Análisis de Algoritmos II

Punto de visibilidad en polígonos simples.

Profesores:

Jorge Urrutia Galicia

Adriana Ramírez Vigueras

Diego Jesús Favela Nava

Adrián Aguilera Moreno.

aguilera@ciencias.unam.mx

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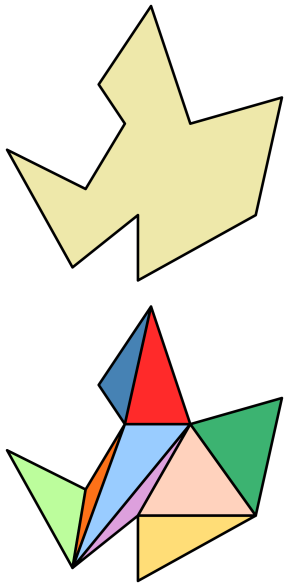
Introducción
Problemas de visibilidad.

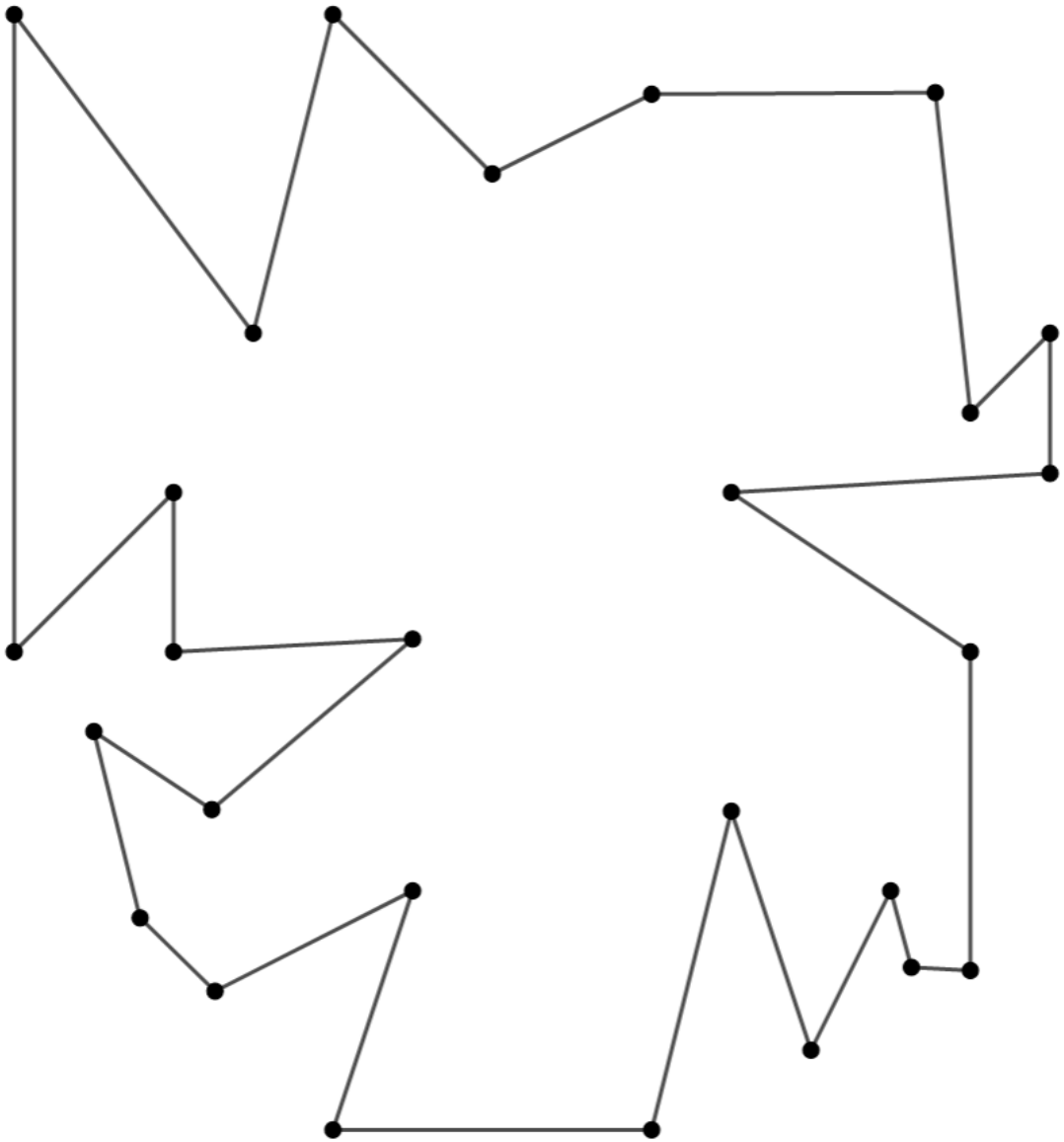
Determinar regiones de visibilidad de un objeto geométrico bajo restricciones es un problema muy estudiado en *Geometría Computacional*.

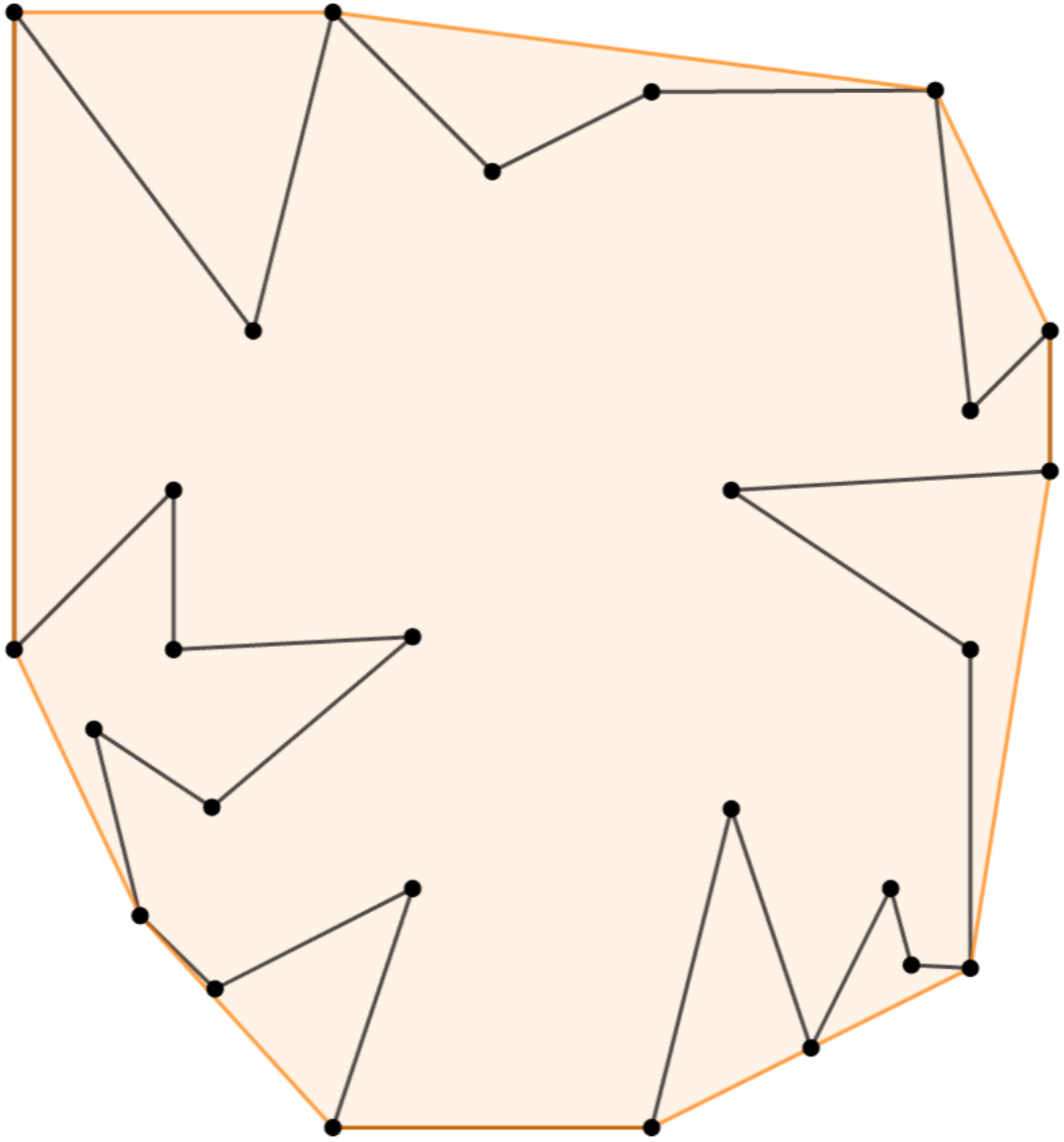


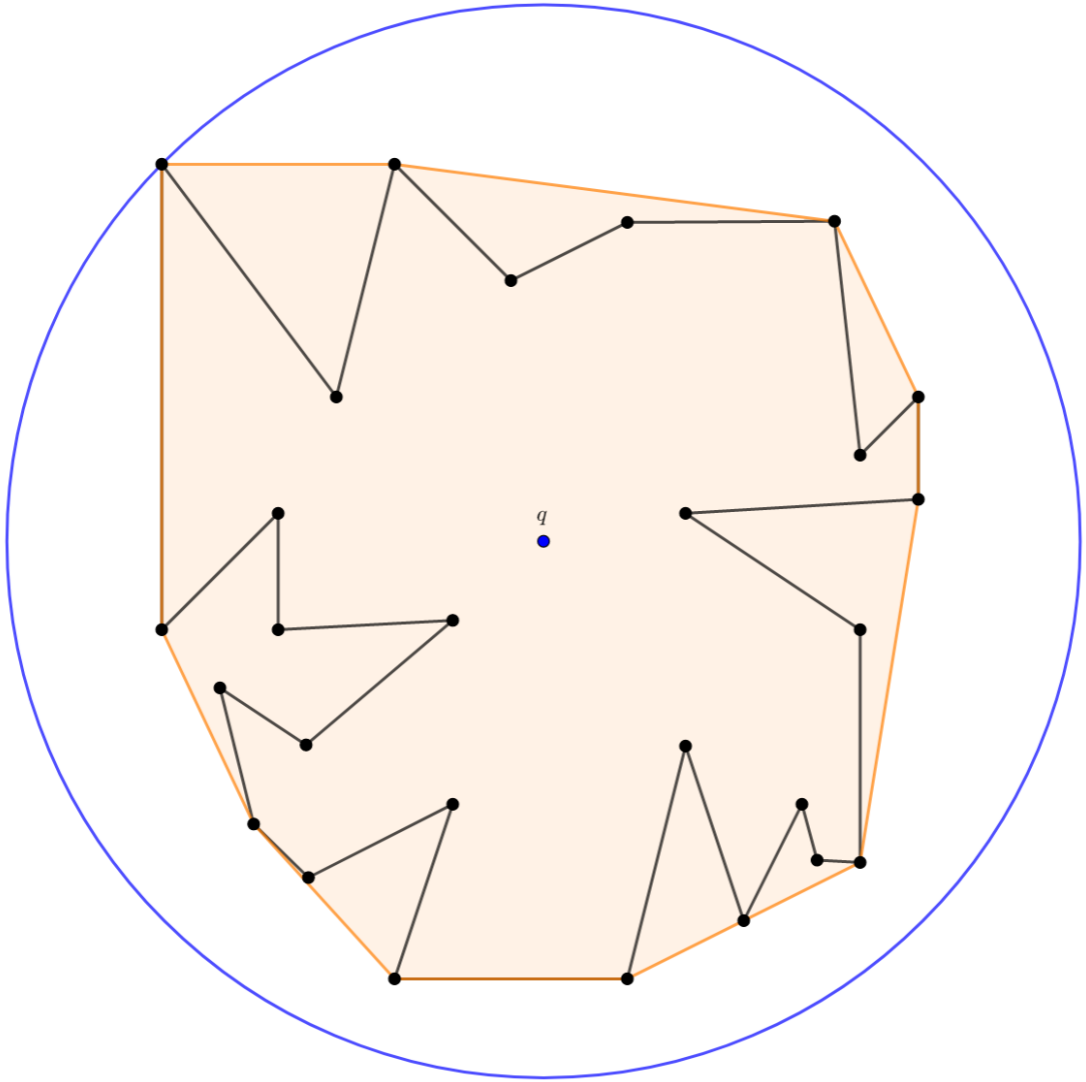
Introducción
Polígono de visibilidad.

Definimos un Polígono de visibilidad como el polígono formado a partir de un punto q dentro de un polígono dado, digamos P. Entonces, el polígono de visibilidad de q definido como

$$V(q) = \{p \in P \,\big|\, q \text{ visualiza a } p\}.$$



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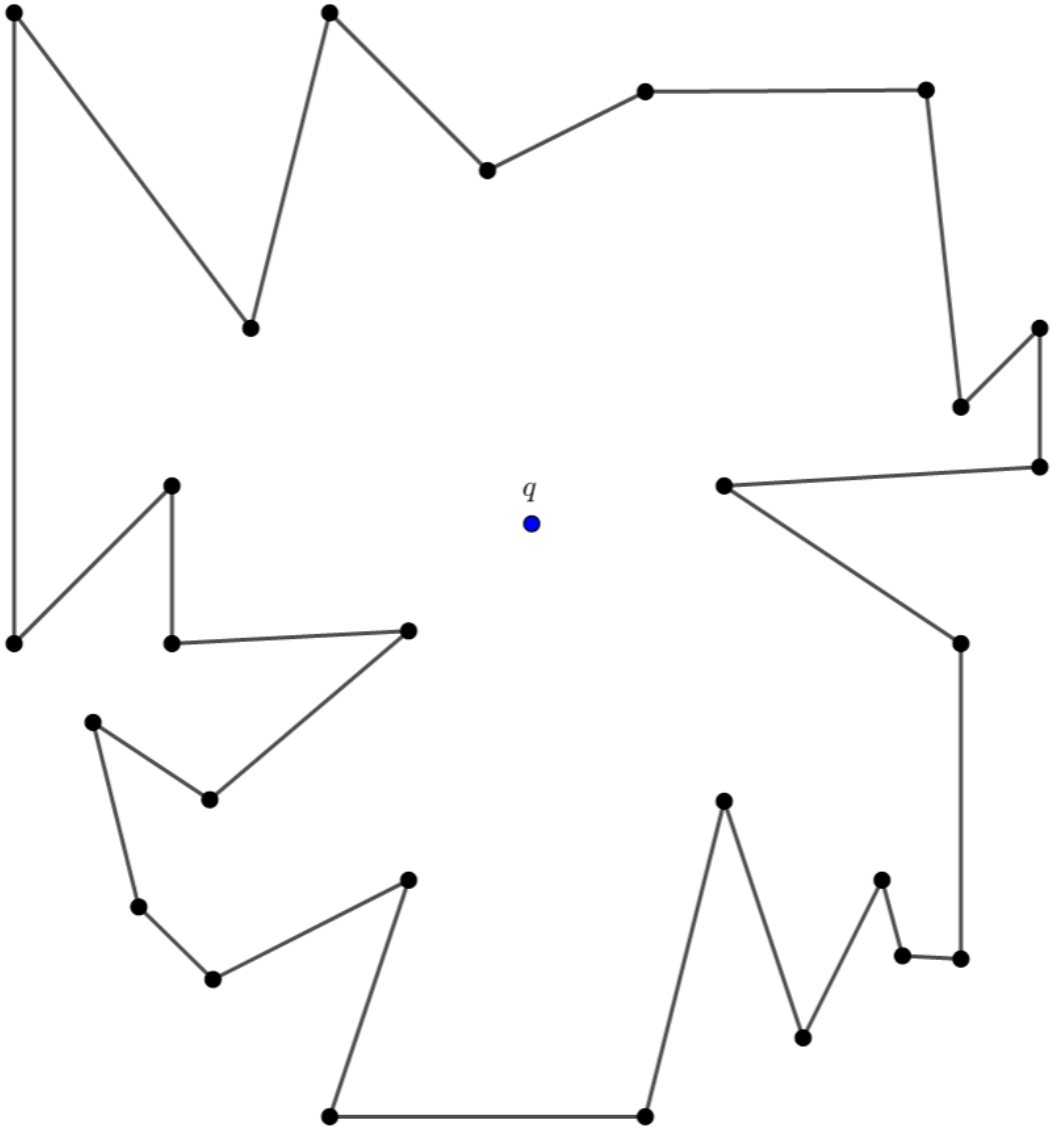
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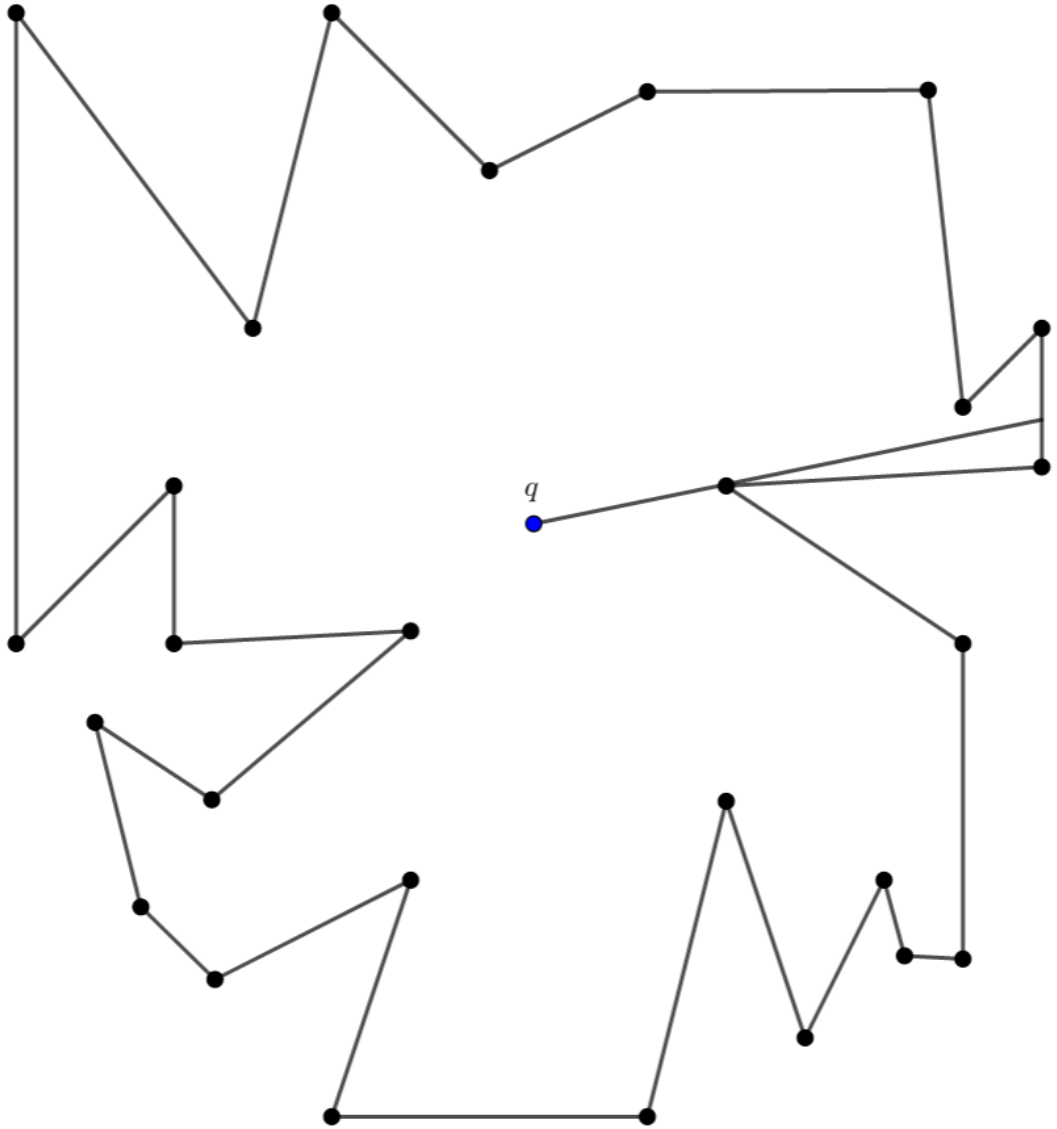
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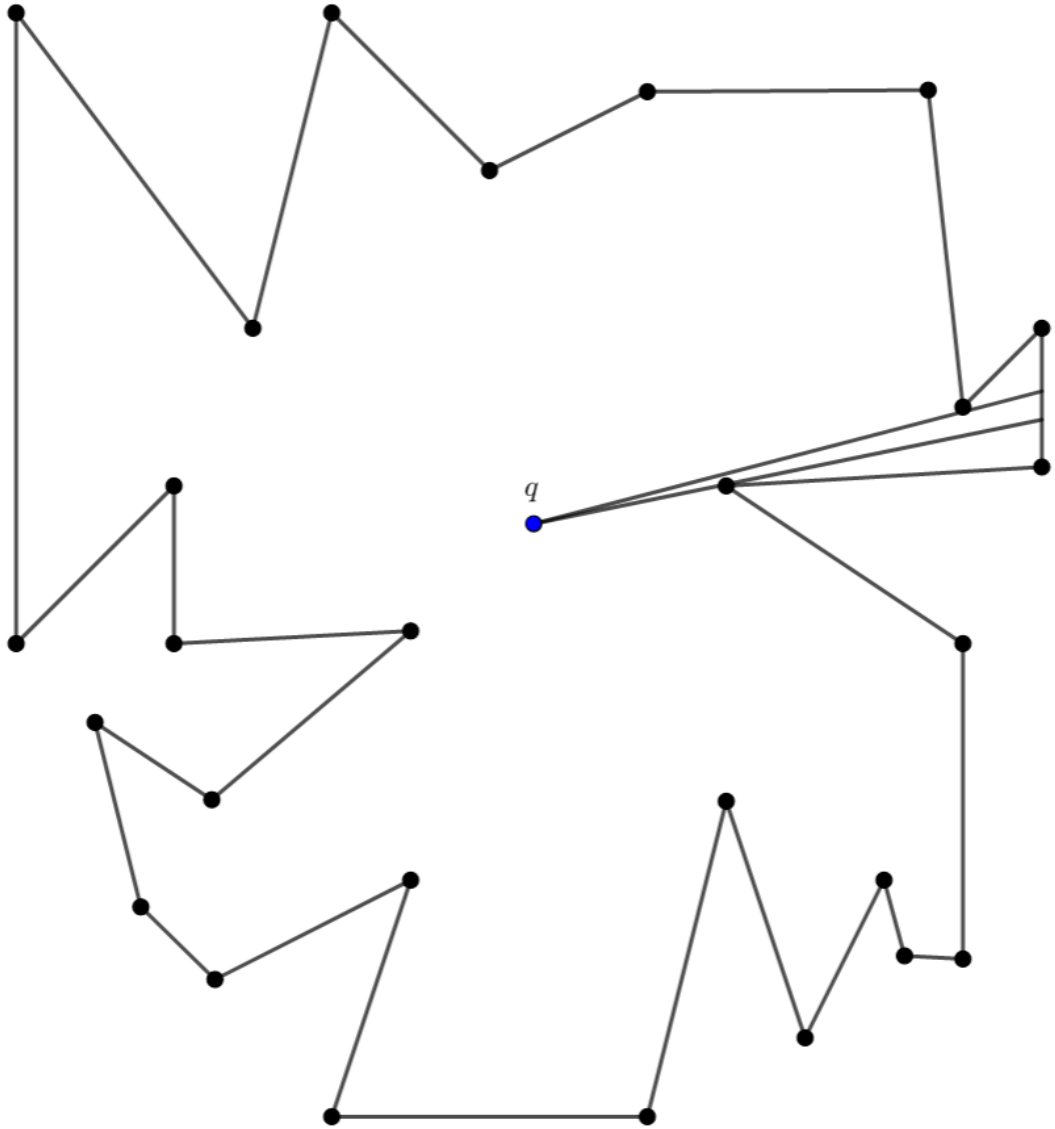
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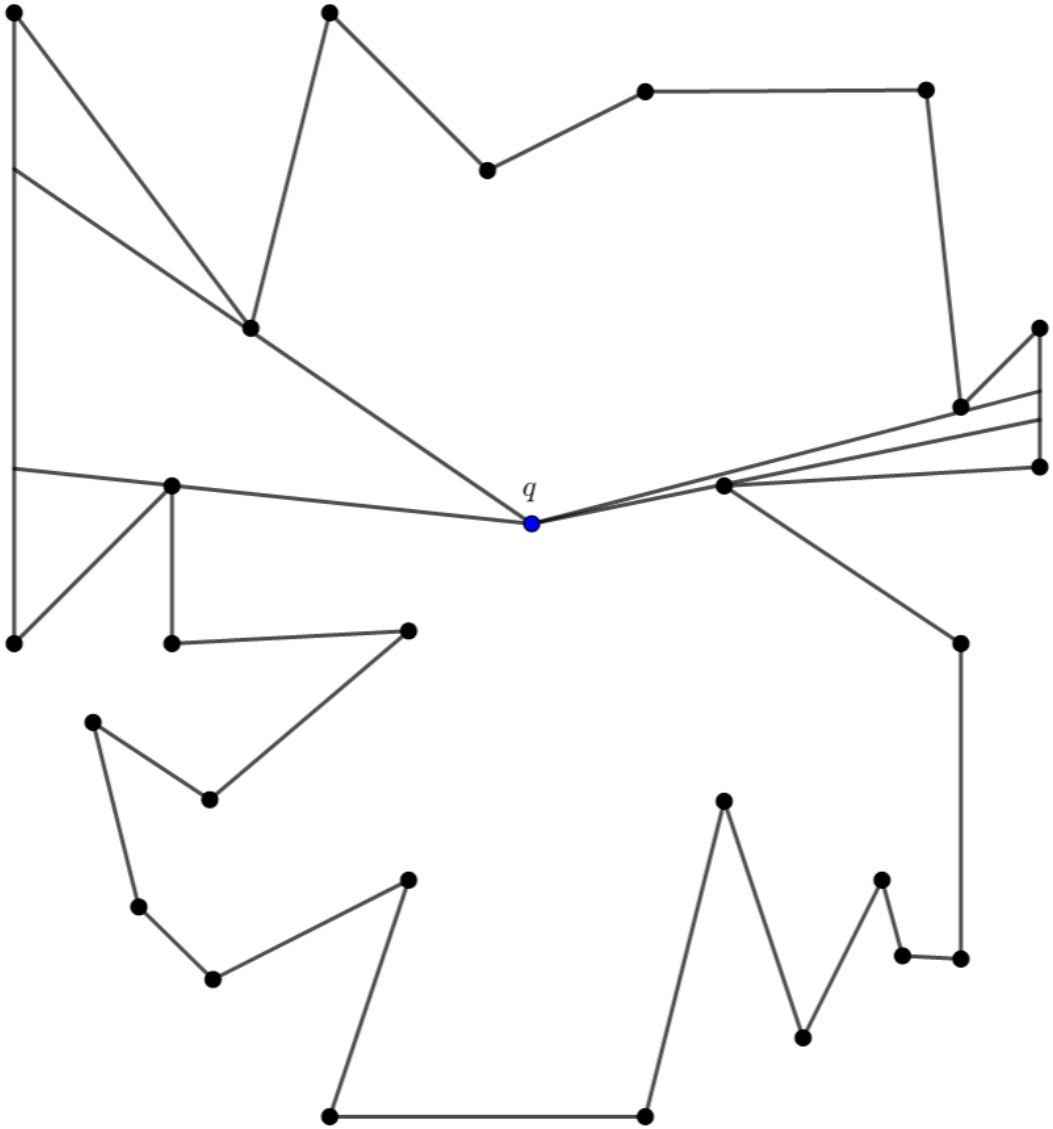
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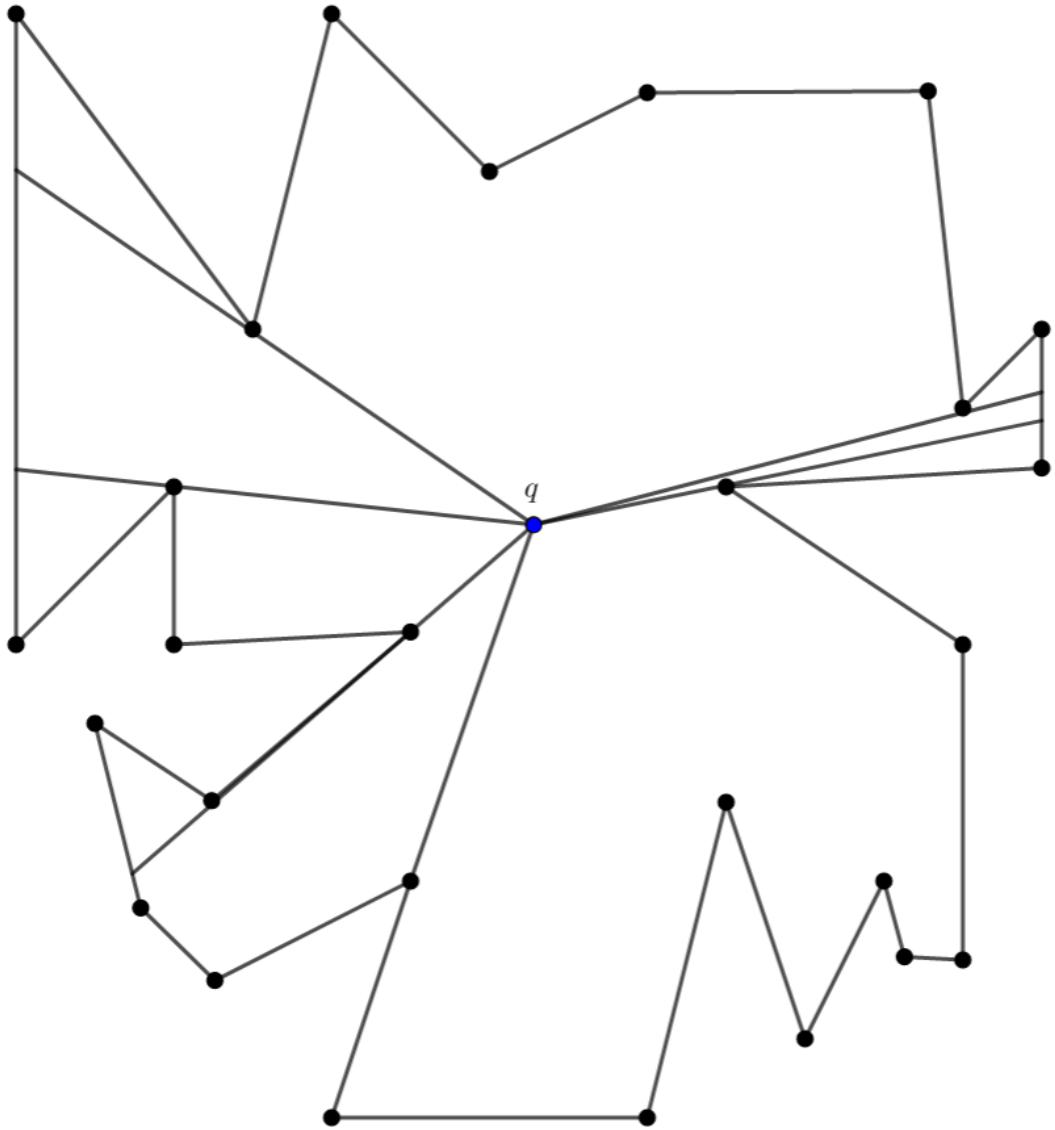
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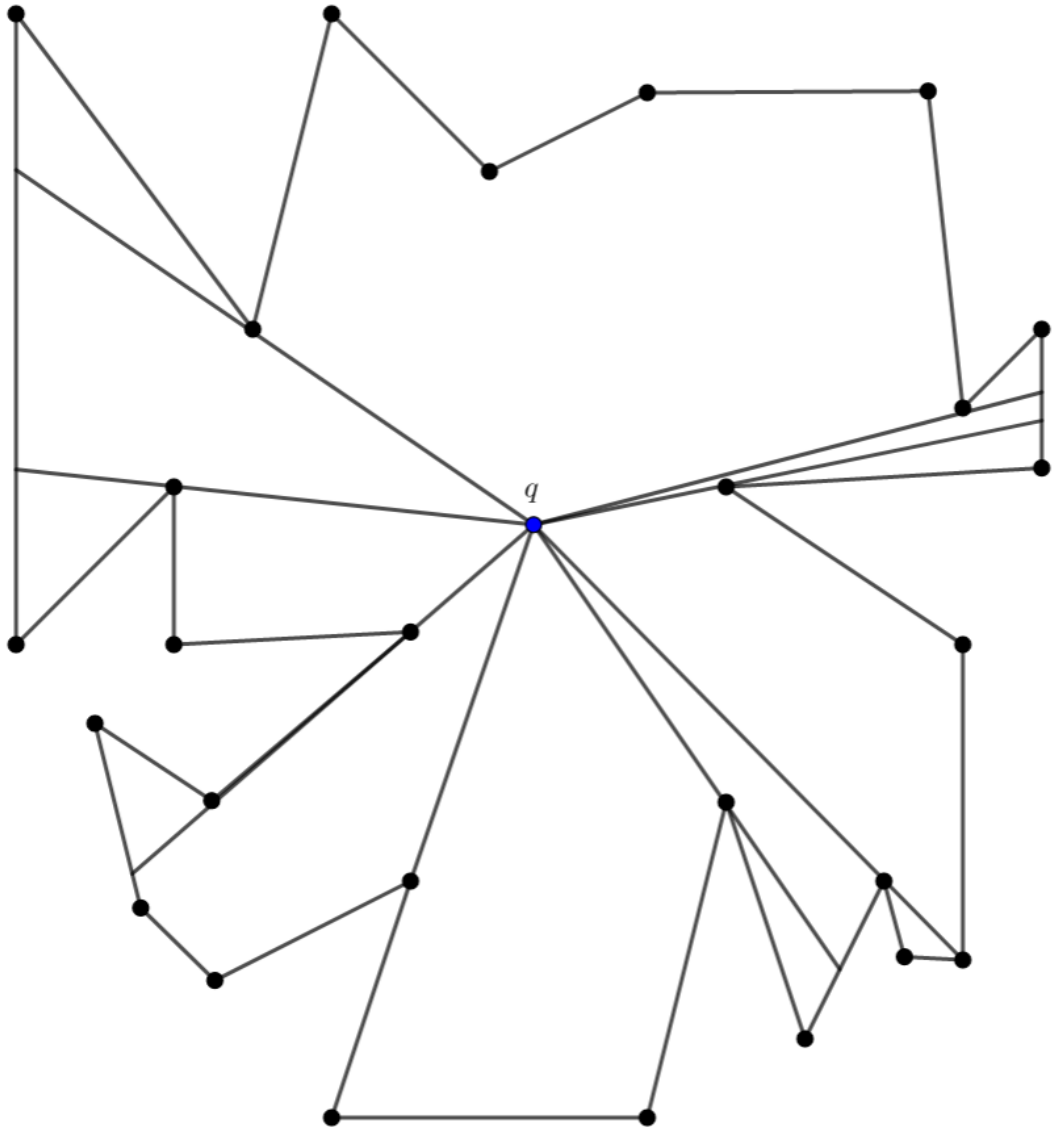
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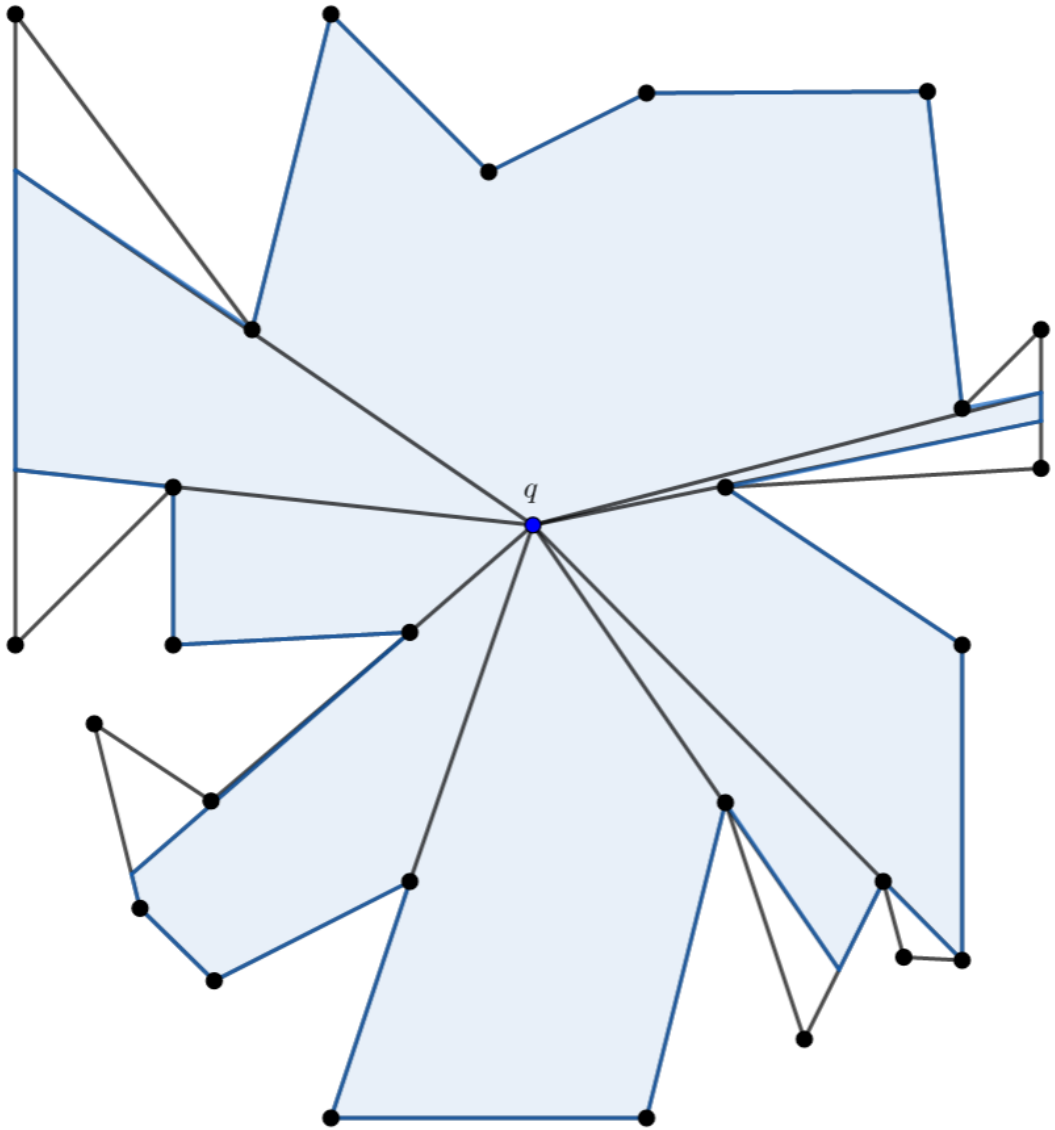
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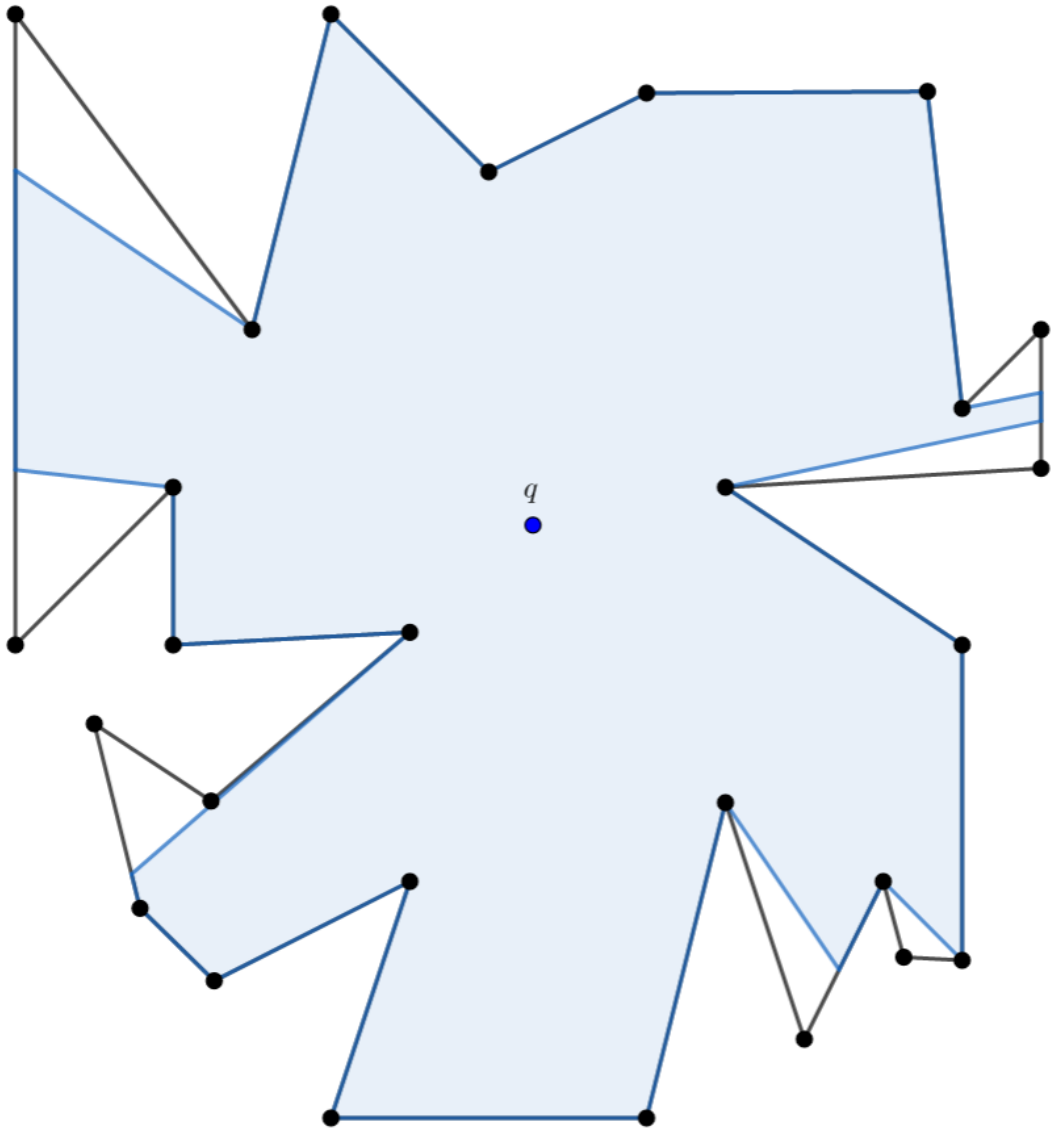
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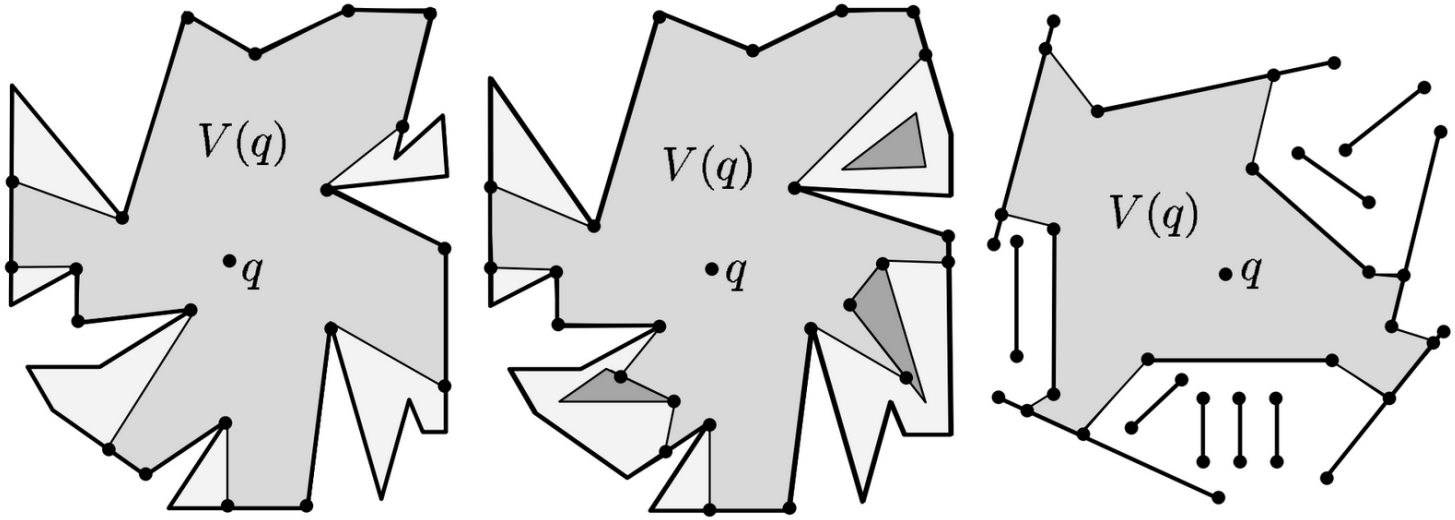
Introducción

