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Time Travel in Gödel's Space[1]

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Abstract

We present an analysis of the motion of test particles in Gödel's universe. Both geodesical and nongeodesical motions are considered; the accelerations for nongeodesical motions are given. Examples for closed timelike world lines are shown and the dynamical conditions for time travel in Gödel's space-time are discussed. It is shown that these conditions alone do not suffice to exclude time travel in Gödel's space-time.

Introduction

In 1949 Kurt Gödel [1] discussed a solution of Einstein's field equations with cosmological constant Λ for incoherent matter which differs from the statical Einstein solution. Among various striking properties of this solution one of the most interesting is the existence of closed timelike world lines (CTW) and past-traveling world lines (PTW). In Gödel's space-time the global serial order with respect to the comparisons "later than" or "earlier than" is no longer given. The possibility of time travel in this space-time has frequently been discussed in both the physical and the philosophical literature [2–15]. In most cases the example of the two cosmological twins is cited: one of them is comoving with the matter along a timelike geodesic; the second one is moving nongeodesically and returning to a previous state of both his own and his twin's past, thereby meeting himself at an earlier stage of his own personal existence. Who does meet whom there at this time and this place? Or even worse, how can we interpret the process that an agent in principle could kill his own grandfather in his (the

[1] Part of this work was performed while the author was a Research Associate at the Center for Philosophy of Science at the University of Pittsburgh.

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grandfather's) childhood, thereby destroying the basic conditions for his own existence?

The proof that in Gödel's space-time such a time travel cannot be performed along the world line of a geodesic has been given by Kundt [16] and independently by Chandrasekhar and Wright [17]. Unfortunately, misinterpreting Gödel's remarks about closed timelike lines, Chandrasekhar and Wright claimed that their results are in contradiction to some of the statements of Gödel. J. Earman [4] suggested and H. Stein [13] proved that this apparent contradiction was based upon a pure misunderstanding. It can be shown in the following how this misunderstanding could arise at all.

The existence of past-traveling but future-pointing world lines for a cosmological solution of the field equations is of basic physical and philosophical interest: The possible causal connectability of each point in this space-time with each other involves relevant questions as to whether our concepts of causal connectibility and causality are compatible with the consequences of this solution. R. Swinburne [14] for instance claims that the possible twofold causal connectibility of two events represents a *logical contradiction* and therefore Gödel's solution has to be excluded. In the same vein S. W. Hawking and G. F. R. Ellis [8] adduce the possibility of *logical paradoxes* and exclude closed timelike world lines by imposing an additional *chronology condition.*

The existence of CTW in general relativity is not restricted to the peculiar case of Gödel's universe. There are several solutions of the field equations which allow for CTW, as for instance the Taub-NUT solution [18, 19], the Kerr solution [20, 21], the Tomimatsu-Sato solution [22, 23]. From a physical point of view more interesting is the low-angular-momentum Kerr geometry ($a^2 < m^2$). This geometry appears to be the unique final state of the gravitational collapse of rotating matter. In this case, however, the region of causality violation lies within an event horizon and could never be observed from the outside of this horizon. In the case $a^2 > m^2$ (a naked singularity), there are no event horizons, and causality violation is global [24, 25]. In addition, F. de Felice and M. Calvani [26, 27] could show that in contrast to the Gödel solution, there even exist closed *null geodesics* in the high-angular-momentum Kerr field. However, causality violation by those null geodesics occurs very close to the singularity and can be experienced by distant observers only under very particular conditions.

The singularity problem does not arise in Gödel's space-time. The group of isometries is five dimensional, and it is transitive, i.e., Gödel's space-time is completely homogeneous (but not isotropic). One could argue that the Gödel solution is of minor physical significance, since it corresponds to a rotating, stationary universe, whereas our actual universe is expanding and apparently nonrotating. However, due to the global regularity of this space-time a discussion of fundamental properties of CTW and PTW—based upon a series of concrete, sufficiently general examples—can be given in a more extensive manner and thus will lead to a more comprehensive understanding of causality in general relativity.

Furthermore, in spite of the fact that our universe is apparently not of the Gödel type, Gödel's space-time can be interpreted as *physical* in the sense that if the right-hand side of the field equations describes a physical energy-momentum tensor (i.e., a possible "reasonable" source) and this right-hand side determines the geometry of space-time and its causal structure, then the solution of the field equations should be interpreted as a physical solution as well. Discussing its global causal structure, one should take into account that durations and distances occurring refer to cosmic durations and distances, respectively. The pictures of the human time travelers quoted above only hold as paradigms. In order to get a more complete interpretation of the results one should first of all investigate under which *dynamical* conditions time travel is possible. One has to answer the following questions: what are the velocities, what are the distances, the proper times, the accelerations, and energies needed for such a time-travel?

The only remark as to how difficult time travel in this universe could be was given by Gödel himself [6]. There are no investigations known which describe the possibilities of either a twin paradox or of a self-encounter on the basis of reasonable cosmic densities of matter. Without detailed evaluation Stein claims that provided *sufficient energy resources* are available, a person can decide to visit the space-time vicinity of any time of his own past, and in addition, this person can do this within an *arbitrarily small lapse of proper time.* In this paper it will be shown what physical obstacles stand against time travel of this kind.

