Introducción a la Inteligencia Artificial

Clase 4

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Clase 5

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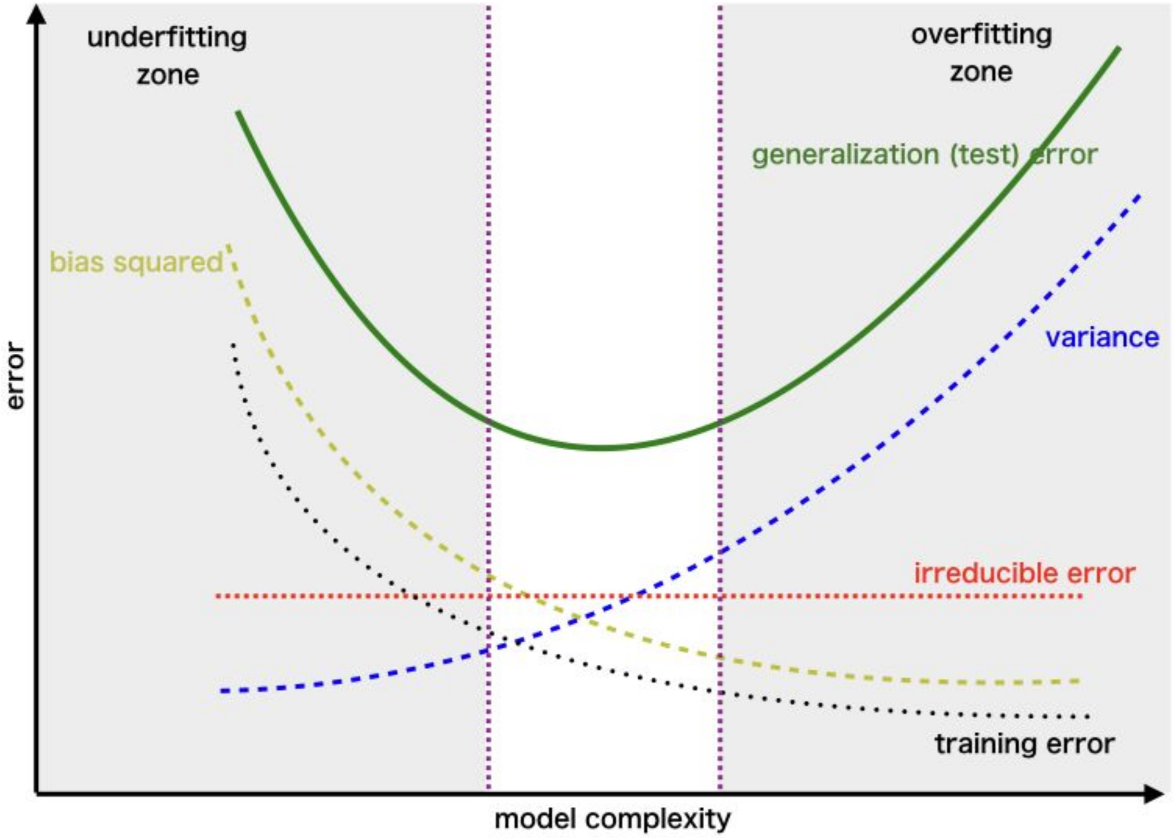
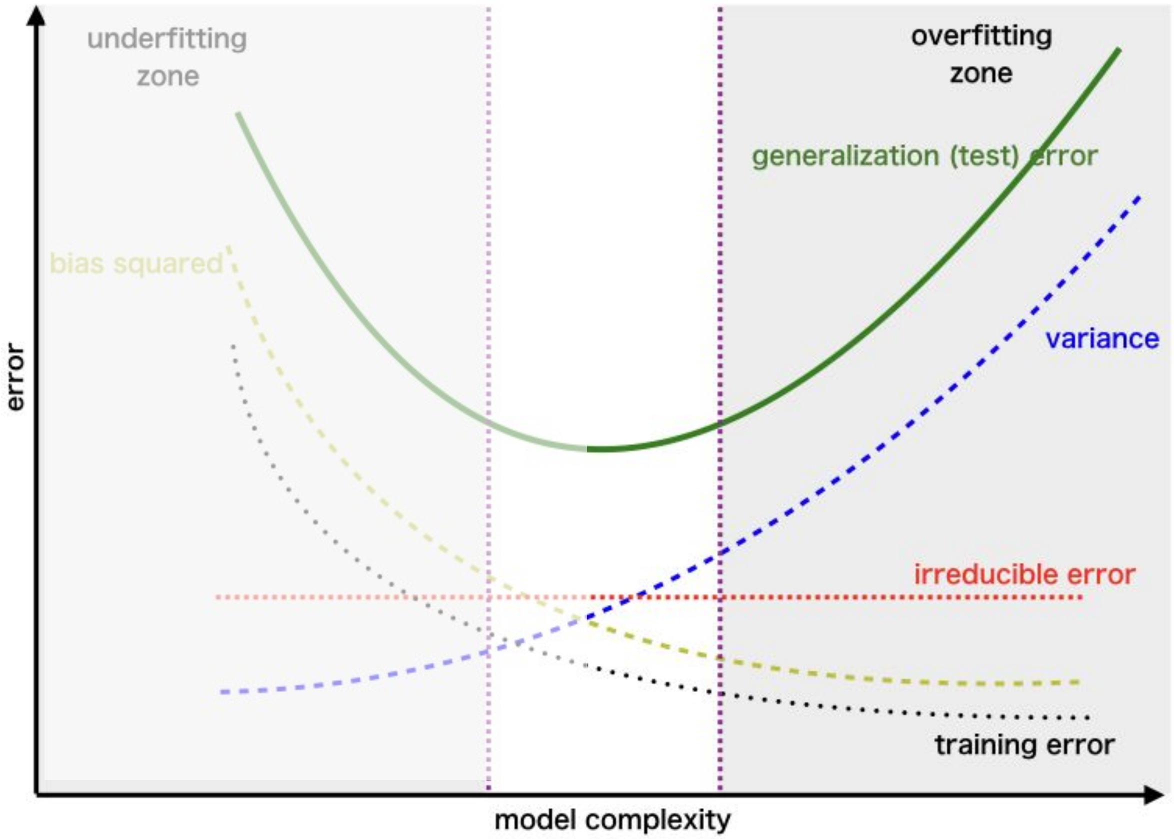
Regularización