En partícular, podemos tener distintos tipos de polígonos.

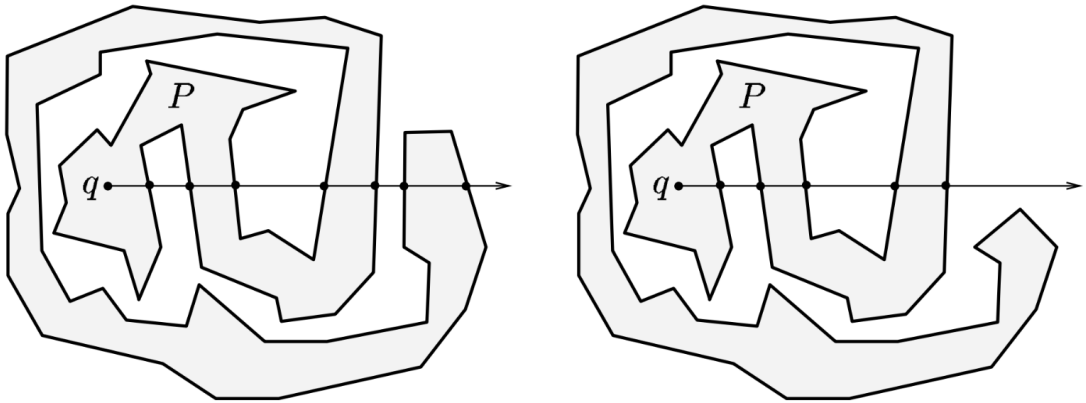


Distintos casos para encontrar polígonos de visibilidad.

Obs. El polígono de visibilidad no, necesariamente, tiene que ser un polígono acotado.

Motivación.

Observemos el siguiente caso partícular de polígono simple:

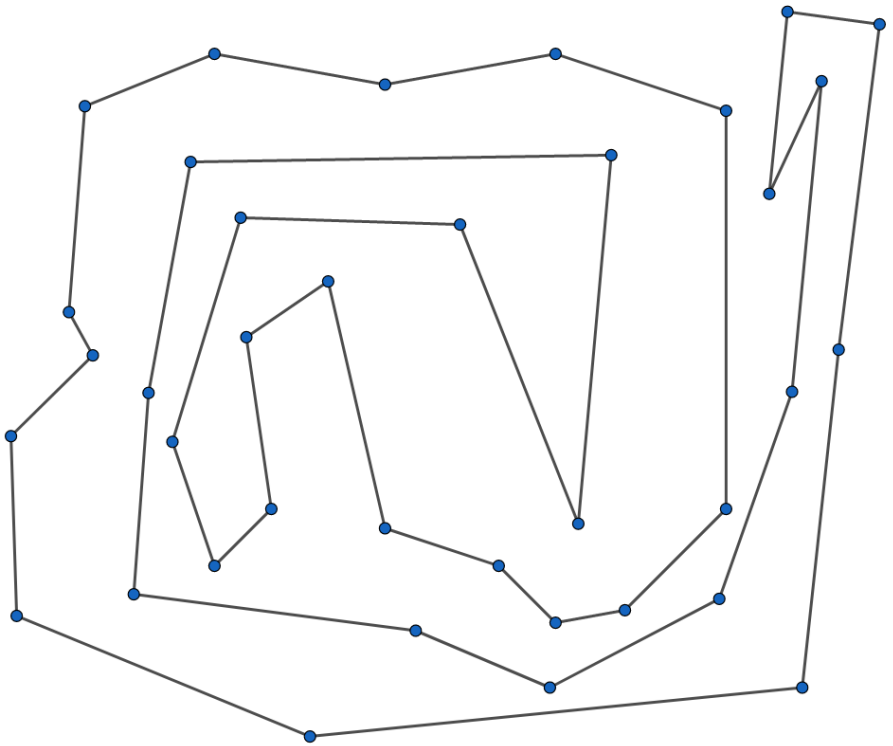


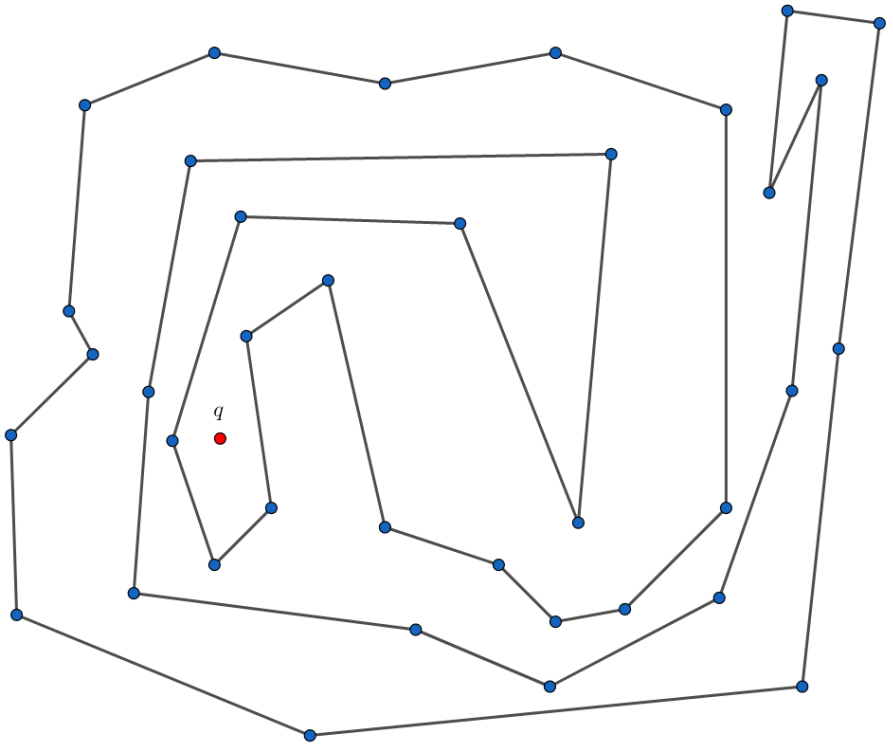
El polígono de la izquierda tiene como número de revoluciones 2 y el polígono de la derecha tiene cómo número de revoluciones 1.

Poda de polígonos simples.

Dado un polígono simple P y un punto $q \in P$, calcular el polígono podado $P_1 \subseteq P$ simple tal que P_1 contiene tanto a $V(q)$ como a q y el número de revoluciones respecto de q es a lo más 1.

Ejecución simple.

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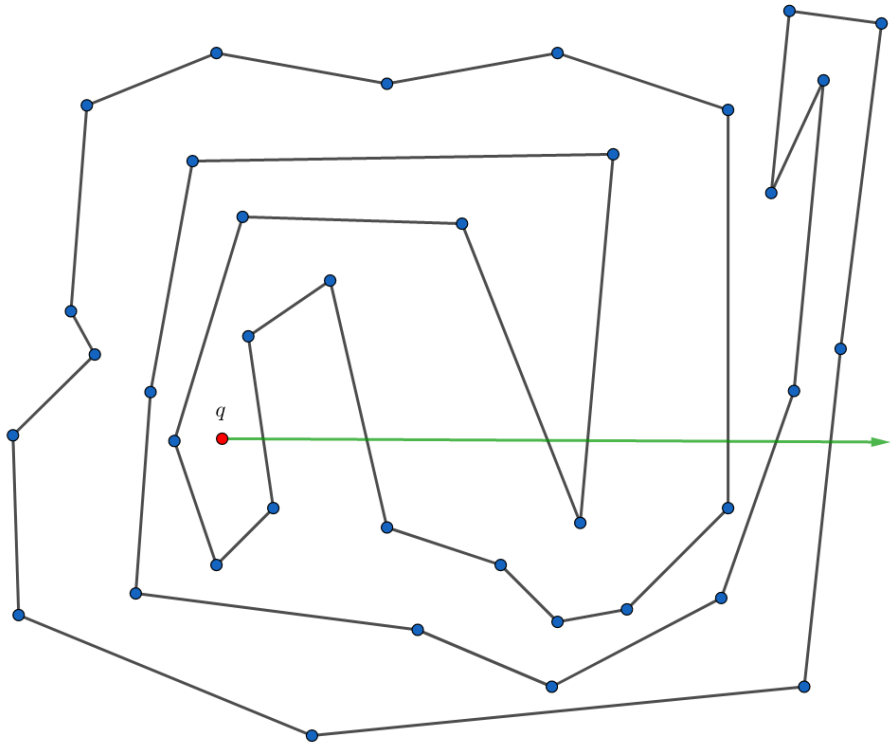
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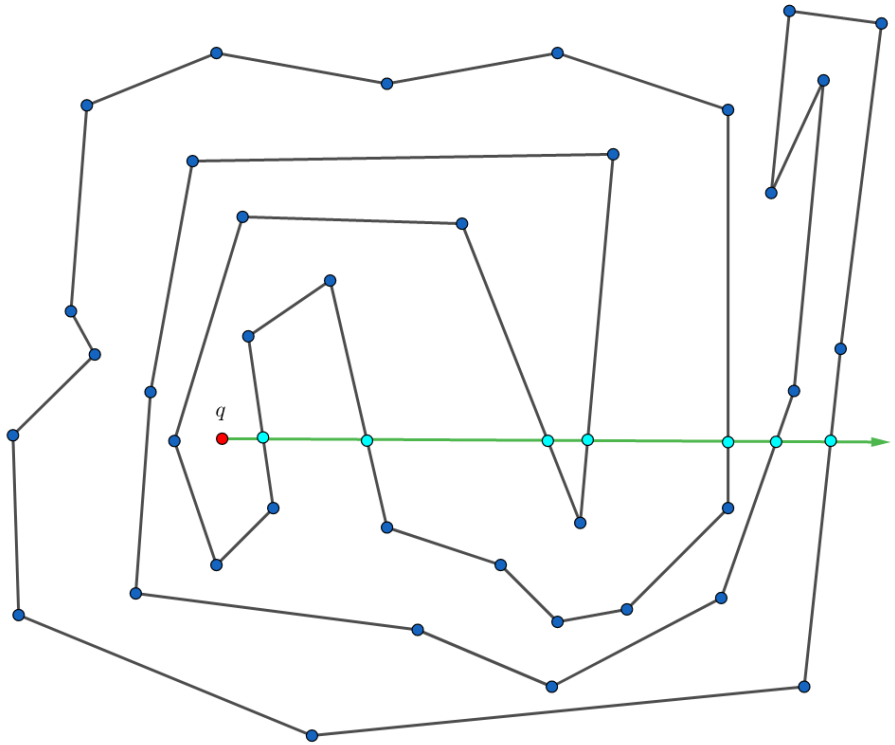
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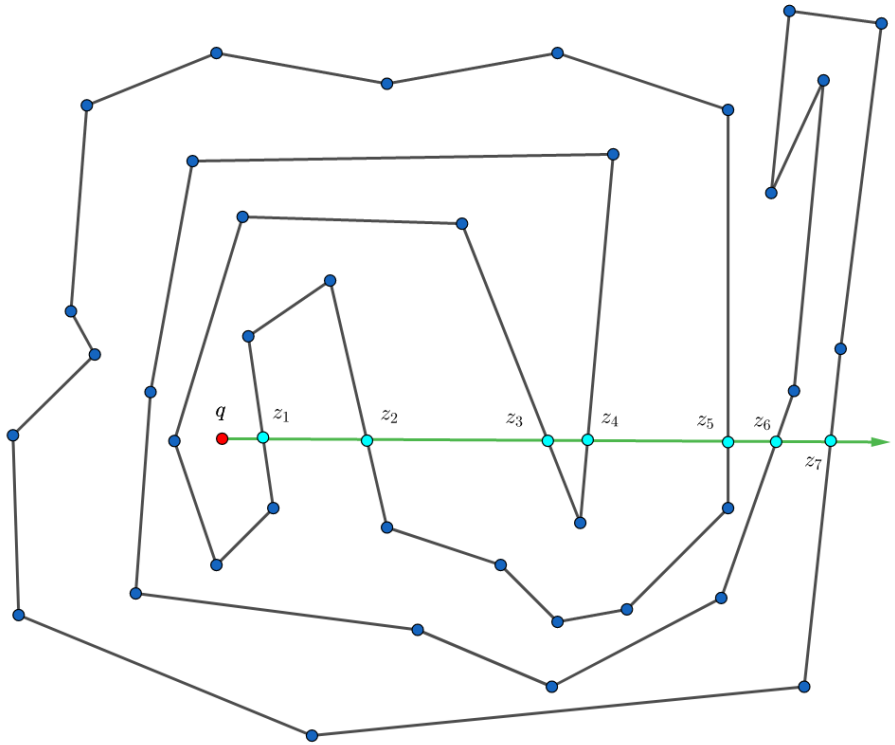
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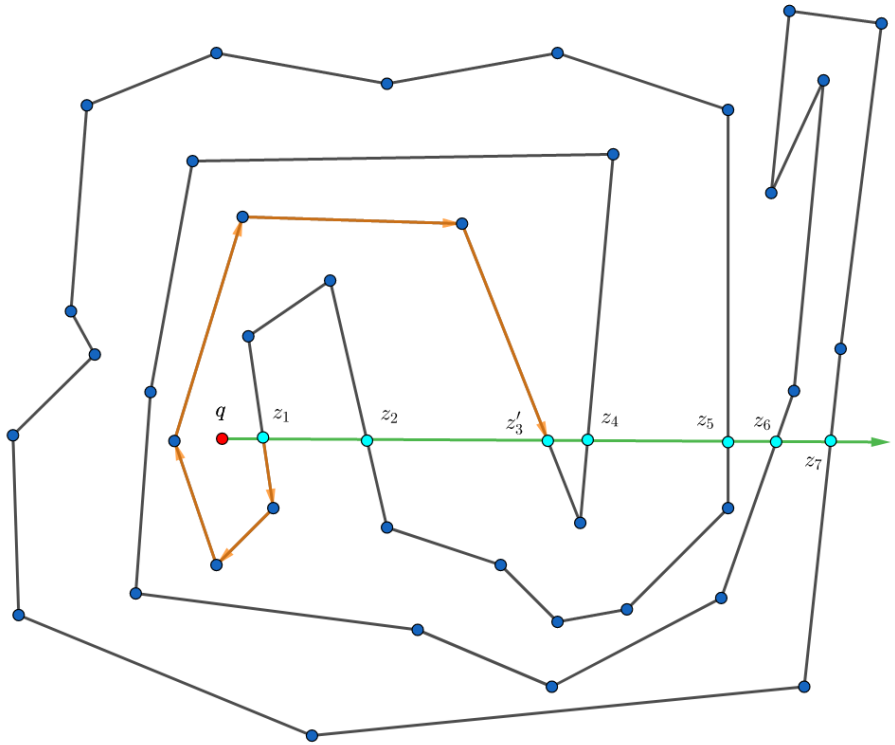
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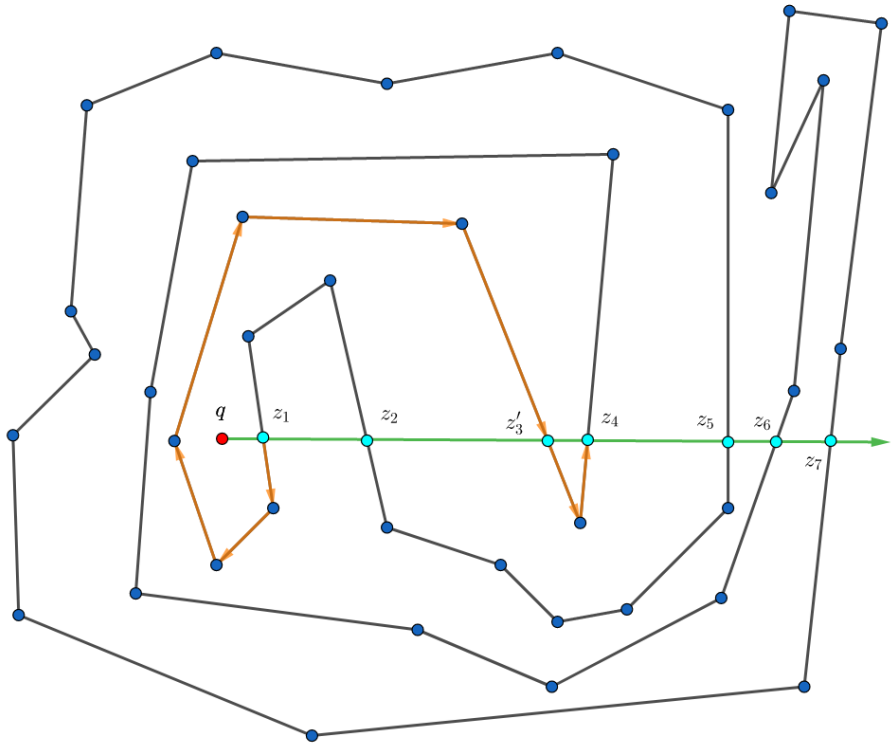
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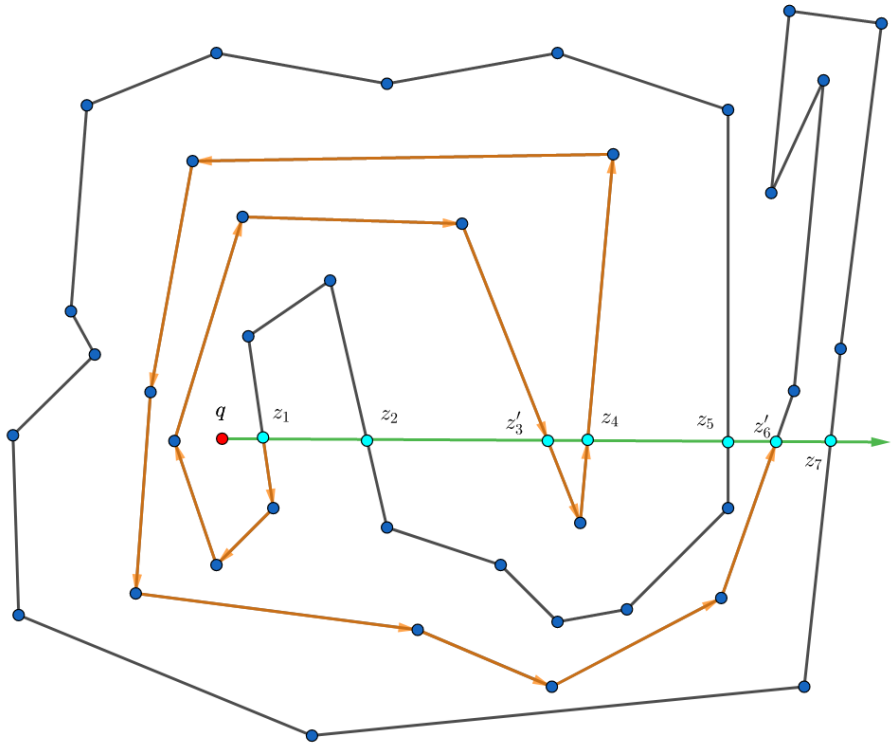
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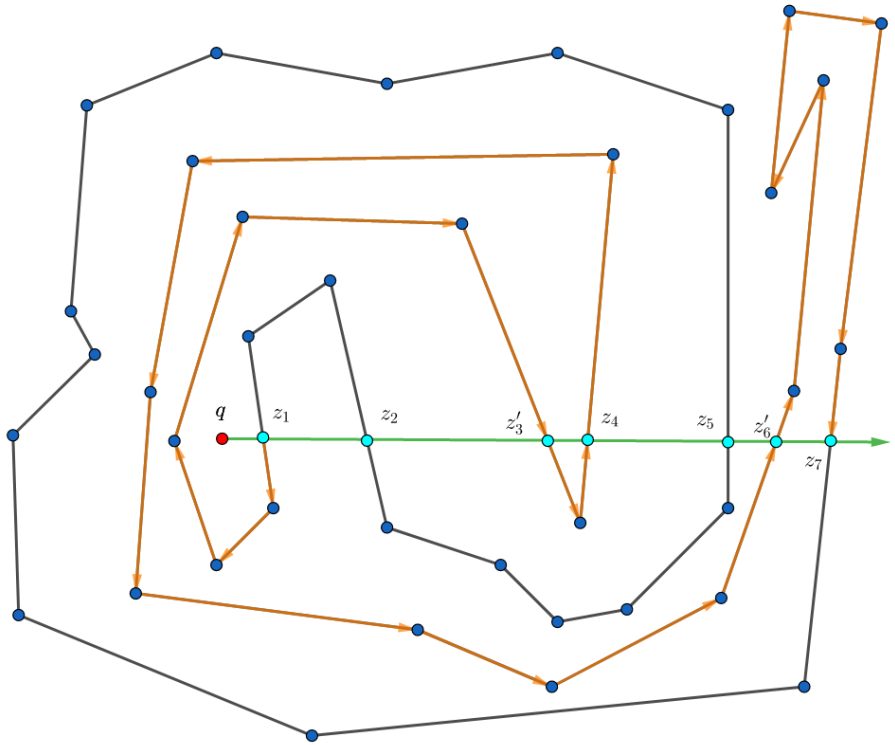
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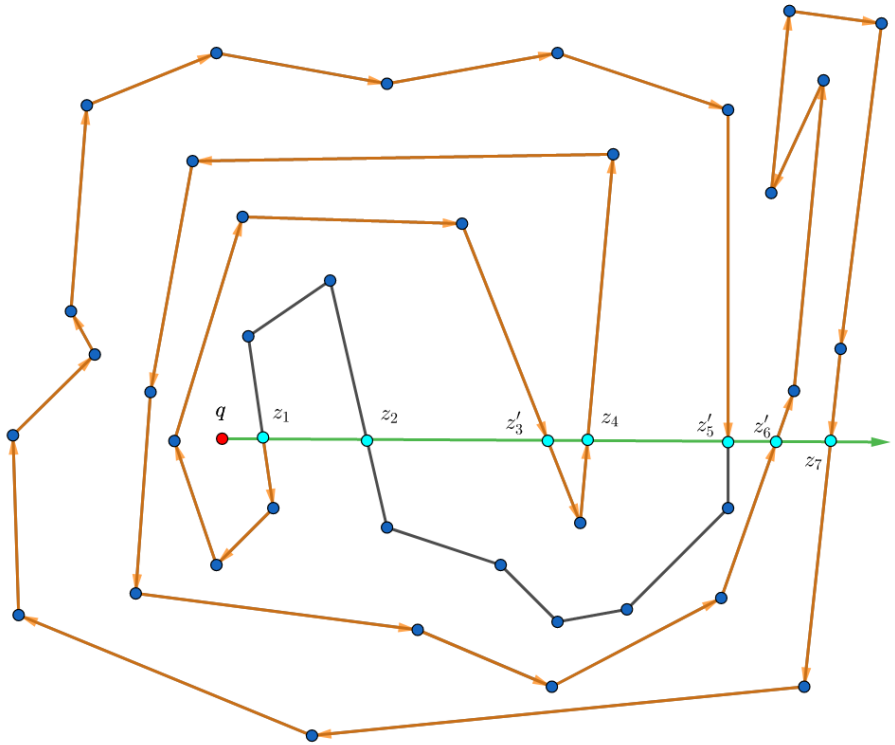
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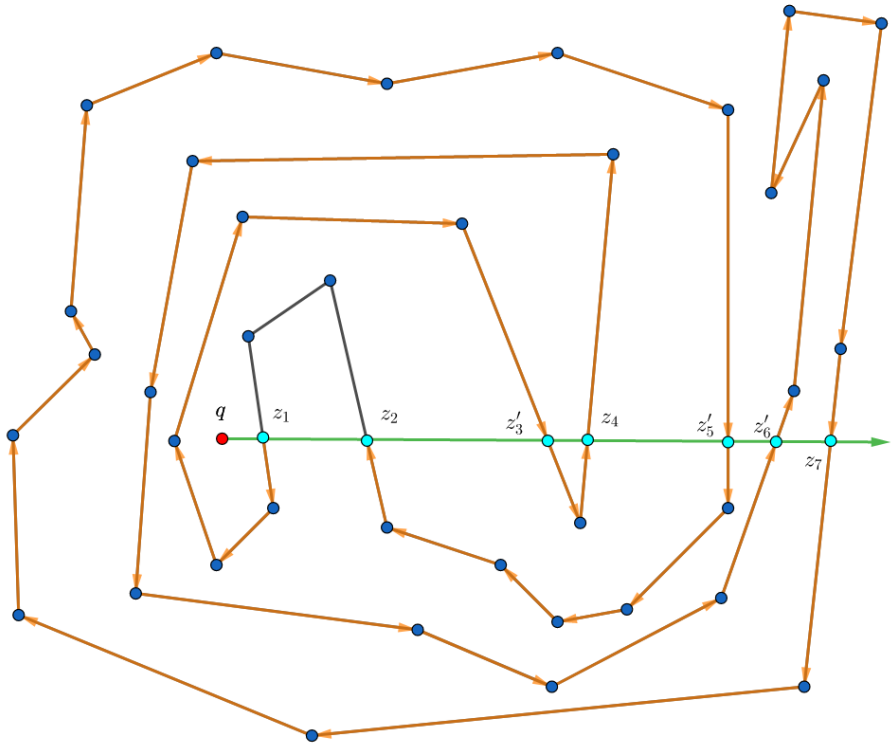
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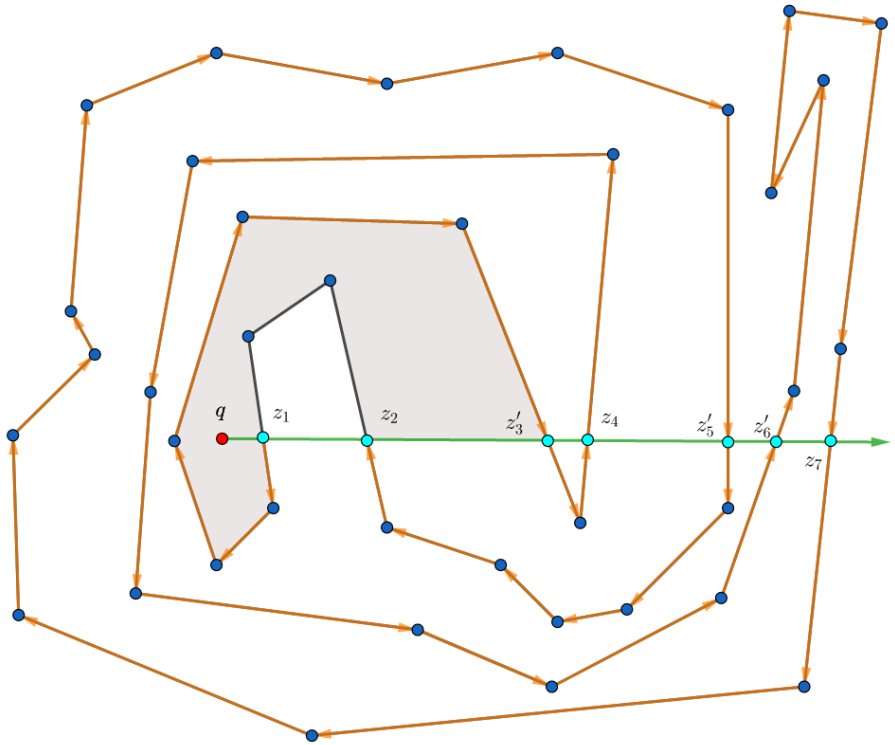
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Análisis de la complejidad.