Although the equations of the geodesics have already been completely integrated by Kundt and by Chandrasekhar and Wright, we start in Section I with a straightforward integration of the geodesics equations by means of a method which differs from those used by Chandrasekhar and Wright or by Lathrop and Teglas [10] and which is mainly based on Kundt's first integrals of the equations of motion. It will be shown that in appropriate coordinates the solutions of the equations of motions have a simple form and are very enlightning, in contradiction to some remarks made by R. Adler, M. Bazin, and M. Schiffer [28]. The review of the integration of the geodesics equation is necessary for two reasons: first there is still some confusion about the geometrical character of the geodesics in the literature: L. Sklar [12] describes their spatial projections as spirals, Kundt as circles; and secondly the evaluation of closed timelike world lines presented here will be primarily based on the behavior of the timelike geodesics and their spatial projections.

In Section II we construct particular timelike world lines which have the following properties:

(a) They travel "faster than the velocity of light" (in a sense which has to be explained), i.e., travel into the future of an observer comoving with the cosmic matter.

(b) They return to the space-time point of the departure (closed timelike world lines).

(c) They travel into the past of a geodesically comoving observer (backwinding timelike world lines).

In Section III we derive the accelerations which are necessary to maintain a travel into the past, and Section IV is concerned with conclusions drawn from our results.

Section I

Gödel's metric is

$$ds^2 = (dx^0 + e^{\alpha x^1} dx^2)^2 - (dx^1)^2 - \tfrac{1}{2} e^{2\alpha x^1} (dx^2)^2 - (dx^3)^2 \tag{1}$$

where $\alpha = (K\mu)^{1/2}$, $K = 8\pi G$, and G is the Newtonian gravitational constant, μ the density of the matter. (The velocity of light is $c = 1$.) The metric (1) satisfies Einstein's field equations

$$R_{\mu\nu} + (\Lambda - \tfrac{1}{2}R) g_{\mu\nu} = - KT_{\mu\nu} \tag{2}$$

with

$$T^{\mu\nu} = \mu u^\mu u^\nu = \mu \delta_0^\mu \delta_0^\nu$$

The coordinate transformation

$$\alpha x^1 =: -\ln(\alpha y), \qquad x^2 =: 2^{1/2} x, \qquad x^3 =: z, \qquad x^0 = x^0$$

was first introduced by Kundt. It brings (1) into the more convenient form

$$ds^2 = [dx^0 + 2^{1/2}/(\alpha y)\, dx]^2 - [(\alpha y)^{-2}(dx^2 + dy^2) + dz^2] \tag{3}$$

The equations for the geodesics are derived by means of the variational problem

$$\delta \int ds = 0, \qquad ds^2 > 0 \text{ or } ds^2 < 0$$

for timelike or spacelike geodesics, respectively, and by means of

$$\delta \int d\lambda = 0, \qquad ds^2 = 0$$

for null geodesics, where λ is an arbitrary parameter.

The first integrals to the geodesic equation read, after some straightforward evaluation and reformulation,

$$
\begin{aligned}
\dot{x}^0 &= -2^{1/2}/C(y' - y) + 2^{-1/2}/Cy' = -2^{1/2}/C(y'/2 - y)\\
\dot{y} &= -(\alpha y)(x'/C - x/C) = (\alpha y)(x - x')/C\\
\dot{x} &= (\alpha y)/C(y' - y) = (\alpha y)(y' - y)/C\\
\dot{z} &= d/C = d/C
\end{aligned}
\tag{4}
$$

(the point denotes derivatives with respect to s) where C, y', x', and d are constants of integration. From the subsidiary condition $ds^2 = \epsilon$ with $\epsilon = 1, 0, -1$ for timelike, null, and spacelike geodesics, respectively, it follows

$$\epsilon = (2^{-1/2} y'/C)^2 - (\alpha y/C)^2/(\alpha y)^2 [(x - x')^2 + (y - y')^2] - d^2/C^2 \tag{5}$$

which can also be written as

$$(x - x')^2 + (y - y')^2 = \tfrac{1}{2} y'^2 - d^2 - \epsilon C^2 =: r^2 \tag{6}$$

r is assumed to be positive and obviously the relation $y'^2 > 2r^2$ always holds for timelike geodesics. These results have already been found by Kundt, and we have essentially followed Kundt's denotation here. From (6) it is seen that the projections of the geodesics into the (xy) coordinate plane are circles.

The metric (3) is the direct sum of the metric g_1 given by

$$ds_1{}^2 = [dx^0 + 2^{1/2}/(\alpha y)\, dx]^2 - (dx^2 + dy^2)/(\alpha y)^2$$

on the manifold $M_1 = R^3$, which is defined by the coordinates (x^0, x, y) and the metric $ds_2^2 = dz^2$ on the manifold $M_2 = R$ defined by the coordinate z alone. Therefore we can restrict ourselves to the considerations of motions in the (xy) plane and completely disregard the "flat-space contributions" due to the component of a motion in the z direction. Without further comment we restrict the following considerations to the metric ds_1^2 and set $z \equiv 0$.

The further integration of the set of equations (4) can easily be performed by means of a single substitution. The integral of the equations of motions for timelike geodesics is

$$y = (y' - r)(1 + \tan^2 \sigma)/(1 - \tan^2 \sigma) \tag{7a}$$

$$x = 2r\varkappa^{1/2} \tan \sigma/(1 + \varkappa \tan^2 \sigma) + x' \tag{7b}$$

$$x^0 = 2^{1/2}/\alpha\{2 \tan^{-1}(\varkappa^{1/2} \tan \sigma) - [(1 + \varkappa)/(2\varkappa^{1/2})]\,\sigma\} + \overset{0}{x}{}^0 \tag{7c}$$

where

$$\sigma := \alpha(y'^2 - r^2)^{1/2}(s - s_0)/2C, \qquad \varkappa := (y' - r)/(y' + r) \tag{8}$$

and s_0 and $\overset{0}{x}{}^0$ are further constants of the integration.

