

**IGNORE THE EFFECTS OF TOLERANCE BUILDUP,  
AND YOU'LL HAVE PROBLEMS AFTER YOU SIGN  
OFF ON YOUR PROTOTYPES.**

# Worst-case circuit design includes component tolerances

**B**UILDING RELIABLE HARDWARE requires that you account for all tolerances during the design stage. Many references discuss active component errors resulting from parameter deviations—showing how to calculate the effects of op-amp offset voltage, input current, and similar parameters—but few of them consider passive-component tolerances. References that do consider component tolerances do it from the scientist's, rather than circuit designer's, point of view.

However, you can understand worst-case design by using basic circuit equations and the component's limit values to calculate the range of worst-case values that a circuit parameter, such as output voltage, can assume over the life of the equipment. Worst-case design lets the components assume a wide range of values, which leads to a wide range of solutions; some of these solutions may be undesirable.

## PASSIVE-COMPONENT-TOLERANCES COUNT

Resistors are fundamental in all electronic circuits, so you should consider them in detail. Resistors are specified with purchase tolerances,  $P$ , which you measure in percent; popular examples include 0.5, 1, 2, 5, and 10%. The purchase tolerance guarantees that the resistor is within its nominal value when you receive it. A resistor's value often is close to its limit when you purchase it, because many manufacturers select and remove the center of the distribution when grading resistors to sell as their tighter tolerance grades.

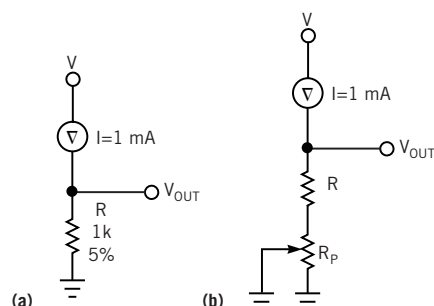
External stresses, such as soldering, cause resistor values to change during the assembly process. Hence, resistor values may change beyond the purchase tolerance before the completed assembly leaves the factory. Component values keep changing during their lifetime, because external stresses, such as temperature, aging, pressure, humidity, mounting, sunlight, and dust, change the component's composition, size, and surface characteristics. Component-value changes experienced during operation are called drift tolerances,  $D$ , and you measure them in percent.

Table 1 shows estimated resistor tolerances. Notice that the purchase tolerance is separate from the drift tolerance, because you can trim to reduce or

eliminate the purchase tolerance, but the drift tolerance occurs during normal operation and causes errors unless the system performs calibration before measurement. The drift tolerance for some resistors is greater than the purchase tolerance.

The resistor-manufacturing process and its operating environment determine the drift tolerance. Vendors manufacture tighter tolerance resistors with stable, controlled methods and materials that resist drift, because excessive drift results in manufacturing rejects. Tight process- and material-control techniques that minimize drift in the factory may not minimize drift in the field. The drift tolerances in Table 1 are worst case for an ambient temperature of  $-25$  to  $+85^{\circ}\text{C}$  and reasonable external stress. Unless the manufacturer states otherwise, temperature drift is unpredictable, and resistor values can increase or decrease due to an increase in temperature. Designers should consult the resistor manufacturer before using lower drift tolerances.

Resistors are usually represented as  $R_1$  or  $R_2$ . Keeping this nomenclature, you can calculate the final resistor value as  $(1 \pm T)R_1$  to obtain the worst-case resistor value, where  $T$  is the total tolerance expressed as a decimal. You use the plus/minus sign, because the tolerance's polarity depends on external conditions, manufacturing methods, materials, and internal stresses. You should assume the worst-case tolerance polarity during your calculations. You must represent individual resistor tolerances as positive or negative, using whichever yields the worst-case calculation, unless the data sheet states that all resistors drift in a prescribed direction. When calculating the absolute worst-case maximum value for  $R_1 = 10\text{ k}\Omega$ , 5%, you should use  $(1 + 0.01(5) + 0.01(5))R_1 = 1.1R_1 = 11\text{ k}\Omega$ . The absolute worst-case minimum value for this resistor is  $(1 - 0.01P - 0.01D)R_1 = (1 - 0.05 - 0.05)R_1 = 9\text{ k}\Omega$ .