1. Lanzar el rayo lo hacemos en tiempo constante.
2. Encontrar el primer punto que intersecta la frontera lo hacemos en $\mathcal{O}(\log n)$ por medio de una búsqueda en las aristas de P.
3. Clasificar las intersecciones lo hacemos en $\mathcal{O}(n)$, pues debemos recorrer el polígono P.
4. Encontrar la primer arista respecto a q tal que tenga sus dos extremos clasificados con diferente denominación $\{\text{abajo},\ \text{arriba}\}$ lo realizamos en $\mathcal{O}(n)$ al recorrer nuevamente nuestro polígono.

Motivación.

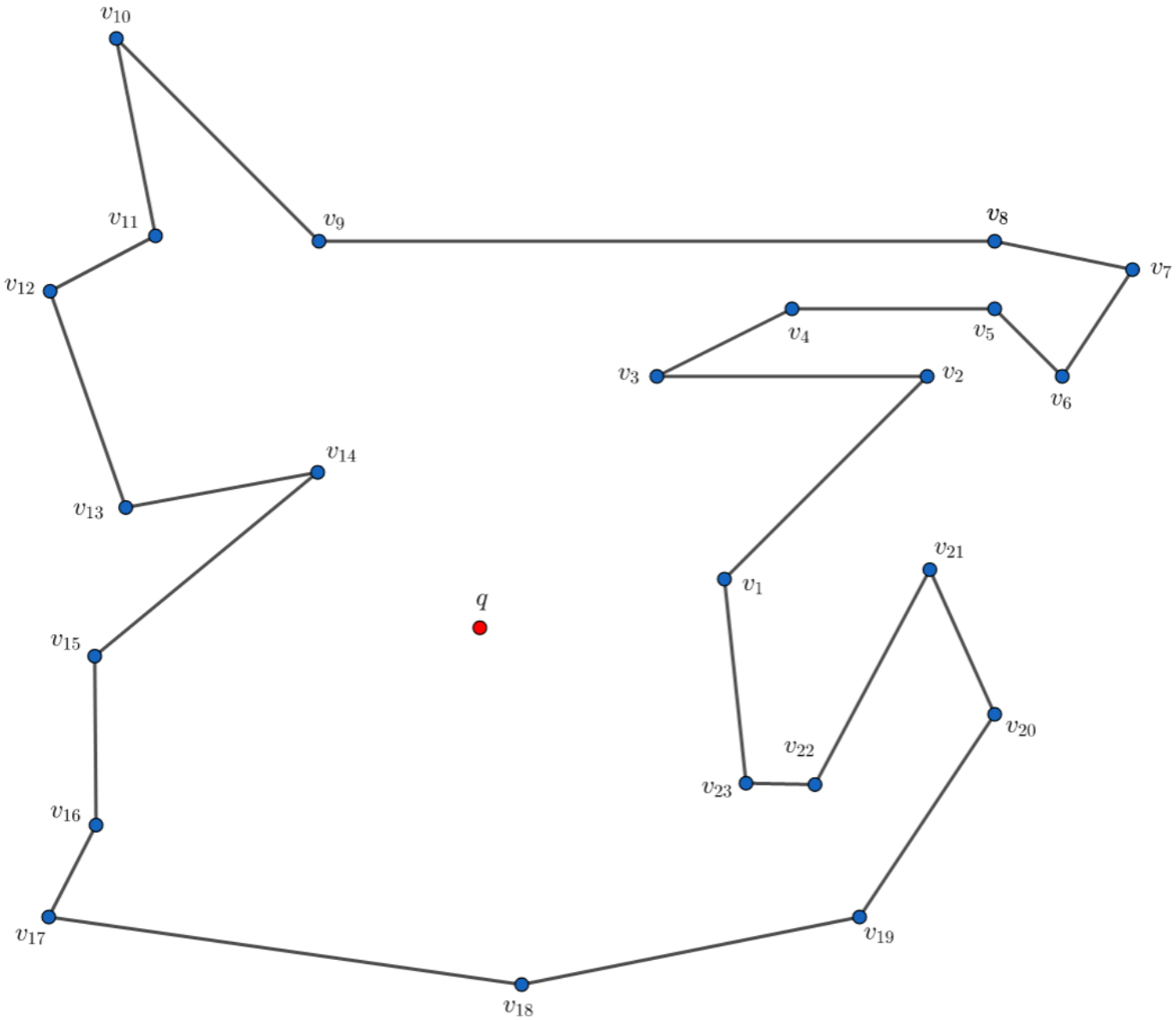
Queremos obtener el polígono de visibilidad $V(q)$ dado un punto q y un polígono simple P.

Obs. Si

$$V(q) = P,$$

entonces P es un polígono estrellado.