Since $y'^2/2r^2 > 1$, i.e., $y'/r > 2^{1/2}$, $\varkappa$ is restricted to the values

$$0.17157 \cdots = (2^{1/2} - 1)^2 < \varkappa \leqslant 1 \tag{9}$$

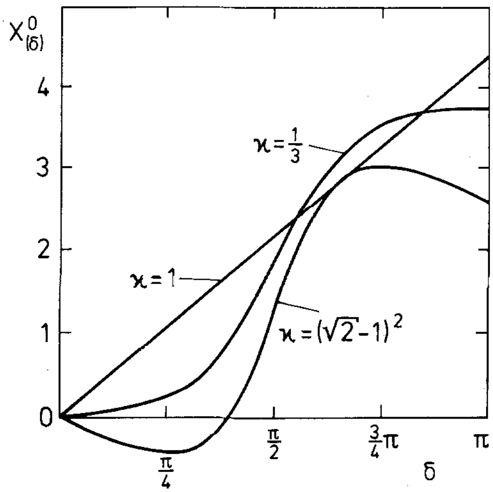

Fig. 1. $x^0(\sigma)$ for different values of $\varkappa$.

The behavior of the graphs of the functions (7) has—for one particular value of $\varkappa$—been discussed by Chandrasekhar and Wright. As is known from (6), the equations (7a) and (7b) represent a somewhat unfamiliar parameter representation of circles and are therefore not as interesting as the interpretation of the equation (7c) for the time coordinate. For some particular values of $\varkappa$ the graphs of $x^0(\sigma)$ are shown in Figure 1. For $\varkappa < 1/3$ the time coordinate starts running backwards; however, this running backwards of x^0 for increasing s has nothing to do with a possible going backward in time or time travel. This effect is a mere consequence of the special choice of the coordinate system. The behavior of the local light cones at the point $(0, 1/\alpha)$ is shown in Figure 2; this figure illustrates that the

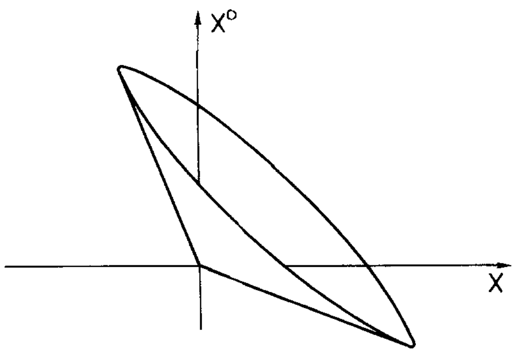

Fig. 2. The local future light cone at the space point $(0, 1/\alpha)$.

light cones intersect the (xy) coordinate plane, which means that the coordinate system violates the condition usually imposed upon gauge transformations, namely, that the coordinate clocks do not run backwards (cf. Moeller [29], p. 352).

The Cartesian coordinate system was very convenient as far as the integration of the geodesics was concerned, and we will use it in the following chapters to investigate various properties of the Gödel metric. However, for particular cases the so-called "standard Gödel coordinate system" is more convenient. One gets the metric in the form

$$ds^2 = (4/\alpha^2)[dt^2 - dr^2 + (\sinh^4 R - \sinh^2 R)\, d\varphi^2 + 2 \cdot 2^{1/2} \sinh^2 R \, d\varphi \, dt] \quad (10)$$

by means of the coordinate transformation

$$\begin{aligned} \alpha y &= (\cosh 2R + \sinh 2R \cos\varphi)^{-1}, \qquad x = (\alpha y) \sinh 2R \sin\varphi \\ \varphi/2 &+ (x^0 - 2t)/2^{3/2} = \tan^{-1}(e^{-2R} \tan \tfrac{1}{2}\varphi) \end{aligned} \quad (11)$$

For the spatial coordinates x and y, we obtain, using the half-angle formulas for the trigonometric functions,

$$\begin{aligned} y &= \frac{e^{-2R}}{(\cos^2(\frac{1}{2}\varphi) + e^{-4R} \sinh^2(\frac{1}{2}\varphi)} \\ x &= \frac{2 \sinh 2R \tan(\frac{1}{2}\varphi)}{e^{2R}[1 + e^{-4R} \tan^2(\frac{1}{2}\varphi)]} \end{aligned} \quad (12)$$

and eliminating φ from (12) we get the simple relation

$$(y - \cosh 2R)^2 + x^2 = \sinh^2 2R \quad (13)$$

If R is a constant with respect to s, then as has been shown by Chandrasekhar and Wright, $\varkappa = e^{-4R}$ and $\varphi = 2\sigma$ and it has to be

$$\coth 2R > 2^{1/2} \quad \text{or equivalently} \quad R < \tfrac{1}{2} \ln(2^{1/2} + 1) \quad (14)$$

for timelike geodesics.

For the time coordinate follows

$$t = 2^{1/2}(1 - \tfrac{1}{2} \cosh 2R)\sigma \quad (15)$$

which is a linear function of σ. Figure 3 shows the different behavior of the time coordinates for null geodesics. In addition, the qualitative behavior of the time coordinate in standard Gödel coordinates is shown if a light ray starts at $R = 0$ and after one orbit returns to this point rather than having R = constant during the whole orbit.