**Figure 1**

**Nonratio circuits must assume the full resistor tolerance because the tolerances do not divide out. The basic calculation is simple (a), and you can narrow the range by adding another resistor to adjust the initial tolerance error (b).**

You handle capacitor tolerances, which these calculations do not discuss in detail, in the same manner. They vary much more between capacitor types than do resistors because of the radically different methods vendors use to manufacture them. Electrolytic capacitors often have purchase tolerances of 80, -20%, but some glass and NPO ceramic capacitors come with purchase tolerances of 1%. In general, it is best to triple all capacitor tolerances unless the manufacturer's data sheets suggest differently. This practice errs on the conservative side but is good judgment when you haven't done your homework.

### NONRATIOMETRIC CIRCUITS

For nonratiometric circuits, you must assume the full resistor tolerance, because the tolerances do not divide out. You calculate the output voltage as  $V_{OUT} = IR$ , where  $I$  is a perfect 1-mA current source, and  $R$  is a 5% resistor (**Figure 1a**).  $V_{OUT} = 1 \text{ mA } (1 \pm 0.05 \pm 0.05) 1 \text{ k}\Omega = (1 \pm 0.05 \pm 0.05) \text{ V}$ . The range of  $V_{OUT}$  is  $0.9 \text{ V} \leq V_{OUT} \leq 1.1 \text{ V}$ , but you can narrow it by adjusting the initial tolerance with another resistor (**Figure 1b**).

You can calculate the adjustable resistor value,  $R_p$ , as follows:

1. Select the closest decade value for  $R$  that is less than the minimum calculated value of  $R = 0.9 \text{ k}\Omega$ ; this value is  $R = 0.82 \text{ k}\Omega$ .

2. Calculate the minimum value of the selected resistor as follows:  $R_{MIN} = (1 - P - D)R = 0.9(0.82) = 0.738 \text{ k}\Omega$ .

3. The variable resistor,  $R_p$ , must make up the difference between  $R_{MIN}$  and  $1 \text{ k}\Omega$ , so  $R_{PMIN} = 1 - 0.738 = 0.262 \text{ k}\Omega$ .

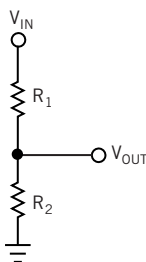
4. The tolerance on potentiometers can be quite high, so  $R_p = R_{PMIN} / (1 - T) = 0.262 \text{ k}\Omega / (1 - 0.8) = 0.328 \text{ k}\Omega$ .

**TABLE 1—RESISTOR TOLERANCES**

Purchase tolerance (P, %)	Drift tolerance (D, %)	Total tolerance (T, %)
0.5	0.25	0.75
1	2	3
2	2	4
5	5	10
10	15	25

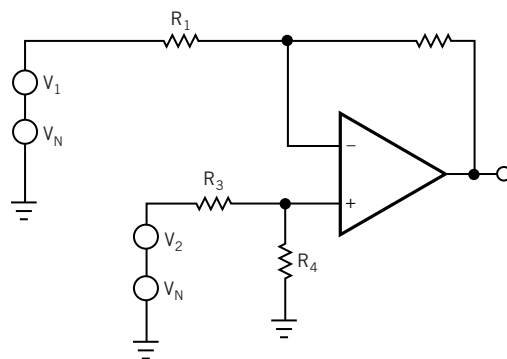
**TABLE 2—GAIN-ERROR CALCULATIONS FOR A VOLTAGE DIVIDER**

	Gain for T=0	Gain for T=1%	Error (%)
$R_1 = R_2$	0.5	0.505	1
$R_1 = 4R_2$	0.2	0.2032	1.6
$R_1 = 9R_2$	0.1	0.1018	1.8



**Figure 2**

Ratio circuits divide out some of the tolerance of the resistor values.



**Figure 3**

The difference amplifier uses ratio resistors to obtain high CMR.

5. Select  $R_p = 500\Omega$ .

The final values are  $R = 820\Omega$  and  $R_p = 500\Omega$ . Some engineers argue that the worst-case design procedures are too stringent and force a large potentiometer value, lower resolution, and higher potentiometer drift error. One possible solution to this problem is to decrease the potentiometer value and take a risk, but a better solution is to use higher precision parts. Nonratio circuits have to account for the full tolerance swing; thus, a 5% purchase tolerance results in a 20% ( $\pm 10\%$ ) overall tolerance.