Asumimos que el polígono a trabajar no tiene un número de revoluciones mayor a 1.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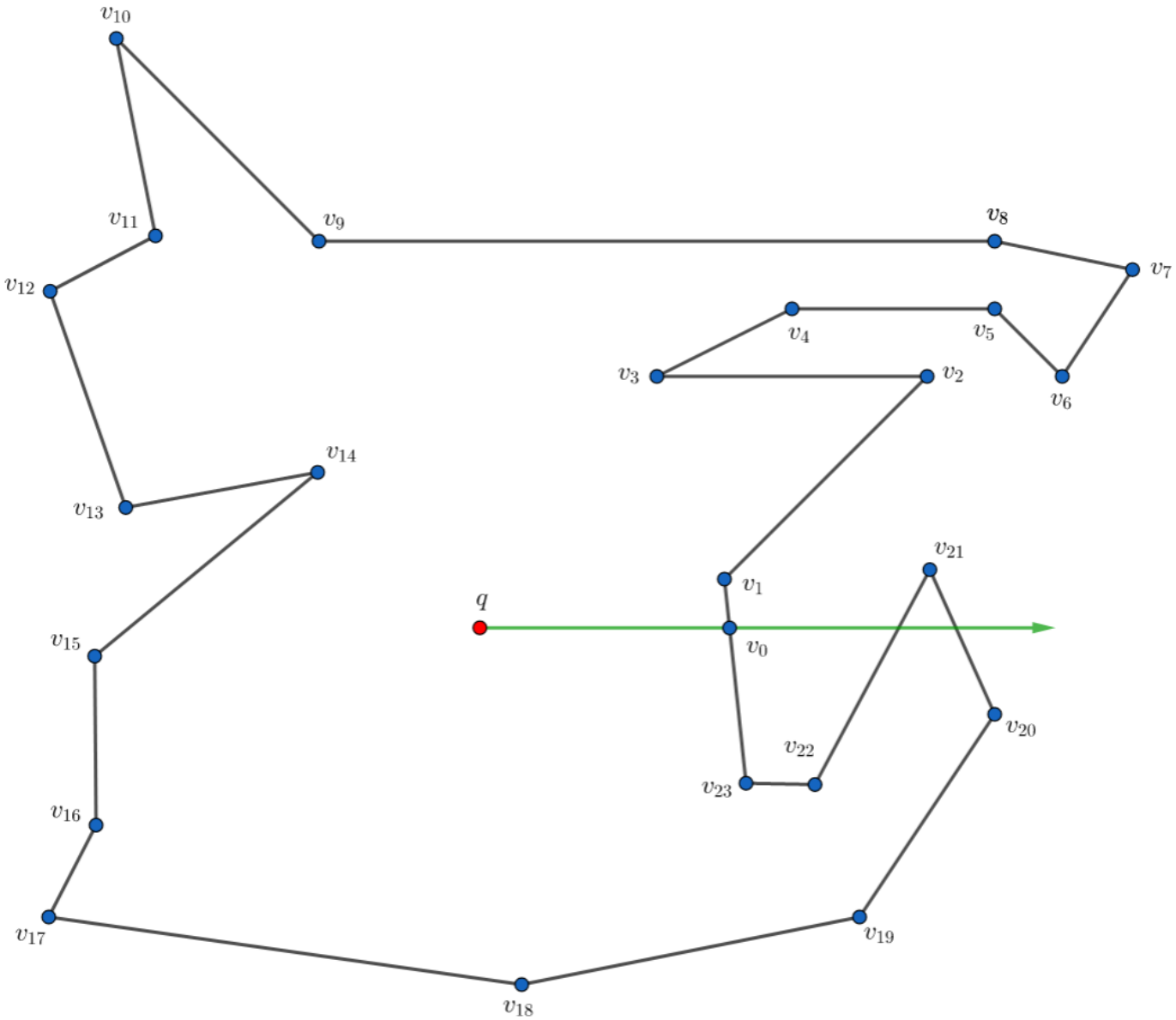
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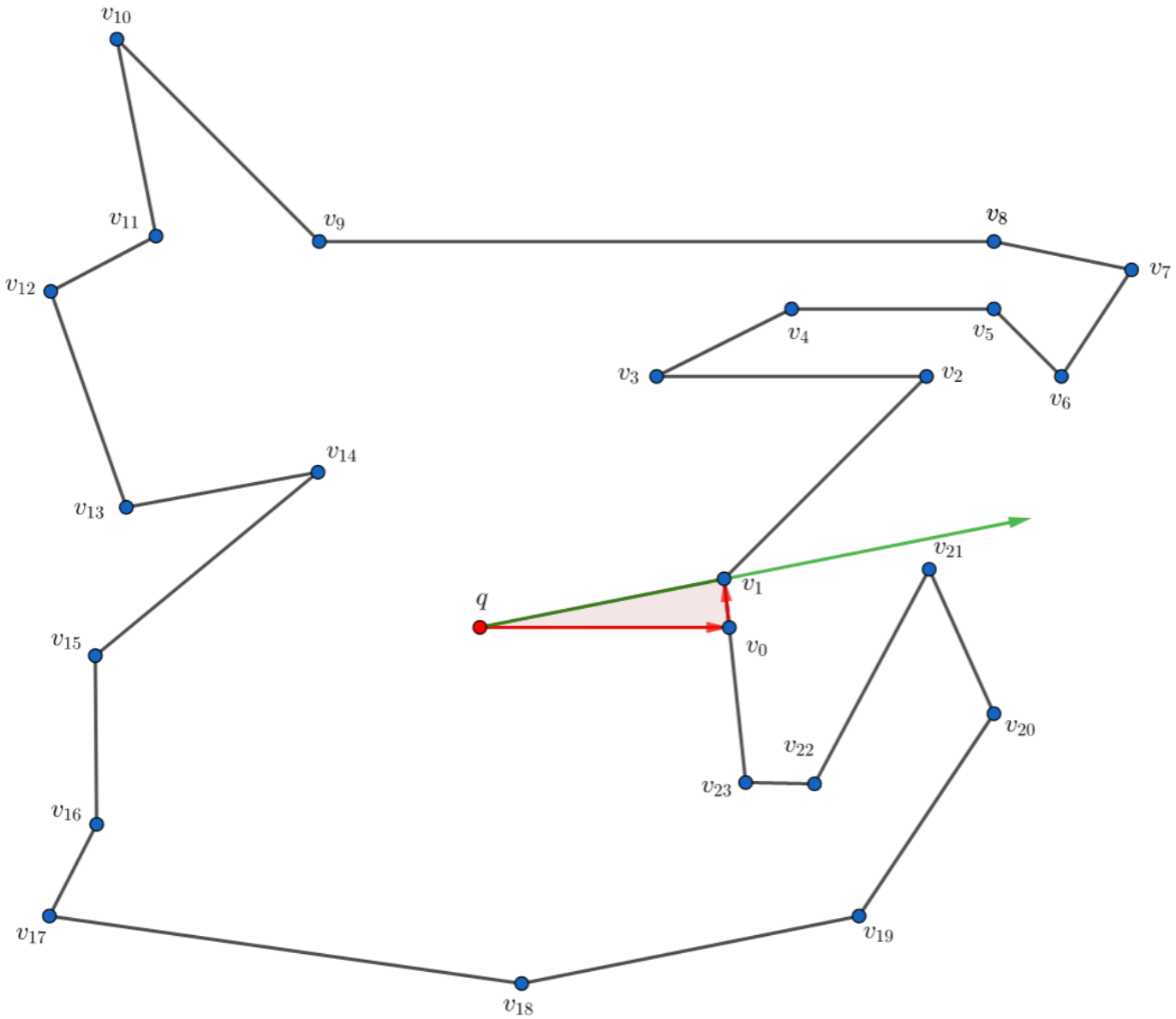
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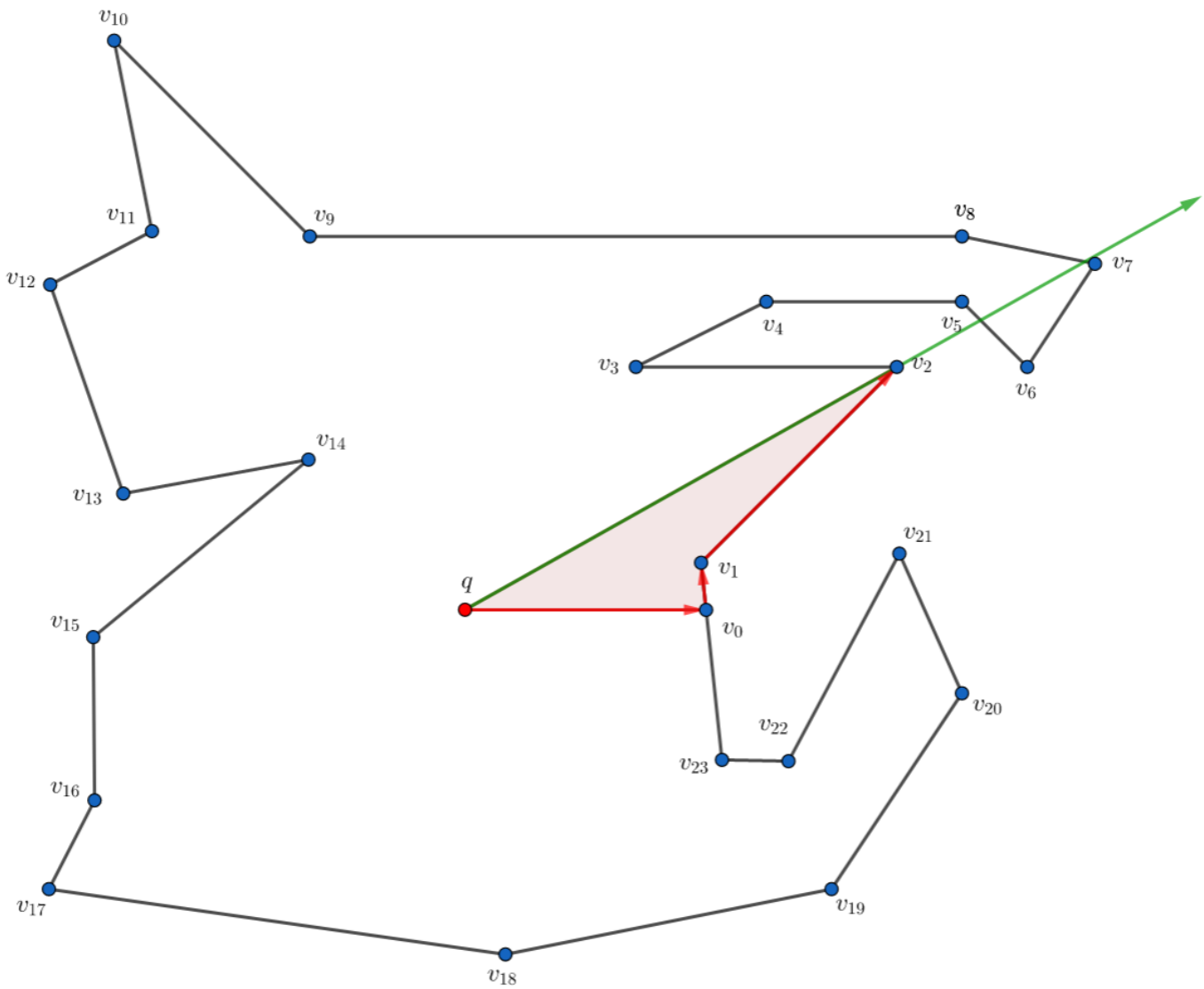
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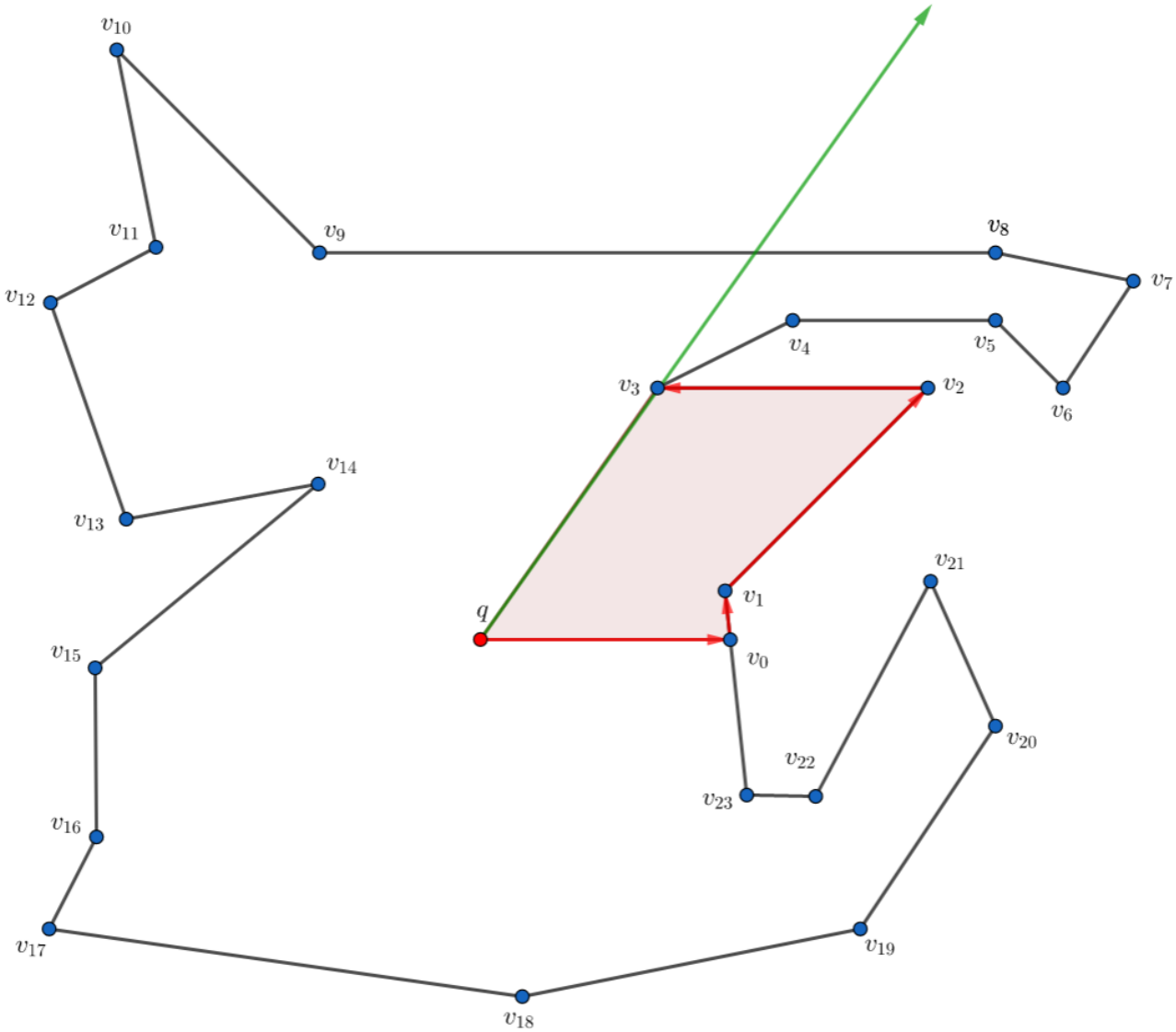
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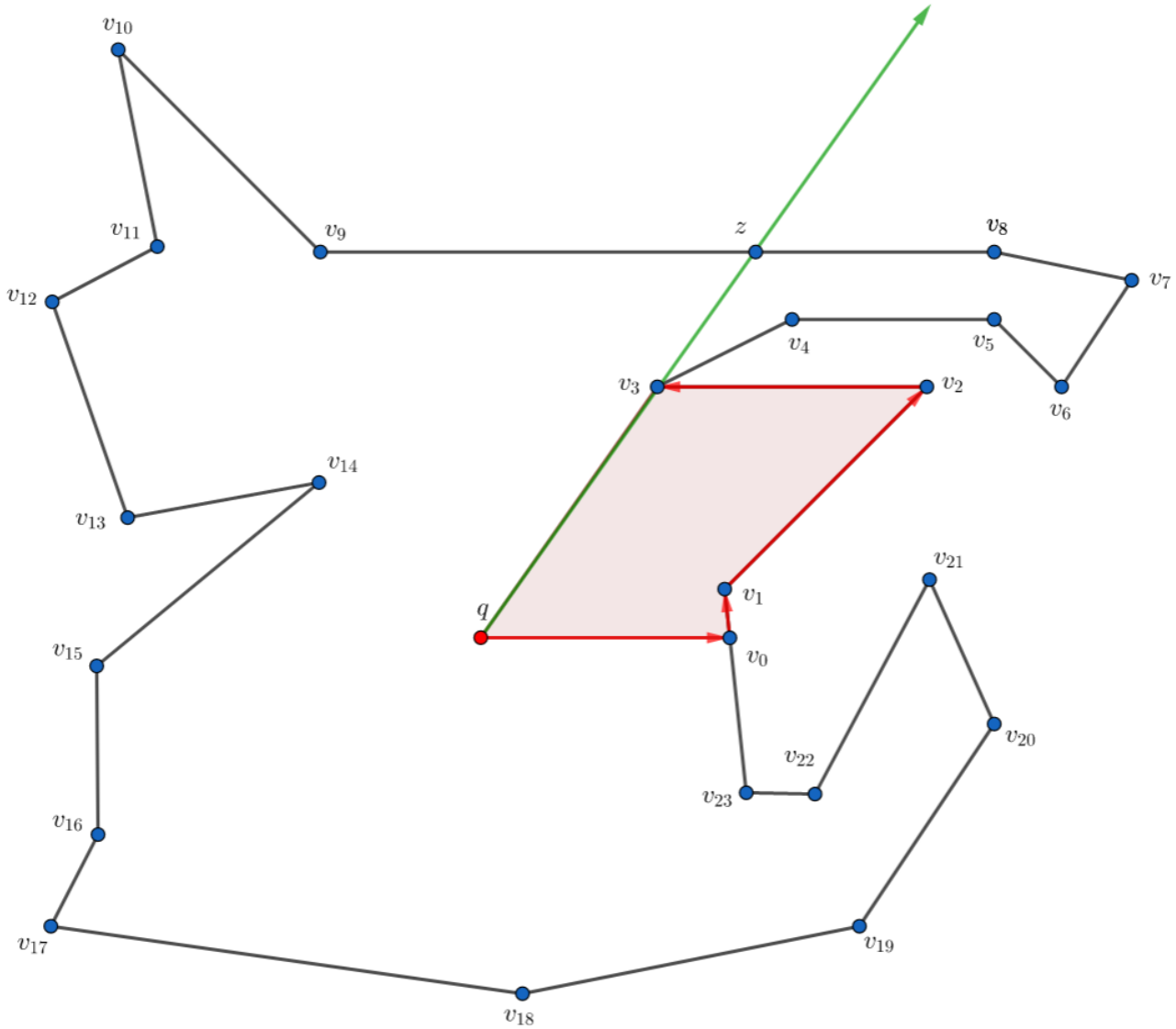
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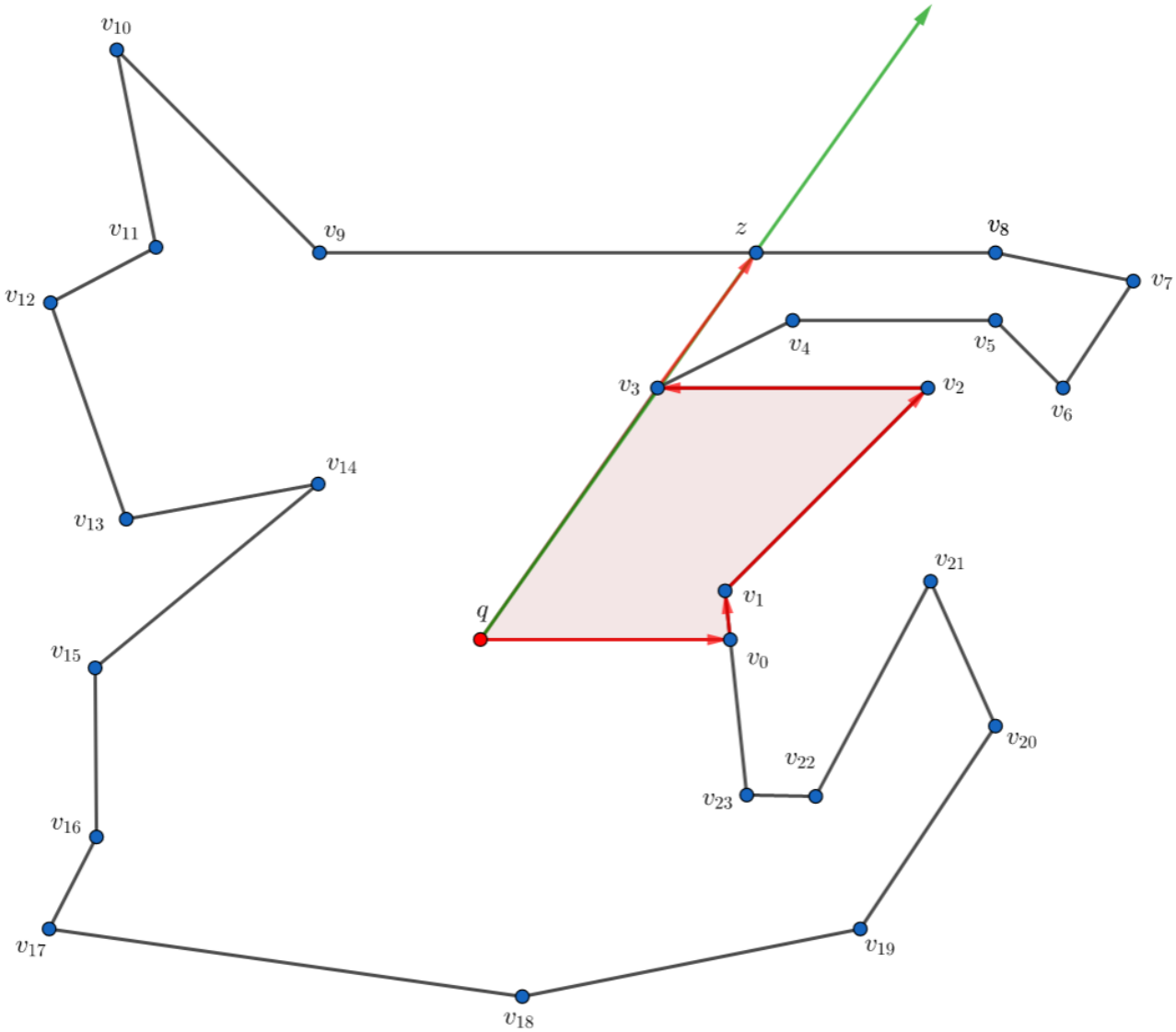
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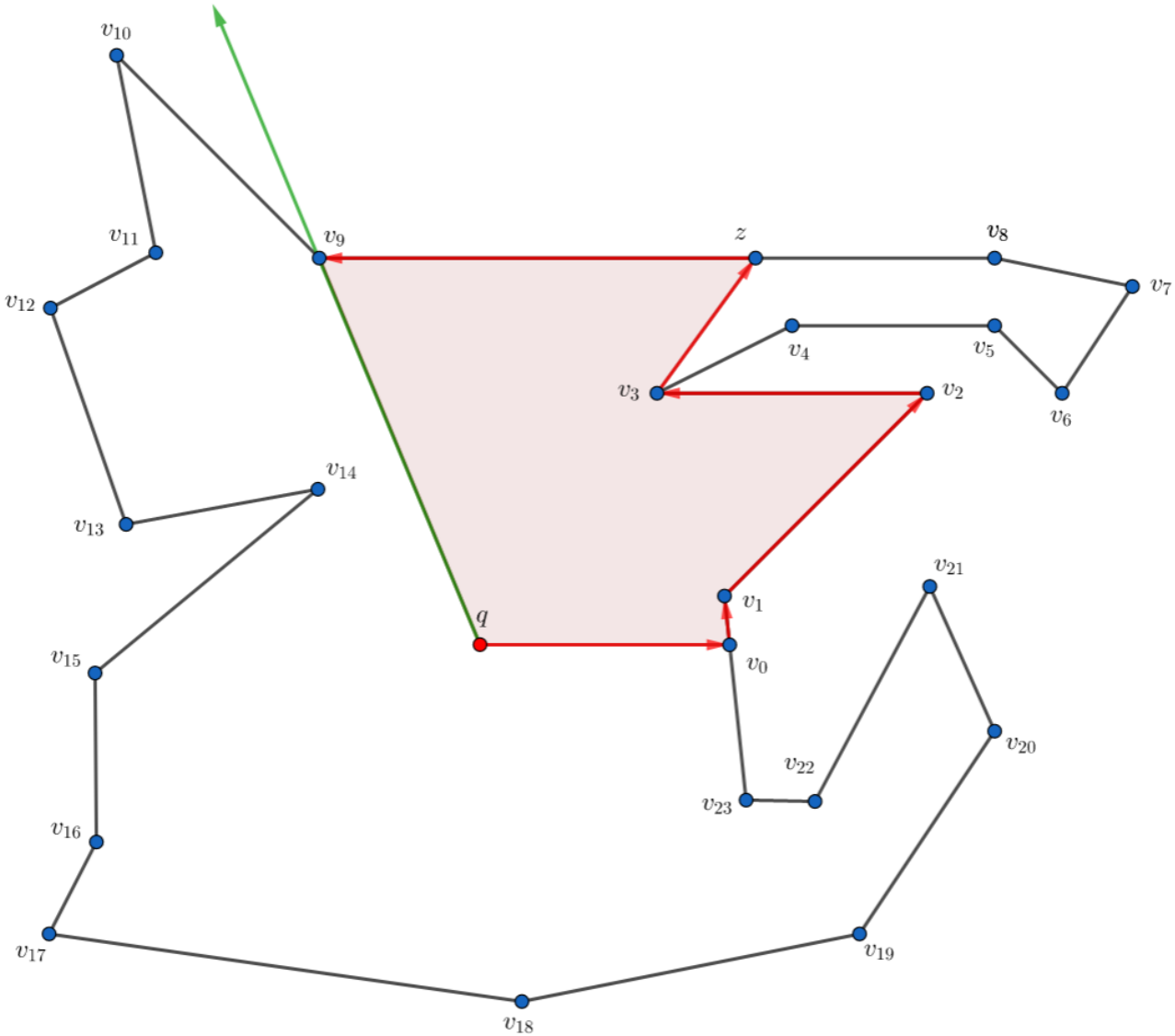
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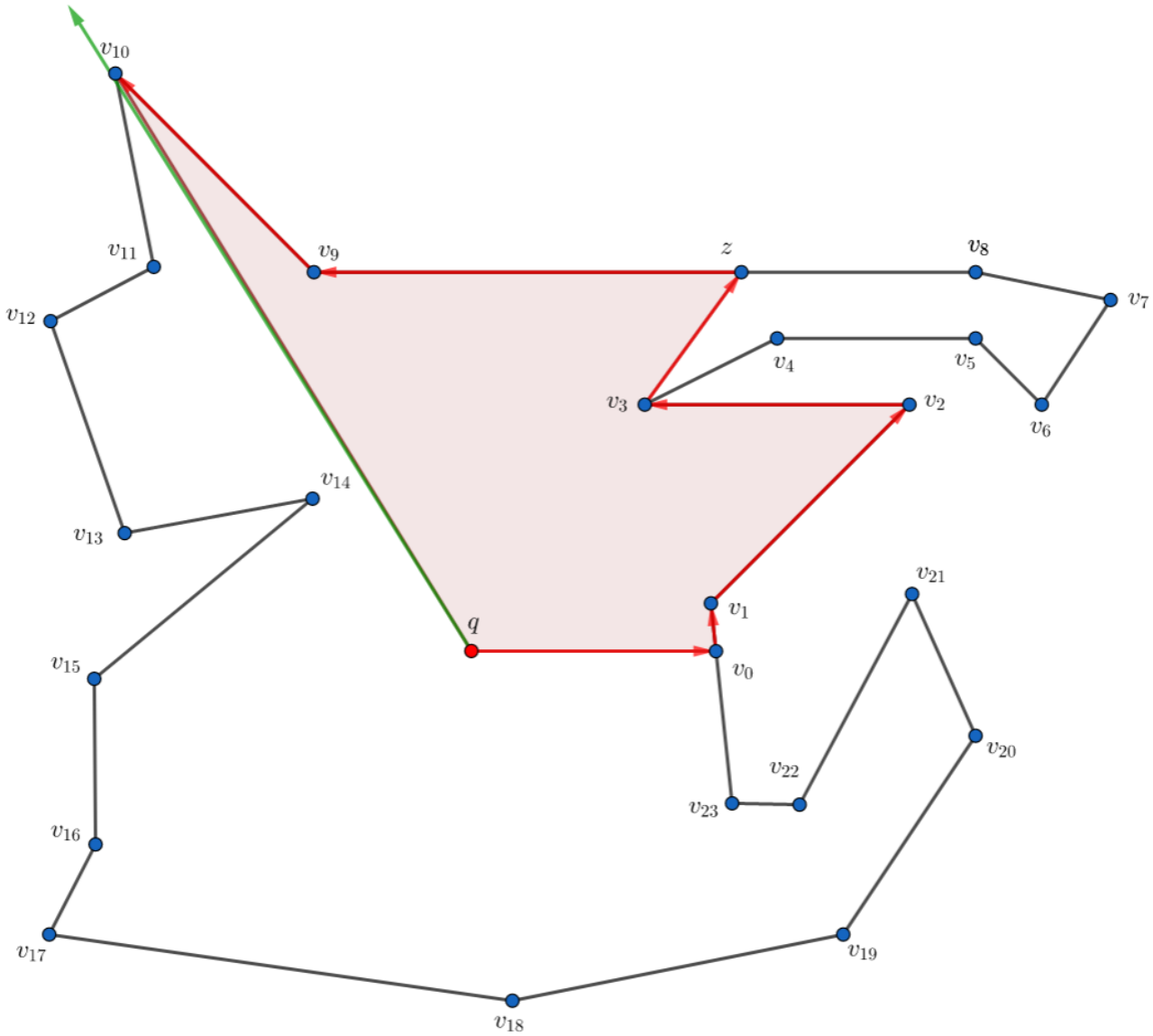
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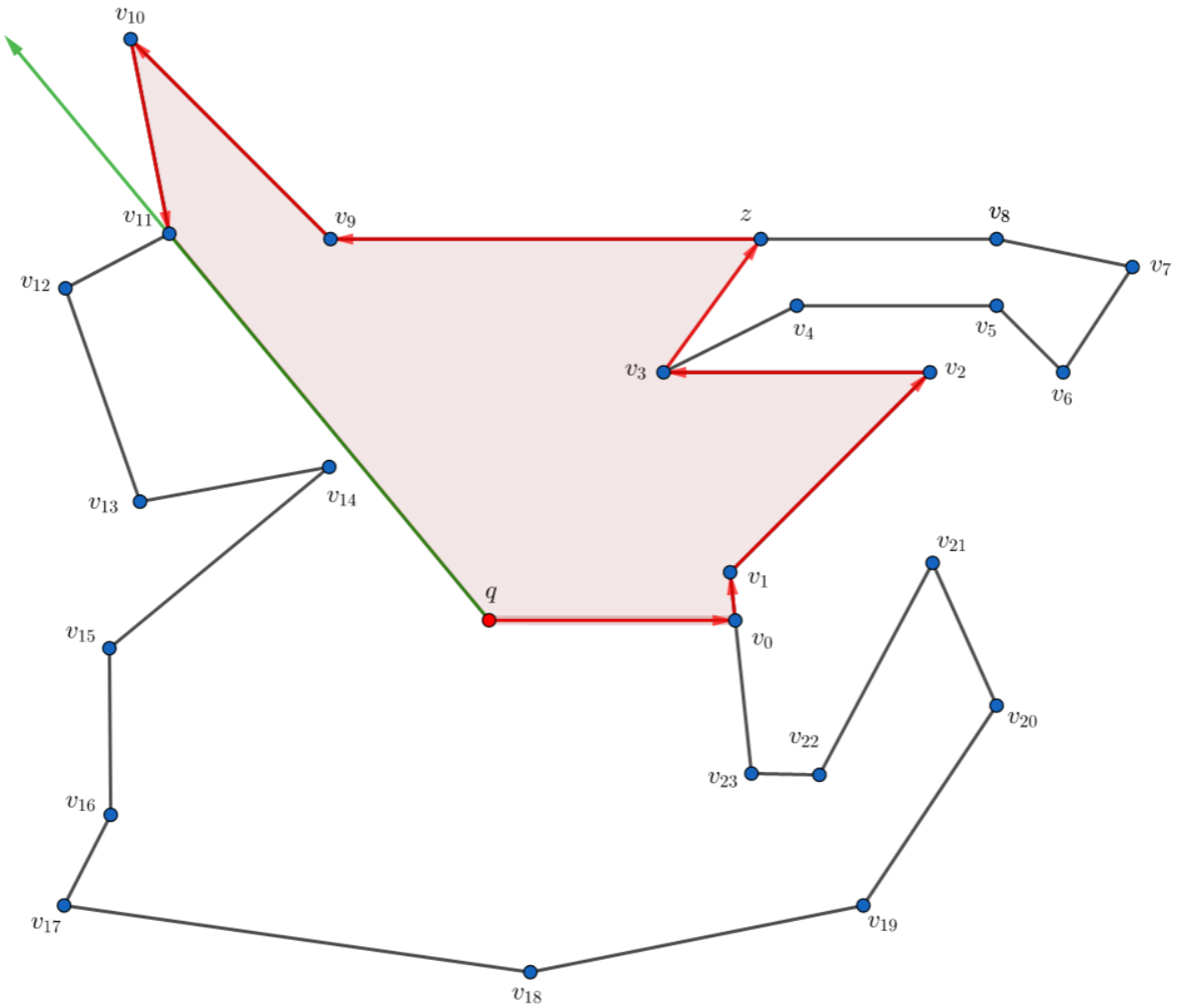
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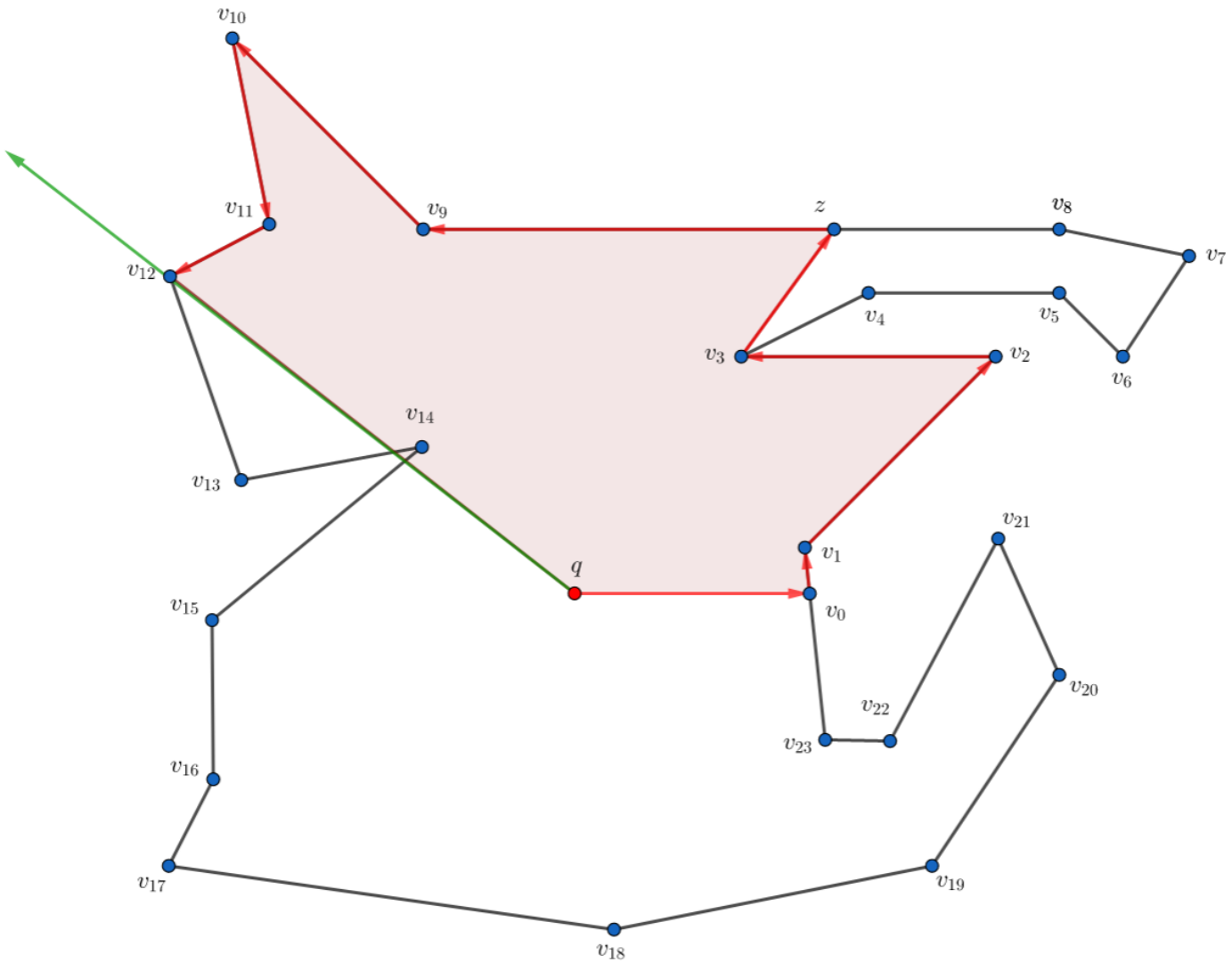
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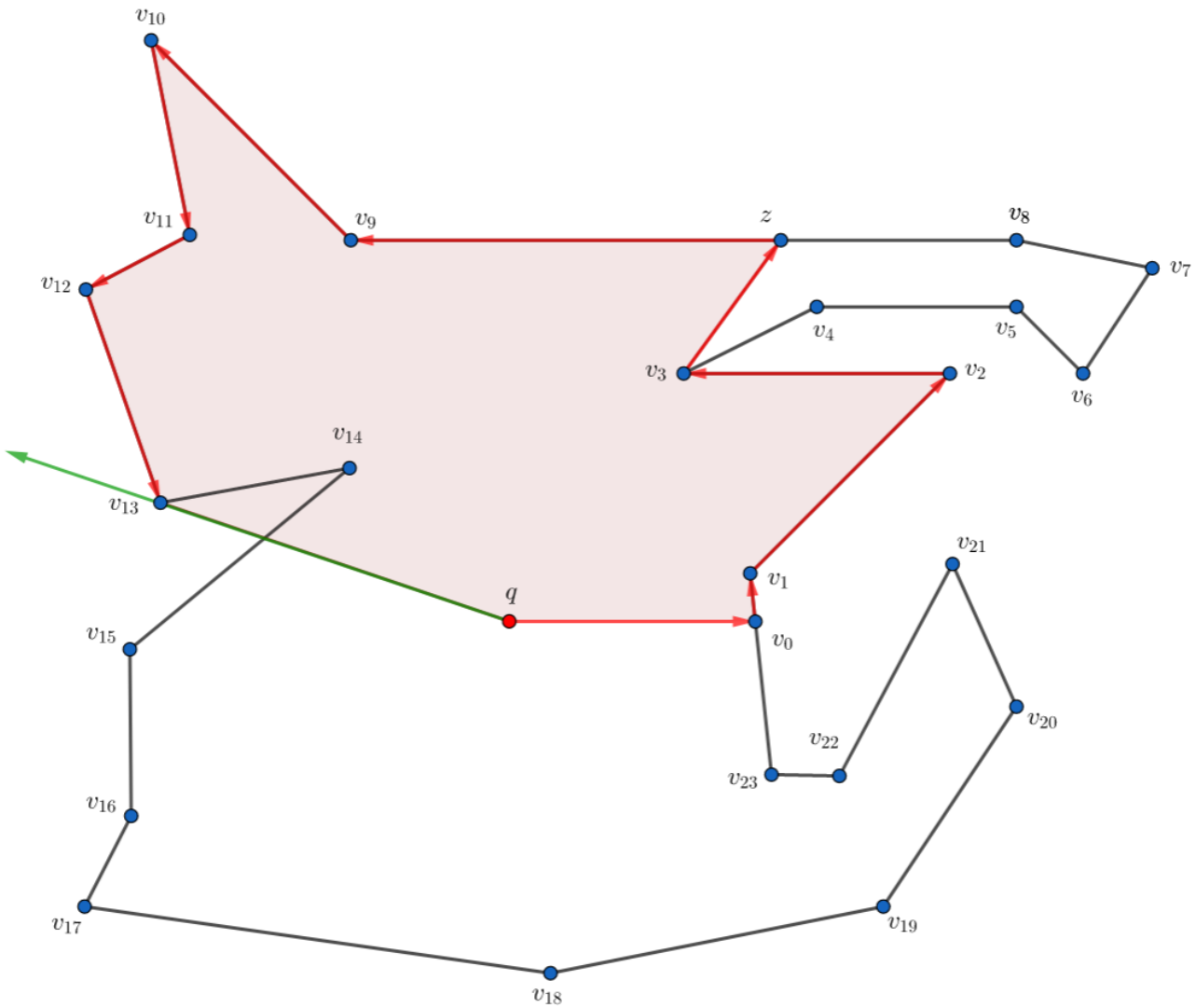
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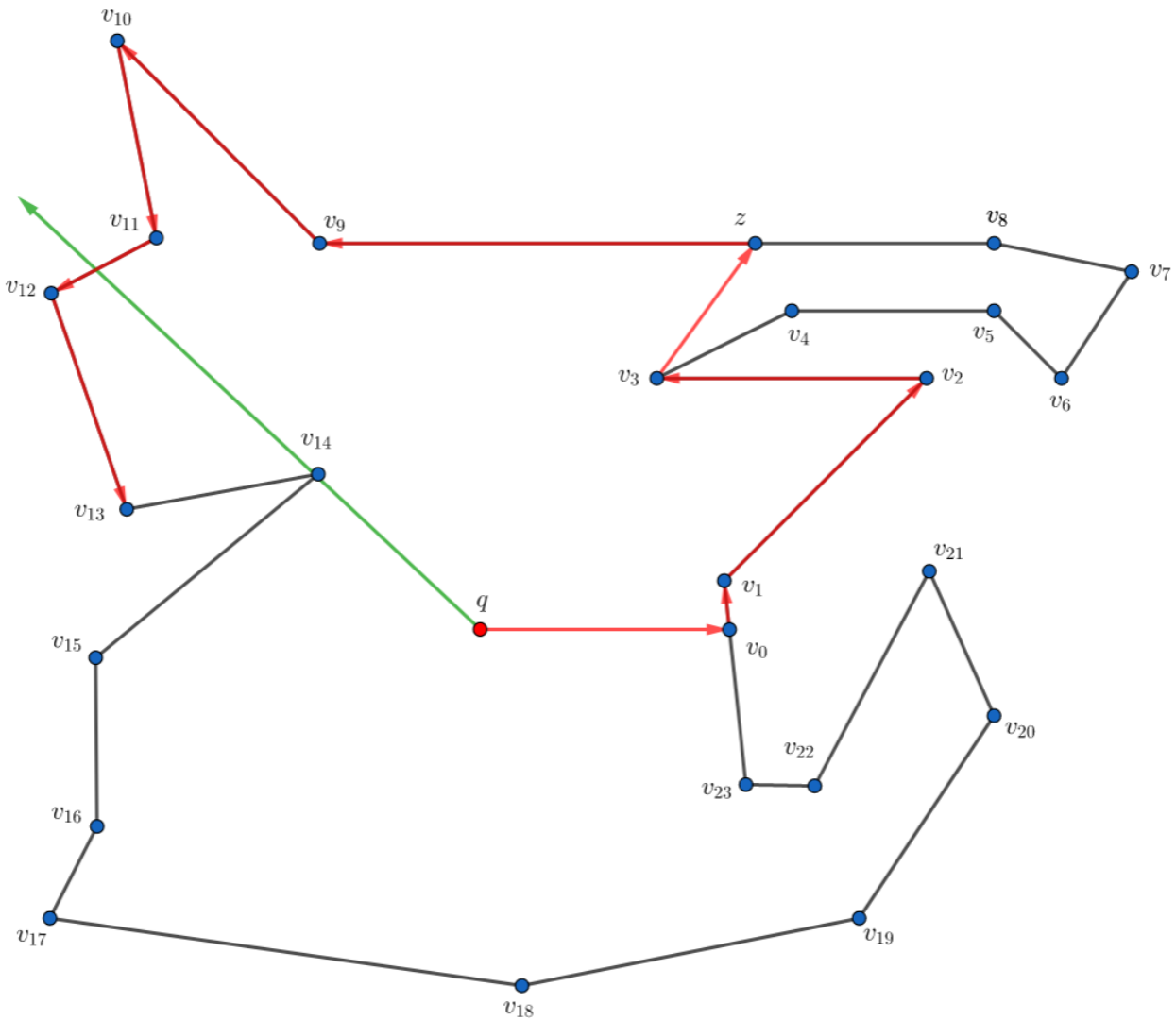
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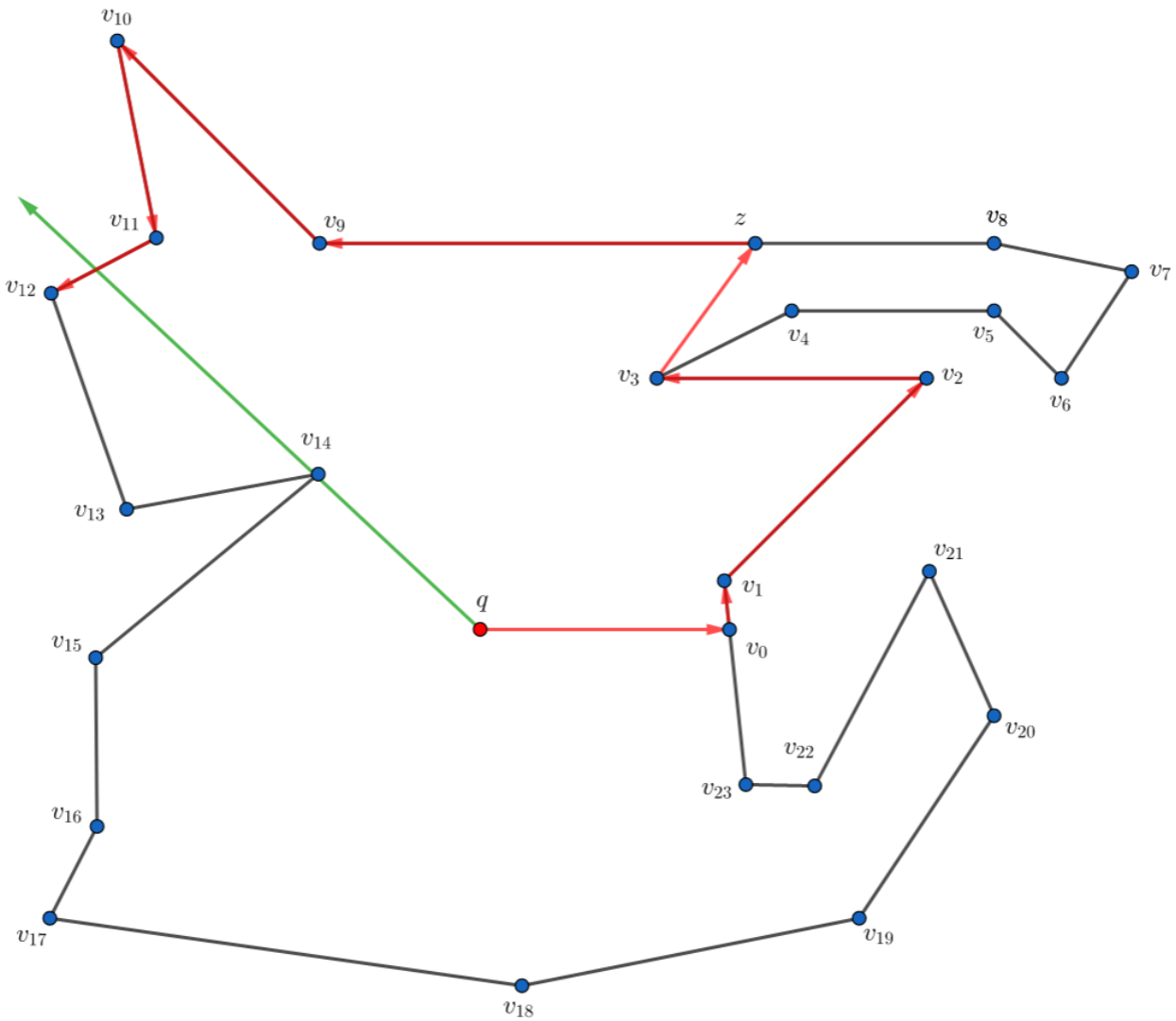
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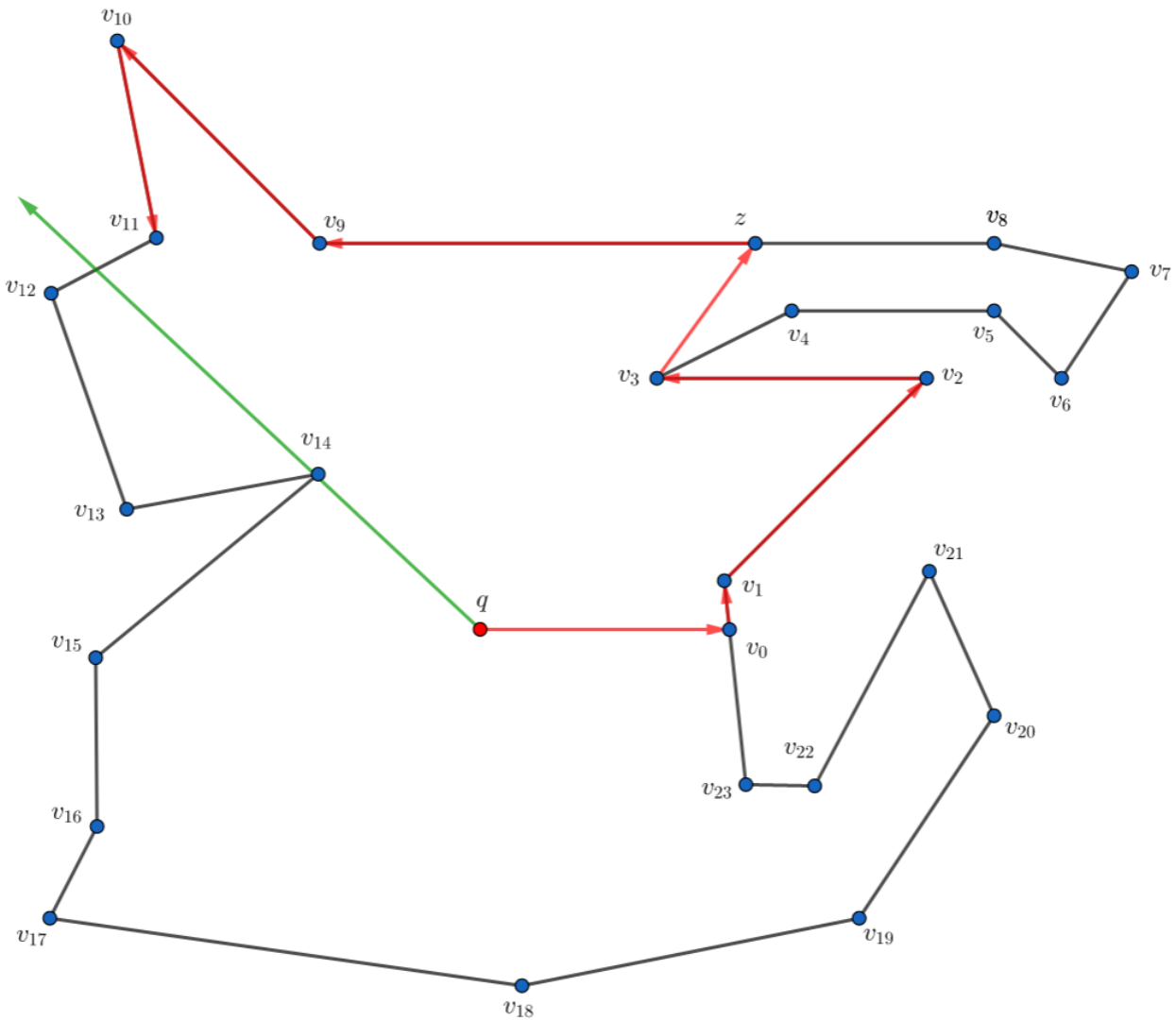
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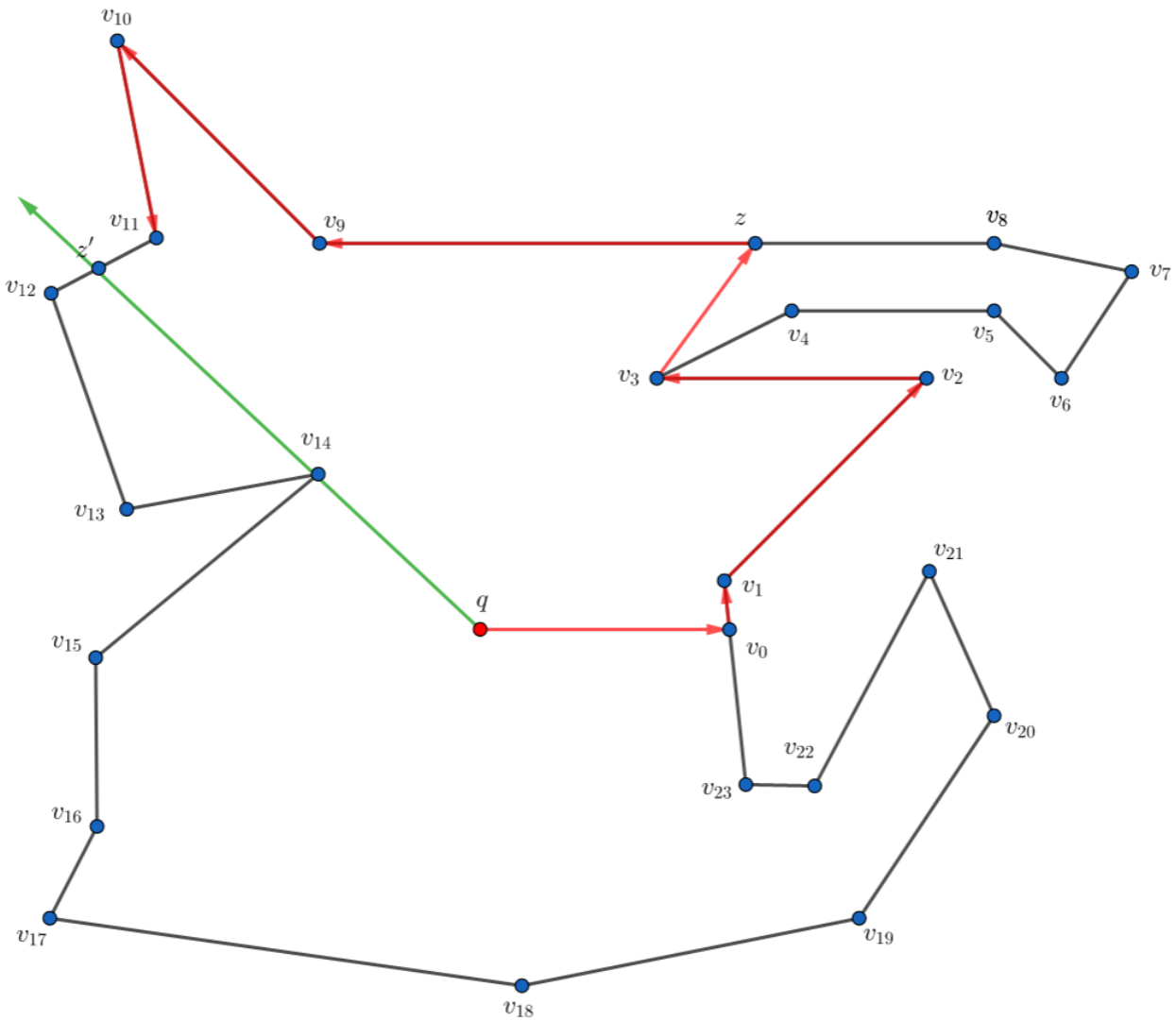
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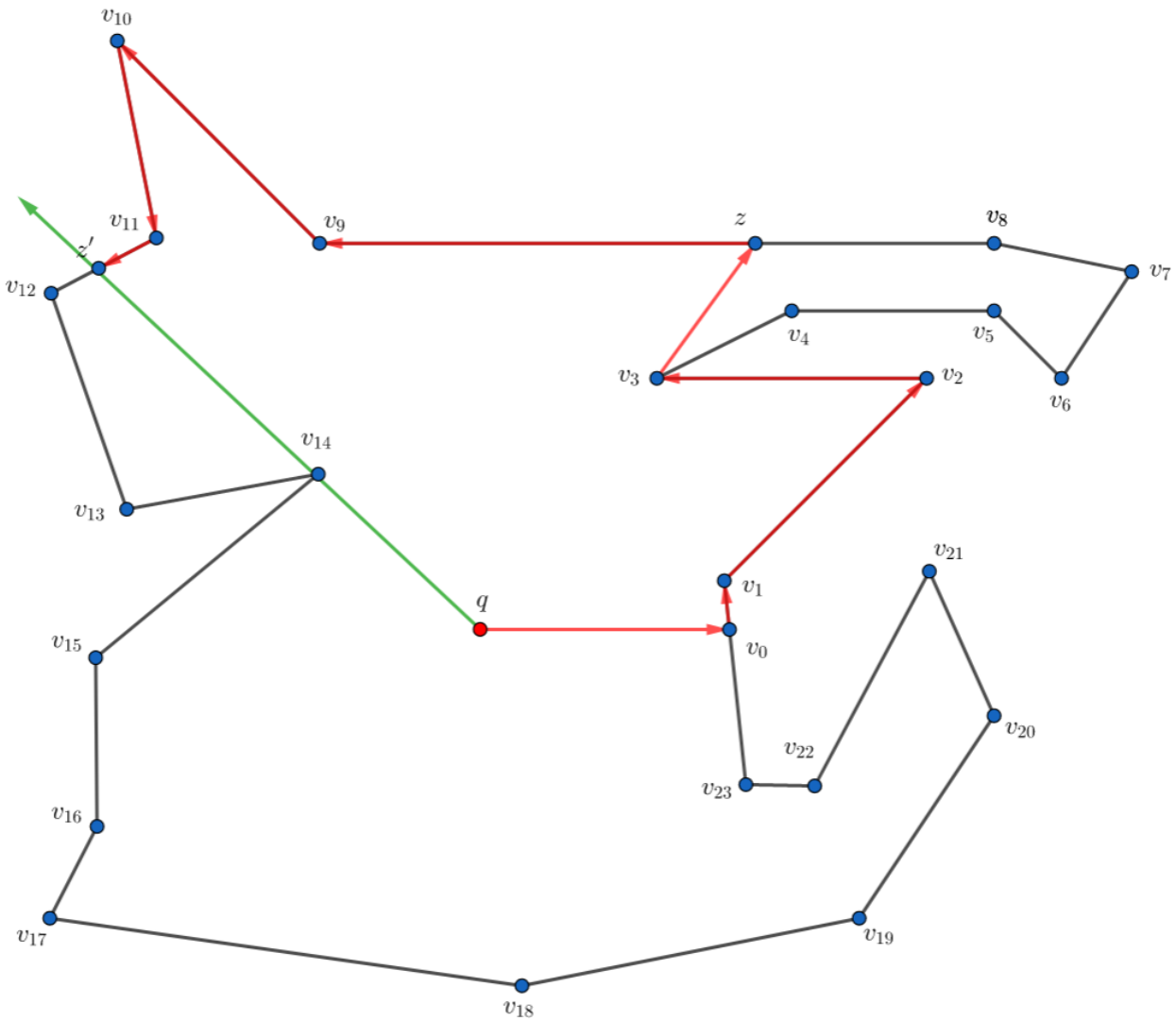
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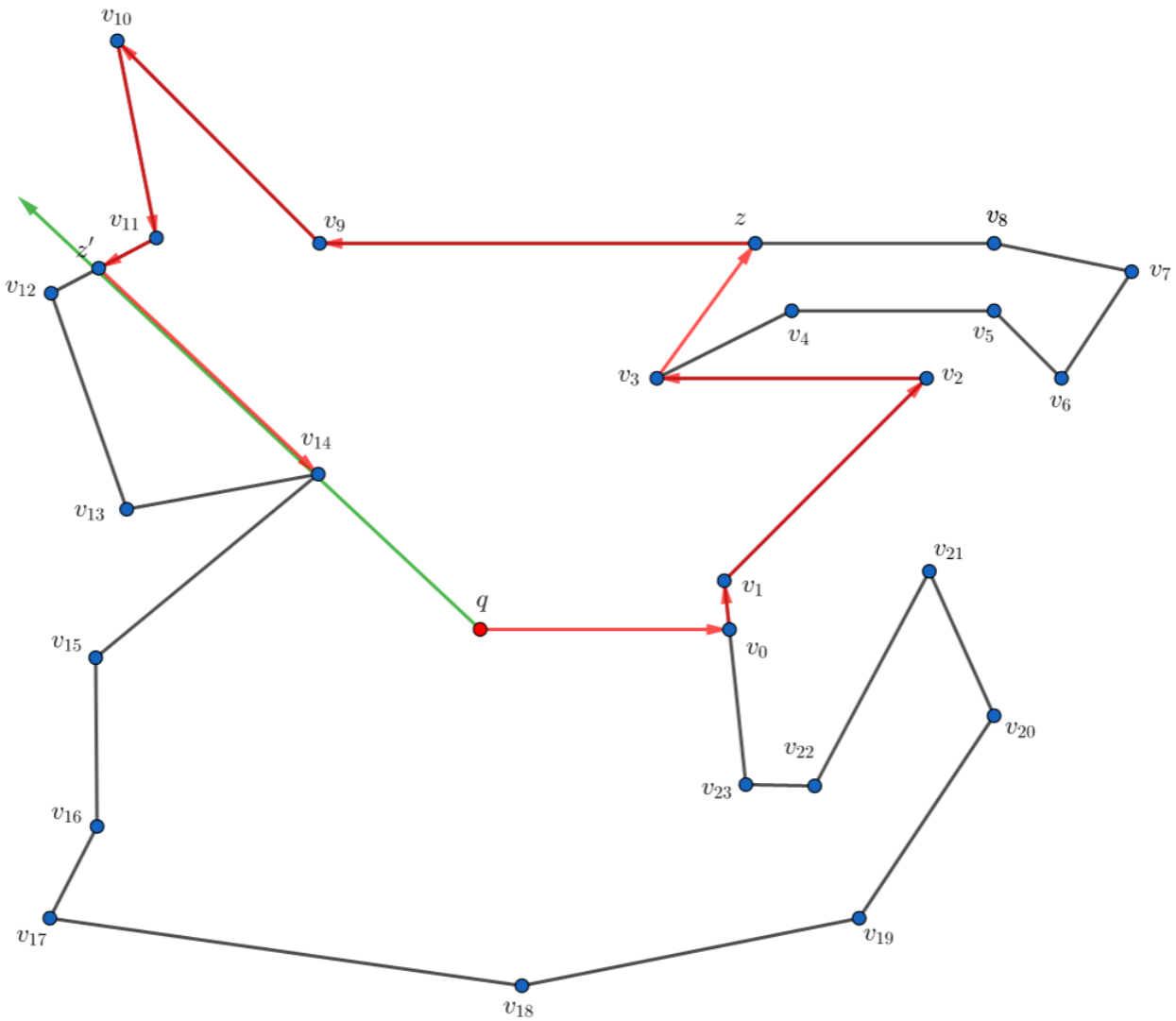
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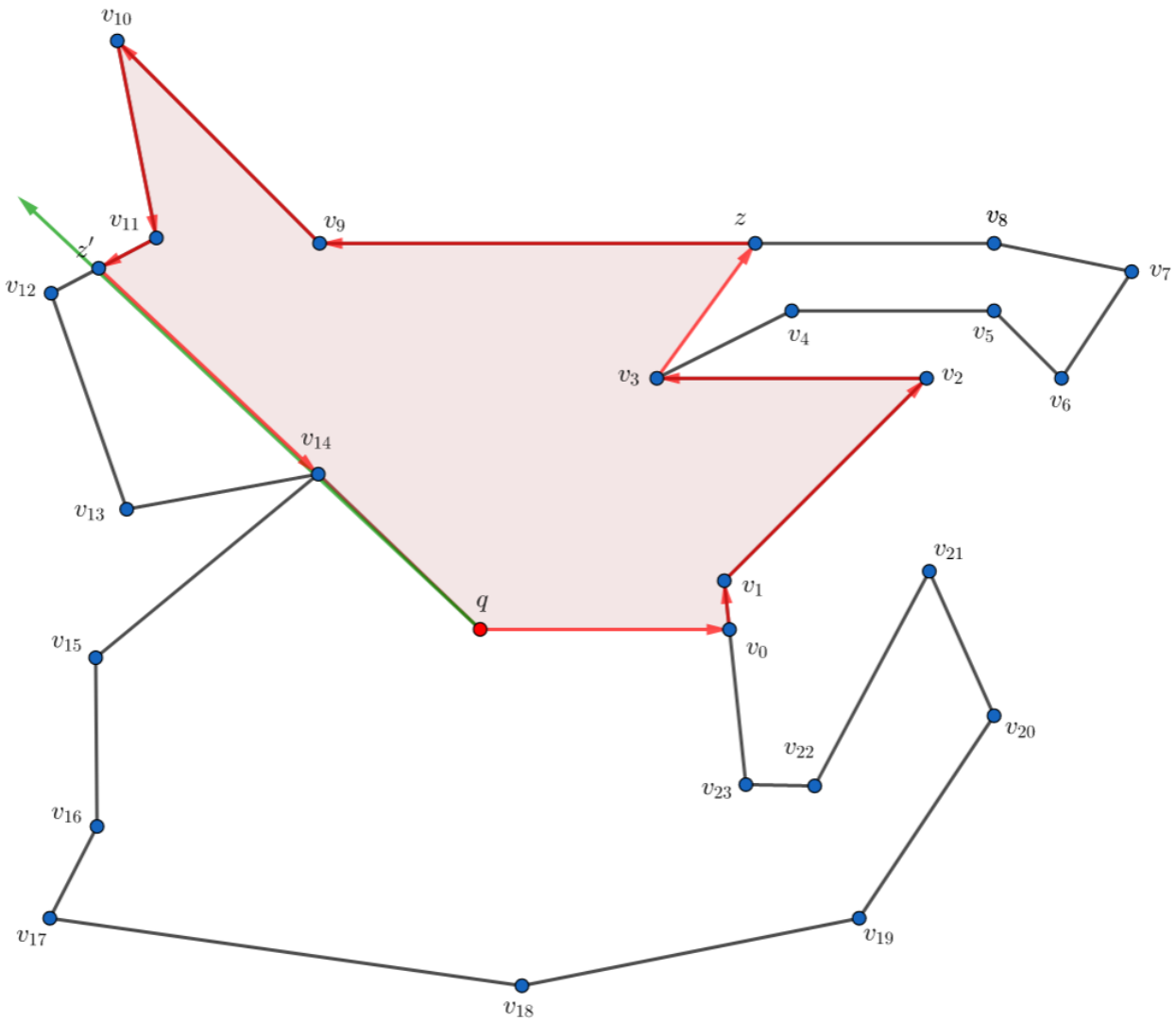
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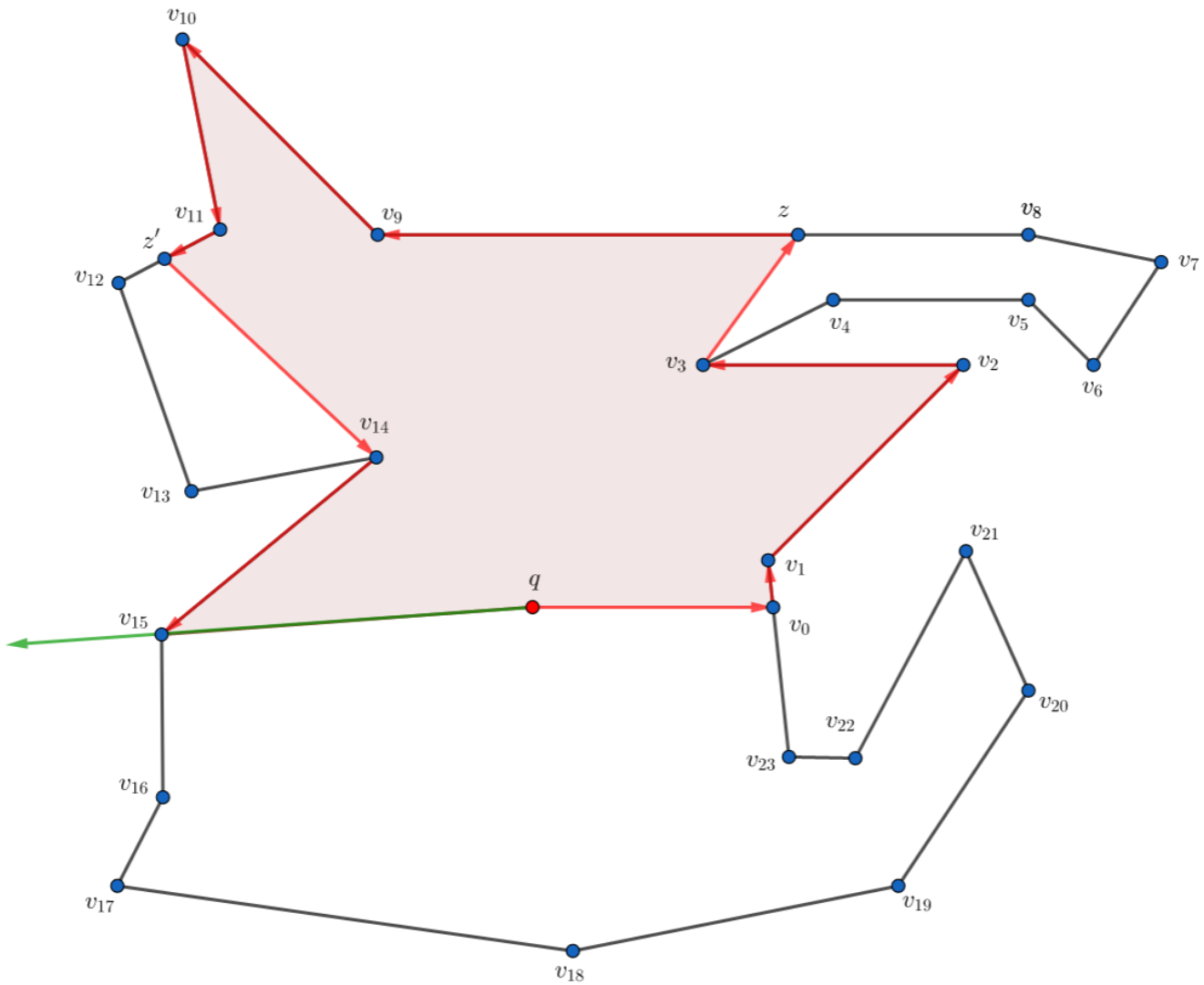
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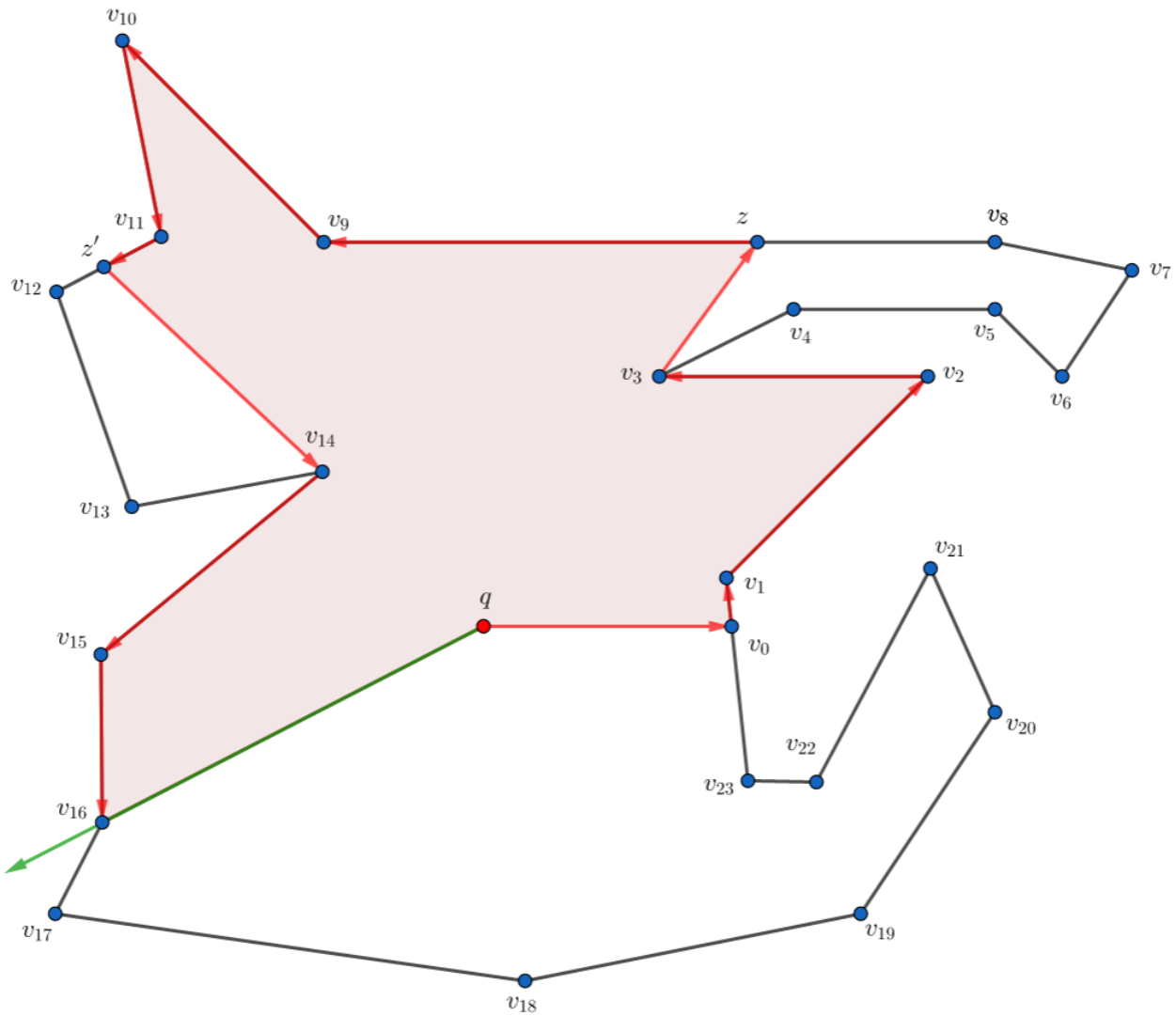
