Review of Surplus Production Models Literature

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1 Haddon (2011)

In this section we summarize the main ideas and conclusions of Chapter 11 (Surplus production models) of Haddon (2011).

1.1 Introduction

Surplus production models are the simplest analytical method available which provides a full fish stock assessment. These models pool the overall effects of recruitment, growth, and mortality (all aspects of production) into a single production function, whereas age and size structure, along with sexual and other differences, are ignored.

The minimum data needed to estimate parameters for such models are time series of an index of relative abundance and of associated catch data. The index of stock abundance is most often catch-per-unit-effort, but could be some fishery-independent abundance index. It is important to mention that too homogeneous data can be uninformative about the dynamics of the populations, to wit, fishing catch and effort information available for a limited range of stock abundance levels can be uninformative in a surplus production model.

Age-stuctured models which are more complex and data demanding are an alternative to surplus production models when the required data is available. However, it is important to keep in mind that surplus production models may produce answers just as useful and sometimes better for management than those produced by age-structured models, at a fraction of the cost.

Surplus production models went out of fashion in the 1980s because in their development it was necessary to assume that the stocks being assessed were in equilibrium given often over optimistic conclusions. Then, posterior advances on surplus production models allowed to avoid the assumption of equilibrium.

1.2 Surplus production

Surplus production, as the name implies, relates to the production from a stock beyond that required to replace losses due to natural mortality. Surplus production, in this case, is the sum of new recruitment and the growth of individuals already in the population minus those dying naturally.

Schaefer (1954) applied the logistic curve as a description of the production, thus, given a known stock biomass, the total production could be predicted thus:

$$B_{t+1} = B_t + rB_t\left(1 - \frac{B_t}{K}\right) \tag{1}$$

where B_t is the stock biomass at time t, r is the population growth rate, and K is the maximum population size for growth to be positive. A property of equation (1) is that the maximum production occurs at $K/2$.

Irrespective of the stock size, it should be possible to take the excess production, above the equilibrium line of replacement ($B_t = B_{t+1}$), and leave the stock in the condition it was in before production and harvesting. An obvious management strategy deriving from this theory would be to bring the stock to a size that would maximize the surplus production and hence the potential yield. There are, of course, many problems with this simplistic view of fisheries production. It assumes that the population is in equilibrium with all of its inputs and outputs (a poor assumption).

Schaefer (1954) curve generates a symmetrical production curve, which can feel to be overly constraining. Pella and Tomlinson (1969) solved this by introducing an asymmetry parameter p, which modifies the logistic as follows.

$$B_{t+1} = B_t + \frac{r}{p} B_t \left(1 - \left(\frac{B_t}{K}\right)^p\right). \tag{2}$$

In Pella-Tomlinson model the stock size at which maximum production occurs is not necessarily at $K/2$ due to the potential asymmetry in the production curve.

1.3 Equilibrium Methods

Although surplus production models based on equilibrium condition are no longer recommended, a review of them can be interested to understand some ideas behind surplus production models intuitively and easily.

Equilibrium methods are best described using the Schaefer (1954) or Pella–Tomlinson (1969) model of stock dynamics minus any catch

$$B_{t+1} = B_t + \frac{r}{p} B_t \left(1 - \left(\frac{B_t}{K}\right)^p\right) - C_t$$

where C_t is the total catch in year t.

Equilibrium methods asssume that the yield taken is always surplus production from a population in equilibrium. More precisely, their assumptions are $B_{t+1} = B_t$=constant, and that the relationship between the index of abundance and stock biomass is exact, to wit, $I = C/E = qB$ where q is the catchability coefficient and E is the associated effort. From this assumptions it is possible to estimate MSY, E_{MSY}, and B_{MSY}, that is, maximum sustainable yield, and associated effort and biomass, respectively.

If we focus on the simplest case $p = 1$, the index $I = C/E$ can be written as $a - bE$ where a and b are new parameters defined from p, q, r, K parameters. Then, we have a simple regression model where the response variable is the index I and the covariable is the effort E. Hence, we can estimate a and b parameters through standard linear regression techniques (minimizing the sum of squares estimates). Then, we can use such values to estimate the reference points, MSY, E_{MSY}, and B_{MSY}, whose expression comes from the above equilibrium catch equation, $C = E\,(a - bE)$. Then the assumption of equilibrium can be overcome using alternative methods to fit the models. Before discussing fitting approaches we review the general Russell's model formulation.

1.4 Russell's model formulation

The general structure of a surplus production model in discrete form is

$$B_{t+1} = B_t + f(B_t) - C_t$$

with

$$\hat{I}_t = C_t/E_t = qB_t,$$

where B_{t+1} is the exploitable biomass at the end of year t or the beginning of year t+1, B_t is the exploitable biomass at the start of year t, C_t is the biomass caught during year t, I_t is the an index of relative abundance for year t, q is the catchability coefficient, $f(B_t)$ the production of biomass function, and the ˆ symbol above a parameter denotes a value estimated from the model. Two well-known possibilities for the production funcion $f(B_t)$ have been mentioned previously. Schaefer (1954),

$$f(B_t) = rB_t \left(1 - \frac{B_t}{K}\right),$$

and Pella and Tomlinson (1969),

$$f(B_t) = \frac{r}{p} B_t \left(1 - \left(\frac{B_t}{K}\right)^p\right).$$

Another popular option not mentioned yet is Fox (1970) which is

$$f(B_t) = Ln(K) r B_t \left(1 - \left(\frac{Ln(B_t)}{Ln(K)}\right)\right).$$

Fox (1970) becomes equal to Pella and Tomlinson (1969) when p tends to zero.

