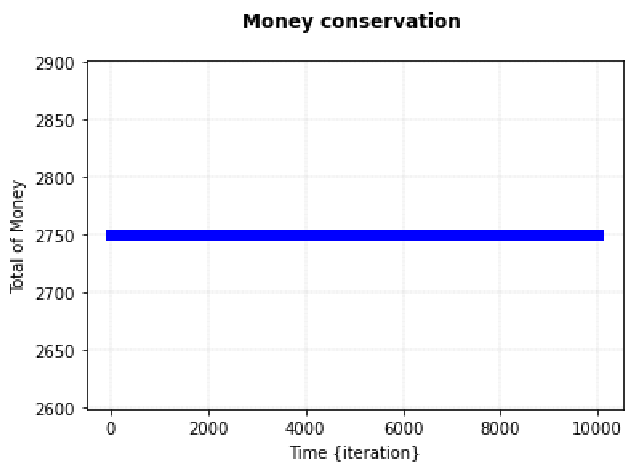


Figure 11: Conservation of money for uniform distribution

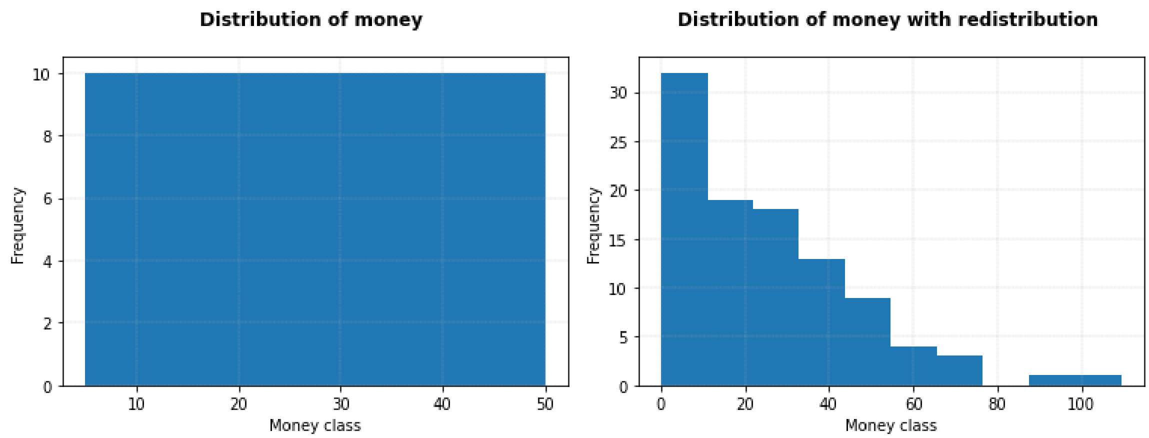
Similarly, when analyzing the temporal evolution of the collective benefit, we observe a behavior similar to that obtained in the case of the Delta distribution. We start at a maximum and it decreases steadily, although in this case we obtained a greater amount of oscillations.

Finally, it is necessary to make sure that the total money in the system does not change, so we graph the sum of the money of all the agents in figure 11, we hope to see a line in *and* = 2750.

3.4. Model with redistribution ion: Uniform Distribution

For this model, we started from the uniform distribution until reaching a non-Boltzmann-Gibbs distribution as can be seen in figure 12.

Similarly, the entropy of the system starts at a minimum and increases as

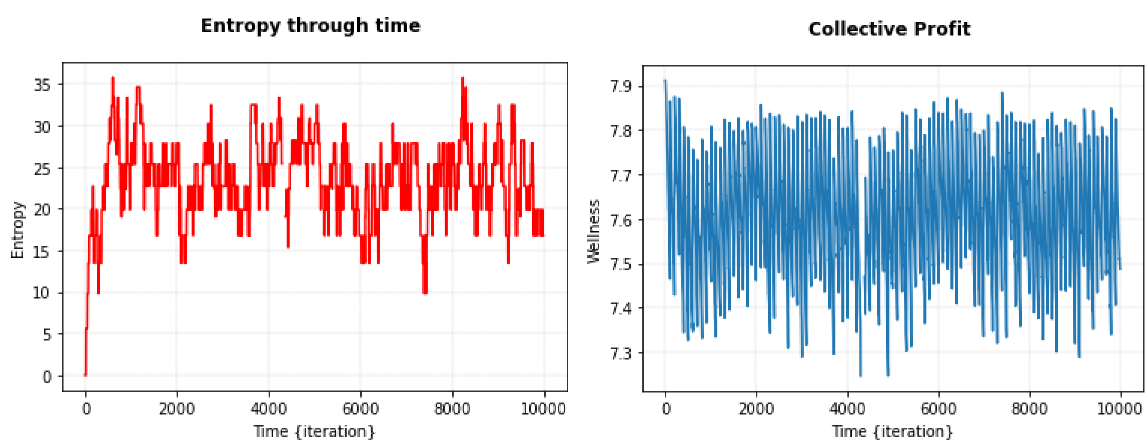


(a) Uniform initial distribution

(b) Final non-Boltzmann-Gibbs distribution

Figure 12: Initial and final distribution

time passes, up to a final value of 16.794 (less than for the case without redistribution). Similarly, the welfare of the system, as expected, begins at a maximum and decreases with each iteration, with the difference of the large number of oscillations that it presents.