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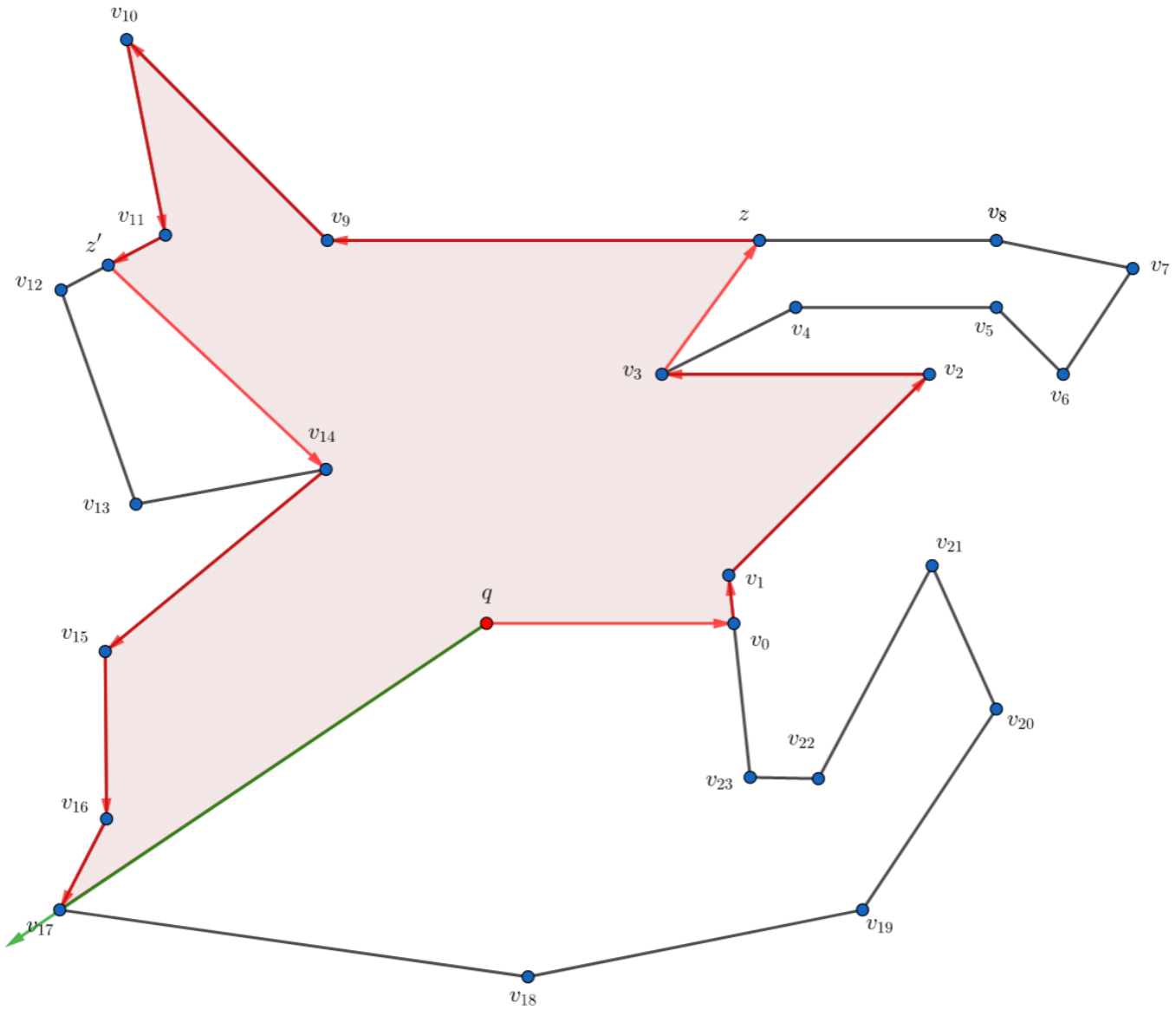
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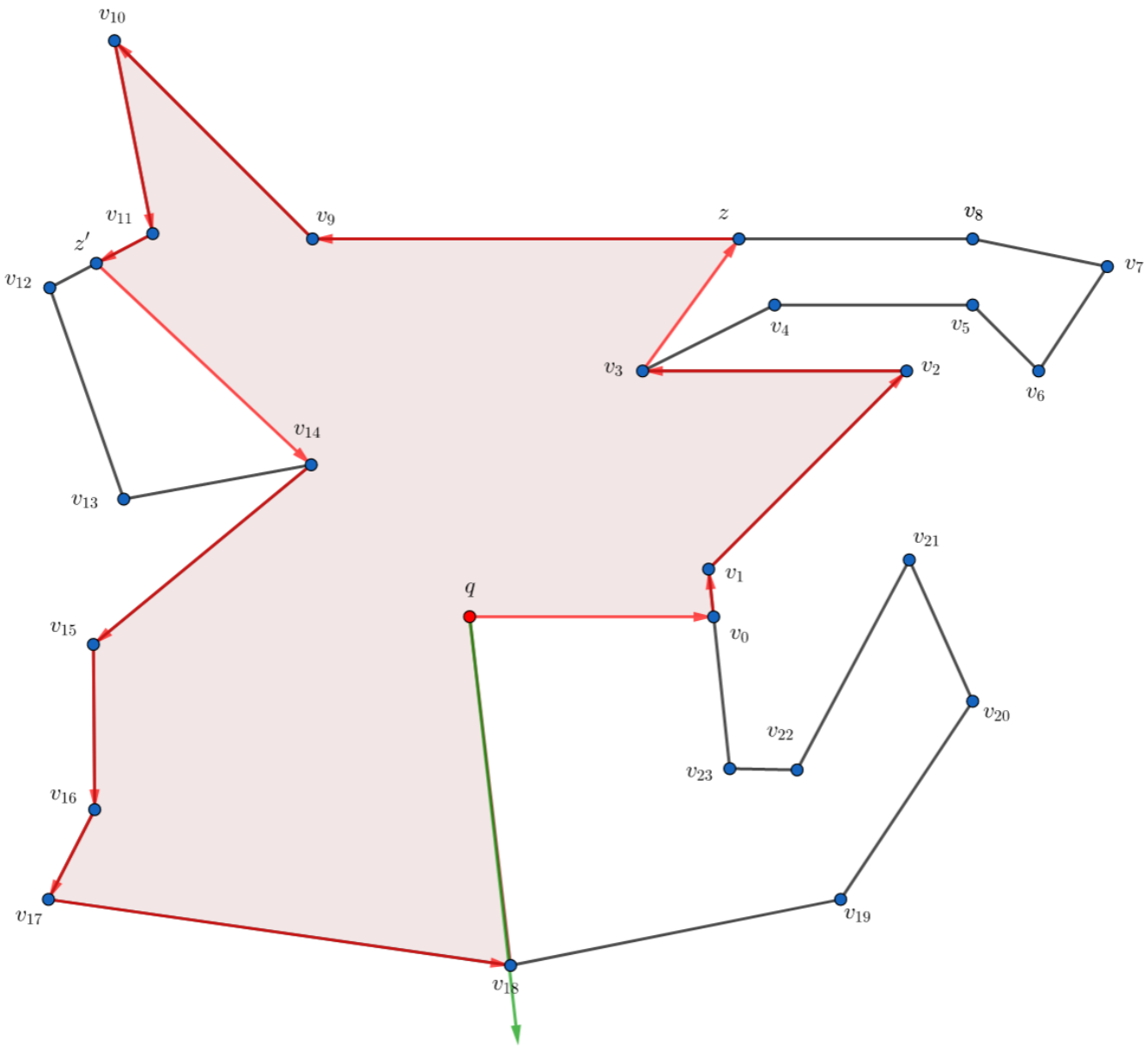
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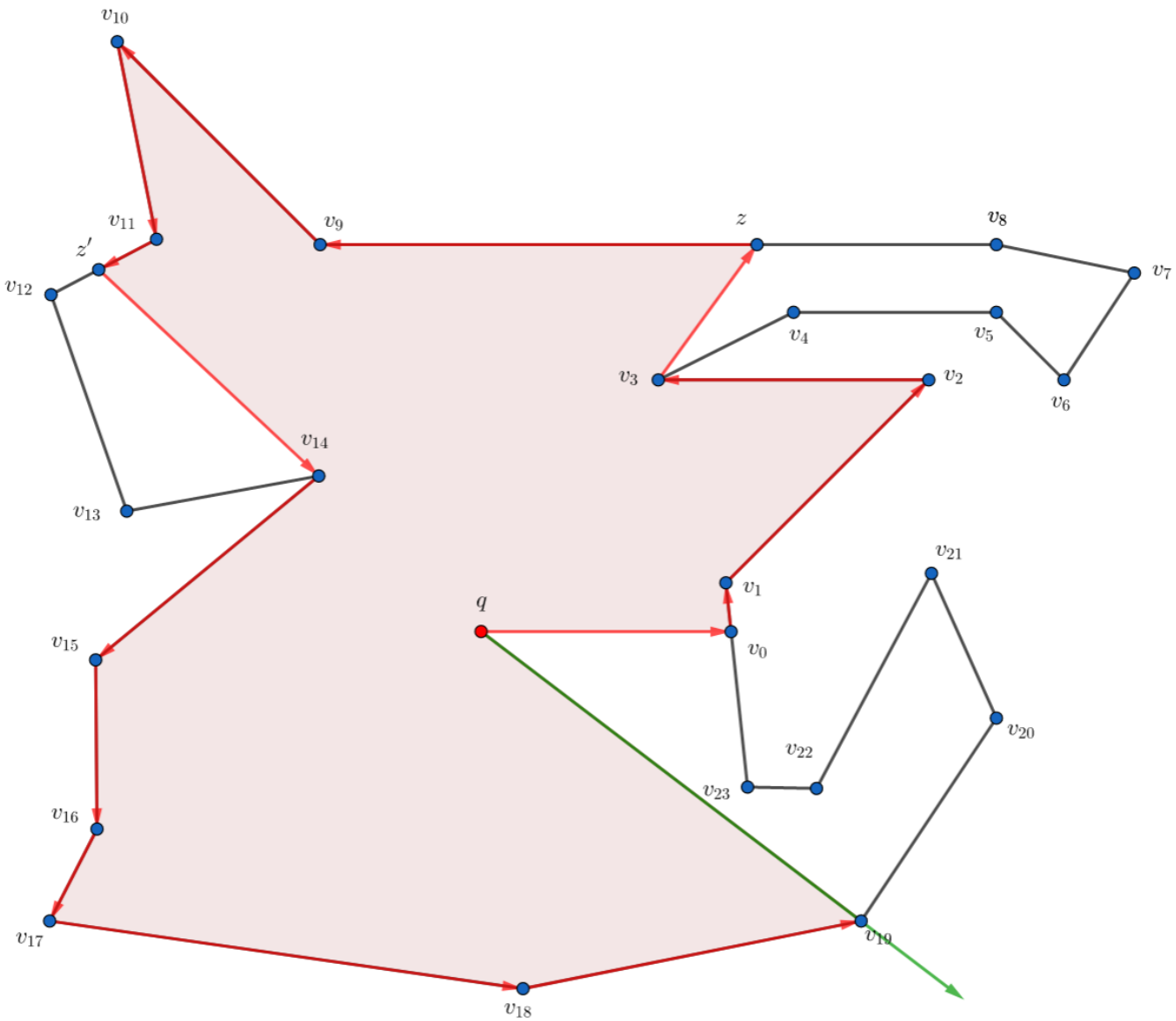
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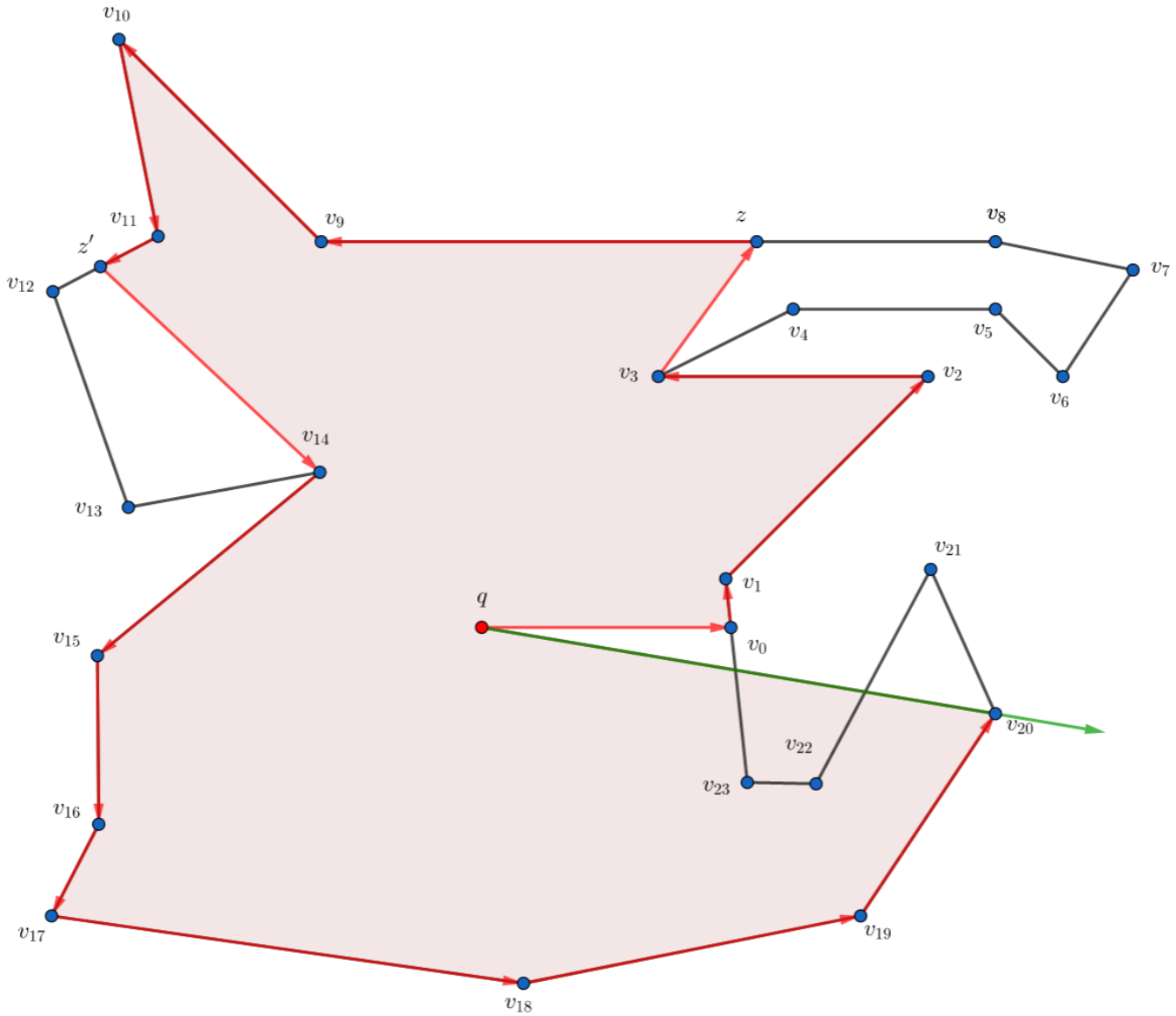
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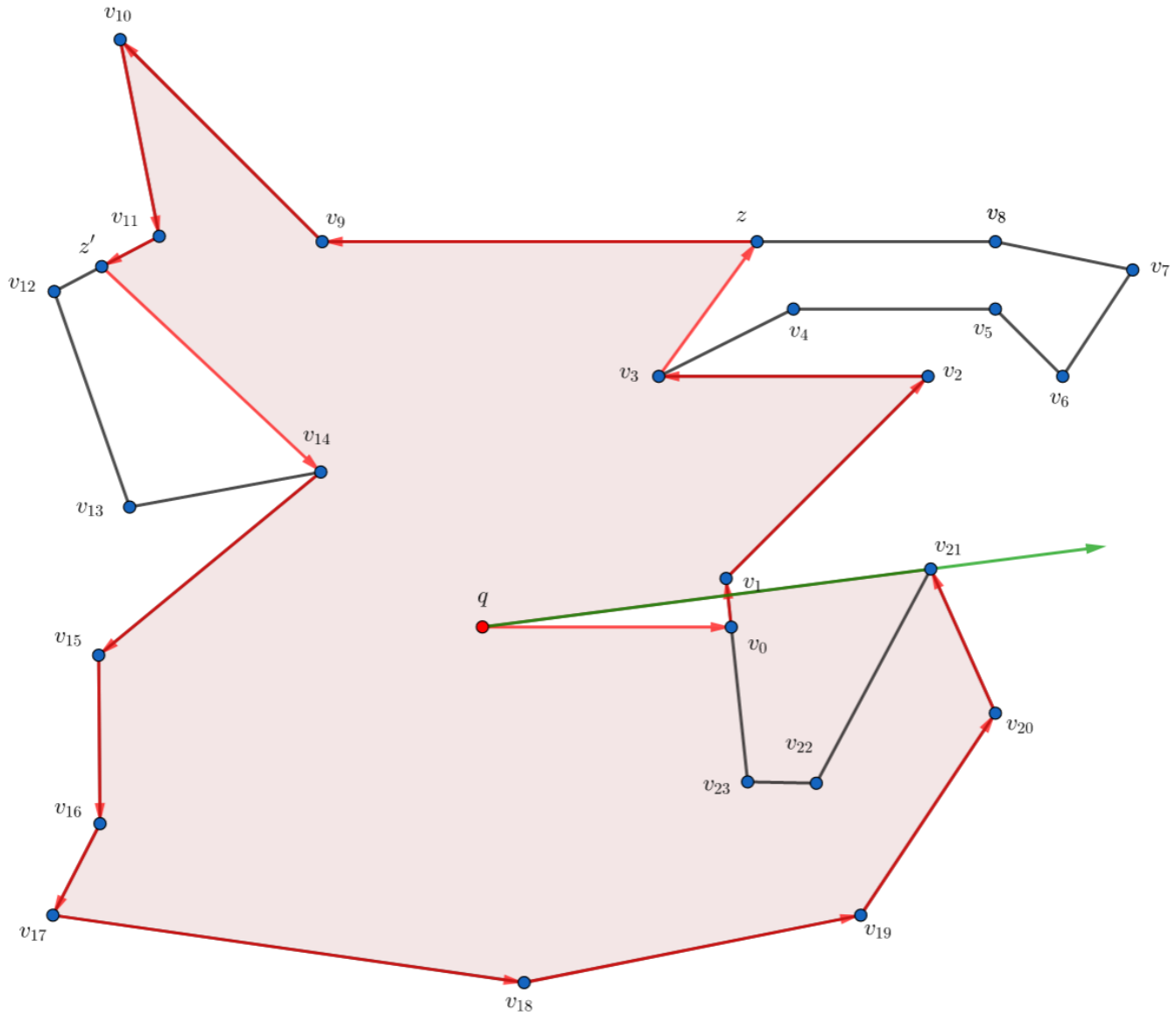
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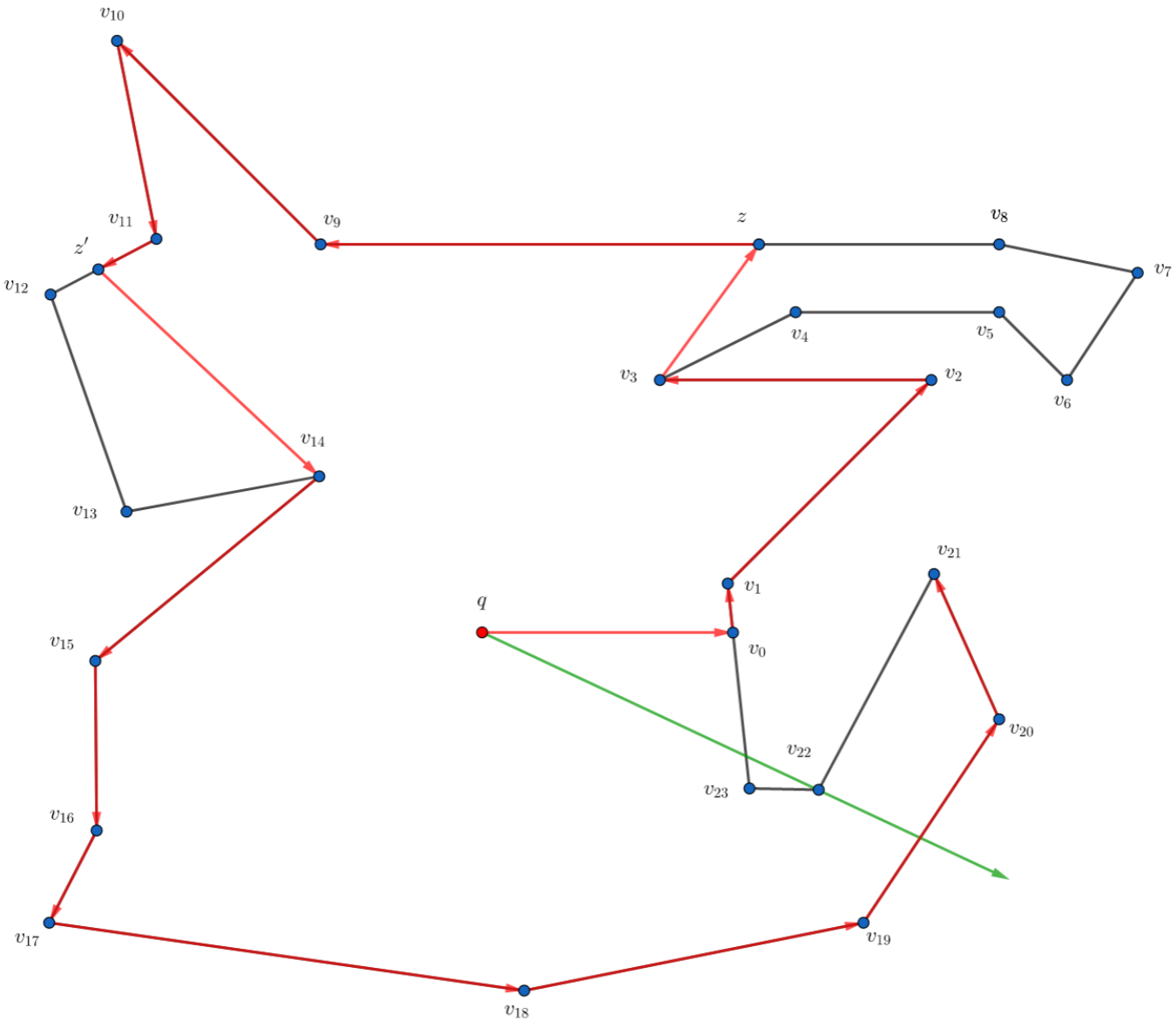
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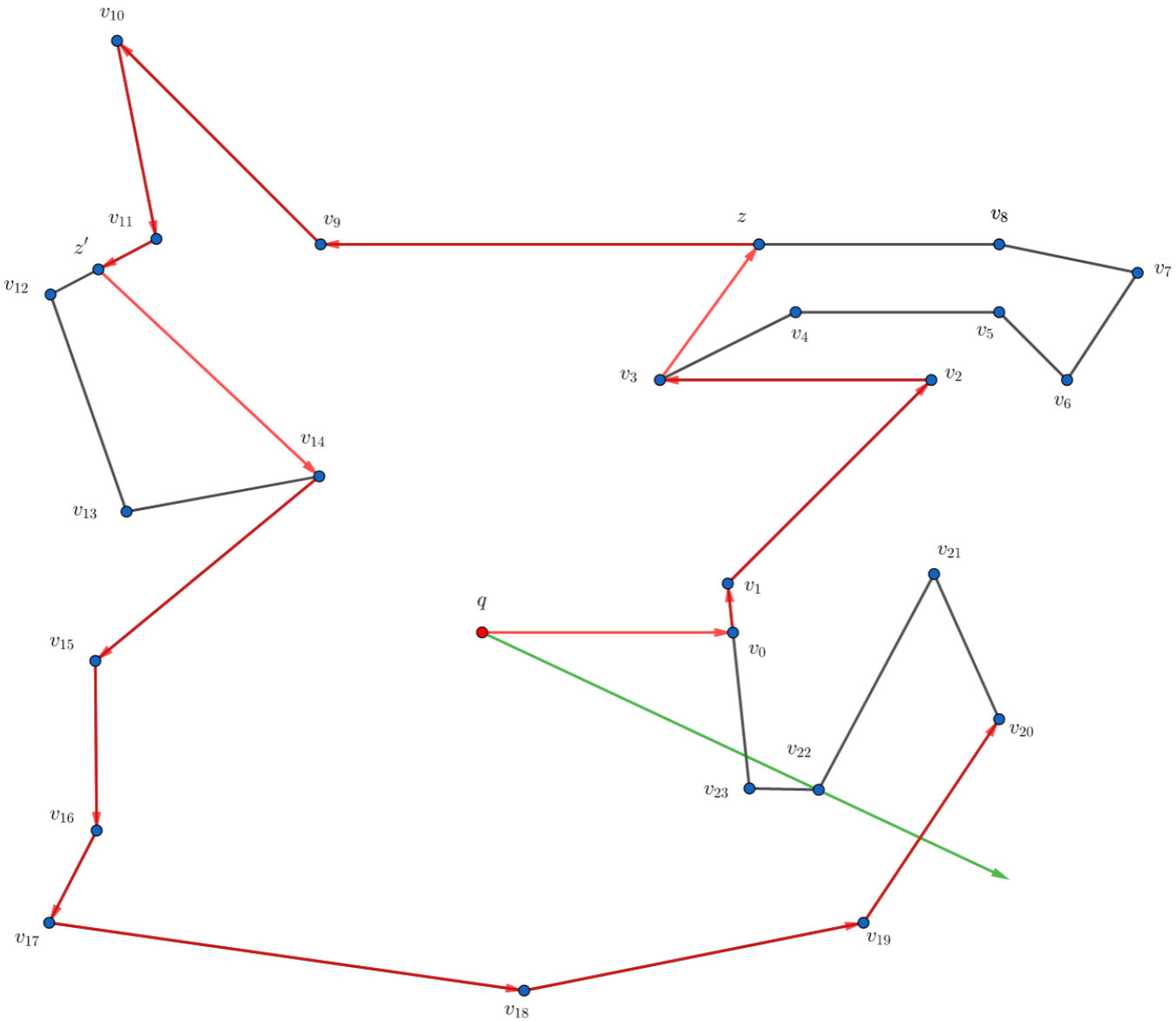
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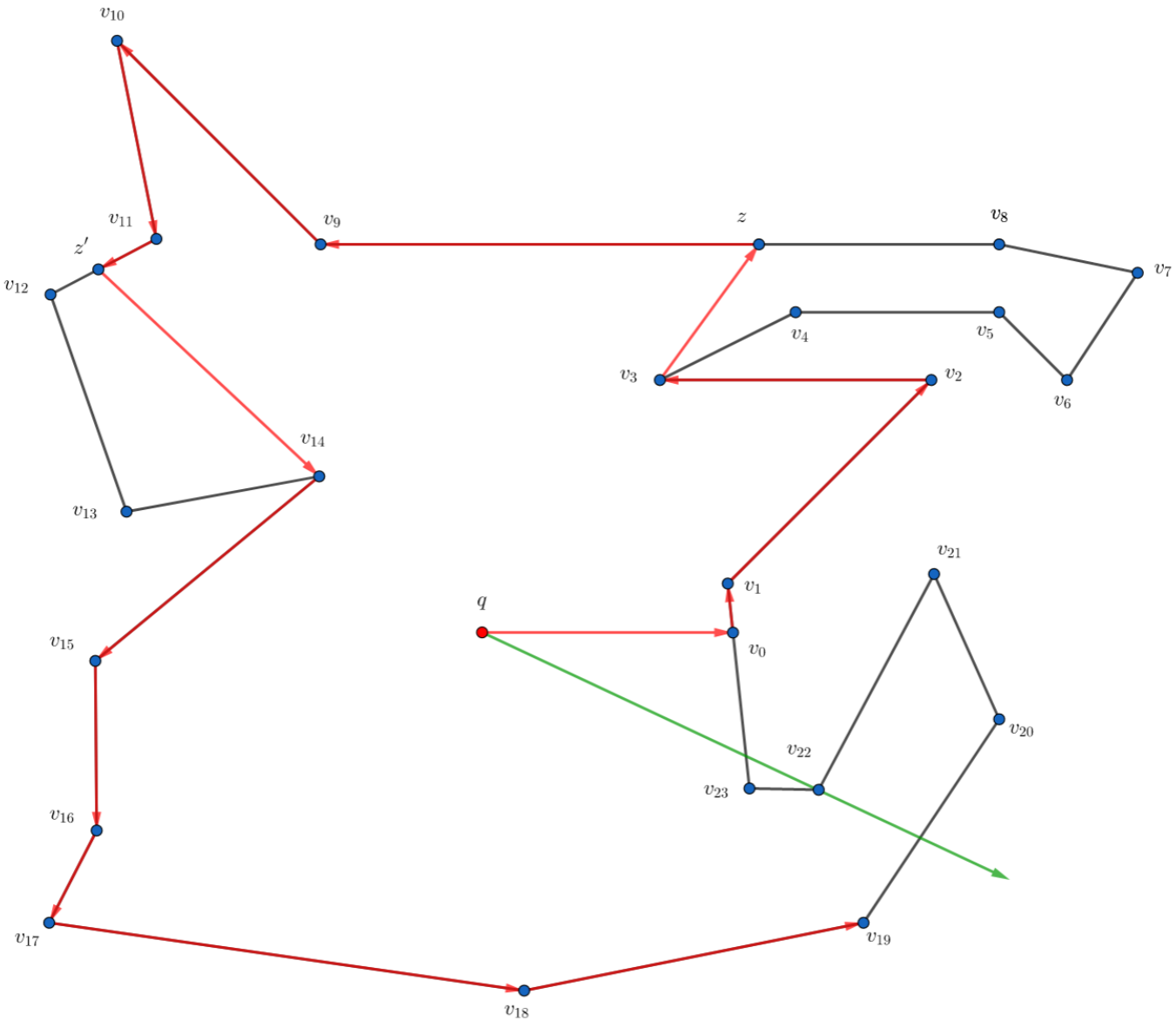
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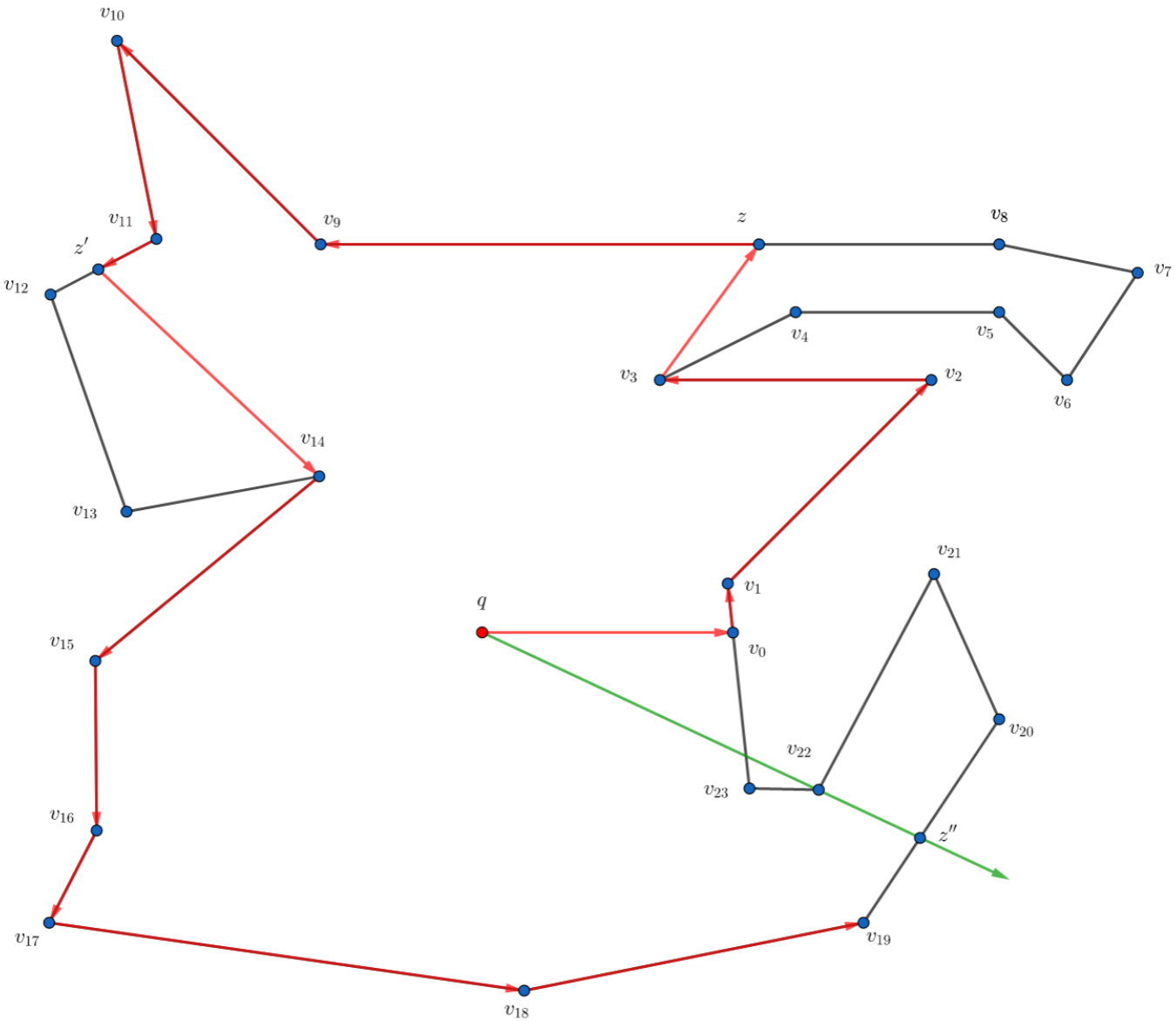
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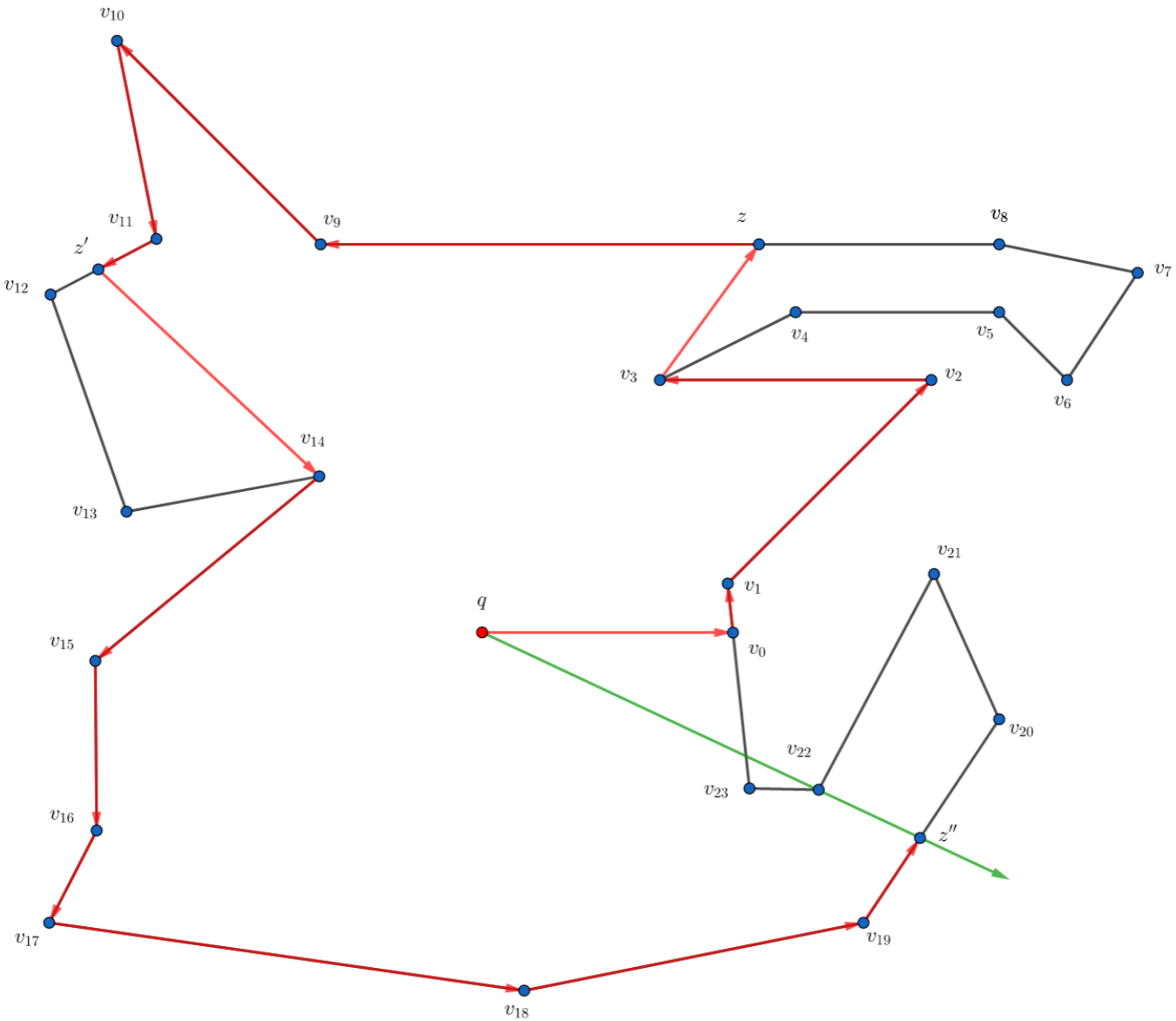
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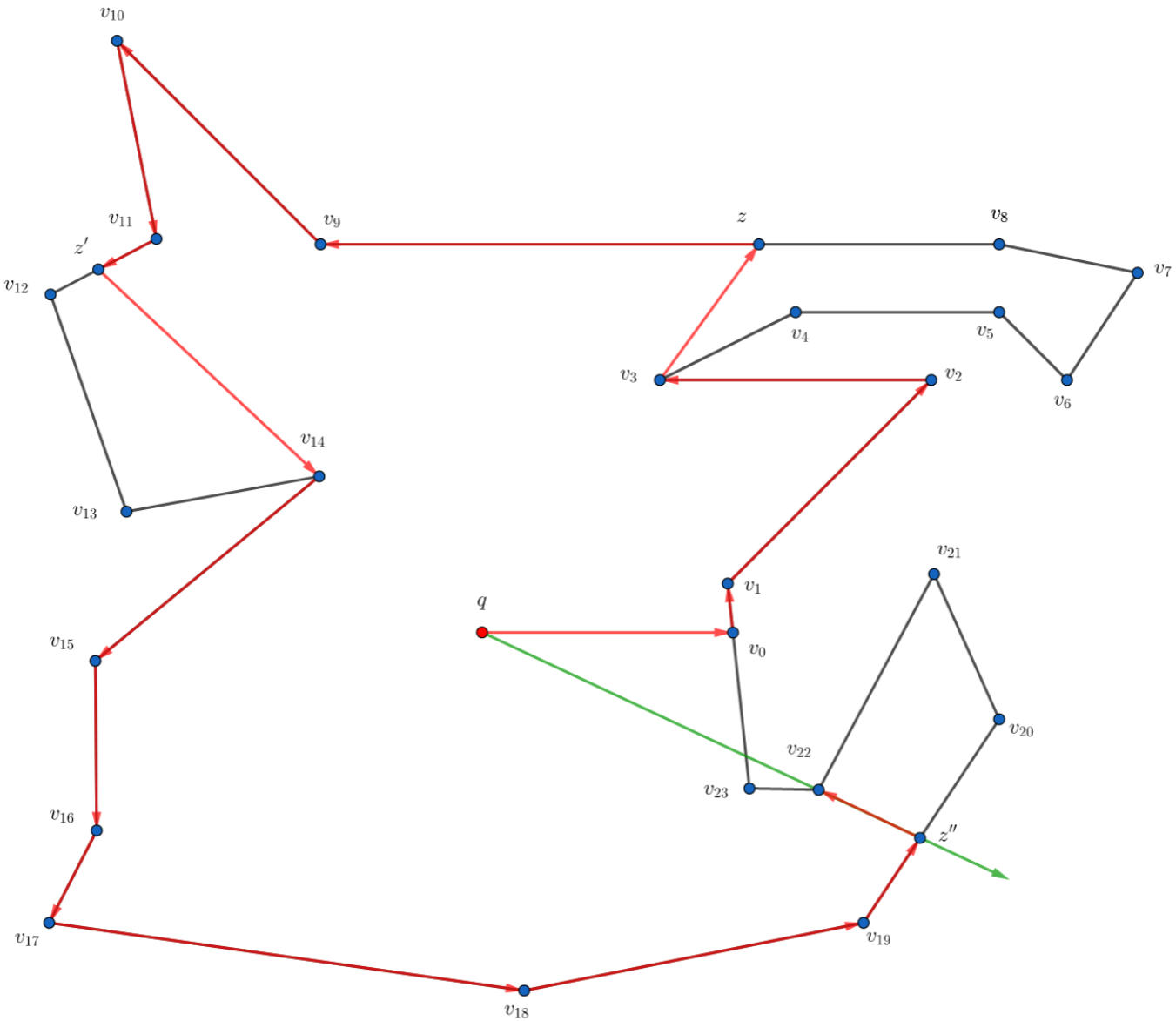
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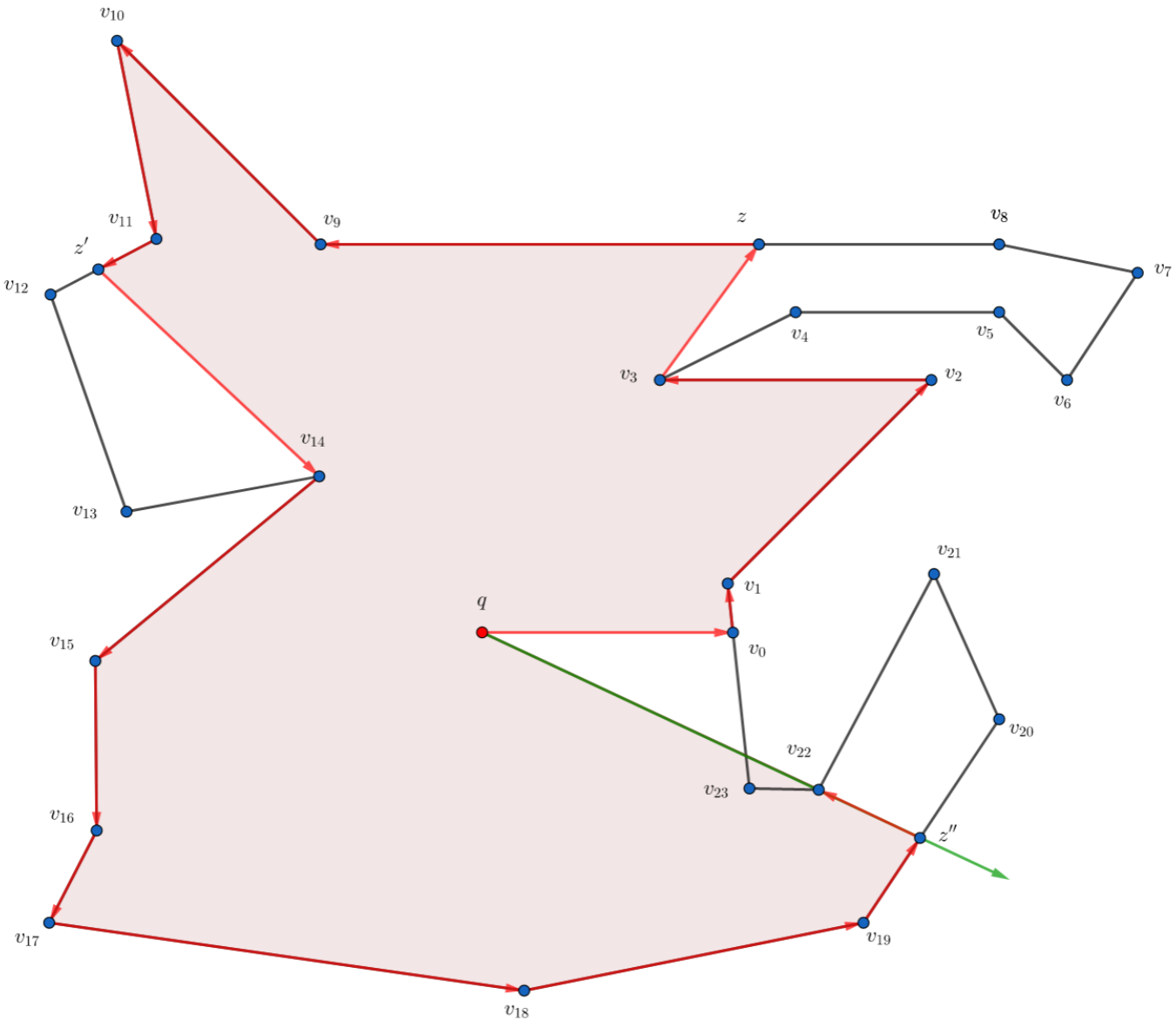
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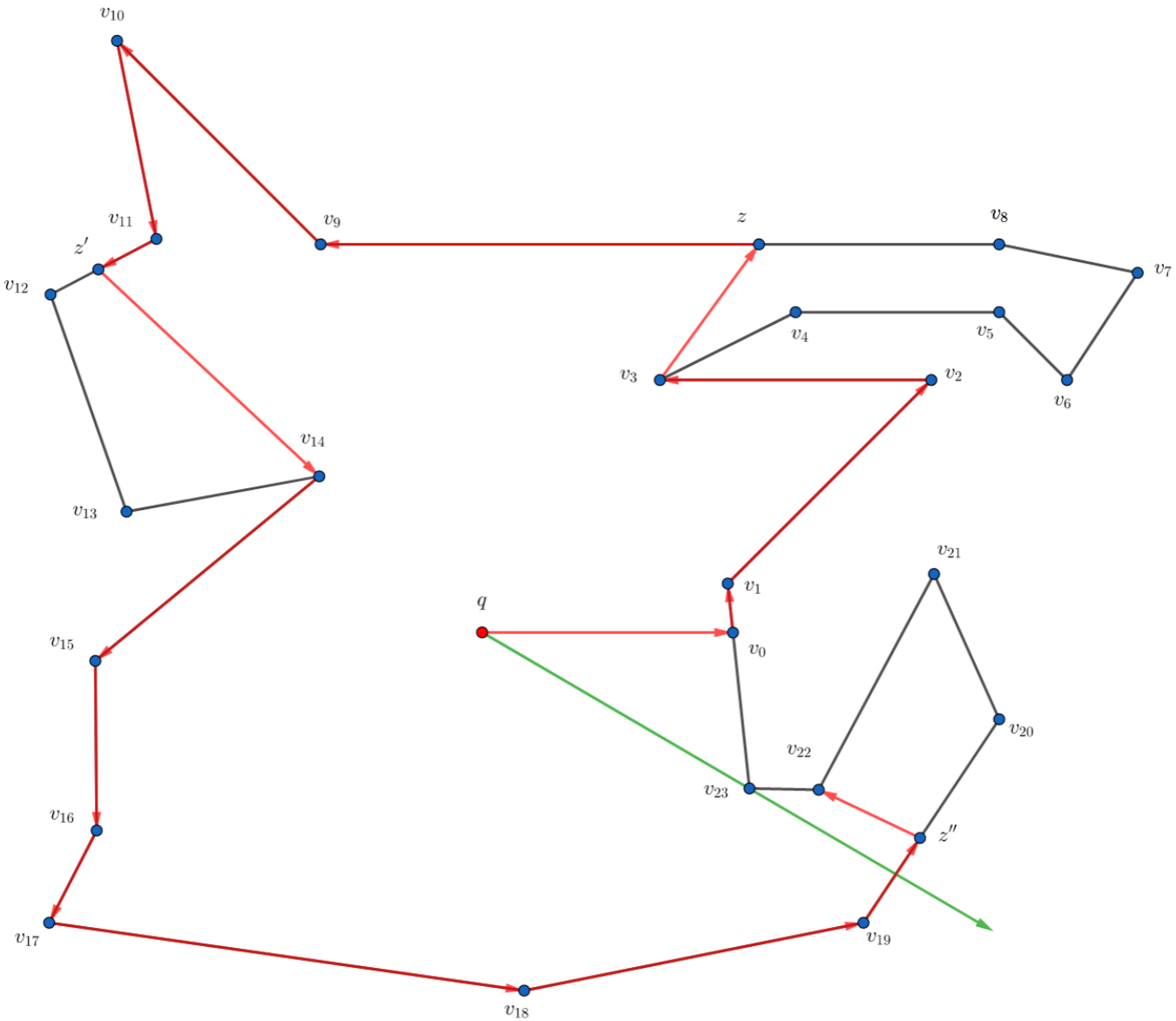
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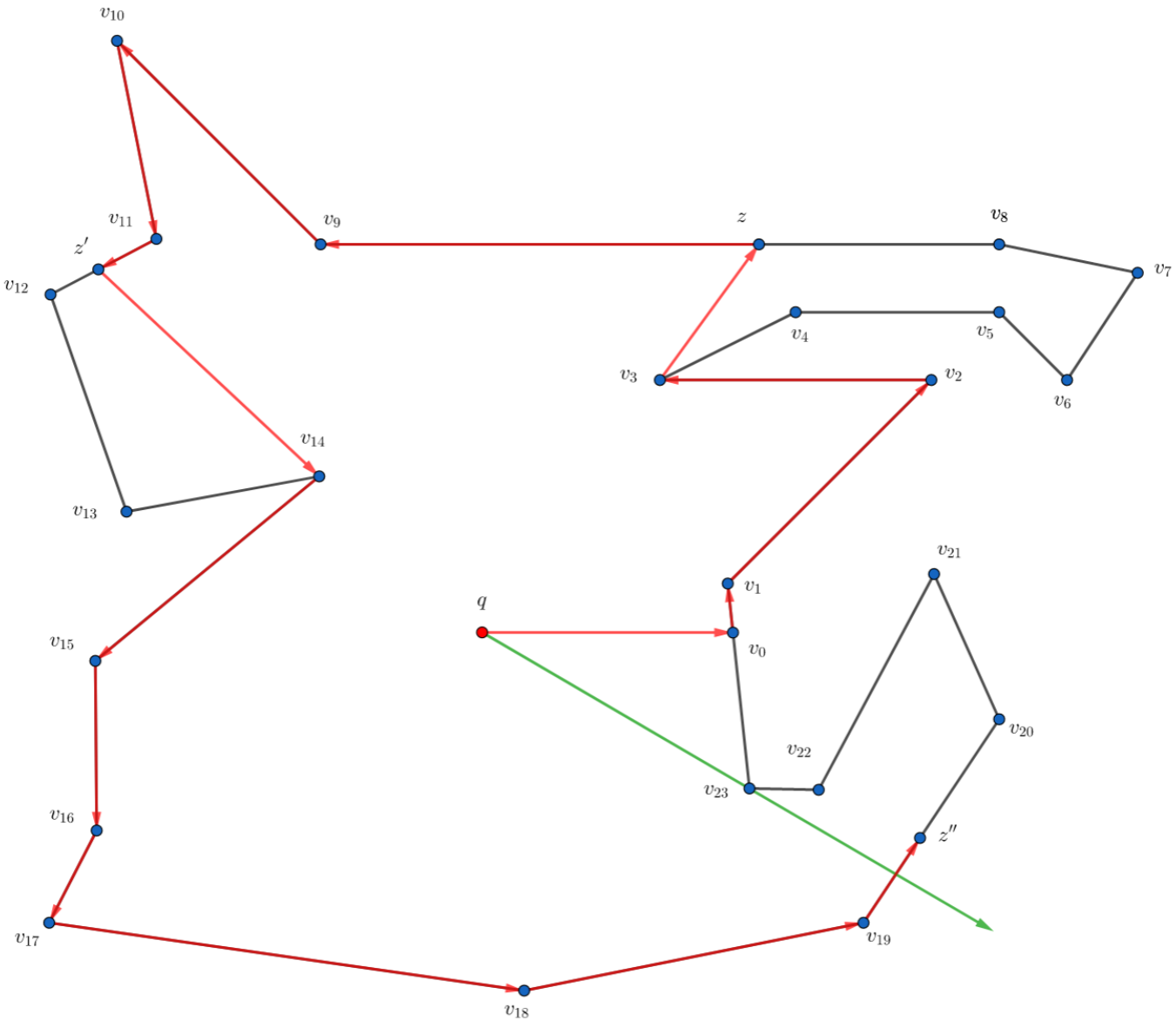
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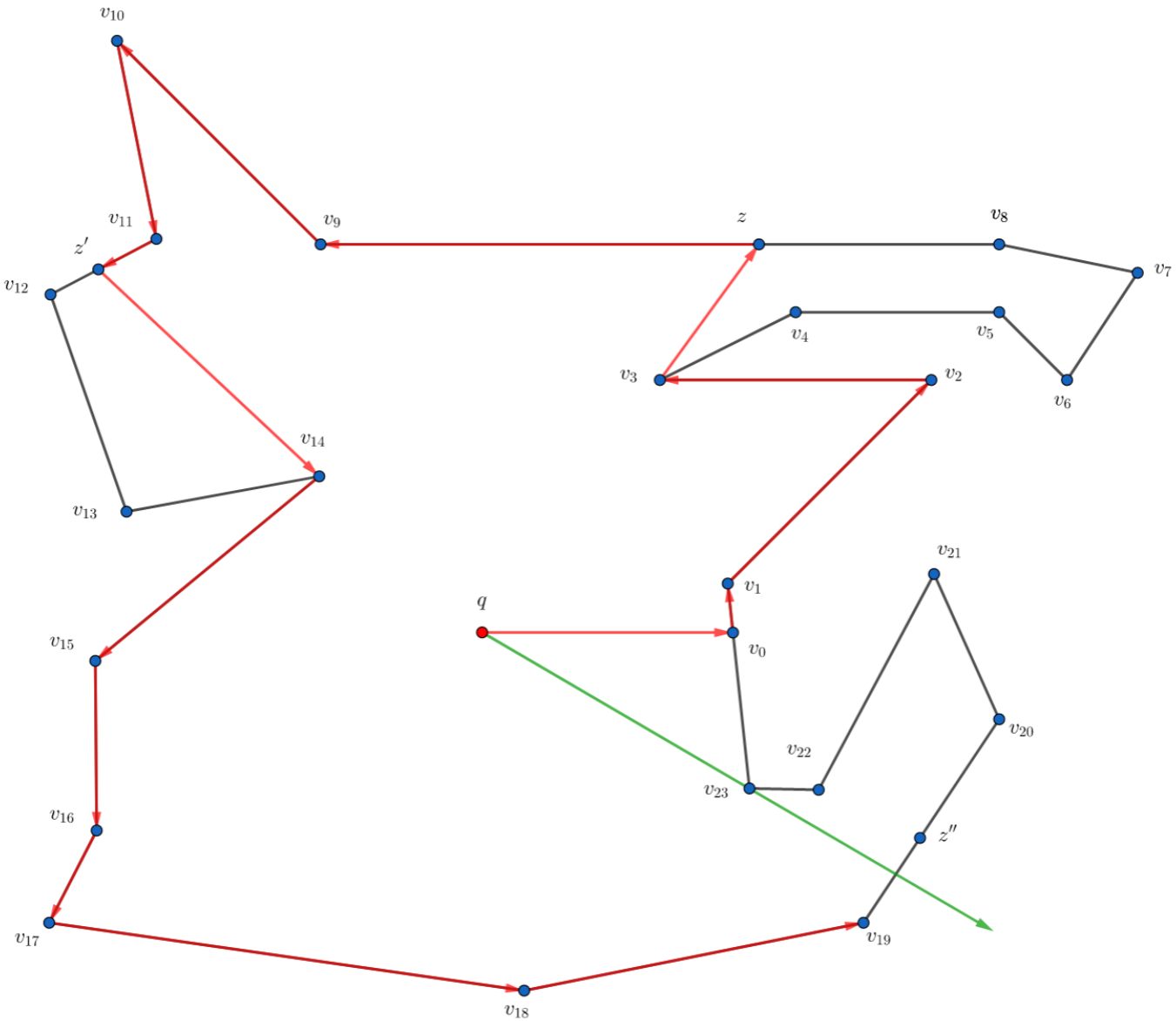
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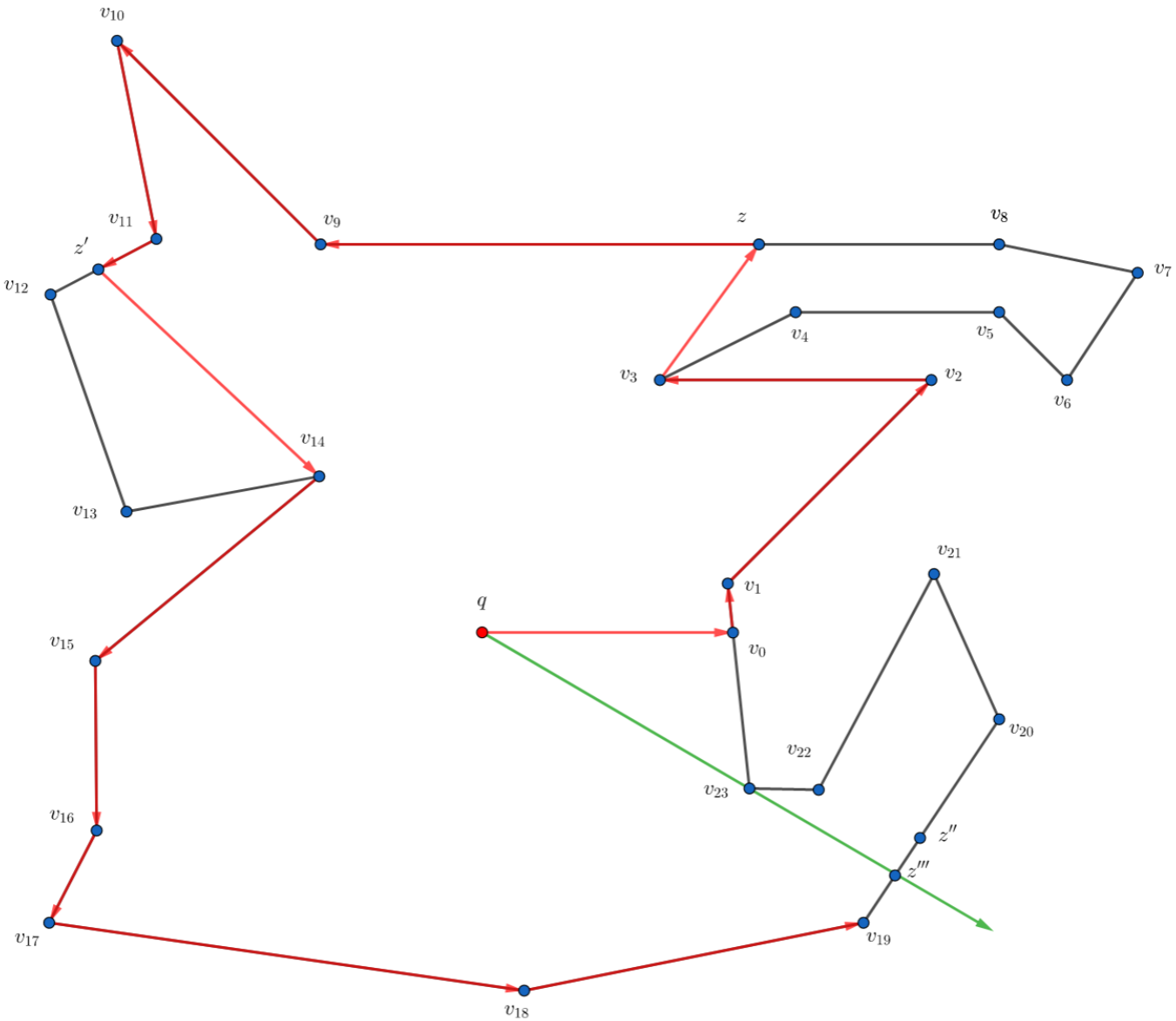
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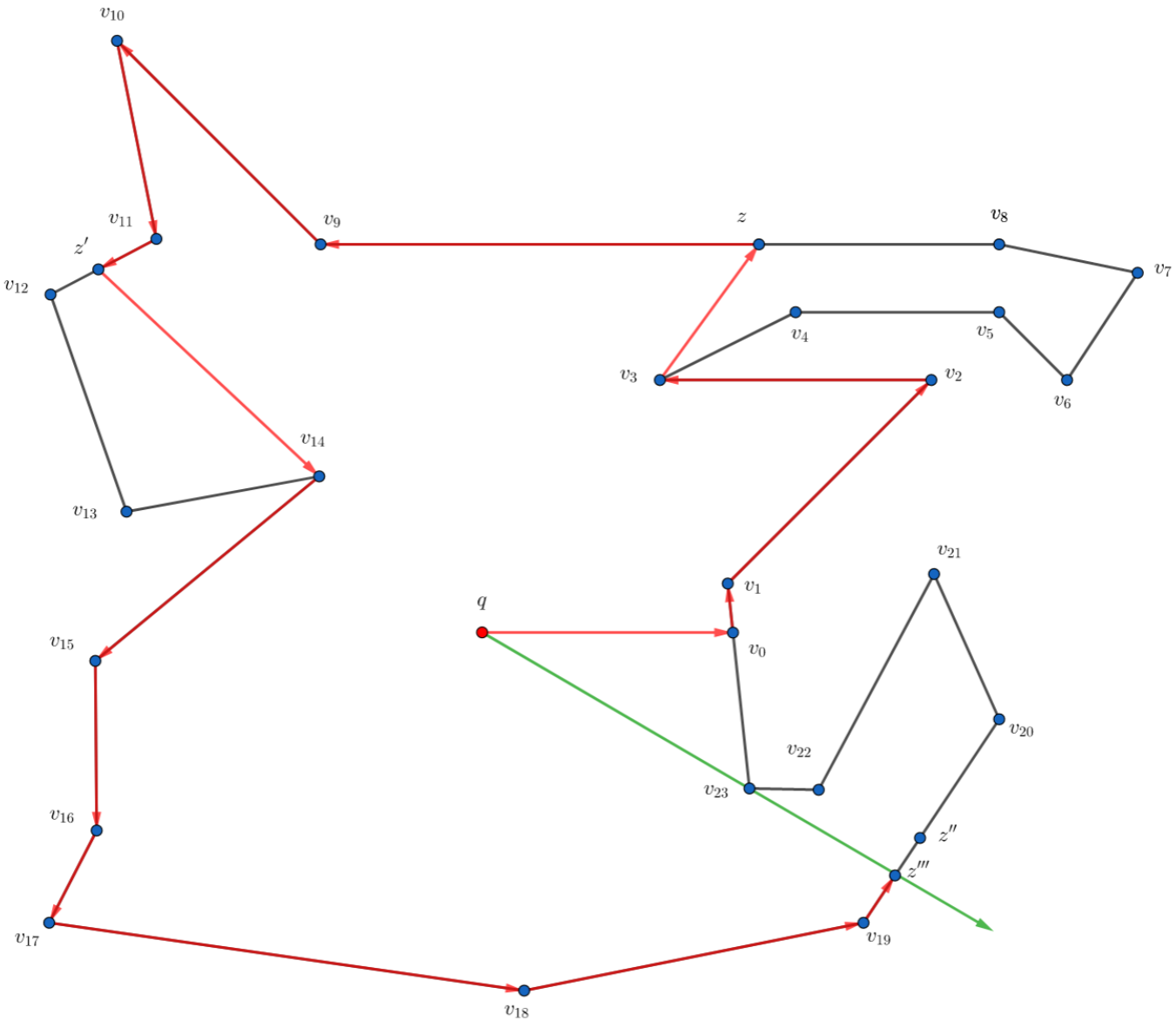
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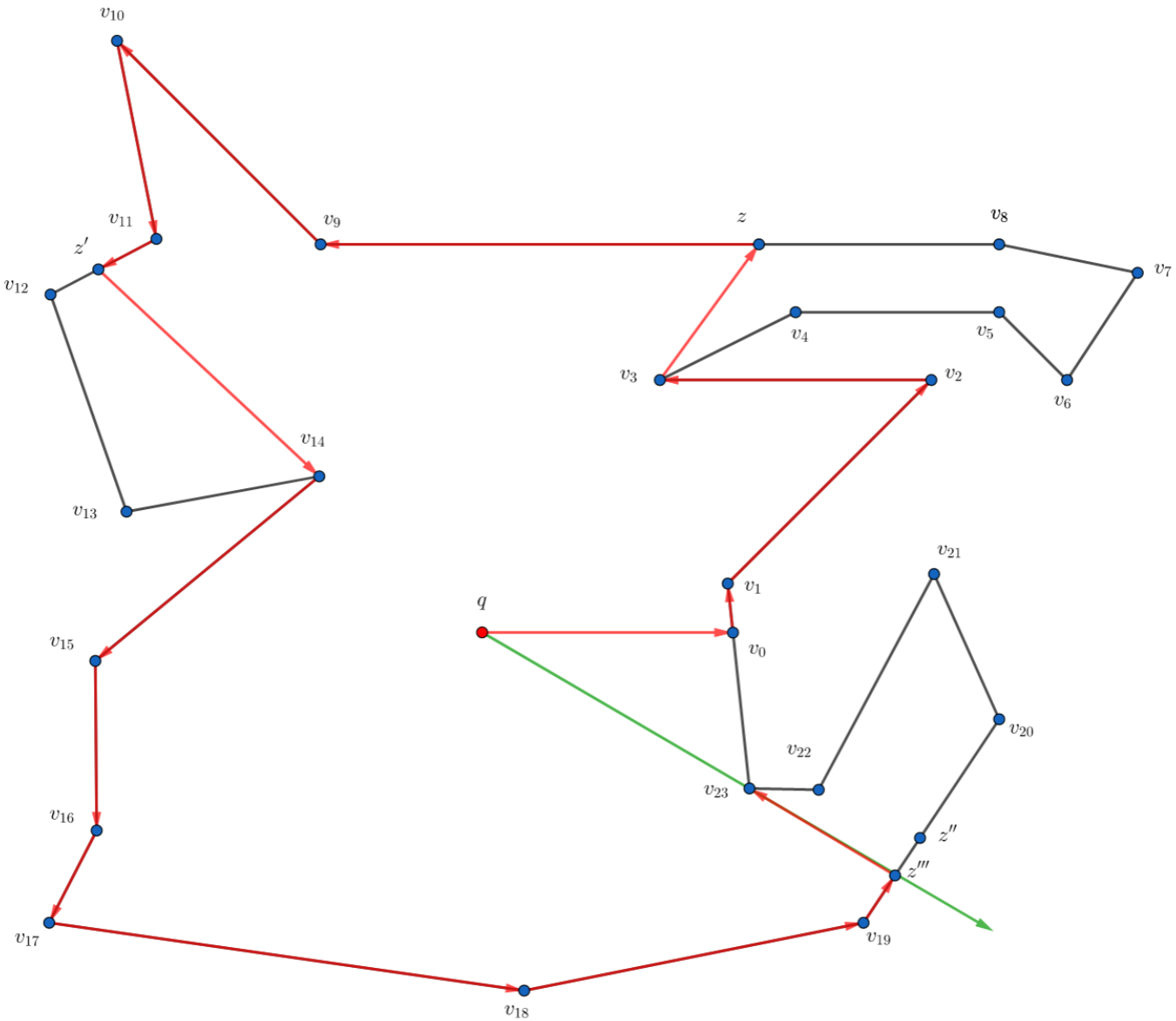
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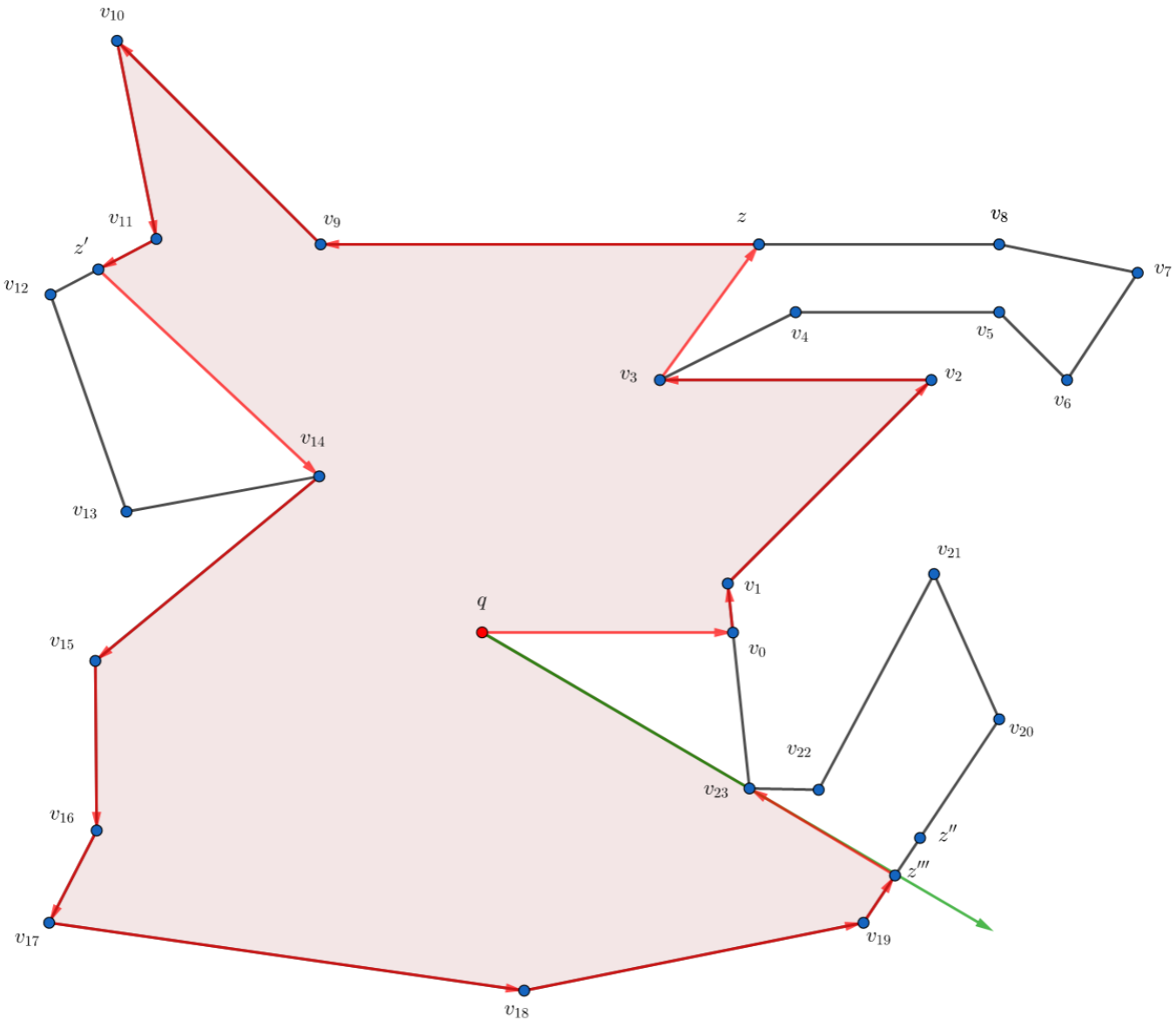
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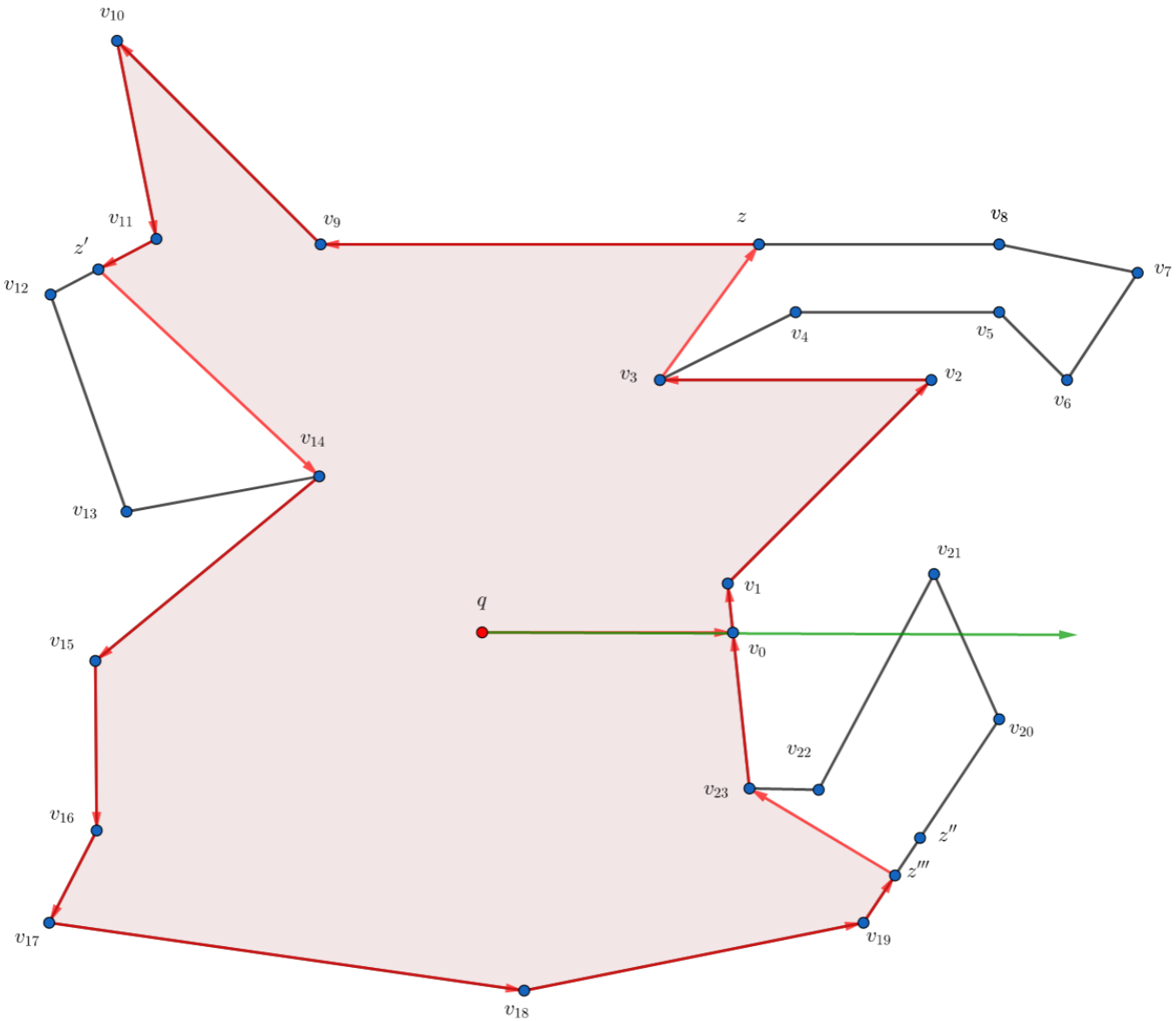
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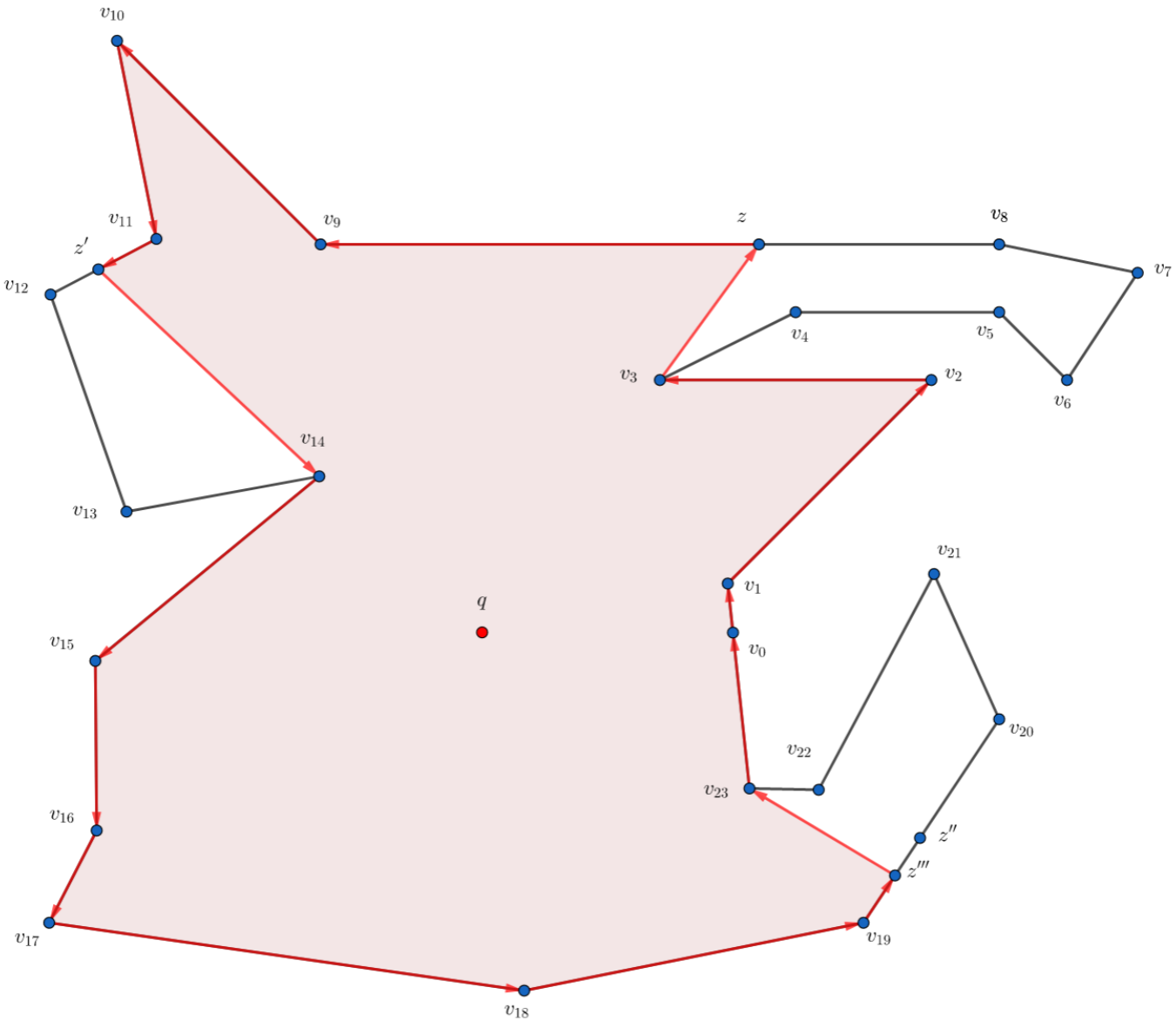
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Análisis de complejidad.