Regularización



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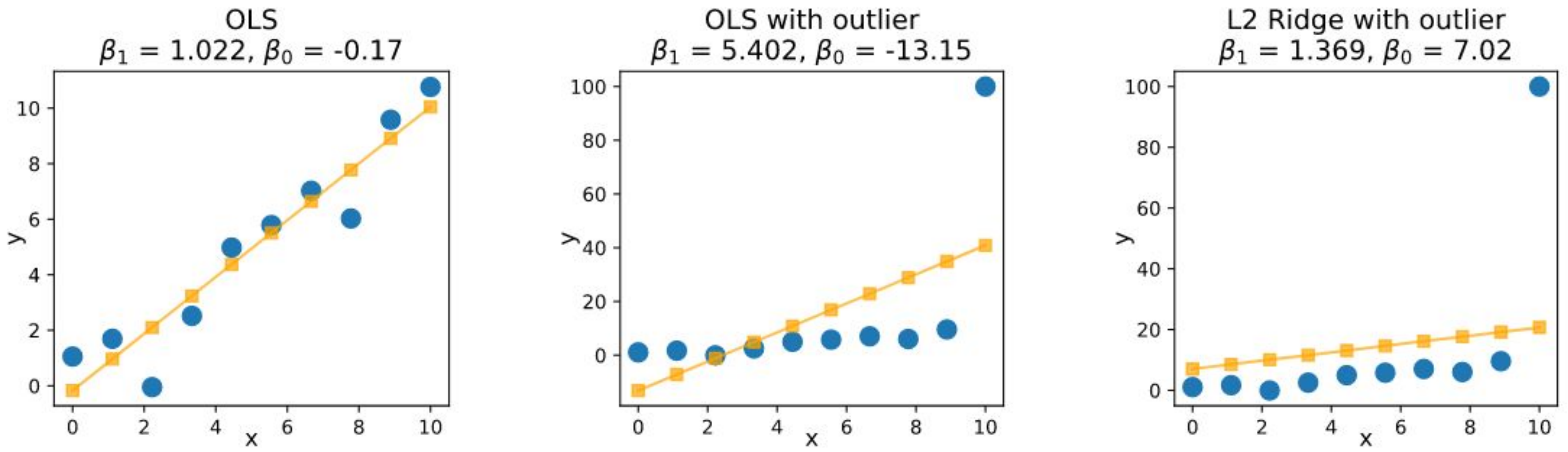
Regularización - Motivación



Regularización

$$E_D(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N}\{t_n - \mathbf{w}^{\mathrm{T}}\phi(\mathbf{x}_n)\}^2$$

Observado **-** Predicción

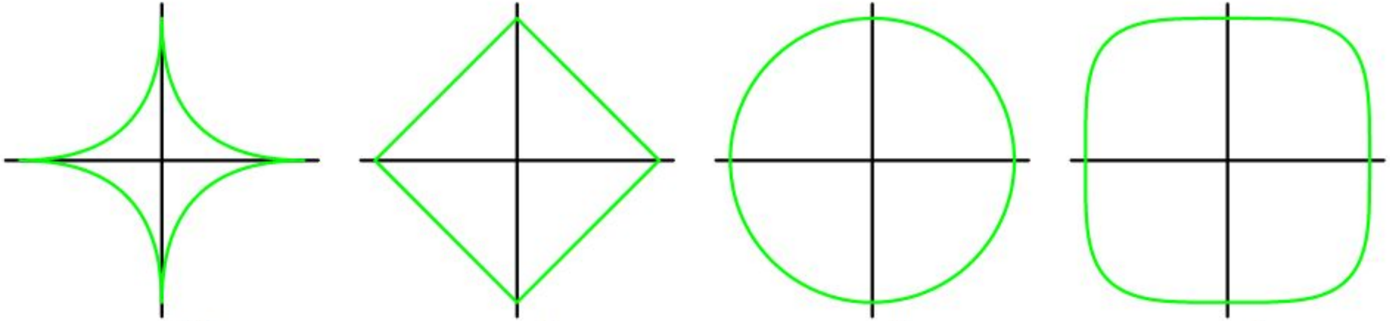
↓

w está "libre"

Regularización

$$\frac{1}{2}\sum_{n=1}^{N}\{t_n - \mathbf{w}^{\mathrm{T}}\phi(\mathbf{x}_n)\}^2 + \boxed{\frac{\lambda}{2}\sum_{j=1}^{M}|w_j|^q}$$

Término de regularización “weight decay” → w afecta la pérdida



$q = 0.5$ $q = 1$ $q = 2$ $q = 4$

Lasso Ridge

$$w = (\Phi^T\Phi + \lambda I)^{-1}\Phi^T y$$

Maximum A Posteriori como regularización

Distribución "a priori" de los parámetros → Observar data → Actualizar distribución (Posterior)

$$p(w) \sim D(\theta)$$

$$(\mathcal{X}, \mathcal{Y})$$

$$p(w|\mathcal{X}, \mathcal{Y}) = \frac{p(\mathcal{Y}|\mathcal{X}, w)p(w)}{p(\mathcal{Y}|\mathcal{X})}$$

$$w_{map} = (\Phi^T\Phi + \frac{\sigma^2}{b^2}I)^{-1}\Phi^T y$$

Gaussian prior con varianza b2

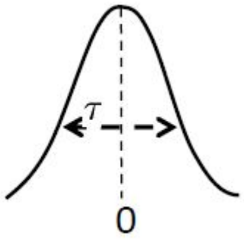
Regularización

Maximum A Posteriori como regularización - Ridge (L2)

$$\widehat{\beta}_{\text{MAP}} = \arg\max_{\beta} \underbrace{\log p(\{Y_i\}_{i=1}^n | \beta, \sigma^2, \{X_i\}_{i=1}^n)}_{\text{Conditional log likelihood}} + \underbrace{\log p(\beta)}_{\text{log prior}}$$

I) Gaussian Prior

$$\beta \sim \mathcal{N}(0, \tau^2 \mathbf{I}) \qquad p(\beta) \propto e^{-\beta^T \beta / 2\tau^2}$$



$$\widehat{\beta}_{\text{MAP}} = \arg\min_{\beta} \sum_{i=1}^{n} (Y_i - X_i\beta)^2 + \underset{\substack{\downarrow \\ \text{constant}(\sigma^2, \tau^2)}}{\lambda} \|\beta\|_2^2$$

