Pemodelan Resiko 2 | Bab 10 No 10.25

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Problem. *Let $x_1, x_2, ..., x_n$ be a random sample from a population with cdf:*

$$F(x) = x^p \quad ,0 < x < 1$$

Determine the method of moments estimate of p.
(Klugman 5th ed, 10.25)

Solution.

Misalkan X adalah distribusi asal dari sampel acak, akan dicari pdf dari X untuk menggunakan metode momen.

$$\begin{aligned} f(x) &= \frac{d}{dx} F(x) \\ &= \frac{d}{dx}\, x^p \\ &= p x^{p-1} \\ \therefore f(x) &= p x^{p-1} \quad ,0 < x < 1 \end{aligned}$$

Akan dicari momen pertama dari X:

$$\begin{aligned} E[X] &= \int_{-\infty}^{\infty} x\, f(x)\, dx \\ &= \int_0^1 x\, p\, x^{p-1}\, dx \\ &= \int_0^1 p\, x^p\, dx \\ &= \left. \frac{p\, x^{p+1}}{p+1} \right|_{x=0} 0^{x=1} \\ &= \left[\frac{p \times 1^{p+1}}{p+1} \right] - \left[\frac{p \times 0^{p+1}}{p+1} \right] \\ &= \left[\frac{p}{p+1} \right] - \left[\frac{0}{p+1} \right] \\ \therefore E[X] &= \frac{p}{p+1} \end{aligned}$$

Diketahui, momen pertama dari sampel adalah:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \mu_1'$$

Dengan menggunakan metode momen, didapat:

$$
\begin{aligned}
&E[X] = \mu_1' \\
\iff &\frac{p}{p+1} = \frac{\sum_{i=1}^n x_i}{n} \\
\iff &pn = (p+1)\sum_{i=1}^n x_i \\
\iff &pn = p\sum_{i=1}^n x_i + \sum_{i=1}^n x_i \\
\iff &pn - p\sum_{i=1}^n x_i = \sum_{i=1}^n x_i \\
\iff &p(n - \sum_{i=1}^n x_i) = \sum_{i=1}^n x_i \\
\iff &p = \frac{\sum_{i=1}^n x_i}{n - \sum_{i=1}^n x_i} \\
\iff &p \approx \hat{p} = \frac{\sum_{i=1}^n x_i}{n - \sum_{i=1}^n x_i} \times \frac{\frac{1}{n}}{\frac{1}{n}} \quad \text{(karena aproksimasi)} \\
\iff &\hat{p} = \frac{\bar{x}}{1-\bar{x}} \therefore \hat{p} = \frac{\bar{x}}{1 - barx}
\end{aligned}
$$

Jadi, dengan metode momen, didapat estimasi dari p adalah $p \approx \hat{p} = \frac{\sum_{i=1}^n x_i}{n - \sum_{i=1}^n x_i} = \frac{\bar{x}}{1-\bar{x}}$