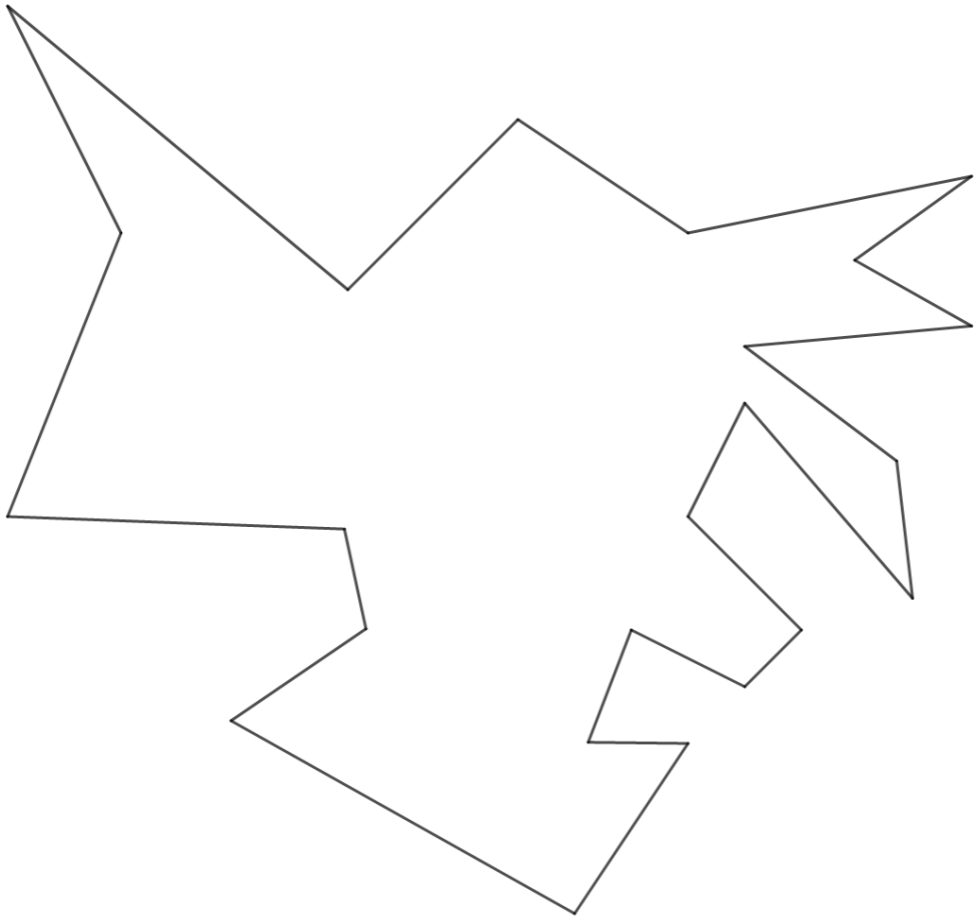
1. Encontrar el punto de intersección del primer rayo a partir de q con la frontera del polígono simple P, lo realizamos en $\mathcal{O}(\log n)$ con una búsqueda en el orden del polígono.
2. En cada paso de la iteración guardamos cada vértice una vez en una pila. Recorrer nuestro polígono nos toma $\mathcal{O}(n)$.
3. Encontrar la intersección del rayo con la arista incidente nos toma $\mathcal{O}(1)$, pues es suficiente hacer pop en la pila usada.
4. Verificar en cada iteración la dirección del vértice siguiente lo podemos realizar en $\mathcal{O}(1)$.