Ridge Regression

$$\widehat{\beta}_{\text{MAP}} = (\boldsymbol{A}^\top \boldsymbol{A} + \lambda \boldsymbol{I})^{-1} \boldsymbol{A}^\top \boldsymbol{Y}$$

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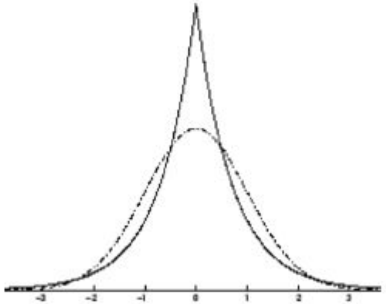
Maximum A Posteriori como regularización - LASSO (L1)

$$\widehat{\beta}_{\text{MAP}} = \arg\max_{\beta} \underbrace{\log p(\{Y_i\}_{i=1}^{n} | \beta, \sigma^2, \{X_i\}_{i=1}^{n})}_{\text{Conditional log likelihood}} + \underbrace{\log p(\beta)}_{\text{log prior}}$$

II) Laplace Prior

$$\beta_i \overset{iid}{\sim} \text{Laplace}(0, t) \qquad\qquad p(\beta_i) \propto e^{-|\beta_i|/t}$$



$$\widehat{\beta}_{\text{MAP}} = \arg\min_{\beta} \sum_{i=1}^{n} (Y_i - X_i\beta)^2 + \underset{\downarrow \atop \text{constant}(\sigma^2, t)}{\lambda} \|\beta\|_1$$

Lasso

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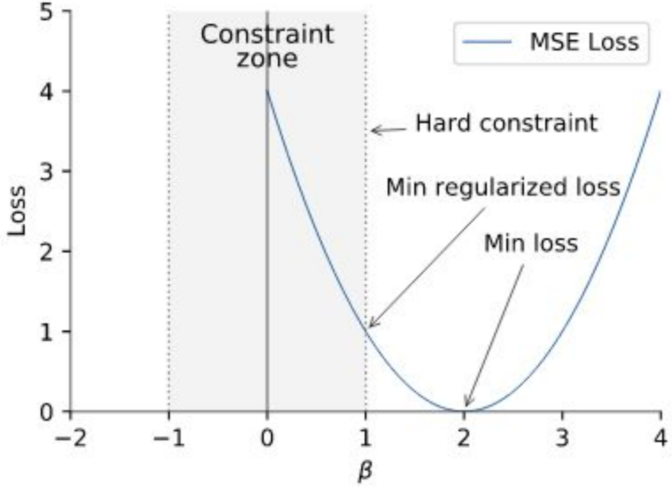
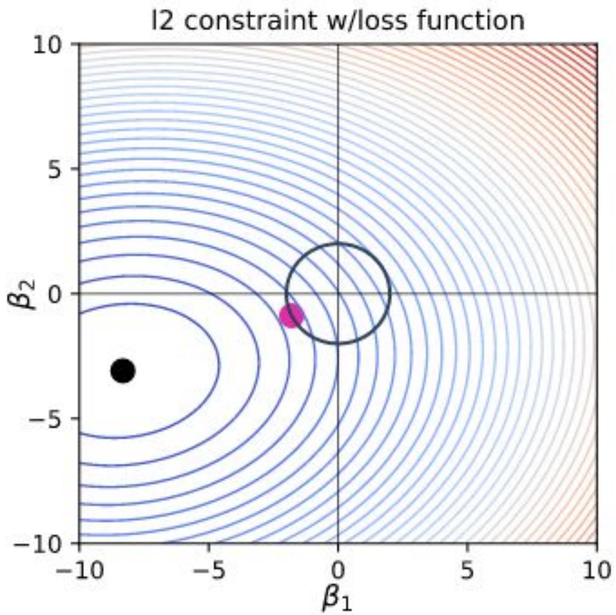
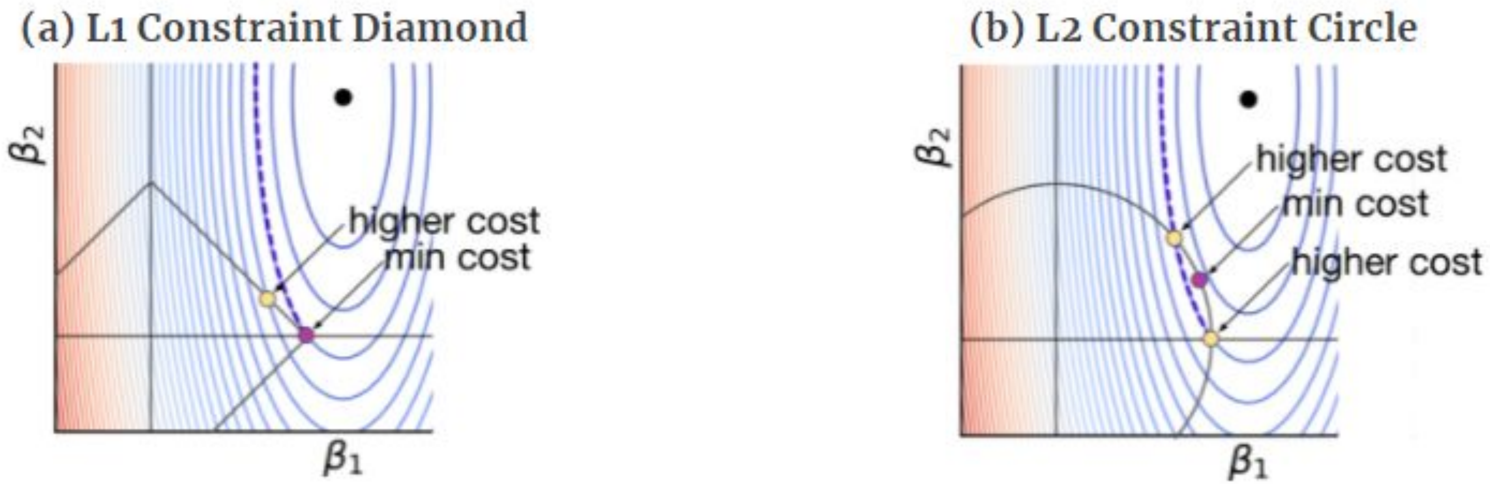
Regularización