### RATIOMETRIC CIRCUITS

The voltage divider of **Figure 2** and **Equation 1** is the classic ratiometric circuit. Referring to the tolerance equation, you can see that some portion of the tolerance divides out of the equation.

$$\frac{V_{OUT}}{V_{IN}} = \frac{R_2}{R_1 + R_2} = \frac{(1 \pm T)R_2}{(1 \pm T)R_1 + (1 \pm T)R_2} \quad (1)$$

To obtain the maximum gain value, you set the tolerance for  $R_2$  high and for  $R_1$  low. Because the tolerance for  $R_2$  is high, it appears as  $(1 + T)R_2$  in all parts of the equation. **Table 2** tabulates the ideal gain, maximum gain, and percent error in four resistor ratios. Notice that the minimum gain error occurs when  $R_1 = R_2$ , and this gain error is equal to the tolerance. The nonratio circuit has to accept twice the tolerance or  $2T$ , but the ratio circuit can have a tolerance of just  $T$ .

When both the resistor tolerances in a voltage divider are simultaneously high or low, the tol-

erances divide out. When the resistor manufacturer guarantees that all resistors drift proportionally and in the same direction during ambient-temperature changes, the temperature tolerance divides out.

### DIFFERENCE AMPLIFIER: ON ITS OWN

Many references say that you can't build a precision differential amplifier that has good CMR (common-mode rejection) using discrete parts. This tolerance analysis of the differential amplifier explains why this statement is true. Consider the differential amplifier circuit in **Figure 3** and **Equation 2** (**Reference 1**), and assume that the amplifier is perfect.

$$V_{OUT} = \frac{(V_2 + V_N)R_4}{R_3 + R_4} \frac{R_2 + R_1}{R_1} - \frac{(V_1 + V_N)R_2}{R_1} \quad (2)$$

The circuit CMR is measured in the absence of signal, so  $V_1 = V_2 = 0.0$ , and **Equation 2** becomes **Equation 3**:

$$\frac{V_{OUT}}{V_{IN}} = \frac{R_4}{R_3 + R_4} \frac{R_2 + R_1}{R_1} - \frac{R_2}{R_1} \quad (3)$$

When  $R_1 = R_3$  and  $R_2 = R_4$ , the gain goes to zero, and the CMR is infinite. In reality, resistor tolerances and op-amp errors always limit the CMR to about 100 dB or less. Rewriting **Equation 3** as **Equation 4** puts your focus on the differential gain and resistor tolerances. **Equation 4** contains four resistor tolerances, so there are 16 possible error factors. If you investigate all the possibilities, you'll see that the error ranges from zero when all resistor tolerances go in the same direction, to  $2T/(1 - T)$ .

$$\frac{V_{OUT}}{V_N} = \frac{(1 \pm T)R_4}{(1 \pm T)R_3 + (1 \pm T)R_4} \cdot \frac{(1 \pm T)R_2 + (1 \pm T)R_1}{(1 \pm T)R_1} - \frac{(1 \pm T)R_2}{(1 \pm T)R_1} \quad (4)$$

The resistor tolerances can cause the CMR to range from as high as the limits of the op amp to as low as  $-34.89$  dB, when the total tolerance is 1% (P+D for 0.5% resistors). Considering purchase and drift tolerances for 1% resistors, your resultant CMR can be as low as  $-24.17$  dB. You calculate this error as a CMR error, but, in the absence of a common-mode voltage and with a differential input signal, it becomes a gain error.

Discrete differential amplifiers are difficult to build and trim, so most designers have gone to IC differential amplifiers with built-in trimmed resistors. Low-cost IC differential amplifiers offer as much as  $-86$ -dB CMR.

When you are analyzing your circuit to ensure both long-term performance and manufacturability, keep in mind that passive components have purchase and drift tolerances, and the drift tolerance may be larger than the purchase tolerance. You can adjust the purchase tolerance at the end of your manufacturing process, but you or your system can adjust the drift tolerance only just before making a measurement. Nonratio circuits assume twice the resistor tolerance, and ratio circuits can reduce the error to the tolerance value. Accurate differential amplifiers are hard to build with discrete devices, but ICs with their resistor trimming and matching capability often obtain  $-90$  dB of CMR. □

#### REFERENCE

1. Mancini, Ron, *Op Amps for Everyone*, Newnes division of Elsevier Science, May 2003.

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