(a) Entropy of the system

(b) Collective welfare of the system

Figure 13: System characteristics

In conclusion, when using a uniform distribution the model presents a greater quantity of oscillations and noise that makes it difficult to observe compliance with the Boltzmann-Gibbs law. However, the results adhere to what is described in Yakovenko's articles.

4. The Wright model

Wright's model allows organizing the system into subsystems, identified in 3 main groups: employees, the unemployed and entrepreneurs. With this, they seek to achieve a model that produces the appearance of two fundamentally different kinds of income.

In figure 14 the interactions between the classes can be observed.

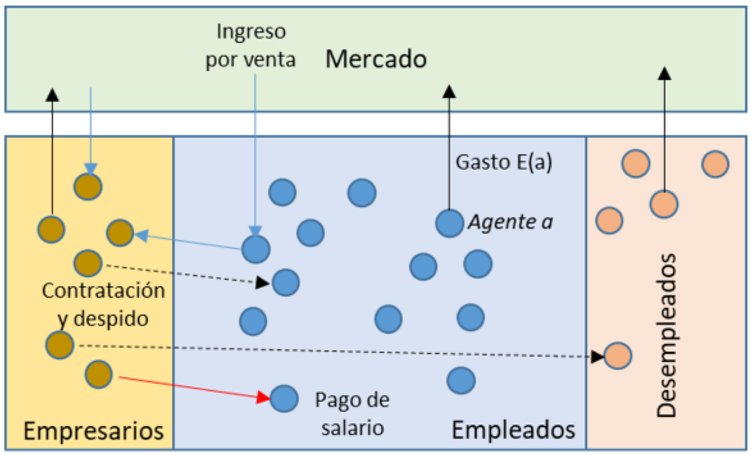


Figure 14: Relationship diagram in Wright's model

The three classes interact directly and through the market. All agents a, in turn, they can make an expense by applying the rule *AND*(*to*), through which a random amount of money is transferred to the market, increasing its value. Only entrepreneurs, considered the only producers of goods and services, can receive income from the market. In the event that an employee makes a sale, then the amount of money is transferred to his employer. Employees stay with their employer until they are laid off. New hires are made only among the unemployed. All employers hire new hires as soon as they have enough money to pay the additional wages; analogously, they lay off the number of employees whose salary they cannot afford. All processes are performed randomly using uniform distributions and certain predefined intervals for each random variable. [4]

4.1. Simulation

For this part a code was made in MATLAB©. With the following parameters.

As we can see in figure 15, a point is reached where entrepreneurs have surpassed the available employees and unemployed, causing a thermal death, that is, the system is not able to continue with the interactions, giving rise to interesting phenomena.

Unlike random exchange systems, the entropy of the class system reaches a maximum value and then begins to decrease. This because the classes increase

Table 3: Parameters with redistribution

Parameter	Value
M	2000
N	100
V	0
t	10,000
Salary	[3 8]