As has already been pointed out by Kundt, each observer comoving with the matter has an optical horizon, i.e., a maximum spatial distance from beyond which he cannot receive light signals. It is the same distance which can be reached

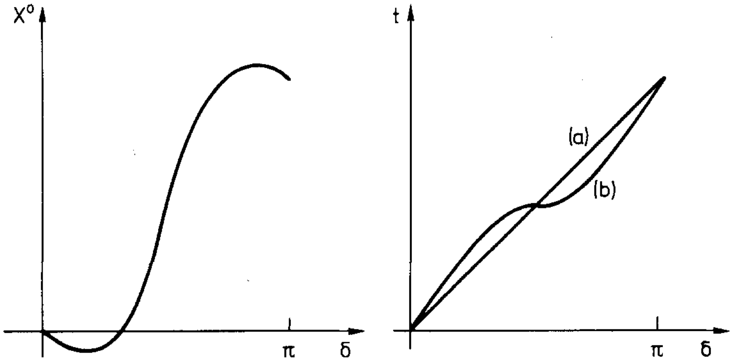

Fig. 3. $x^0(\sigma)$ and $t(\sigma)$, (a) for constant $R(\sigma)$, (b) for variable R.

by geodesically traveling and originating from a comoving piece of matter. This distance (with respect to the spatial metric) is (cf. again Kundt)

$$\rho_H = (2/\alpha)\ln(2^{1/2} + 1)$$

Within the region with a spatial distance $\rho < (2/\alpha)\ln(2^{1/2} + 1)$ from every comoving observer no acausal or anormal behavior of *any world line* is possible with respect to the origin. All the local forward light cones within this region point forward with respect to the local forward light cone at the origin. And since any timelike world line is bound to stay within these light cones, time travel is not possible within the region around the origin characterized by the spatial distance $\rho = \rho_H$ (see Fig. 4).

Section II

A spatial distance $\rho_{\text{light}} = (2/\alpha)\ln(2^{1/2} + 1)$ from a space point P cannot be reached by means of geodesic motion. Such distances, however, can be reached with nongeodesical timelike world lines. We will restrict ourselves to a class of timelike world lines whose projections into the (xy) plane of the quasi-Cartesian coordinate system are circles again and thus do not deviate too much from the structure of the timelike geodesics. In particular we choose the equations (4b) and (4c)

$$\begin{aligned} \alpha^{-1} C\dot{y} &= y(y' - y) \\ \alpha^{-1} C\dot{x} &= y(x - x') \end{aligned} \tag{16}$$

as well as the condition

$$(x - x')^2 + (y - y')^2 = r^2$$

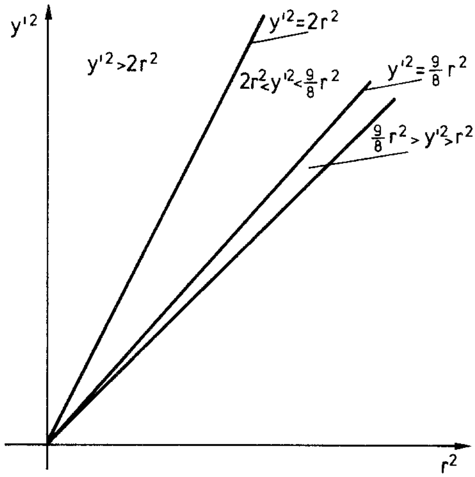

Fig. 4. The classification of timelike world lines with respect to their time behavior.

where x', y', C, and r are now some constants. They correspond to the constants of integration in (5) and (6), but they are in general not identical with them.

The differential equation for x^0 will be obtained from the metric condition for timelike world lines

$$[dx^0 + 2^{1/2}/(\alpha y)\,dx]^2 - (dx^2 + dy^2)/(\alpha y)^2 = ds^2 > 0 \qquad (17)$$

or

$$[\dot{x}^0 + 2^{1/2}/(\alpha y)\dot{x}]^2 = 1 + r^2/C^2$$

which by means of (16) leads to

$$C\dot{x}^0 = 2^{1/2}(y - y') + (C^2 + r^2)^{1/2} \qquad (18)$$

and replaces the formula (4a). Simple integration gives for x^0

$$x^0 = 2 \cdot 2^{1/2}/\alpha\{\tan^{-1}(\varkappa^{1/2} \tan\sigma) - [y' - 2^{-1/2}(C^2 + r^2)^{1/2}]/(y'^2 - r^2)^{1/2}\sigma\} + \overset{0}{x}{}^0 \qquad (19)$$

For the particular choice $y'/2^{1/2} = (C^2 + r^2)^{1/2}$ we get again the well-known x^0 for a timelike geodesic. By definition of the coordinate system, y and y' always have to be positive; from this and the circle condition it follows that $y'^2 > r^2$ for all possible timelike world lines whose projections into the xy plane are circles.

Because of the particular behavior of the $\tan^{-1}(\varkappa^{1/2} \tan\sigma)$ function the time behavior of x^0 is determined by the coefficient of σ in (19). We get $y'^2 > 2r^2$ for timelike geodesics, for light in particular $y'^2 = 2r^2$, for $y'^2 > (9/8)r^2$ we get closed timelike world lines, and for $y'^2 = (9/8)r^2$ closed null curves. These re-

TABLE I

Timelike geodesics	$y'^2 > 2r^2$	$R < \frac{1}{2}\ln(2^{1/2}+1)$
Null geodesics	$y'^2 = 2r^2$	$R = \frac{1}{2}\ln(2^{1/2}+1)$
Closed timelike world lines	$y'^2 < (9/8)r^2$	$R > \ln(2^{1/2}+1)$
Closed null curves	$y'^2 = (9/8)r^2$	$R = \ln(2^{1/2}+1)$