It is important to keep in mind that the addition of parameter p modifies the interpretation of the other parameters, that is, the parameter values of the three models can not be compared directly. Furthermore, it is also relevant to stand out that we have imposed a strong assumption above, the catch rates are assumed linearly related to stock biomass ($\hat{I}_t = C_t/E_t = qB_t$).

1.5 Fitting Methodology

In the development of fisheries methods a number of contrasting algorithms or strategies have been used for **fitting** stock production models to observed data on catches and catch rates. As we mentioned before, at first point these methods differ with regard to whether or not they assume the population to be in equilibrium with the exploitation rate. Furthermore, they can also differ with regard to where the residual errors are attributed between the model and the data.

As mentioned before equilibrium assumption is not realistic and then it must be avoided in the fitting process. Hence, now, the main difference between the methods regard the type of error assumed.

- **Process error estimators**. Assume that all observations were made without error and that all error was in the equation describing changes in population size ($B_{t+1} = B_t + f(B_t) - C_t$).
- **Observation error estimators**. Assume that all residual errors are in the catch rate or biomass observations and that the equation describing the time series of biomass values is deterministic and without error.

More recently, attempts have been made to create estimators that use both forms of error. But this remains generally intractable. One way in which both types of error could be modelled would arise if there were an estimate of the ratio of the respective variances of the two processes. In practice, it is extremely difficult to work with both forms of error in an estimation model.

In the case of observation error, it is assumed that $I_t = qB_te^{\epsilon}$ where $\epsilon = N(0, \sigma^2)$, indicates that the residual errors are assumed to be lognormally distributed. Then estimates of the model parameters (if $p = 1$; r, K, q and B_0 which is the initial starting biomass) can be obtained by maximizing the appropriate likelihood function.

There is more than one method available for determining B_0. For example, it can be set equal to K or B_0 can be estimated directly as a separate parameter. Some studies mentioned that even in situations where B_0/K was substantially different from unity, estimation performance was better when B_0/K was set at unity than when attempts were made to estimate B_0 separately.

1.6 Model Outputs

There are many **outputs** possible from most **fishery models**. The two classical performance measures which derive from surplus production modelling are the maximum sustainable yield (MSY) and the corresponding effort (E_{MSY}). In addition, we could consider outputs as the current estimated biomass, the ratio of the current biomass with K or B_0, and possibly include an estimate of fishing mortality rate, F. Note that an equilibrium is now assumed to be unlikely in a fished population, so the interpretation of MSY is more like an average, long-term expected potential yield if the stock is fished optimally.

1.7 Beyond Simple Models

Previous sections have been reviewed some of the fundamental ideas relating to the population dynamics. Now, in this section we focus on some specific problems and possible extensions of the simple models.

First of all, some considerations of parameter p are needed. When $p = 1$, Pella and Tomlinson (1969) model is algebraically identical to the Schaefer (1954) model. Although some studies suggest that the simple Schaefer (1954) model must not be used. The difficult to generate an acceptable estimate of the asymmetry parameter, p, implies that the well-known and simpler model has advantages. In practice, a sensitive analysis of this parameter is recommended.

On the other hand it is important to stand out that one major assumption in the use of surplus production models is that the relationship between catch rates and stock biomass is constant ($C/E = qB$). This relationship implies that the catchability coefficient, q, remains constant through time. In fact, because fishers tend to be good at what they do, there tend to be continual improvements to fishing gear and fishing practices such that the effectiveness of each unit of effort increases through time. Clearly, the assumption that q is a constant is rather an oversimplification. Hence, some studies assume a linear increase in catchability through time or a constant proportional increase in catchability each year.

Another important question, is how to measure the uncertainty of parameter estimates. One of the procedures used to provide confidence intervals around the parameter estimates is the likelihood profile method (Polacheck et al. 1993 applied this criterion). There are disadvantages to using likelihood profiles. These include the complexity of implementing the method when there are many parameters. Fortunately, alternative methods exist for fitting confidence intervals around parameter estimates. A common approach is to use a bootstrap strategy. This approach resamples the residuals from the optimum fit to generate new bootstrap samples of the observed time series. The model is fitted to many replicate bootstrap samples and the outputs are stored so that percentile confidence intervals can be determined, as is usual with bootstrap methods. The confidence intervals generated can be asymmetric and synthesize the effects of all the parameters varying at once. If we wished to take into account any bias in the parameter estimates, we would do best to calculate bias-corrected percentile confidence intervals (see details on Section 11.6.2 of Haddon, 2011).

To finish this section some comments of risk assessment projections are provided. Invariably, there will be many sources of error and uncertainty that are not accounted for in the model. Determining the uncertainty in an analysis only tells us that we need to be careful when attempting to interpret the model outcomes; it cannot inform resource managers about the risk level associated with a particular management option. To answer such questions a risk assessment is required. Risk assessment implies projecting the population dynamics model into the future under the constraint of different management option. Given the selected catch or effort, we need to be able to model the projected recruitment levels in a stochastic manner, with the variability of that recruitment reflecting the stock dynamics observed in the available time series of data. The problem, when using surplus production models, is to generate these **stochastic recruitments**. There are several alternatives to carry out risk assessment like, bootstrap projections, projections with set catches or with set effort (details in Section 11.7 of Haddon, 2011).