Regularización

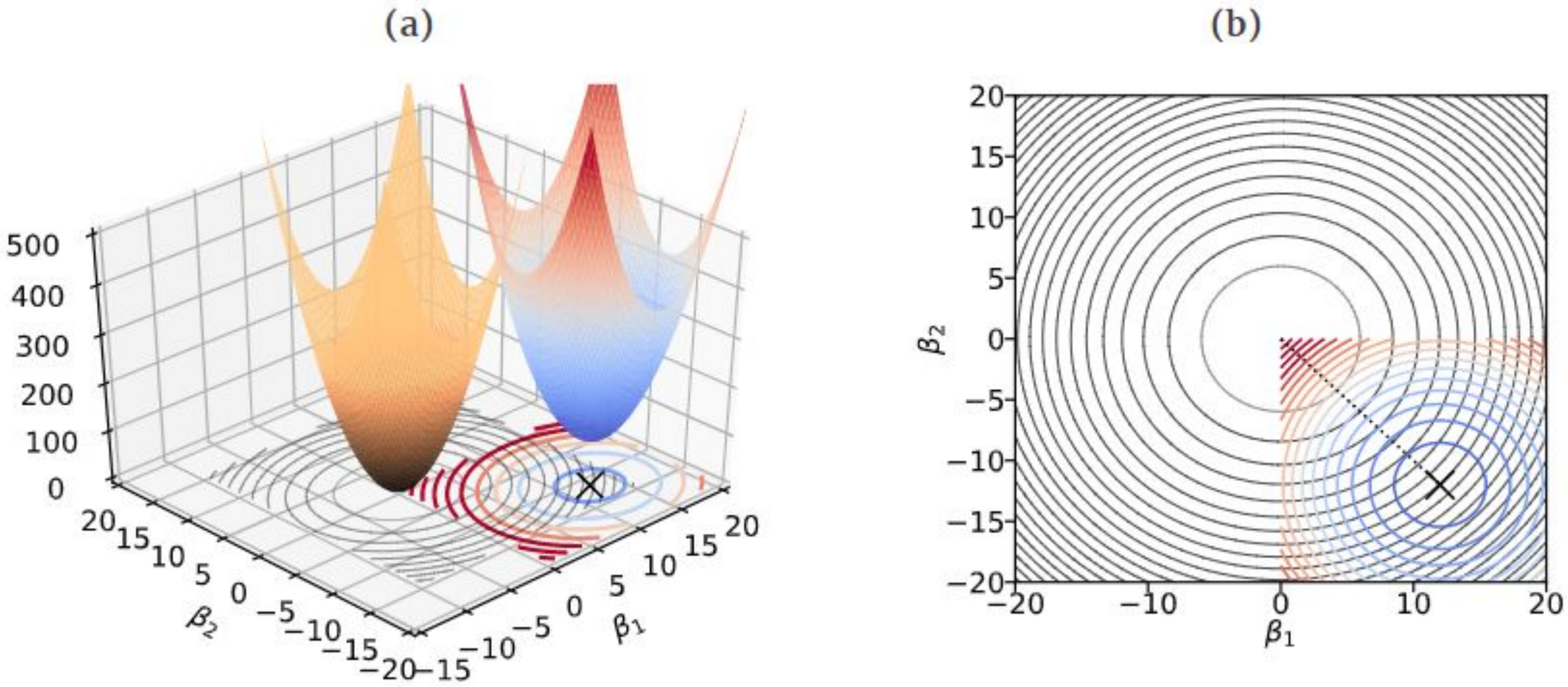


ElasticNet

$$(\alpha\lambda||\beta||_1 + \tfrac{1}{2}(1-\alpha)||\beta||_2^2)$$

¿Qué β se reduce más?

