Penurunan Persamaan Lagrange Double Pendulum

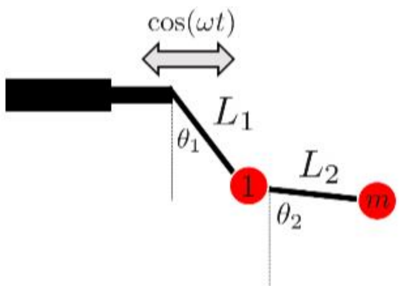
$$L = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2) - m_1 g y_1 - m_2 g y_2$$

$$\frac{L}{A^2 m_1} = \frac{1}{2}\left(\left(\frac{\dot{x}_1}{A}\right)^2 + \left(\frac{\dot{y}_1}{A}\right)^2\right) + \frac{1}{2}\frac{m_2}{m_1}\left(\left(\frac{\dot{x}_2}{A}\right)^2 + \left(\frac{\dot{y}_2}{A}\right)^2\right) - \frac{g}{A}\frac{y_1}{A} - \frac{m_2}{m_1}\frac{g}{A}\frac{y_2}{A}$$

$$\frac{L}{A^2 m_1} = \frac{1}{2}(\dot{x'}_1^2 + \dot{y'}_1^2) + \frac{1}{2}m(\dot{x'}_2^2 + \dot{y'}_2^2) - g'y'_1 - mg'y'_2$$

Perhatikan bahwa menghasilkan persamaan gerak yang sama dengan L (untuk sembarang C, CL menghasilkan persamaan gerak yang sama dengan. Dengan demikian kita dapat menyelesaikan masalah Lagrangian di atas, dan kemudian memasukkan nilai A dan saya yang kita inginkan setelahnya, dan menskalakan dengan tepat.

Soal Double Pendulum :



and thus

- $x_1 = \cos(\omega t) + L_1 \sin(\theta_1)$
- $x_2 = \cos(\omega t) + L_1 \sin(\theta_1) + L_2 \sin(\theta_2)$
- $y_1 = -L_1 \cos(\theta_1)$
- $y_2 = -L_1 \cos(\theta_1) - L_2 \cos(\theta_2)$

$$L_1 \cos(\theta_1(t)) \frac{d}{dt}\theta_1(t) - \omega \sin(\omega t) \qquad \frac{\partial L}{\partial \theta} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = 0$$

$$L_1 \left(-L_1 m \frac{d^2}{dt^2}\theta_1(t) - L_1 \frac{d^2}{dt^2}\theta_1(t) - L_2 m \sin(\theta_1(t) - \theta_2(t)) \left(\frac{d}{dt}\theta_2(t)\right)^2 - L_2 m \cos(\theta_1(t) - \theta_2(t)) \frac{d^2}{dt^2}\theta_2(t) \right.$$
$$\left. + \omega^2 m \cos(\omega t) \cos(\theta_1(t)) + \omega^2 \cos(\omega t) \cos(\theta_1(t)) - gm \sin(\theta_1(t)) - g \sin(\theta_1(t)) \right)$$

$$L_2 m \left(L_1 \sin\left(\theta_1(t) - \theta_2(t)\right) \left(\frac{d}{dt} \theta_1(t) \right)^2 - L_1 \cos\left(\theta_1(t) - \theta_2(t)\right) \frac{d^2}{dt^2} \theta_1(t) - L_2 \frac{d^2}{dt^2} \theta_2(t) \right.$$

$$\left. + \omega^2 \cos(\omega t) \cos\left(\theta_2(t)\right) - g \sin\left(\theta_2(t)\right) \right)$$

$$\frac{L_2 m \left(-L_1 \sin\left(\theta_1(t) - \theta_2(t)\right) \left(\frac{d}{dt} \theta_1(t) \right)^2 - \omega^2 \cos(\omega t) \cos\left(\theta_2(t)\right) + g \sin\left(\theta_2(t)\right) \right) \cos\left(\theta_1(t) - \theta_2(t)\right)}{-L_1 L_2 m \cos^2\left(\theta_1(t) - \theta_2(t)\right) - L_2 \left(-L_1 m - L_1\right)}$$

$$- \frac{L_2 \left(L_2 m \sin\left(\theta_1(t) - \theta_2(t)\right) \left(\frac{d}{dt} \theta_2(t) \right)^2 - \omega^2 m \cos(\omega t) \cos\left(\theta_1(t)\right) - \omega^2 \cos(\omega t) \cos\left(\theta_1(t)\right) + g m \sin\left(\theta_1(t)\right) + g \sin\left(\theta_1(t)\right) \right)}{-L_1 L_2 m \cos^2\left(\theta_1(t) - \theta_2(t)\right) - L_2 \left(-L_1 m - L_1\right)}$$

$$L_1 \left(-L_1 m \frac{d^2}{dt^2} \theta_1(t) - L_1 \frac{d^2}{dt^2} \theta_1(t) - L_2 m \sin\left(\theta_1(t) - \theta_2(t)\right) \left(\frac{d}{dt} \theta_2(t) \right)^2 - L_2 m \cos\left(\theta_1(t) - \theta_2(t)\right) \frac{d^2}{dt^2} \theta_2(t) \right.$$

$$\left. + \omega^2 m \cos(\omega t) \cos\left(\theta_1(t)\right) + \omega^2 \cos(\omega t) \cos\left(\theta_1(t)\right) - g m \sin\left(\theta_1(t)\right) - g \sin\left(\theta_1(t)\right) \right)$$