1.8 Conclusions

From this review of surplus production model chapter 11 of Haddon (2011) some conclusions are derived. Surplus production models have developed over the years and now the initial equilibrium assumption have been overcome providing a useful tool in the assessment of stocks, specifically for which there is only limited information available. This models now have surprising flexibility and can be used in risk assessments and to produce management advice. Furthermore although they are usually used in the assessment of data limited stocks, they can be also applied even if more information is available and more complex and realistic models can be implemented to act as a contrast.

2 Prager (1992)

Prager (1992) describes a simple non-equilibrium approach ASPIC (A Surplus-Production Model Incorporating Covariates) to fitting a logistic production model to catch and effort data. Although ASPIC framework could be applied to Pella and Tomlinson (1969) or Fox (1970) production functions he focus on Schaefer (1954) logistic function since for this model it is possible to obtain a closed form for the yield. For Pella and Tomlinson (1969) function such equation is not possible and numerical integration must be used. ASPIC provides estimates of the two logistic parameters r and K, the catchability parameter q, and the initial starting biomass B_0 using their general optimization process similar to that used in Pella and Tomlinson (1969) but ASPIC uses an analytical solution of the yield equation, as we mentioned previously. He considered a loss function equal to the sum of the squares of the residuals of yield, however due to this function can be insensitive to the estimate of B_0 producing values of B_0 above K contradicting logistic production theory, an additional penalty term is introduced in the loss function to avoid unrealistic results on the B_0 estimate. The simulation results shows that the penalty constrains the estimate of B_0 reasonable without degrade the other parameter. In spite of this B_0 is known as the more complicated parameter for estimating accurate. Estimates of MSY, E_{MSY}, B_{MSY} and F_{MSY} (maximum sustainable yield, and associated effort, biomass and fishing mortality) can be made from K, q and B_0 estimates. Another refinement of ASPIC is that it included bootstrap estimates of variability, more precisely, using bootstrap method with resampled residuals, standard errors and confidence intervals can be obtained for each parameter estimate, and for the restropective estimates of biomass in any year. On the other hand the method allows missing data since years with effort and yield equal to zero ara treated correctly in ASPIC since the loss function is based on yield. Furthermore, ASPIC is flexible handling different patterns of fishing, for example, time trends in catchability and two or more simultaneous fisheries with different types of gear. In the last case, the fishing instantaneous mortality in year t is modelled as $F_t = \sum_{j=1}^{J} q_j E_j$ being q_j the catchability coefficient of fishery j and E_j the corresponding effort. The method also consider time trends in catchability, for example, linear trend, a simple way of parameterizing this model woul be to estimate first and last years and then generate intermediate years using interpolation.

3 Polacheck et al. (1993)

Three approaches are commonly used to fit surplus production models to observed data: effort-averaging methods; process-error estimators; and observation-error estimators. Polacheck et al. (1993) compare these approaches using real and simulated data sets, and conclude that they yield substantially different interpretations of productivity. Effort-averaging methods assume the stock is in equilibrium relative to the recent effort; this assumption is rarely satisfied and usually leads to overestimation of potential yield and optimum effort. Process-error estimators produce much less reliable estimates than observation-error estimators. The observation-error estimator provides the lowest estimates of maximum sustainable yield and optimum effort and is the least biased and the most precise (shown in Monte-Carlo trials). Hence, Polacheck et al. (1993) suggest that observation-error estimators must be used when fitting surplus production models, whereas effort-averaging methods must be abandoned, and that process-error estimators should only be applied in simulation studies and/or when the practical experience suggests that they will be superior to observation-error estimators. In this article the uncertainty associated with an estimate is quantified computing its confidence bounds using likelihood profiles.

4 Prager (1994)

Prager (1994) follows and extends the ideas in Prager (1992). Prager (1994) reviews the logistic production model (Schaefer, 1954) starting with the basic differential equation and continuing with a description of the model development without the equilibrium assumption. A brief description of the non equilibrium version of the logistic production model proposed by Prager (1994) is provided in this section.

The population's rate of increase or decrease is

$$\frac{dB_t}{dt} = (r - F_t)\, B_t - \frac{r}{K} B_t^2,$$

where B_t is the population biomass at time t, F_t is the fishing mortality, r is the the stock's intrinsic rate of increase and K is the maximum population size (the carrying capacity). The previous equation notation is simplified as

$$\frac{dB_t}{d_t} = \alpha_t B_t - \beta B_t^2, \tag{3}$$

defining $\alpha_t = r - F_t$ and $\beta = r/K$.

An easy way to solve the above equation is to assume that F_t is contant, however this is a poor assumption, then the time is divided in periods of constant or nearly constant F_t periods and a soluction is found for each period. For simplicity, assume that there are T equal time periods, indexed by $\tau = \{1, 2, \ldots, T\}$, and that a period is one year in duration. In spite of this simplification to explain easily the model a geneal framework considering general periods from $t = h$ to $t = h + \delta$ is also provided in Prager (1994).

Prager (1994) provides equations for biomass for each time period solving equation (3). Then the steps of the parameter estimation algorithm are the following ones.

- Obtain starting guesses for the parameters.
- Beginning with the current estimate of B_0, project the population through time according to the biomass equations. For each year of the projection, compute estimated yield using the closed equation of yield derived from $Y_\tau = \int_{t=\tau}^{\tau+1} F_\tau B_t d_t$ where Y_τ is the yield during period starting at $t = \tau$ and ending at $t = \tau + \delta$, and F_τ is the (constant) instantaneous rate of fishing mortality during such time period.
- Compute the objective function to be minimized. Assuming a multiplicative error structure in yield, this is

$$\sum_{\tau=1}^{T} \Big[\log(Y_\tau) - \log(\hat{Y}_\tau)\Big]^2,$$

  where T is the total number of time periods.
- Monitor the objective function for convergence. If achieved, end. Otherwise, revise the parameter estimates (using a standard minimization scheme) and continue at step 2.