Regularización



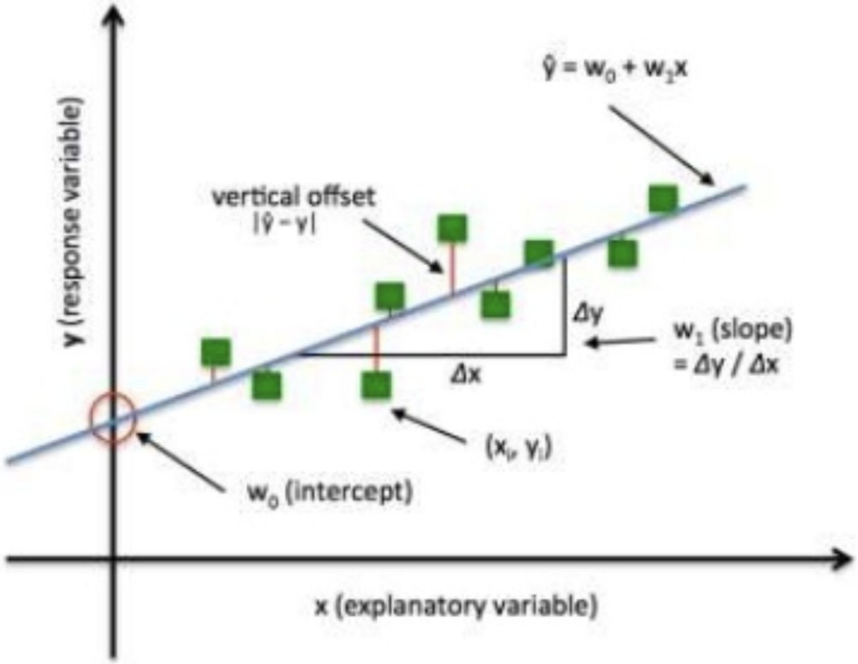
Gradiente Descendente

Implementación de Gradiente Descendente

Solucion analitica

$$\min_{W} \|Y - XW\|_2^2$$

$$W = (X^T X)^{-1} X^T Y$$

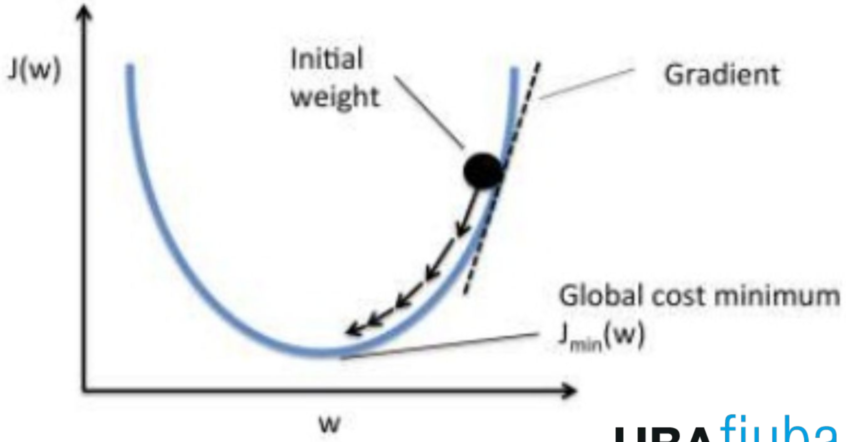


Implementación de Gradiente Descendente

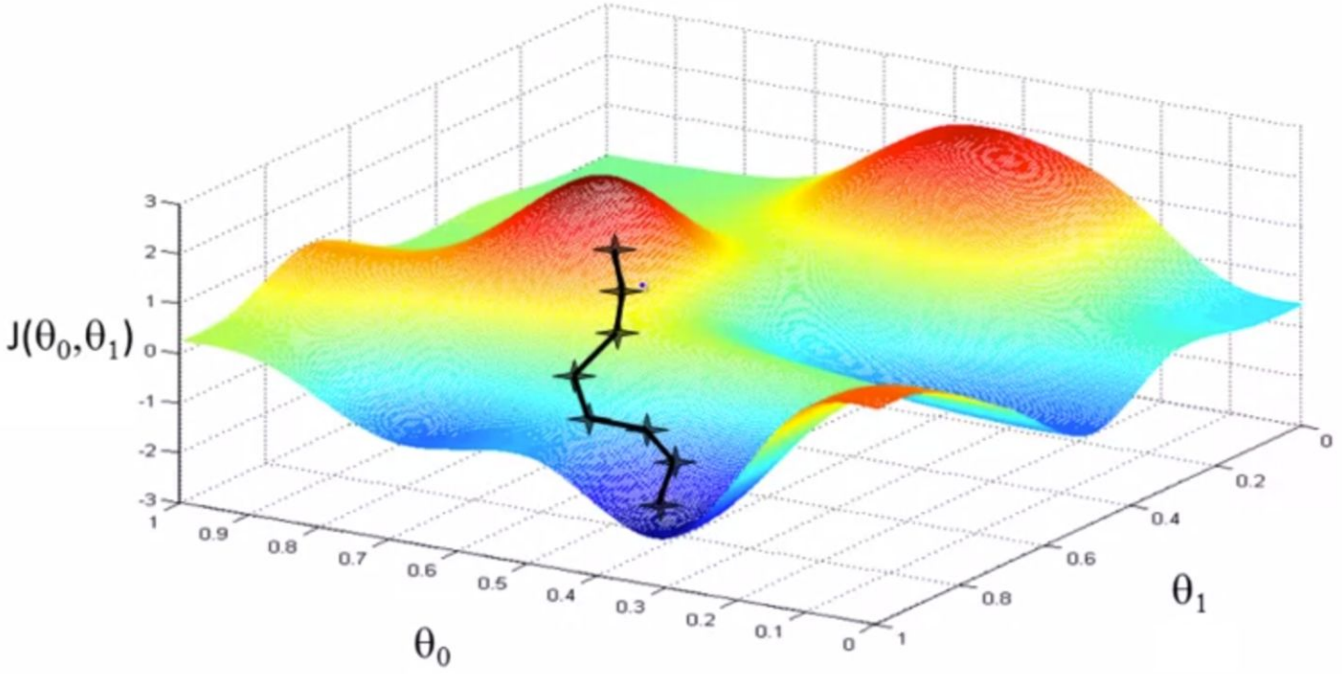
Solución numérica

$$\min_W \|Y - XW\|_2^2 \implies \min_W \sum_i \left(y_i - X_i \cdot W\right)^2$$

$$W \longleftarrow W - \alpha \nabla \left(\sum_i (y_i - X_i \cdot W)^2 \right)$$



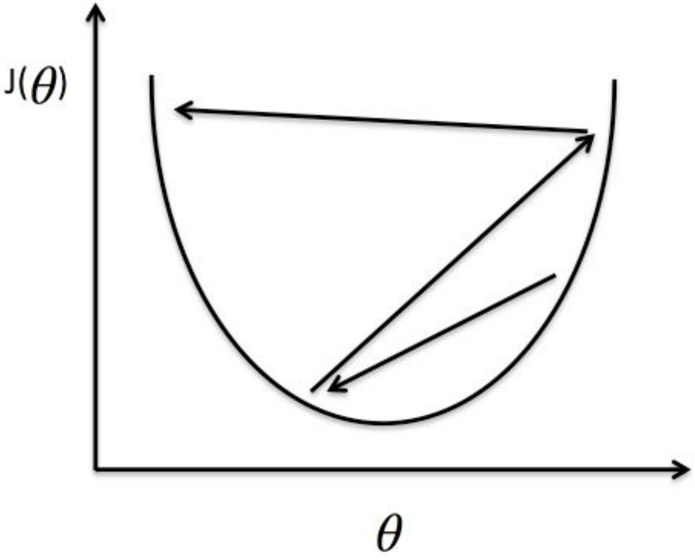
Gradiente Descendente



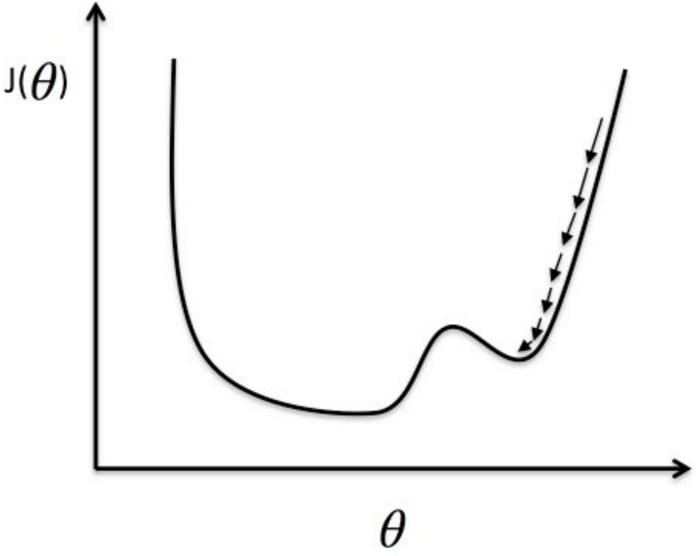
Andrew Ng

Gradient Descent

Gradiente Descendente



Large learning rate: Overshooting.