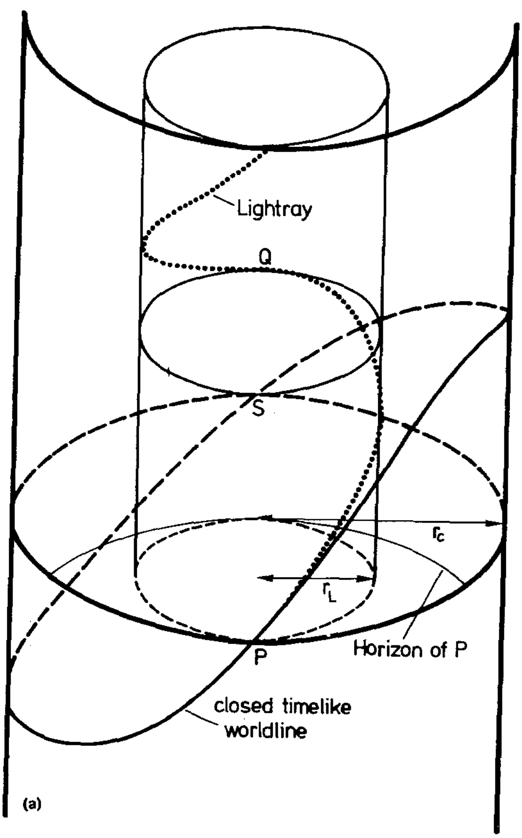

Fig. 5. (a) Example for a closed timelike world line with high velocity parameter $\beta \approx 1$. In addition, the trajectory of a light ray (dotted line) is given for comparison. (b) Projections of the world lines in Figure 5a into the $(R - \varphi)$ plane; (a) refers to the geodesic of a light ray, (b) to a closed null curve, (c) to the closed timelike world line with $\beta \approx 1$.

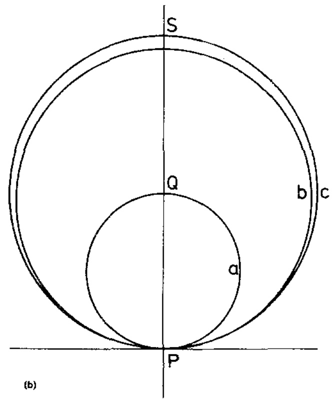

Fig. 5. Continued.

sults are summarized in Table I, where in addition the corresponding value for R in standard Gödel coordinates is given (R constant).

Figure 4 illustrates this table. World lines which are characterized by the relation $2r^2 > y'^2 > (9/8)r^2$ allow a maximum distance which exceeds that of a light ray; nevertheless a traveler going along this world line will arrive earlier than a light ray which has been emitted at the traveler's departure. The condition $y'^2 > (9/8)r^2$ is a necessary condition for "backward winding" timelike world lines, as well. An example for a closed timelike world line is shown in Figure 5.

The "faster-than-light" world lines have hitherto been disregarded in the literature. However, these world lines were meant by Gödel when he talked about "traveling into the future": Within finite proper time a traveler can reach a space-time point in the future of the world line of a comoving piece of matter which will be reached much later by a light ray which started at the same time from the same place.

It can be taken from Table I or Figure 5 that it is necessary to go at least the spatial distance $\rho_0 = 2\rho_{\text{light}}$ in order to get a closed timelike world line.

Denoting the velocity with respect to the comoving matter by β, we can write

$$C^2 = r^2/\beta^2(1 - \beta^2) \tag{20}$$

By means of this relation (19) can be written as

$$x^0 = 2 \cdot 2^{1/2}/\alpha\{\tan^{-1}(\varkappa^{1/2} \tan \sigma) - [y' - r/(\beta 2^{1/2})]/(y'^2 - r^2)^{1/2} \sigma\} + \overset{0}{x}{}^0 \tag{21}$$

We now compare the proper times of two observers: One is comoving with the matter, as the other one travels along a timelike world line characterized by y', r, and β (we omit the irrelevant constant x' here). As $\sigma \in [0, \pi]$ denotes one orbit, we require that the two observers first meet at a space point with $\sigma = 0$ and once again at the same space point with $\sigma = \pi$. This corresponds to a proper time of the comoving observer of

$$\Delta S_1 = x^0\big|_{\sigma=0}^{\sigma=\pi} = (2 \cdot 2^{1/2}/\alpha)\pi[1 - \{(y' - r)/(\beta 2^{1/2})\}/(y'^2 - r^2)^{1/2}] \quad (22)$$

For the traveling observer we get

$$\Delta S_2 = (2C/\alpha)(y'^2 - r^2)^{1/2} \int_0^\pi d\sigma = [2\pi r/(\alpha\beta)] \cdot (1 - \beta^2)^{1/2}/(y'^2 - r^2)^{1/2} \quad (23)$$

As long as we do not intend to meet particular spatial points we can restrict ourselves to such world lines for which $y'^2 - r^2 = 1$. As we know from (13) these world lines are described by constant R in standard Gödel coordinates, and the two proper times read

$$\begin{aligned} \Delta S_1 &= (2 \cdot 2^{1/2}/\alpha)\pi[1 - y' + r/(\beta 2^{1/2})] \\ \Delta S_2 &= [2r/(\alpha\beta)]\pi(1 - \beta^2)^{1/2} \end{aligned} \quad (24)$$

For the particular case of geodesical motion we get, using $y'/2^{1/2} = r/\beta$

$$\begin{aligned} \Delta S_1 &= (2 \cdot 2^{1/2}/\alpha)\pi(1 - y'/2) \\ \Delta S_2 &= (2 \cdot 2^{1/2}/\alpha)\pi(1 - \beta^2) \end{aligned} \quad (25)$$

In particular we get for a light ray ($\Delta S_2 = 0$)

$$\Delta S_1^{\text{light}} = (2\pi/\alpha)(2^{1/2} - 1) \quad (26)$$

and for the limit of slow motion ($\beta \ll 1, y' \approx 1$)

$$\Delta S_1^{(0)} = 2^{1/2}\pi/\alpha = \Delta S_2 \quad (27)$$

obviously the minimum round-trip time for timelike geodesics—as measured by an observer who is comoving with the cosmical matter—is that of a light ray, and from (25) we derive the relation

$$\Delta S_1 \geqslant \Delta S_2 \quad (28)$$

showing that the proper time of the comoving observer always exceeds that of an observer performing a geodesical round trip. However, there is no contradiction or clock paradox: Whereas the comoving observer has constant distances with respect to the comoving matter, the traveler measures distances to the cosmic masses that vary with time. Therefore, it can be determined uniquely which one of the two observers is moving with respect to the cosmical background matter.