Note that the model fitted process leads to an observation-error estimator since the logarithmic objective function assumes multiplicative errors with constant variance.

The estimation method just described uses the recorded effort in each year to estimate yield. Alternatively, one could use the recorded yield in each year to estimate the fishing mortality rate. Prager (1994) also proposed an algorithm based on this idea, however estimating effort from yield introduces two small practical difficulties. The first difficulty is that not an explicit solution for effort is provided an hence so it must be solved iteratively. The second difficulty involves a fundamental difference between predicting yield and predicting effort. For a given starting biomass and effort, one can always compute the corresponding yield. For a given starting biomass, however, there are some yields that can never be obtained, no matter how high the effort. On the other hand, in fisheries contexts, yield is usually observed more precisely than fishing effort; for that reason, it seems preferable on statistical grounds to use the second approach, estimating effort from yield, rather than estimating yield from effort.

The above explanation presents the basic continuous non equilibrium surplus production model implemented as an observation error estimator presented by Prager (1994). Furthermore than the basic framework, several extensions of the model as "tunning" the model to a biomass index, partitioning fishing mortality by gear, time or area, variability in catchability (and associated tests), computation of confidence intervals through bootstrap, and making projections are also considered in Prager (1994). Some of them have been mentioned and described previously in Prager (1992) section above, for other questions see Prager (1994) page 376.

5 Prager et al. (1996)

Prager et al. (1996) tried to solve questions of general interest in the surplus production model area. The first question was if surplus production model should be applied to a stock that exhibits pronounced age structure, whereas the second focus on asking if the used of surplus production models can be recommended if there are changes in selectivity. They focus in the assessment of North Atlantic sword fish Xiphias gladius, and they simulated an age-structured population, with fishery, similar to that of swordfish in the North Atlantic. The production model used was the dynamic logistic surplus production model described by Prager (1994). The model was fitted under the assumption of no process error, but lognormal observation error in the annual abundance index. Thus, implementing an observation-error estimator in the sense of Polacheck et al. (1993), who found such estimators superior to others for this application.

They concluded that the selectivity changes simulated in their study resulted in relatively small changes in the MSY obtainable from the stock, and in B_{MSY} and F_{MSY}. Nothing in their results suggests that selectivity changes of this magnitude should reject the use of a production model or cause problems in estimation for a stock similar to North Atlantic swordfish. Similarly, they found nothing to indicate that production models cannot or should not be used on strongly age-structured stocks. To the contrary, in most cases the production model produced useful assessment results, and the estimates of MSY were usually near the underlying "true" values derived from the characteristics of the simulated population, including age and size.

Hence, for stocks similar to swordfish, the presence of strong age structure and moderate changes in selectivity should not lead to reject the application of simple production models

6 Prager and Goodyear (2001)

This paper focus on a underlying assumption of surplus production models and check the effects on the outputs of the model through a simulation study. The assumption studied is that the model's two basic data streams (indices of abundance and records of catch) are expressed in the same metric, either biomass or numbers. For lack of data, that assumption is sometimes violated. The conclusion of the study is that the resulting estimates of maximum sustainable yield and stock status were surprisingly robust to the use of mixed-metric data.

7 Prager (2002)

Prager (2002) focus on solving the concers related to effects derived from fixing the shape parameter p equal to 1 using in this way Schaefer (1954) production function. Then, in this article, the results derived from both models, Schaefer (1954) and Pella Tomlinson (1969), applied to real and simulated data have been compared. This comparactive study shows that the generalized model (Pella-Tomlinson) with estimated exponent was quite sensitive to outliers whereas the logistic model was much less sensitive. For that reason, the logistic production model can be recommended as a central approximation that will likely provide more precise and stable estimates, or provide estimates when none can be obtained from the generalized model. On the other hand it seems likely that better estimates could be obtained from the generalized model by specifying a priori the correct value of p. The difficulty, of course, is knowing the correct value of p.

8 Willians and Prager (2002)

Willians and Prager (2002) focus on showing clearly because estimators derived from the equilibrium asssumption must not be used. More precisely, they pointed out that the parameter estimation for Schaefer (1954) production model is commonly done using an observation-error estimator combining nonlinear-function minimization and forward projection of estimated population state. However, to fit the generalized (Pella–Tomlinson, 1969) production model, an equilibrium approximation method is often used, despite calls in the literature to abandon equilibrium estimators. Hence, they considered an extensive simulation study with widely varying population characteristics, covered wide ranges of stock productivity, shape of the production curve, and observation error, while also considered a broad range of possible time trajectories of stock biomass. Through this study, they showed that observation error estimator provides more accurate and precise estimates than the equilibrium approach. They mentioned that their study was the most comprehensive study about this question demonstrating clearly why equilibrium estimators should be abandoned. Their final message was that equilibrium methods played an important role when computer power was far less available, but there is no reason to continue using them; concluding that equilibrium methods should be abandoned completely.

9 Lucey et al. (2012)

Current single species reference points are derived from assessments that consider each species in isolation from the ecosystem; however, individual species catches cannot be considered independently in multi-species fisheries as a result of both biological and fishery interactions. When such interactions are ignored, the sum of single species maximum sustainable yields (MSYs) is often greater than that sustainable by the ecosystem. Evaluations of overall potential yield from multispecies fisheries assessments suggest that system or aggregate level MSY is generally less than the sum of the individual species MSYs.