Small learning rate: Many iterations until convergence and trapping in local minima.

Implementación de Gradiente Descendente

Solución numérica

$$
\begin{aligned}
\nabla_w J(w) &= \nabla_w \Big(\sum_i (y_i - X_i W)^2 \Big) \\
&= \sum_i \Big(\nabla_w (y_i - X_i W)^2 \Big) \\
&= \sum_i \Big(\nabla_w (y_i - (x_{i1} w_1 + x_{i2} w_2 + \cdots + x_{im} w_m))^2 \Big) \\
&= \sum_i \Big(-2 (y_i - \hat{y}_i) x_{ij} \Big) \qquad \forall j \in (1 \cdots m)
\end{aligned}
$$

Implementación de Gradiente Descendente

Solución numérica

$$\nabla \left(\sum_{\text{all samples}} (y_i - f_W(X_i))^2 \right)$$

Gradient Descent algorithm

for epoch in n_epochs:

- compute the predictions for **all the samples**
- compute the error between truth and predictions
- compute the gradient using **all the samples**
- update the parameters of the model

Implementación de Gradiente Descendente Estocástico

Solución numérica

$$\nabla \left((y_i - f_W(X_i))^2 \right)$$

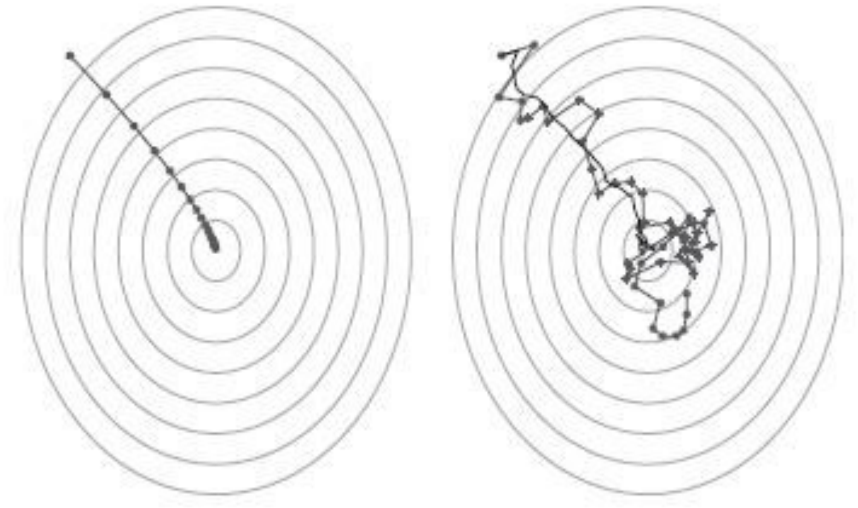
Stochastic Gradient Descent algorithm

for epoch in n_epochs:

- shuffle the samples
- for sample in n_samples:
    - compute the predictions for the sample
    - compute the error between truth and predictions
    - compute the gradient using the sample
    - update the parameters of the model

Implementación de Gradiente Descendente Estocástico

Solución numérica



Implementación de Gradiente Descendente Mini-Batch

Solución numérica

$$\nabla \left(\sum_{\text{batch samples}} (y_i - f_W(X_i))^2 \right)$$

Mini-Batch Gradient Descent algorithm

for epoch in n_epochs:

- shuffle the batches
- **for batch in n_batches:**
  - compute the predictions for **the batch**
  - compute the error for the batch
  - compute the gradient for **the batch**
  - update the parameters of the model

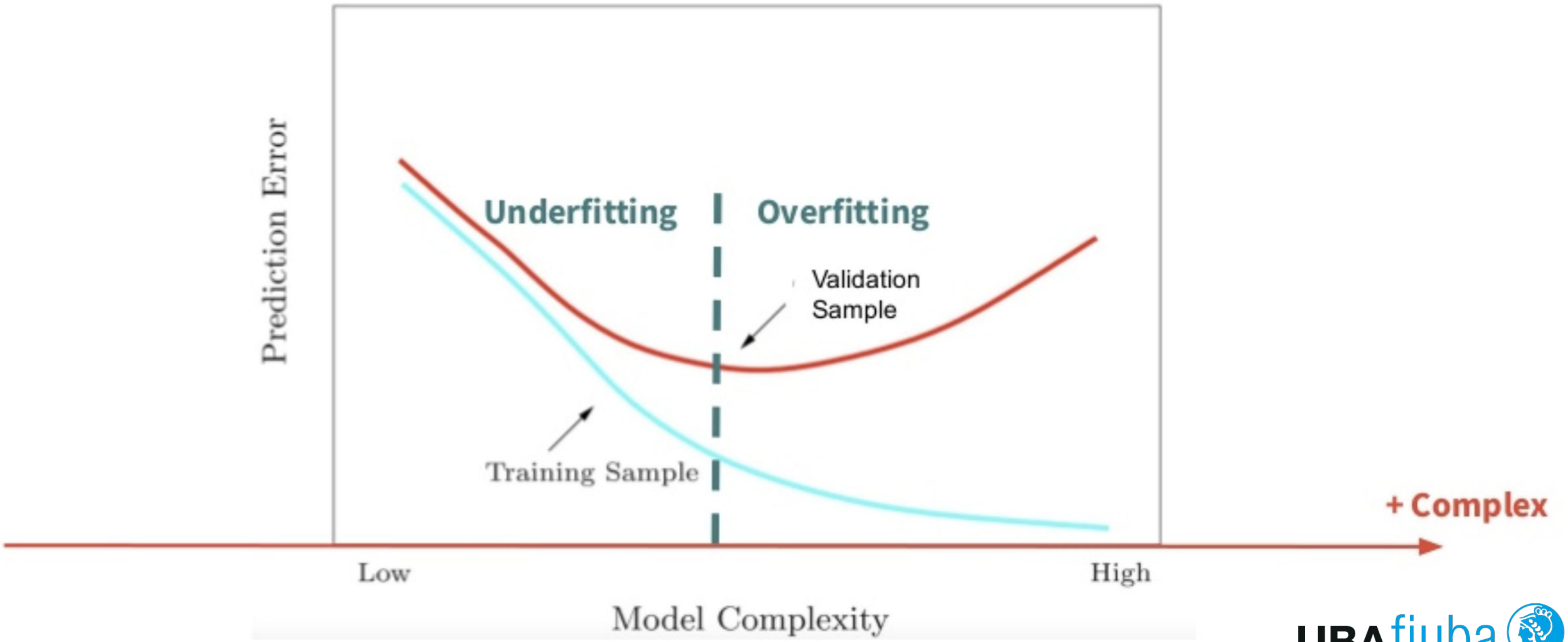
\sum_i \Big(-2 (y_i - \hat{y}_i) x_{ij} \Big)