Casos especiales.

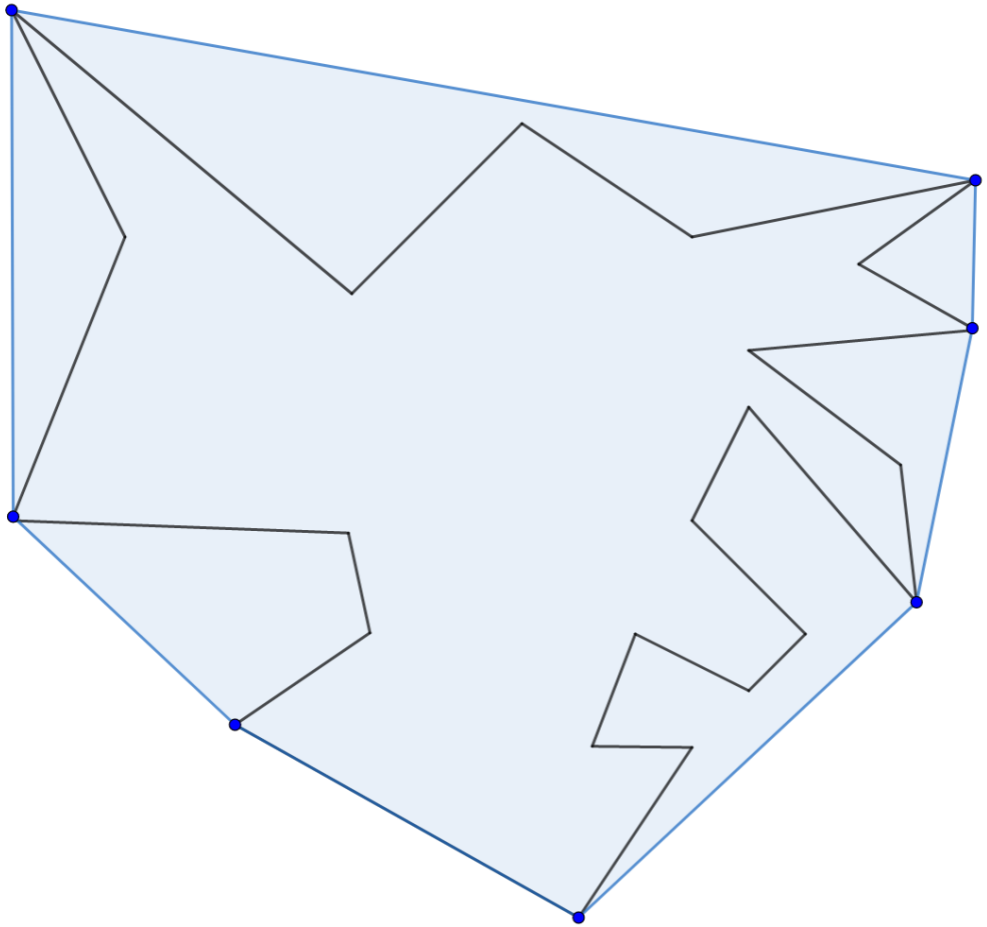
Existen dos casos espciales para encontrar $V(q)$ de P, estos son