There are multiple reasons for using an aggregated approach to evaluate production at the ecosystem level. First, the energy available from lower trophic levels is limited and shared by the entire suite of living marine resources. Second, because fish stocks have different productivities, it is often difficult to simultaneously attain single stock objectives in multispecies fisheries. Third, there are biological and/or technological interactions that may not always be directly accounted for in single species assessments. However, aggregate models account for all of these interactions without having to explicitly estimate them as in multispecies models.

Lucey et al. (2012) explore, compare and contrast production model outputs across both various aggregation schemes and multiple ecosystems. They aggregated species using 3 different aggregation types: habitat, feeding guild and size class.

Aggregate annual surplus production (ASP) was calculated for each aggregate group (agg) and ecosystem (j) as:

$$ASP_{agg,j,t} = B_{agg,j,t+1} - B_{agg,j,t} + C_{agg,j,t}$$

where $B_{agg,j,t}$ is the total biomass of all species within aggregation in ecosystem j for year t and $C_{agg,j,t}$ is the corresponding total catch. They fit both a null model and a Graham-Schaefer surplus production model. The null model assumed that aggregate annual surplus production was linearly related to the aggregate biomass, whereas Graham-Schaefer model estimated surplus production as a quadratic function of biomass and additive error (first order autocorrelation error structure $\epsilon_t = \phi\epsilon_{t-1} + V_t$ where V_t follows a normal distribution of zero mean). Note that this model assumes that observations are made without error and that all of the error occurs in the change in population size (process error).

Lucey et al. (2012) fit such surplus models to different types of aggregations for 12 marine ecosystems. They compared and contrasted model outputs across the various types of aggregations and among ecosystems. They have shown that, with a few exceptions, estimated aggregate group BRPs are relatively invariant among ecosystems when data are aggregated to reflect habitat, trophodynamic or allometric affinities. Furthermore, regardless of the type of aggregation, aggregate production never exceeded 6 t/km^2 and was generally less than 3 t/km^2. Patterns of production varied among ecosystems with no particular pattern with respect to ocean basin, latitude or component species. The comparative ecosystem approach is important for revealing commonalities and differences across ecosystems.

10 Fogarty et al. (2012)

Fogarty et al. (2012) deals as Lucey et al. (2012) with aggregate surplus production model. They fitted surplus production models for 12 demersal fish species in the Gulf of Maine at the single-species and aggregate-species levels. Summed single-species production model reference points were higher than estimates from the aggregate surplus production model. The equilibrium yield (maximum sustainable yield, MSY) and biomass at MSY (B_{MSY}) levels for the summed single-species production model reference points exceeded the aggregate model results by 28.0 and 27.5%, respectively. Biological interactions such as predation and competition are potential reasons for differences between the aggregate and summed results. Not accounting for biological interactions may result in overly optimistic predictions of long-term sustainable yield and unrealistically high estimates of B_{MSY}. The previous conclusions derived from Fogarty et al. (2012) have been mentioned also in Lucey et al. (2012).

11 Prager (2016)

Prager (2016) is a guide describing Version 7 of ASPIC Suite, a set of computer programs to fit non-equilibrium surplus production models to fisheries data. A general technical description of the theory behind ASPIC was given by Prager (1994).

ASPIC 7 fits the logistic production model (Schaefer, 1954), in which the production curve is symmetrical around B_{MSY}; the generalized model of Pella and Tomlinson (1969) and the Fox exponential yield model (Fox, 1970). ASPIC incorporates several notable features:

- Analysis of up to 12 data series, which may represent different gears or different periods of time. Data accepted include annual catches, indices of relative abundance, and estimates of absolute biomass.
- Bootstrapping to provide nonparametric confidence intervals on estimated quantities. A resampled data set is generated by combining each saved predicted datum with a randomly chosen residual. The model is then refit to the resampled data.
- Fitting statistically conditioned on yield, fishing effort, or relative abundance. Yield usually is known more precisely than effort or relative abundance; therefore, conditioning on yield is recommended for most analyses. When conditioning on yield, an iterative solution of the catch equation is used, and computation is slower than when conditioning on fishing effort.
- ASPIC is a continuous-time, observation-error model that assumes that observation errors are lognormally distributed. Four methods of estimation are provided: least squares, least absolute values, maximum likelihood, and maximum a posteriori. Regardless of estimation method, fitting is based on minimizing residuals in some way. Residuals are formed from different types of data depending on the statistical conditioning chosen and the type of data series. Because observation errors are assumed lognormal, residuals are formed after log transformation, in all estimation methods, and with all data types.

In relation to the required data, ASPIC needs a series of observations on yield (catch in biomass) and one or more corresponding series of relative abundance. Data on fishing effort rate can be used instead of relative abundance, and if so, they are assumed to represent effective (standardized) effort. In addition to data, ASPIC requires starting guesses of its estimated parameters. The leading parameters are MSY (maximum sustainable yield), F_{MSY} (fishing mortality rate under which MSY can be attained), $B_0 = K$ (ratio of stock biomass at the beginning of the analysis to K, the carrying capacity). For each data series, indexed by j, a value of the catchability coefficient q_j is also required.

Finally, it is important to mention that ASPIC has two modes of operation, or program modes:

- In FIT mode, ASPIC fits the model and computes estimates of parameters and other quantities of management interest, including time trajectories of fishing intensity and stock biomass.
- In BOT mode, ASPIC fits the model and computes bootstrapped confidence intervals on estimated quantities.