Comparativa de gradientes

	Gradient Descent	Stochastic Gradient Descent	Mini-Batch Gradient Descent
Gradient	$\nabla \left(\sum_{\text{all samples}} (y_i - f_W(X_i))^2 \right)$	$\nabla \left((y_i - f_W(X_i))^2 \right)$	$\nabla \left(\sum_{\text{batch samples}} (y_i - f_W(X_i))^2 \right)$
Speed	Very Fast (vectorized)	Slow (compute sample by sample)	Fast (vectorized)
Memory	O(dataset)	O(1)	O(batch)
Convergence	Needs more epochs	Needs less epochs	Middle point between GD and SGD
Gradient Stability	Smooth updates in params	Noisy updates in params	Middle point between GD and SGD

Selección de modelos

Selección de modelos



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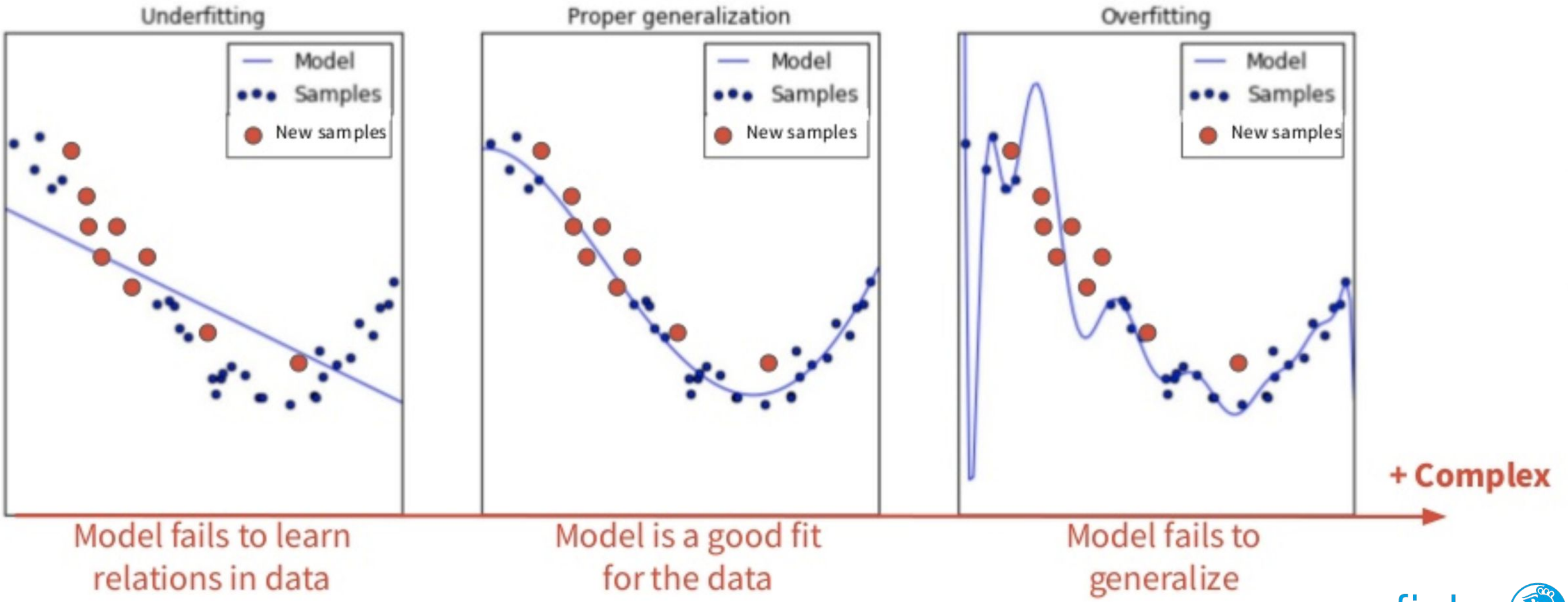
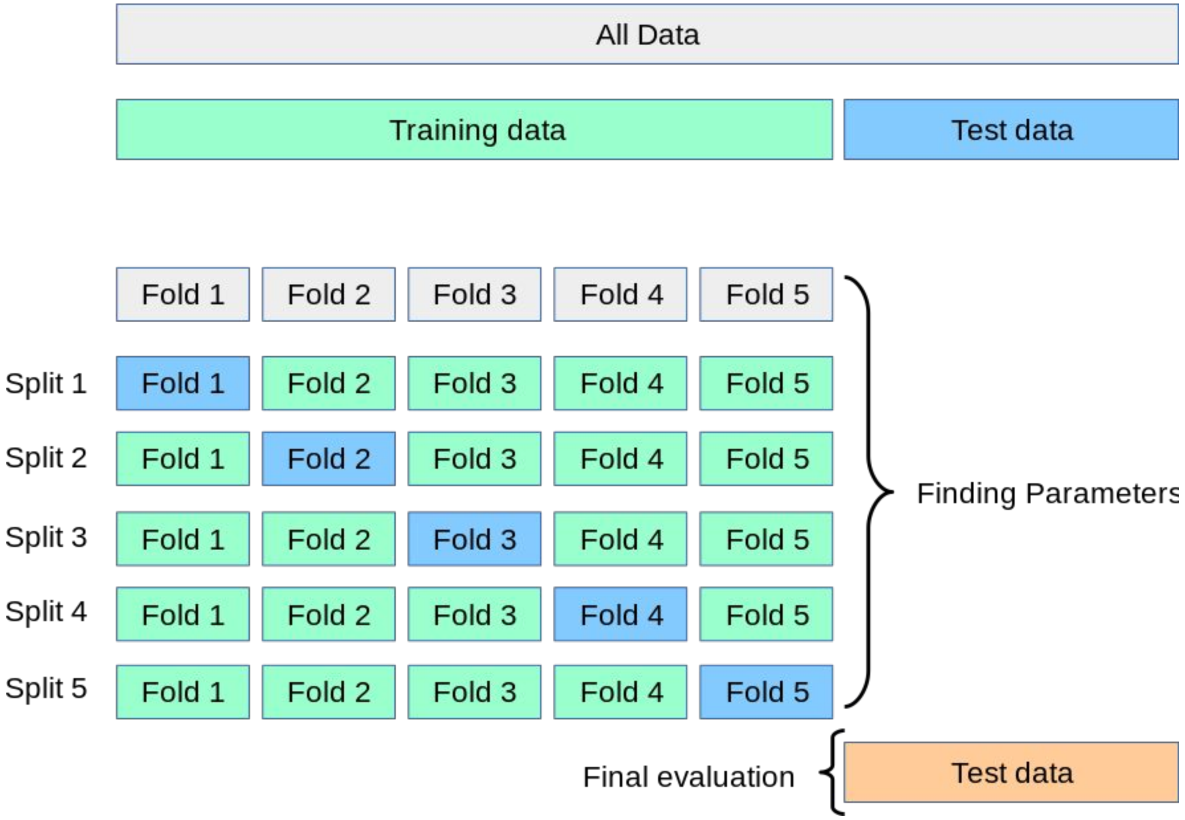
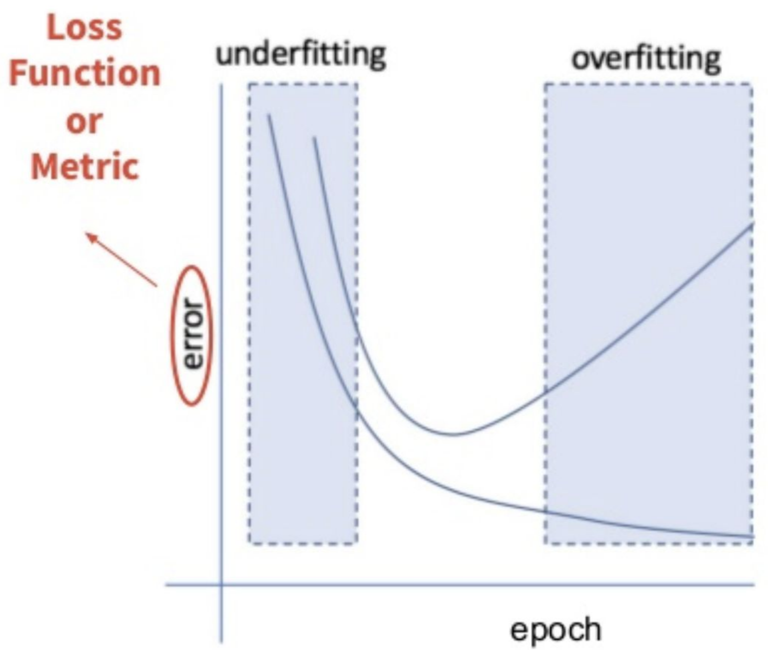