- q está fuera de P y se encuentra dentro de la envolvente convexa del polígono simple P.
- q está fuera de P y se encuentra fuera de la envolvente convexa del polígono simple P.

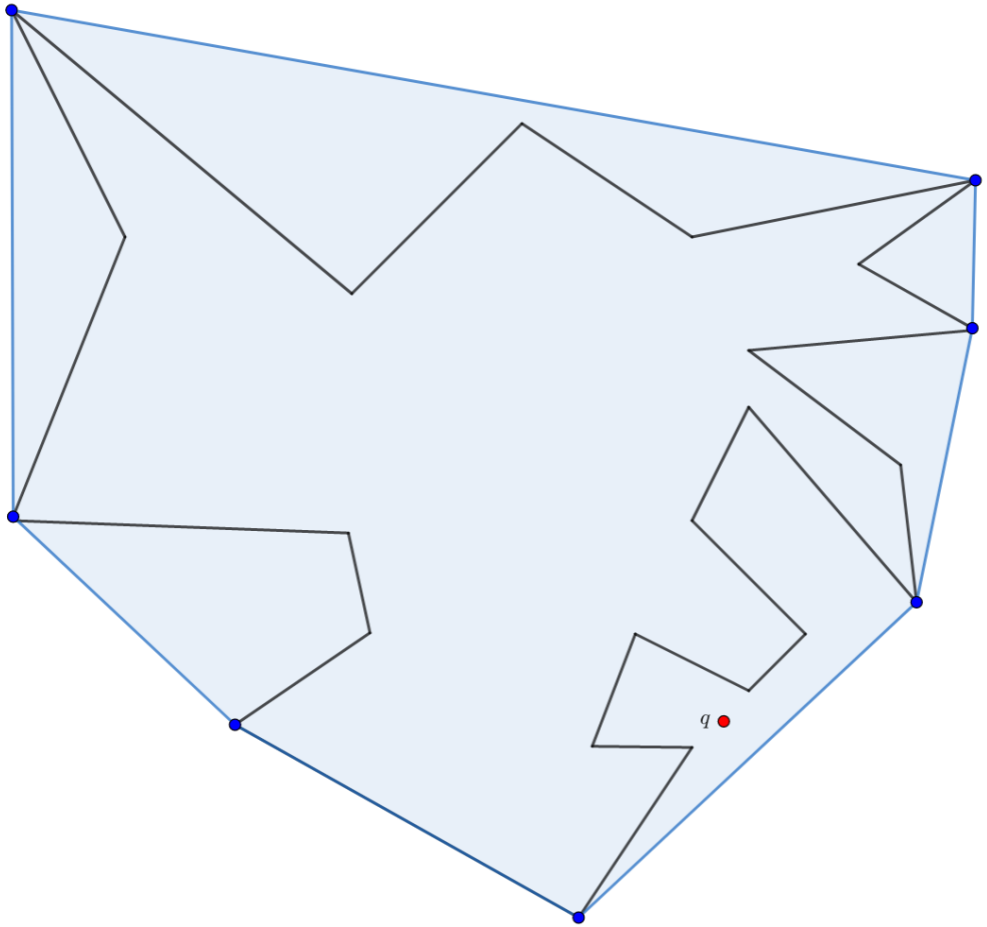
Caso 1.



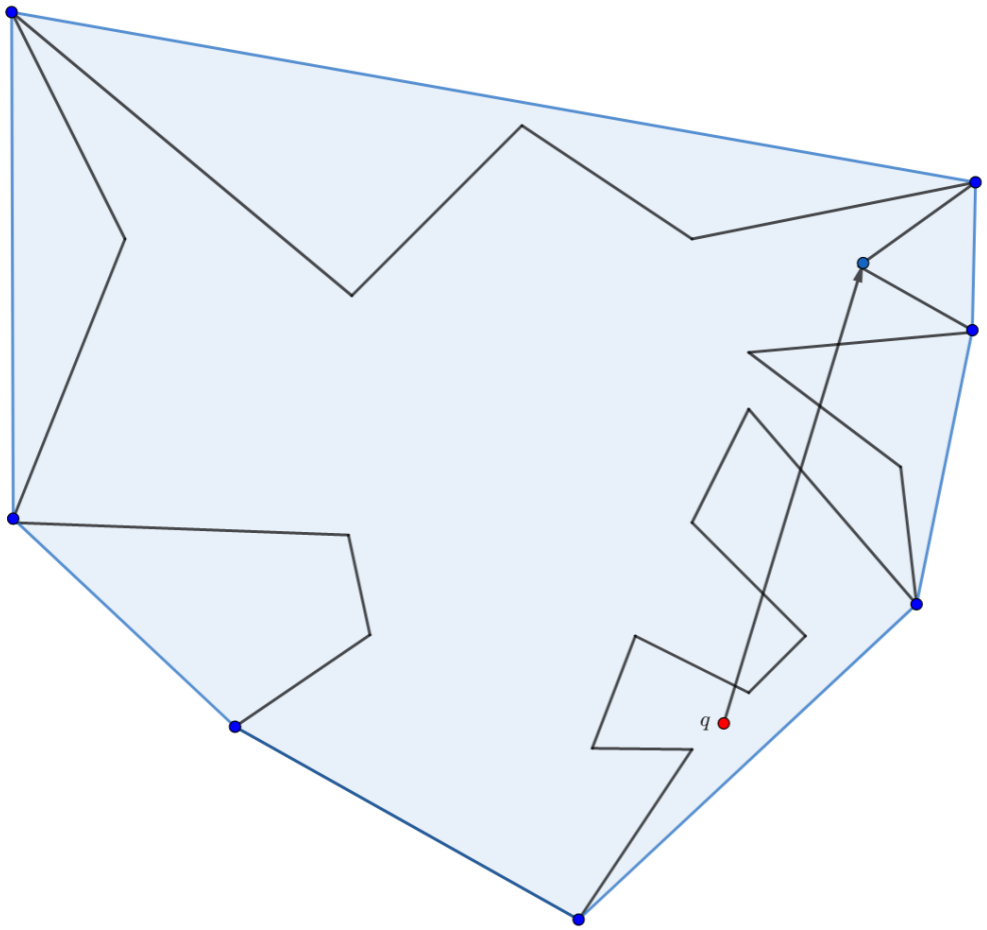
Caso 1.



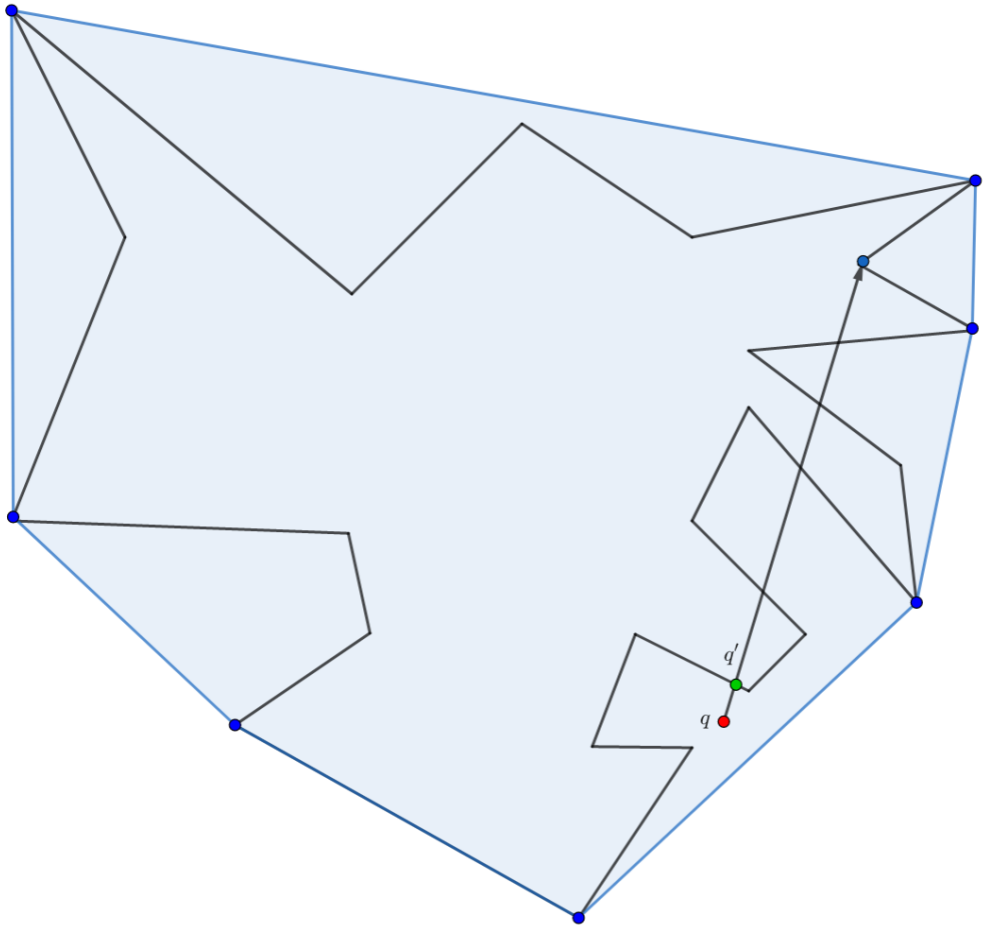
Caso 1.



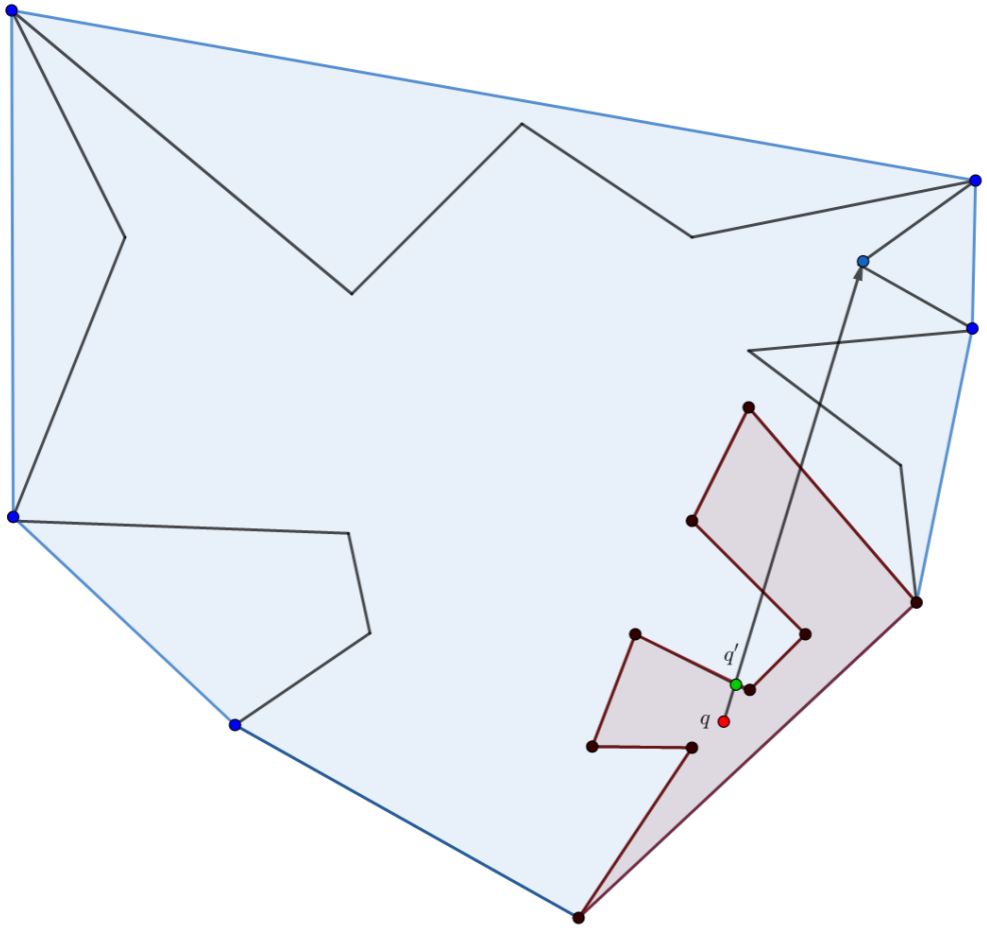
Caso 1.



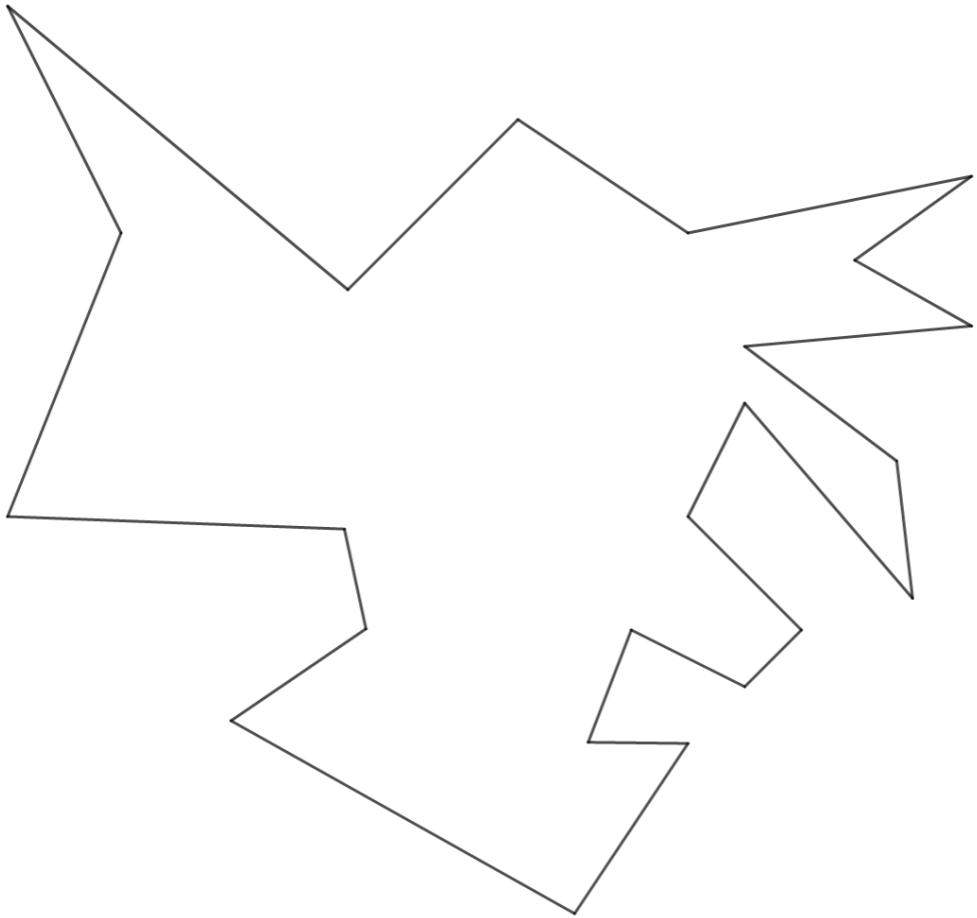
Caso 1.



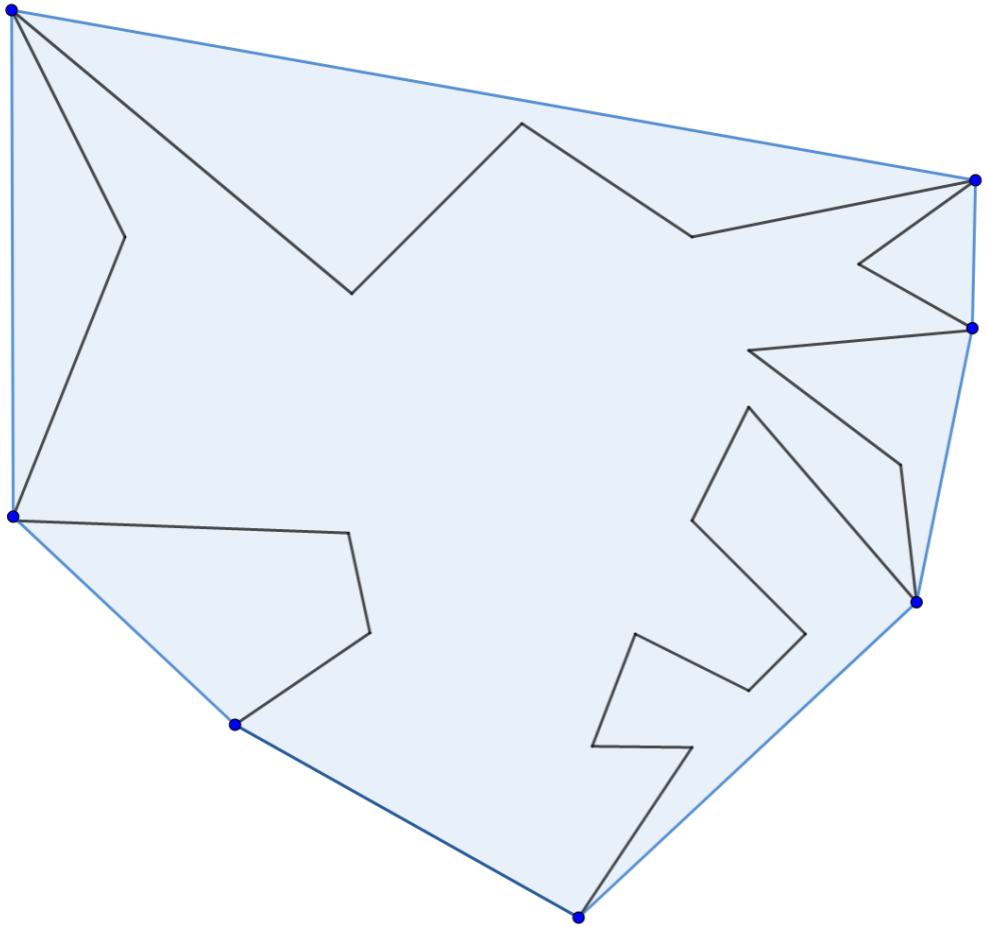
Caso 1.



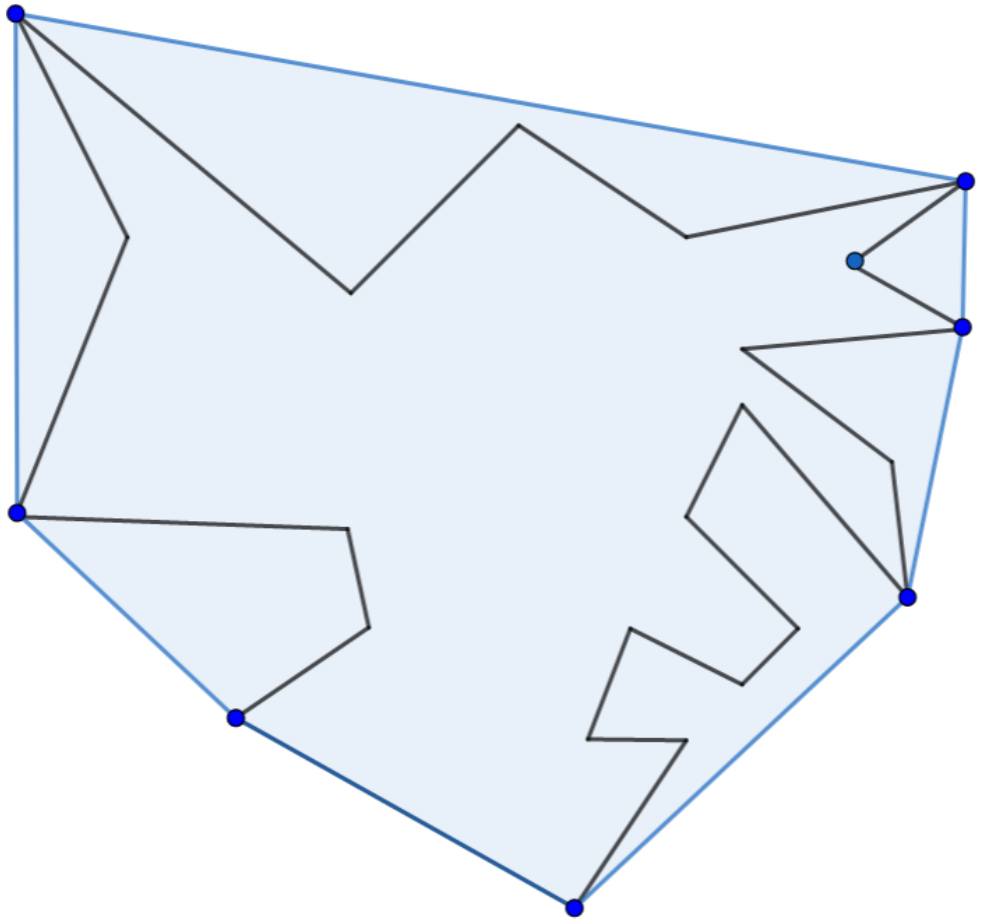
Caso 2.



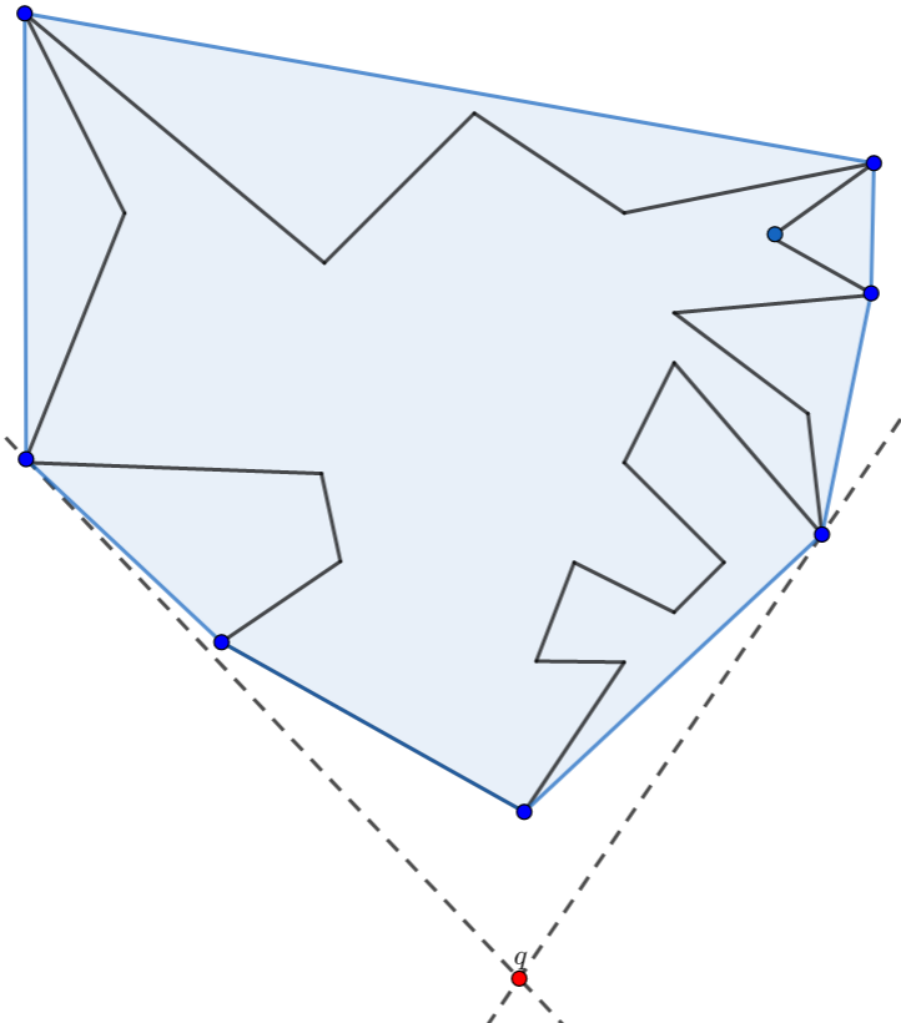
Caso 2.



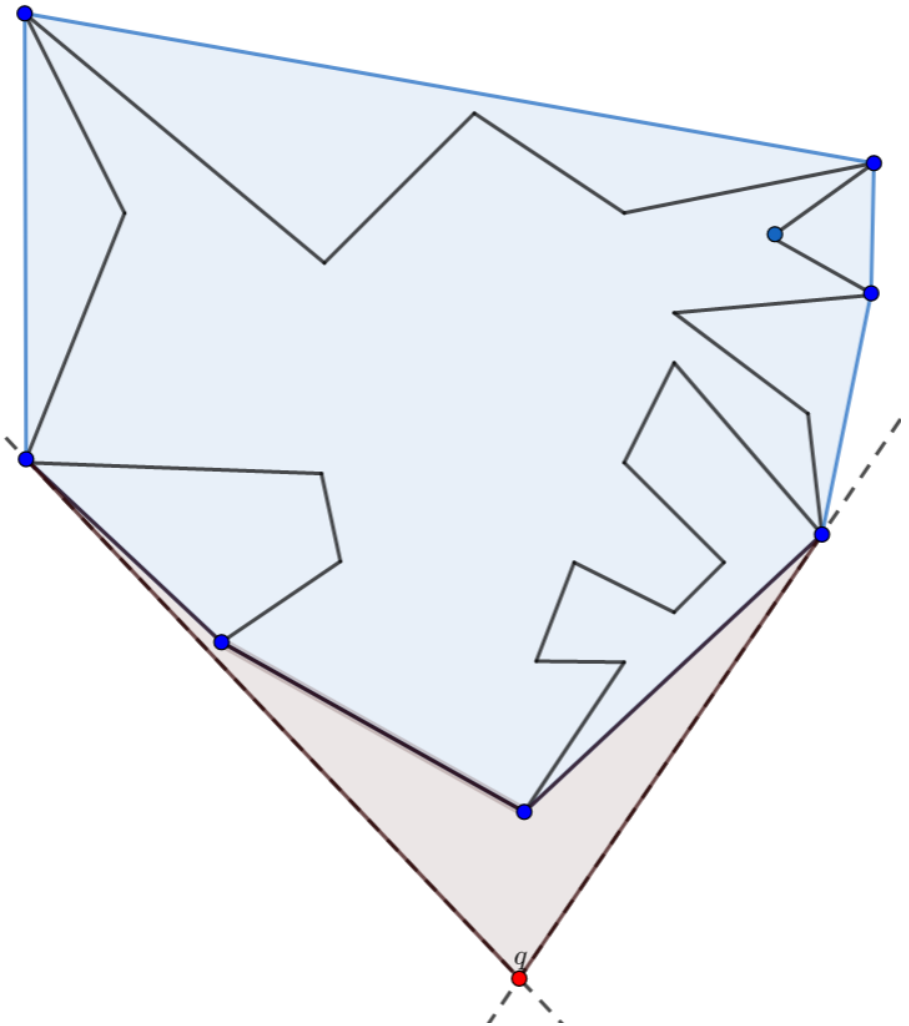
Caso 2.



Caso 2.



Caso 2.



Análisis de complejidad.

1. Encontrar la envolvente convexa de P por medio del algoritmo de Graham-Jao nos toma $\mathcal{O}(n)$.
2. Encontrar q' o en su defecto las tangentes a P nos toma $\mathcal{O}(\log n)$.
3. Unir las fronteras encontradas nos toma a lo más $\mathcal{O}(n)$.

Gracias