From (23) and (24) it can be seen that time travel can indeed be done in principle within an arbitrarily small lapse of proper time, only the velocity has to be close enough to the velocity of light.

Whereas a timelike geodesic was uniquely defined by the velocity parameter β, once the constants y' and r have been chosen, both β and y' are independent parameters if we proceed to nongeodesic motions. Using (21) we can make the following distinctions:

If y' and β are chosen such that

$$y' = [2^{1/2}\beta^2 + (1 - \beta^2)]/(2\beta^2 - 1) \tag{29}$$

then a traveler arrives simultaneously with a light ray which has been emitted at the place and time of the traveler's departure. ($\beta^2 > 1/2$ because of $y' > 0$.)

For the choice

$$1 > y' - r/(2^{1/2}\beta) > 2^{1/2}/2 \tag{30}$$

traveling "faster than the velocity of light" can be done. In addition the proper times of the comoving and the nongeodesically moving observer can be made equal.

If in particular

$$y' - r/(2^{1/2}\beta) = 1$$

or equivalently

$$y' = (2\beta^2 + 1)/(2\beta^2 - 1) \tag{31}$$

a traveler returns to the starting event in space-time (closed timelike world line). In the limiting case $\beta \to 1$ we get the values $y' = 3$ and $r = 2 \cdot 2^{1/2}$ and thus are led to the closed null curve already discussed in Table I. For any $\beta > 2^{-1/2}$ there exists a $y' > 3$ which satisfies the condition (31).

If furthermore

$$y' - r/(2^{1/2}\beta) > 1$$

or

$$y' > (2\beta^2 + 1)/(2\beta^2 - 1)$$

past travel is possible.

All these considerations confirm the comments on Gödel's results already given by himself. However, we should keep in mind that we are dealing with a cosmological solution whose cosmological character is apparent in the factor α^{-1} with the proper times and with the distances. Assuming a cosmic matter density of

$$\mu = k^{-1} \times 10^{-30} \text{ g/cm}^3 \tag{32}$$

(k is a correcting factor), we get for α^{-1}

$$\alpha^{-1} = k^{1/2} \times (2.27) \times 10^{10} \text{ light years} \tag{33}$$

For instance, this means that a light ray returns to its point of emission after

$$k^{1/2} \times (2.27) \times 2\pi(2^{1/2} - 1) \text{ yr} \approx 10^{11} \text{ yr}$$

The slowest geodesical motion lasts about 1.7 times longer.

Because of the high value of α^{-1} very high velocities are necessary in order to get reasonable proper times for the traveler. Besides the problem of reaching these velocities as initial conditions one has to take into account that the motion throughout the whole world line is of nongeodesic character, i.e., a permanent acceleration has to be imposed. Thus, besides the topological and kinematical possibilities for time travel being warrented, the dynamical problem still has to be solved.

Section III

In the last section two proper times which are relevant for time travel have been discussed: the proper time of the traveling observer and the proper time of the observer who is comoving with the matter or, in the case of past travel, the amount of time on the world line of the comoving matter which the traveling observer claims to go backwards. We now discuss the dynamical quantities for these world lines.

If u^μ denotes the 4-velocity of the traveling observer $dx^\mu/d\tau$, then the 4-acceleration A^μ is defined by

$$A^\mu = Du^\mu/d\tau \tag{34}$$

where $Du^\mu/d\tau$ is the absolute derivative

$$Du^\mu/d\tau = du^\mu/d\tau + \Gamma^\mu_{\alpha\beta}u^\alpha u^\beta$$

In the standard Gödel coordinate system the covariant components of this 4-acceleration are

$$A^\mu = [2^{1/2}\beta/(1 - \beta^2)]\,[(y'/2^{1/2}r) - 1]\,\delta^R_\mu \tag{35}$$

However, rather than the components of the 4-acceleration, we need for a dynamical analysis the acceleration with respect to the locally comoving Lorentz frame whose time axis coincides with u^μ. Up to a negligible effect due to the rotation of the spatial axes this acceleration is (see Moeller [29], p. 327)

$$g^L_{\hat{R}} = [2^{1/2}\beta/(1 - \beta^2)]\,[y'\beta/(2^{1/2}r) - 1]\,\alpha \tag{36}$$

In order to illustrate the dependence of $g^L_{\hat{R}}$ on the two relevant proper times mentioned above we introduce the following dimensionless parameters:

$$\tau := \Delta S_2 \alpha/(2\pi \cdot 2^{1/2}) = r(1-\beta^2)^{1/2}/(2^{1/2}\beta) \tag{37a}$$

$$T := \Delta S_1 \alpha/(2\pi 2^{1/2}) = 1 - y' + r/(2^{1/2}\beta) \tag{37b}$$

and insert them into (36):

$$g_{\check{R}}^{L} = (2^{1/2}/4)(r^2 + 2\tau^2)[(1-T)2^{1/2}(r^2+2\tau^2)^{1/2} - 1]\alpha/\tau^2 \tag{38}$$