12 Pedersen and Berg (2017)

It is widely recognized that the model structure of surplus production models is too simple to adequately describe the population dynamics of a real-world stock subject to variability in size-structure, species interactions, recruitment, catchability, selectivity, environmental conditions, etc. To mitigate this, it is common to include a random error term in the equation governing the biomass dynamics as a proxy for unmodelled variability (process error). Similarly, it is often assumed that the biomass index is subject to error in sampling that causes the observed values to deviate from the true. This variability is incorporated by including an observation error term in the equation describing how the index data relate to the biomass. The majority of subsequent model extensions, with the exception of ASPIC (Prager, 1994), adopted the discrete-time form including state-space models that simultaneously estimate both process and observation error. ASPIC, while free of discrete-time average approximations, lacks flexibility owing to its deterministic population dynamics.

The increased flexibility of state-space models, which are the variants of surplus production models most commonly applied today, entails a need for informative data. Thus, data situations with short time series or limited contrast may require that some model parameters are constrained or fixed to obtain model identifiability. For example, the ratio of process to observation noise is often fixed.

Pedersen and Berg (2017) present a stochastic surplus production model in continuous time (SPiCT), which incorporates dynamics in both biomass and fisheries and observation error of both catches and biomass indices. Seasonal extensions to the fisheries dynamics component of the state-space model are also developed facilitating the use of subannual data that contain seasonal patterns. To understand correctly the model we review it from the simplest (initial) equation to its final parametrization.

It is well known that generalized surplus production models in the form of Pella and Tomlinson (1969) can be written as

$$\frac{dB_t}{dt} = \frac{r}{n-1} B_t \left(1 - \left(\frac{B_t}{K}\right)^{n-1}\right) - F_t B_t,$$

where B_t is the exploitable stock biomass (ESB), F_t is the instantaneous fishing mortality rate, r is the intrinsic growth rate of the population, K is the carrying capacity, and $n > 0$ is a unitless parameter determining the shape of the production curve. Note that in Haddon (2011) the shape parameter is p equal to $n - 1$.

Although the above parametrization is easy to interpret biologically, is difficult to estimate owing high correlation between r and K. Then, they used a more stable parametrization.

$$\frac{dB_t}{dt} = \gamma m \frac{B_t}{K} - \gamma m \left(\frac{B_t}{K}\right)^n - F_t B_t, \tag{4}$$

where $\gamma = n^{n/(n-1)}/(n-1)$ and $m = \frac{rK}{n^{n/(n-1)}}$.

Estimating m instead of r results in a more well-defined optimum because of the separate roles of m and K in defining the production curve with K representing the width of the biomass range and $m = MSY^d$ representing the maximum sustainable yield (maximum attainable surplus production). The superscript d indicates that these are deterministic reference points that do not account for random variability.

Equation (4) is a simplified and deterministic description of biomass dynamics. In reality, many additional factors (e.g. species interactions, environmental variability) influence biomass dynamics. In the absence of specific data pertaining to these processes, one can model their influence using a stochastic process noise term. Including process noise in equation (4) results in

$$dB_t = \left(\gamma m \frac{B_t}{K} - \gamma m \left(\frac{B_t}{K}\right)^n - F_t B_t\right) dt + \sigma_B B_t dW_t,$$

where σ_B is the standard deviation of the process noise, and W_t is Brownian motion. Representing surplus production by a stochastic differential equation acknowledges the presence of random and unmodelled process variability, while retaining the property that the process is defined continuously in time and not restricted to specific discrete time points.

Multiplicative noise terms can, in terms of numerical implementation and model fitting, lead to instability problems. Hence, they transform the model to obtain an additive noise term.

$$dZ_t = \left(\frac{\gamma m}{K} - \frac{\gamma m}{K}\left(\frac{e^{Z_t}}{K}\right)^{n-1} - F_t - \frac{1}{2}\sigma_B^2\right) dt + \sigma_B dW_t,$$

being $Z_t = log(B_t)$.

The majority of existing production models leave the process of commercial fishing, F_t, unmodelled. An some of them it is assumed that the catch is observed without error. If present, observation error in the catch will therefore propagate directly to F_t and influence conclusions regarding the current fishing pressure. Hence, Pedersen and Berg (2017) proposed an alternative approach to model F_t as a separate and unobserved process in the same sense that B_t is unobserved, which allows F_t to be estimated at any time even when a catch observation is unavailable. Our general model for F_t is the product of a random component G_t and a seasonal component S_t

$$F_t = S_t G_t$$

$$\mathrm{d}\log G_t = \sigma_F dV_t$$

where dV_t is standard Brownian motion and σ_F is the standard deviation of the noise. Given subannual data, they suggest two models for seasonal variation in the fishing. The first model represents seasonal variation using a cyclic spline, while the second model uses a system of SDEs (stochastic differential equations) whose solutions oscillate periodically. Note that a spline-based model is not able to adapt to changes in amplitude and timing (phase) of the real seasonal fishing pattern, whereas the second proposal can do that but at the cost of increasing model complexity and therefore potential difficulties in obtaining model convergence.

An observation of commercial catch is reported as the cumulative catch C_t over a time interval Δ_t. Given B_t and F_t the observed catch in log can therefore be written as an integral in continuous time plus noise.

$$\log(C_t) = \log\left(\int_t^{t+\Delta_t} F_s B_s ds\right) + \epsilon_t,$$

where the catch observation errors $\epsilon_t \sim N(0, \sigma_C^2)$ are independent and σ_C is the standard deviation of the catch observation error. This formulation allows the noise of the F_t process to be separated from the observation noise of C_t.

