Pemodelan Resiko 2 Bab 11 No 11.2
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Problem. *You are given the five observations 521, 658, 702, 819, and 1,217. Your model is the single-parameter Pareto distribution with distribution function:*

$$F(x) = 1 - \left(\frac{500}{x}\right)^{\alpha} \quad , x > 500\,, \alpha > 0$$

Determine the maximum likelihood of α

(Klugman 5th ed, 11.2)

Solution. *Akan dicari* $f(x|\alpha)$

$$\begin{aligned}
f(x|\alpha) &= \frac{d}{dx}F(x) \quad \textit{secara implisit, } F(x) \textit{ bergantung } \alpha \\
&= \frac{d}{dx}\left(1 - \left(\frac{500}{x}\right)^{\alpha}\right) \\
&= 0 - \frac{d}{dx}\left(\frac{500}{x}\right)^{\alpha} \\
&\textit{misalkan } u = \frac{500}{x} \implies \frac{du}{dx} = -500x^{-2}\textit{, dengan aturan rantai:} \\
&= -\frac{d}{du} \times (u)^{\alpha} \times \frac{du}{dx} \\
&= -\alpha \times u^{\alpha-1} \times \left(-500x^{-2}\right) \\
&= \alpha \times \left(\frac{500}{x}\right)^{\alpha-1} \times \frac{500}{x^2} \\
&= \frac{500\alpha}{x^2} \times \frac{500^{\alpha-1}}{x^{\alpha-1}} \\
&= \frac{\alpha \times 500^{\alpha}}{x^{\alpha+1}}
\end{aligned}$$

AKan dicari fungsi likelihoodnya:

$$\begin{aligned}
L(\alpha) &= f(521|\alpha) \times f(658|\alpha) \times f(702|\alpha) \times f(819|\alpha) \times f(1217|\alpha) \\
&= \frac{\alpha \times 500^{\alpha}}{(521)^{\alpha+1}} \times \frac{\alpha \times 500^{\alpha}}{(658)^{\alpha+1}} \times \frac{\alpha \times 500^{\alpha}}{(702)^{\alpha+1}} \times \frac{\alpha \times 500^{\alpha}}{(819)^{\alpha+1}} \times \frac{\alpha \times 500^{\alpha}}{(1217)^{\alpha+1}} \\
&= \frac{\alpha^5 \times 500^{5\alpha}}{(521 \times 658 \times 702 \times 819 \times 1217)^{\alpha+1}} \quad \text{hasil perkalian akan dimisalkan sebagai } K \\
&= \alpha^5 \times 500^{5\alpha} \times K^{-(\alpha+1)}
\end{aligned}$$

Note: $(K = 2.398696e + 14)$

Akan di-ln-kan fungsi likelihoodnya agar lebih mudah mencari nilai maksimumnya:

Misal $l(\alpha) = ln(L(\alpha))$

$$\begin{aligned} l(\alpha) &= ln\left(\alpha^5 \times 500^{5\alpha} \times K^{-\alpha-1}\right) \\ &= ln(\alpha^5) + ln(500^{5\alpha}) + ln(K^{-\alpha-1}) \\ &= 5\,ln(\alpha) + 5\alpha\,ln(500) + (-\alpha - 1)\,ln(K) \end{aligned}$$

Akan dicari nilai maksimum dari

$$\begin{aligned} & \frac{d}{d\alpha}\,l(\alpha) = 0 \\ \iff & \frac{d}{d\alpha} 5\,ln(\alpha) + 5\alpha\,ln(500) + (-\alpha\,ln(K) - 1\,ln(K)) = 0 \\ \iff & \frac{5}{\alpha} + 5\,ln(500) - ln(K) = 0 \\ \iff & \frac{5}{\alpha} + ln(500^5) - ln(K) = 0 \\ \iff & \frac{5}{\alpha} + ln(\frac{500^5}{K}) = 0 \\ \iff & \frac{5}{\alpha} = -ln(\frac{3125 \times 10^{10}}{K}) \\ \iff & \alpha = \frac{5}{-ln(\frac{3125 \times 10^{10}}{K})} \\ \iff & \alpha = 2.453294 \\ & \therefore \hat{\alpha} = 2.453294 \end{aligned}$$

Maka, maximum likelihood dari α adalah $\hat{\alpha} = 2.453294$

Answer Key: 2.45