Remembering that r^2 has to be larger than 8 for time travel and keeping in mind that for $\Delta S_1 \cong 100$ years we get $T \cong 10^{-9}$ as well as for $\Delta S_2 \cong 100$ years τ gives $\tau \cong 10^{-10}$, we recognize that for sufficiently small T the contributions due to the additional T component in (38) can be neglected as long as $T \ll 1$. In particular this means that from a dynamical point of view it makes no difference in this case whether a closed timelike world line is required or past travel is wanted: We can neglect T and very easily express $g_{\check{R}}^{L}$ depending on, for instance β alone,

$$g_{\check{R}}^{L} = [2^{1/2}\beta/(1-\beta^2)](2\beta^2 - 3)\alpha/4 \tag{39}$$

for ΔS_2, depending on β, we get

$$\Delta S_2 = 4\pi 2^{1/2}(1-\beta^2)^{1/2}/[\alpha(2\beta^2 - 1)] \tag{40}$$

For the energy necessary to maintain such a travel the product of ΔS_2 with $g_{\check{R}}^{L}$ is important:

$$\eta := \Delta S_2 \cdot g_{\check{R}}^{L} = 2\pi\beta(2\beta^2 - 3)(2\beta^2 - 1)^{-1}(1-\beta^2)^{-1/2} \tag{41}$$

The graph of this mapping for the interesting values of β is given in Figure 6.

$g_{\check{R}}^{L}$ is the acceleration which has to be imposed upon a test particle of constant rest mass m_0 which is guided along a closed or backward winding timelike world line characterized by the particular value of β.

Gödel [6] himself gave a rough estimate of how much "fuel" would be needed by a rocketship performing time travel. He claims that the amount of fuel would be of the order of $10^{22}/t^2$ times the mass of the ship which returns after the trip, provided the fuel is expelled with the velocity of light. t here denotes the proper time measured in the rocket ship. Obviously this ratio corresponds to our $(\alpha \cdot \Delta S_2)^{-2}$ above. However, this ratio only denotes the amount of fuel necessary to accelerate the rocketship to the desired initial velocity and to decelerate it after the trip. Illustrating this we take the mass-ratio formulas from special relativity (see, e.g., Arzeliès [30], p. 282)

$$\gamma_{(a)} = [(1 + v/c)/(1 - v/c)]^{1/2}$$

or (42)

$$\gamma_{(a)} = (1 + v/c)/(1 - v^2/c^2)^{1/2}$$

$\gamma_{(a)}$ here denotes the ratio of the initial and final values of the masses of a rocketship whose final velocity is v and whose fuel is ejected with the velocity of light.

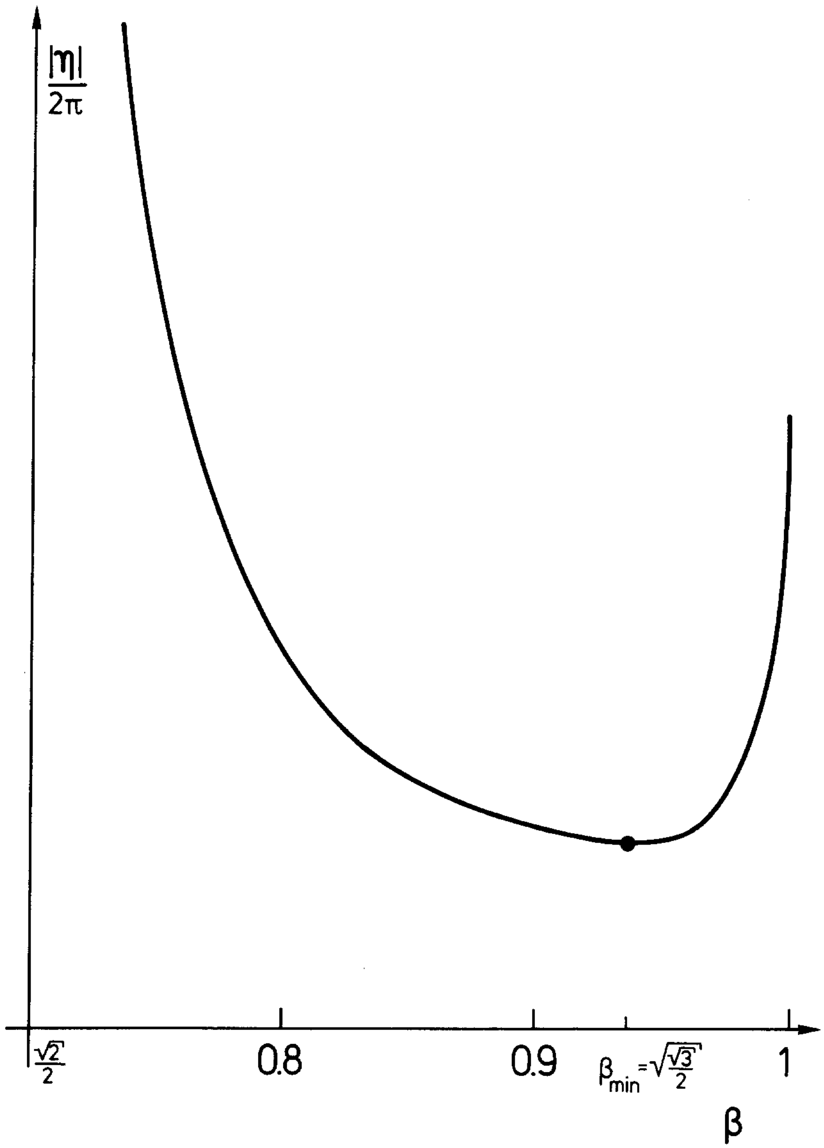

Fig. 6. $|\eta|$ as a function of β.