In addition to catch observations, they assume to have observations of N_i series of indices of exploitation biomass $I_{t,i}$, $i = 1, \ldots, N_i$, which could be commercial or scientific catch-per-unit-ofeffort data or other biomass indicators. Contrary to catch observations that are aggregated over a period of time, $I_{t,i}$ are regarded as "snapshots" related to the time point t given as

$$\log(I_{t,i}) = \log(q_i B_t) + e_{t,i},$$

where $e_{t,i} \sim N(0, \sigma_{I,i}^2)$ are independent normal deviates and $\sigma_{I,i}$ is the standard deviation of the i-th index observation error, and q_i is a catchability parameter for the i-th index. In the common situation where only one series of biomass indices is available ($N_i = 1$).

They also define the ratios between observation and process errors $\alpha = \sigma_I/\sigma_B$ and $\beta = \sigma_C/\sigma_F$. In cases where it is not possible to separate process and observation error, a common simplification is to fix $\alpha = 1$ and $\beta = 1$.

Extreme observations or outliers in index and catch are commonly encountered problem in fisheries data. Such outliers are poorly modelled when using the normal distribution for observation errors, which may lead to bias of parameter estimates. Common approaches to mitigate the influence of outliers include objective outlier detection and subsequent residual rescaling, or robust estimation using fat-tailed error distributions. Perdersen and Berg (2017) takes a simple approach to robust estimation where the observation error follow the mixture distribution $pN(0, \sigma^2) + (1 - p)N(0, [w\sigma]^2)$, where p and w are parameters controlling the fatness of the tails.

In summary, SPiCT is a state-space model for surplus production containing unobserved processes for B_t and F_t and observed quantities C_t and $I_{t,i}$, which include observation noise. In addition to the usual parameters of deterministic production models, process and observation variance parameters, $\sigma_{I,i}$, σ_B, σ_C, σ_F, are estimated from data (if possible), while the unobserved processes B_t and F_t are treated as random effects.

Auxiliary information can, if available, be incorporated in a Bayesian estimation framework using so-called informative priors, which are probability distributions that narrow the range of the model parameters they target. Including priors typically stabilizes model fitting, and reduces uncertainty of estimated quantities. It is, of course, imperative that priors are only included if their specification relies on a solid foundation such as meta-analyses or independent data. Particular caution is required if informative priors are specified for n, r, m or K, as these are the main parameters determining management quantities.

Both frequentist and Bayesian inference of model parameters are possible. In a frequentist framework, model parameters are estimated by maximizing the log-likelihood function. In case of available a priori information, prior distributions are multiplied with the likelihood function to obtain the posterior distribution. Bayesian maximum a posteriori parameter estimates are thus located at the maximum of the posterior distribution. Hence, confidence and credible intervals can be derived from both frequentist and Bayesian inference, respectively.

To finish, SPiCT model is a full state-space model in that both biomass and fishing dynamics are modelled as states, which are observed indirectly through biomass indices and commercial catches sampled with error.

Comments about the simulation study:

The first purpose of simulation study was to quantify the estimation performance of SPiCT in terms of estimation stability (proportion of converged runs), estimation precision (expressed by the coefficient of variation, CV, of estimates), the coverage of 95% CIs (proportion containing the true value) and the median bias of estimates. These quantities were evaluated for eight variants of SPiCT and ASPIC version 7.02 (Prager, 1994), with particular focus on the influence of fixing and misspecifying the parameters n, α, and β, which can be difficult to estimate.

The second purpose was to assess the difference in estimation performance between a continuous-time model fitted to quarterly data containing within-year seasonal variation and a discrete-time model ($dt_{Euler} = 1$ year, the inverse of this parameter is the number of intervals per year) fitted to annual data obtained by aggregating the quarterly data.

They don't focus on comparing with ASPIC in the simulation study. Note that SPiCT is implemented in the R package spict and that a manual and some guidelines for its use are available at https://github.com/DTUAqua/spict.

13 Kathena et al. (2018)

Kathena et al. (2018) is an SPiCT application on two Namibian stocks: (i) the data-rich Cape monkfish Lophius vomerinus, where results are compared to a new data-rich assessment using a state–space assessment model (SAM); and (ii) the data-moderate west coast sole Austroglossus microlepis, which is an important bycatch species in the Cape monkfish fishery, but currently unassessed. SPiCT and SAM gave largely consistent estimates of relative fishing mortality and relative exploitable biomass for the Cape monkfish stock, although with some discrepancies. Differences in the biomass estimates between the two assessments suggest that further investigation is required to understand the cause, and that some caution is necessary when considering the biomass of the stock. SPiCT shows that the west coast sole may be overexploited, although the confidence bounds were too wide for a firm conclusion.

14 Winker et al. (2018)

Winker et al. (2018) present a new, open-source modelling software entitled "Just Another Bayesian Biomass Assessment" (JABBA). The name is a reference to JAGS (Just Another Gibbs Sampler), which is the language in which the bayesian algorithm is executed. JABBA is a generalized bayesian state-space surplus production model and represents an innovative approach to biomass dynamic modelling. The motivation for developing JABBA was to provide a user-friendly R to JAGS interface for fitting generalized bayesian state-space surplus production models to generate reproducible stock status estimates and diagnostics for a wide variety of fisheries.