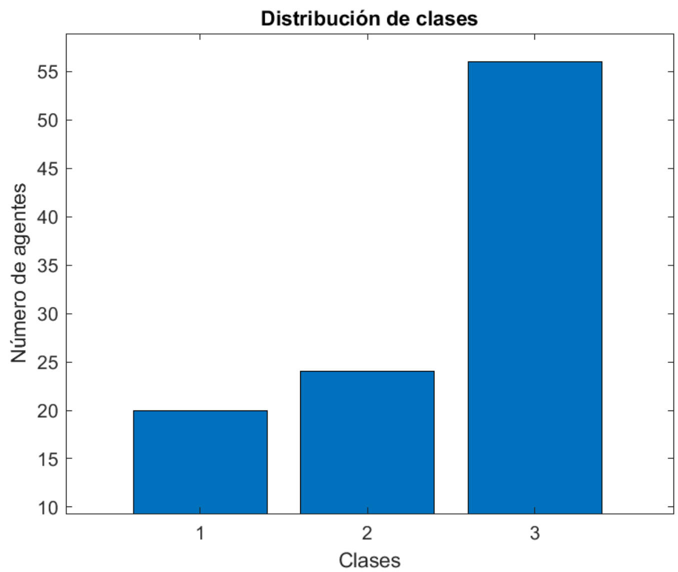


Figure 15: Final distribution of classes

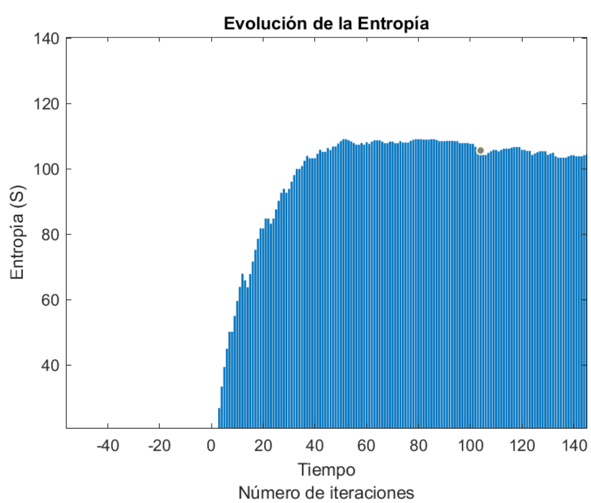


Figure 16: System entropy

and they decrease randomly and the combinations are reduced as the actors move between the classes.

The collective welfare is quite similar to the results obtained with the dynamics of random exchanges. However, it is necessary to point out that in Wright's dynamics these

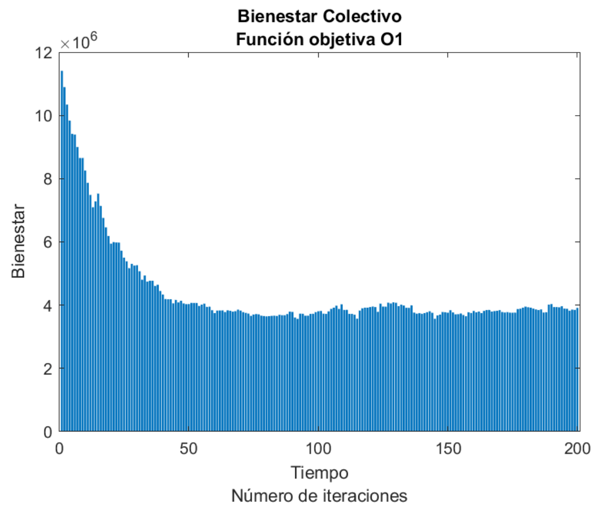


Figure 17: Collective well-being using a linear function

they reach the quasi-static state more quickly. This indicates that the greater the number of interactions, the greater the entropy the system has. Giving a glimpse of the complexity of today's society.

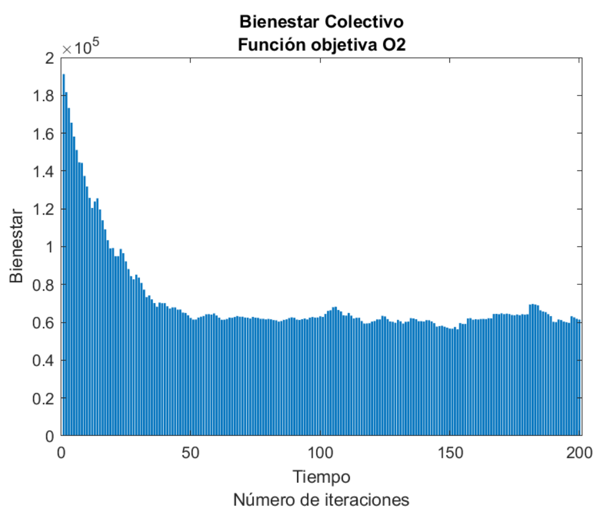


Figure 18: Collective well-being using an exponential function

5. Conclusion

Economic systems are complex and abstract entities, with endless dynamics that are difficult to model. As we saw, the Boltzmann-Gibbs distributions have their limitations in terms of the diversity of phenomena that they can describe, being the restriction of temporal symmetry the most important to determine their validity or not. Furthermore, in Wright's dynamics we observe the emergence of a long tail in the distribution of income, which would correspond to the super-rich or the highest social classes.

Finally, landing on the Mexican context, we must remember that Mexico is a country with great inequalities. However, the statistical information available has led to an underestimation of inequality and an overestimation of poverty. This is due to two main reasons: true household income is higher than reported, and households with income much higher than reported are not included in the sample. Similarly, the discussion about economic inequality between individuals and households, both wealth and income, has recently returned to the center of attention of politicians and academics. Authors such as Piketty propose that uncontrolled inequality is not only a reflection of an unjust economic system, but ultimately it is a brake on efficiency and economic growth. In our country, the book Extreme Inequality in Mexico: Concentration of Economic and Political Power (2015) by Gerardo Esquivel, triggered a wave of reflections and a renewed interest in inequality in Mexico. [1] In this way, much work remains to be done to achieve a modeling of income distribution that allows us to design better strategies to combat inequality and end poverty.

References

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