By means of (37a), (40) we get

$$\gamma_{(a)} \approx (1+\beta)/(1/2\tau) \approx 10^{11}\ \mathrm{yr}/\Delta S_2$$

and since the process of slowing down is ruled by the same law the complete mass ratio is given by

$$\gamma_{(a+d)} = (M_0^{\text{initial}}/M_0^{\text{final}}) \approx 10^{22} \text{ yr}^2/(\Delta S_2)^2 \tag{43}$$

In Gödel's footnote 11 in [6] indeed the impression arises that it would be sufficient for a time travel into the past to accelerate a rocketship to the proper velocity, let it make a *geodesical* round trip, and decelerate it again. A suspicion of this kind was formulated by Stein [13, p. 593] when he said "Rockets might be required only to initiate and terminate a ballistic trajectory." It is clear from Gödel's remarks that he already used the particular properties of spatially closed timelike world lines in order to get an estimation of the amount of fuel. Without these properties a vessel would have to be accelerated to the velocity v_0, then decelerated to $v = 0$, accelerated to $-v_0$, and finally decelerated to $v = 0$. This would lead to a final mass ratio of $\gamma_f = \gamma_{(a+d)}^2 = (10^{22}/(\Delta S_2)^2)^2$.

Assume a rocketship ejects fuel with the velocity w with respect to the spaceship and thus obtains an acceleration which is necessary to perform time travel. The ratio of the initial and final rest masses of the rocketship then gives

$$\gamma = M_0^{\text{initial}}/M_0^{\text{final}} = \exp(|g_{\check{R}}^{L} \cdot \Delta S_2/w|) \tag{44}$$

For $w = 1$, $|\eta|$ of (41) has a minimum at

$$\beta_{\min} = (3^{1/2}/2)^{1/2} = 0.93 \ldots .$$

where $|\eta|$ has the value $|\eta| = 27.67$. Whence follows

$$\gamma_{\min} \approx 10^{12} \tag{45}$$

and the corresponding value for ΔS_2 reads $\Delta S_2 \approx 35\alpha^{-1} \approx 8 \times 10^{12}$ yr for the density mentioned above. $\gamma_{\min}$ denotes the mass ratio for the most "inexpensive" way of time travel.[2] In order to get sufficiently small travel times one has to use extremely high accelerations, which, according to (39), lead to huge values for the mass ratio.[3]

Section IV

The above considerations could not generally rule out the possibility of time travel in Gödel's universe by means of dynamical arguments. They could exclude the possibility of past travel "within an arbitrarily small lapse of proper time" when this proper time is supposed to be of the order of the lifetime of human beings. Thus any anthropomorphic reference with respect to the famous two

[2] To illustrate this result let us assume the earth as a whole to be the rocketship; after the time travel we would end up with a sphere of radius $r_E \approx 630$ m. Acceleration and deceleration to and from $\beta_{\min}$ gives an additional $\gamma_{(a+d)} \approx 28$. It would reduce the radius of the returning sphere to 210 m.

[3] For small ΔS_2 we get $|\eta| \sim (\Delta S_2\alpha)^{-4}$ and $\gamma \sim \exp(\Delta S_2\alpha)^{-4}$.

cosmical twins can only be regarded as an *illustration* for the *topological* properties of this space-time.

In Gödel's space-time the *local temporal and causal order* will not differ from the one we are familiar with in our world; deviations can only occur for global distances. Given two events P and Q on a geodesic comoving with the cosmic matter, and supposing the arrow of time points from Q to P, then the verification of the second possible temporal connection between P and Q with the respective direction of the arrow of time succumbs to severe dynamical restrictions originating from the global properties of the space-time.

Owing to the physical theory upon which cosmology is based, the existence of closed timelike world lines only describes the topological features of the space-time, namely, that it is possible to transform one space-time point into itself along such a curve. This does not automatically imply the possibility of a transformation of a complete physical state into itself. Suppose instead of the point we take a box filled with a gas in nonequilibrium state and we guide this box along a closed timelike world line. At the end of the travel we will expect two copies of the same gas in different states with different values of entropy, and there is no need to cite self-identity problems or logical paradoxes.

The treatment turns out to be even more difficult if a quantum mechanical problem has to be solved. In order to be sure that a quantum mechanical state guided along such a world line is transformed into itself one has to prepare the initial states precisely and uniquely. Within the Gödel universe this is not possible since there are no *global spacelike hypersurfaces*. It is not known which kind of quantum mechanics is able to describe physical processes without reference to global Cauchy surfaces.

It is known from experience, of course, that our universe is expanding and the compass of inertia is not rotating with respect to the compass of matter. These are already sufficient reasons to exclude the Gödel solution as a model for our universe. What strikes us as rather precarious is the exclusion of Gödel's space-time—as desired by several authors—only because of the existence of closed timelike world lines. A gravitational theory based on the dynamics of incoherent matter is too incomplete a physical theory to describe the totality of physical processes. Rather than rule out such solutions by means of reference to logical paradoxes or philosophical self-identity problems it should be investigated whether physical reasons can be presented which require an exclusion.

During the last 30 years since Gödel's discovery his solution could not be excluded by cosmological arguments alone. It seems that the "physical grounds" for excluding this solution which Einstein mentioned in his reply to Gödel have to be looked for beyond the theory of gravity.

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