The surplus production function is formulated with the generalized three parameters surplus production model by Pella and Tomlinson (1969).

$$SP_t = \frac{r}{m-1} B_t \left(1 - \left(\frac{B_t}{K}\right)^{m-1}\right), \tag{5}$$

where r is the intrinsic rate of population increase at time t, K is the carrying capacity, B_t is stock biomass at time t, and m is a shape parameter. NOTE that m is called n in SPiCT.

The shape parameter m can be directly translated into the biomass level where MSY is achieved,

$$\frac{B_{MSY}}{K} = m^{(-1/(m-1))}, \tag{6}$$

furthermore, r can be expressed as

$$r = \frac{MSY}{B_{MSY}} \frac{m-1}{1-m^{-1}}. \tag{7}$$

Equations (6) and (7) emphasize the potential of translating estimates of MSY/B_{MSY} and B_{MSY}/K into r and m parameters, respectively. This presents a crucial bridge for parameters derived from age-structured equilibrium models to be implemented in surplus production models.

The parametrization of the surplus production function used in (5) has the biologically undesirable property of surplus production per unit biomass approaching to infinity even as biomass approaches zero. To address this anomaly, JABBA provides the additional option of combining the surplus production with a generic "hockey stick" recruitment function. The hockey-stick, assumes that recruitment potential becomes increasingly impaired below a given biomass ratio level ($P_{lim} = B_{lim}/K$), with P_{lim} ranges of 0.2–0.25 having been widely adopted as thresholds for recruitment overfishing. The linear decrease of the underlying hockey-stick between 1 and 0 is implemented by introducing the multiplier to the surplus production function, so that for values of $B/K < P_{lim}$:

$$SP_t = \frac{r}{m-1} B_t \frac{B_t}{KP_{lim}} \left(1 - \left(\frac{B_t}{K}\right)^{m-1}\right)$$

JABBA is formulated on the bayesian state-space estimation framework. The biomass B_y in year y is expressed as a proportion of K (i.e. $P_y = B_y/K$) to improve the efficiency of the estimation algorithm. The model is formulated to accommodate multiple CPUE series i. The initial biomass in the first year of the time series is scaled by introducing model parameter ϕ to estimate the ratio of the spawning biomass in the first year to K. The stochastic form of the process equation is given by:

$$\begin{aligned}
P_y &= \phi e^{\eta_y} \text{ if } y = 1 \\
P_y &= \left(P_{y-1} + \frac{r}{m-1} P_{y-1}\left(1 - P_{y-1}^{m-1}\right) - \frac{\sum_f C_{f,y-1}}{K}\right) e^{\eta_y} \text{ if } P_{y-1} \geqslant P_{lim} \\
P_y &= \left(P_{y-1} + \frac{r}{m-1} P_{y-1}\left(1 - P_{y-1}^{m-1}\right) \frac{P_{y-1}}{P_{lim}} - \frac{\sum_f C_{f,y-1}}{K}\right) e^{\eta_y} \text{ if } P_{y-1} < P_{lim}
\end{aligned}$$

where η_y is the process error, with $\eta_y \sim N(0, \sigma_\eta^2)$, and $C_{f,y}$ is the catch in year y by fishery f. Note that the corresponding biomass for year y is $B_y = P_y K$, then the observation equation is given by

$$I_{i,y} = q_i B_y e^{\epsilon_{y,i}}, \ \ y = 1, 2, \ldots, n.$$

where q_i is the estimable catchability coefficient associated with the abundance index i, and $\epsilon_{y,i}$ is the observation error, with $\epsilon_{y,i} \sim N(0, \sigma_{\epsilon,y,i}^2)$.

The full bayesian state-space surplus production model projected over n years requires a joint probability distribution over all unobservable hyper-parameters $\{K, r, \phi, \sigma_\eta^2, q_i, \sigma_{\epsilon,y,i}^2\}$ and the n process errors relating to the vector of unobserved states $\eta = \{\eta_1, \eta_2, \ldots, \eta_y\}$, together with all observable data in the form of the relative abundance indices i, $I_i = \{I_{i,1}, I_{i,2}, \ldots, I_{i,y}\}$.

JABBA is run in JAGS to estimate the Bayesian posterior distributions of all quantities of interest by means of a Markov Chains Monte Carlo (MCMC) simulation. The model is based on the assumption that catch observations are error free, which will usually not be true. It is possible to address this issue by modelling harvest rates as a separate and unobserved process. Further developments of JABBA should allow annual estimates of uncertainty in catch to be included in the assessment model fitting, which is then reflected in the uncertainty of estimated model parameters and management quantities.

An interesting research perspective is to potentially improve the comparability between age-structured models and JABBA by linking the parameterization of the production and shape parameters to dynamic pool models with integrated spawner-recruitment relationship.

15 Winker et al. (2020a)

They introduce "JABBA-Select", an extension of the JABBA software (Just Another Bayesian Biomass Assessment; Winker et al., 2018), that is able to overcome some of the shortcomings of conventional surplus production models and allows a direct comparison to age structured production models. JABBA-Select incorporates life history parameters and fishing selectivity and distinguishes between exploitable biomass (used to fit indices given fishery selectivity) and spawning biomass (used to predict surplus production). Applying JABBA-Select involves using an age-structured equilibrium model to convert the input parameters into multivariate normal priors for surplus-production productivity parameters. JABBA-Select performed in the simulation study was at least as well as the age structured production models in accuracy for most of the performance metrics and best characterized the stock status uncertainty. The results indicate that JABBA-Select is able to accurately account for moderate changes in selectivity and fleet dynamics over time and to provide a robust tool for data-moderate stock assessments.

A clearly example prior generation for this surplus production parameters is provided by Winker et al. (2020b)

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