Problem. *A 1000 par value bond pays annual coupons of 80. The bond is redeemable at par in 30 years, but is callable any time from the end of the 10th year at 1050. Based on her desired yield rate, an investor calculates the following potential purchase prices, P :*

- *Assuming the bond is called at the end of the 10th year, P = 957*
- *Assuming the bond is held until maturity, P = 897*

The investor buys the bond at the highest price that guarantees she will receive at least her desired yield rate regardless of when the bond is called. The investor holds the bond for 20 years, after which time the bond is called. Calculate the annual yield rate the investor earns.

Solution. *Diketahui:*

- *F = 1,000 (par value)*
- *Kupon Annual (tahunan) sebesar 80* $\implies F \times r = 80$
- *30 tahun dengan kupon tahunan* $\implies n = 30$
- *Callable bond sejak tahun ke-10, Jika di call* $\implies C = 1050$
- *Jika di hold sampe mature* $\implies C = 1000$
- *Bond di hold selama 20 tahun, lalu di call*

Ditanya: Annual yield rate investor
Jawab:

Akan dicari tingkat bunga kupon (r):

$$\begin{aligned} F \times r = 80 &\iff 1000 \times r = 80 \\ &\iff r = \frac{80}{1000} \\ &\iff r = 0.08 = 8\% \end{aligned}$$

Misal diasumsikan bond di call pada tahun ke-10, maka $P = 957$ *Dengan metode Bond Salesman:*

$$
\begin{aligned}
i &\approx \frac{g - \frac{m}{n}}{1 + \frac{n+1}{2n}m} \quad , g = \frac{Fr}{C} = \frac{80}{1050} \quad , m = \frac{P-C}{C} = \frac{957-1050}{1050} \\
&= \frac{\frac{80}{1050} - \frac{\frac{957-1050}{1050}}{n}}{1 + \frac{n+1}{2n} \times \frac{957-1050}{1050}} \\
&= \frac{\frac{80}{1050} - \frac{-93}{1050\times 20}}{1 + \frac{21}{40} \times \frac{-93}{1050}} \\
&= \frac{\frac{80}{1050} - \frac{-93}{1050\times 20}}{1 + \frac{21}{40} \times \frac{-93}{1050}} \\
&= \frac{3386}{40,047} \approx 0.08455065298 = 8.455065298\%
\end{aligned}
$$

Misal diasumsikan bond di call pada tahun ke-30, maka P = 897

$$
\begin{aligned}
i &\approx \frac{g - \frac{m}{n}}{1 + \frac{n+1}{2n}m} \quad , g = \frac{Fr}{C} = \frac{80}{1050} \quad , m = \frac{P-C}{C} = \frac{897-1050}{1050} \\
&= \frac{\frac{80}{1050} - \frac{\frac{897-1050}{1050}}{n}}{1 + \frac{n+1}{2n} \times \frac{897-1050}{1050}} \\
&= \frac{\frac{80}{1050} - \frac{-153}{1050\times 20}}{1 + \frac{21}{40} \times \frac{-153}{1050}} \\
&= \frac{\frac{80}{1050} - \frac{-153}{1050\times 20}}{1 + \frac{21}{40} \times \frac{-153}{1050}} \\
&= \frac{3506}{38,787} \approx 0.09039111 = 9.03911\%
\end{aligned}
$$

Jadi, annual yield rata yang akan diterima oleh investor adalah setidaknya $i \approx 8.455065298\%$ *jika ia membeli di harga 957 dan sebesar* $i \approx 9.03911\%$ *jika ia membeli dengan harga 897*

Answer Key: 9.24%

Will attempt another try, from another perspective

Misal diasumsikan bond di call di tahun ke-10, maka P = 957

Yield rate jika hal tersebut terjadi:

$$
\begin{aligned}
P &= (Fr)a_{\overline{n}|i} + Cv_i^n \\
&\iff 957 = 80 \times \frac{1-(1+i)^{-10}}{i} + 1050 \times (1+i)^{-10} \\
&\iff i \approx 0.0899911 \quad (courtesy\ of\ wolfram\ my\ sunshine)
\end{aligned}
$$

Misal diasumsikan bond di call di tahun ke-30, maka P = 897

Yield rate jika hal tersebut terjadi:

$$\begin{aligned} P &= (Fr)a_{\overline{n}|i} + Cv_i^n \\ \iff 897 &= 80 \times \frac{1-(1+i)^{-30}}{i} + 1000 \times (1+i)^{-30} \\ \iff i &\approx 0.0900281 \quad (courtesy\ of\ wolfram\ my\ sunshine) \end{aligned}$$

Back to jawaban:
$P = 957$

$$\begin{aligned} P &= (Fr)a_{\overline{n}|i} + Cv_i^n \\ \iff 957 &= 80 \times \frac{1-(1+i)^{-20}}{i} + 1050 \times (1+i)^{-20} \\ \iff i &\approx 0.0855902 \quad (courtesy\ of\ wolfram\ my\ sunshine) \end{aligned}$$

$P = 897:$
Attempt to solve analyticallt with wolfram:

$$\begin{aligned} P &= (Fr)a_{\overline{n}|i} + Cv_i^n \\ \iff 897 &= 80 \times \frac{1-(1+i)^{-20}}{i} + 1050 \times (1+i)^{-20} \\ \iff i &\approx 0.0924304 \quad (courtesy\ of\ wolfram\ my\ sunshine) \end{aligned}$$

... (Finan, why?)