Pemodelan Resiko 2 Bab 10 No 10.28
Andhika N. (19 Feb 2025)

Problem. *The random variable X has pdf:*

$$f(x) = \beta^{-2}\, x\, e^{\frac{-0.5x^2}{\beta^2}} \quad ; x, \beta > 0$$

For this random variable, $E[X] = \frac{\beta}{2}\sqrt{2\pi}$ and $Var(X) = 2\beta^2 - \frac{\pi\beta^2}{2}$. You are given the following five observations: 4.9, 1.8, 3.4, 6.9, *and* 4.0

Determine the method of moments estimate of β.
(Klugman 5th ed, 10.28)

Solution.

Akan digunakan *raw moment* untuk estimasi, maka akan dicari terlebih dahulu nilai $E[X^2]$ dari variabel acak X (Tidak diperlukan karena hanya ada 1 variabel dalam ekspektasi)

$$\begin{aligned}
Var(X) &= E[X^2] - (\,E[X]\,)^2 \\
\iff E[X^2] &= Var(X) + (\,E[X]\,)^2 \\
&= \left(2\beta^2 - \frac{\pi\beta^2}{2}\right) + \left(\frac{\beta}{2}\sqrt{2\pi}\right)^2 \\
&= 2\beta^2 - \frac{\pi\beta^2}{2} + \frac{\beta^2}{4}2\pi \\
&= 2\beta^2 - \frac{\pi\beta^2}{2} + \frac{\pi\beta^2}{2} \\
\therefore E[X^2] &= 2\beta^2
\end{aligned}$$

Raw moment dari sampel adalah:
Momen pertama:

$$\begin{aligned}
\mu_1' &= \frac{\sum_{i=1}^{5} x_i}{5} \\
&= \frac{4.9 + 1.8 + 3.4 + 6.9 + 4.0}{5} \\
&= \frac{21}{5} \\
\therefore \mu_1' &= 4.2
\end{aligned}$$

Dengan metode momen:

$$\begin{aligned} & E[X] = \mu_1' \\ \iff & \frac{\beta}{2}\sqrt{2\pi} = 4.2 \\ \iff & \beta\sqrt{2\pi} = 8.4 \\ \iff & \beta = \frac{8.4}{\sqrt{2\pi}} \approx 3.35111515537 \end{aligned}$$

Jadi, dengan metode momen didapat $\beta \approx \hat{\beta} = \frac{8.4}{\sqrt{2\pi}} \approx 3.35111515537$

Answer Key: $\hat{\beta} = 3.35112$