Cross-Validation



Entrenamiento numérico del modelo seleccionado - Obtención de parámetros

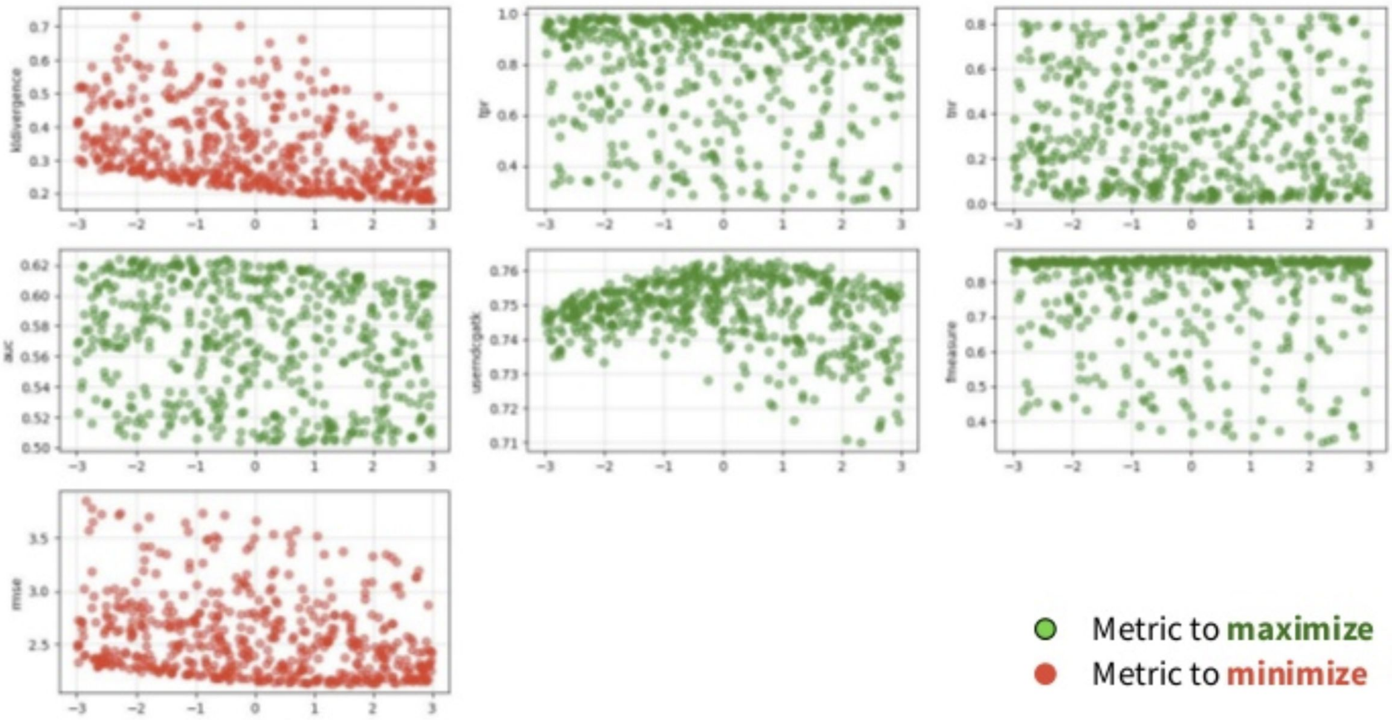


Mini-Batch Gradient Descent

for epoch in n_epochs:

- shuffle the batches
- for batch in n_batches:
  - compute the predictions for the batch
  - compute the error for the batch
  - compute the gradient for the batch
  - update the parameters of the model
- plot error vs epoch

Selección de los hiper parámetros



Grid Search

Random Search

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