Exercises chapter 1

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1. All horses are the same color; we can prove this by induction on the number of horses in a given set. Here's how: "If there's just one horsethen it's the same color as itself, so the basis is trivial. For the induction step, assume that there are n horses numbered 1 to n. By the induction hypothesis, horses 1 through $n-1$ are the same color, and similarly horses 2 through n are the same color. But the middle horses, 2 through $n-1$, can't change color when they're in different groups; these are horses, not chameleons. So horses 1 and n must be the same color as well, by transitivity. Thus all n horses are the same color; QED." What, if anything, is wrong with this reasoning?

Solution:
In the use of the condition $p(n-1) \to p(1)$. While it's true that we can say that if $p(n-1)$ then all horses numbered from 1 to $n-1$ have the same colour, we can't say that horses from 2 to n are the same colour, as the hourse numbered n is not considered as part of $p(n-1)$, only the first $n-1$ ones.

2. Find the shortest sequence of moves that transfers a tower of n disks from the left peg A to the right peg B, if direct moves between A and B are disallowed. (Each move must be to or from the middle peg. As usual, a larger disk must never appear above a smaller one.)

Solution:
Let's check first simple cases.

Assuming $T'(n)$ is the minimum number of steps required under the new rules and that the towers are set in the order $A \to C \to B$, we have that $T'(1) = 2$, given that we need to do the following sequence of movements: $A \to C$, $C \to B$.

Then we have that $T'(2) = 8$. The steps are "simple:" $A \to C$, $C \to B$ (up to here we move the smallest disk to B), $A \to C$ (move the largest disk to the middle tower), $B \to C$, $C \to A$ (move the smallest disk back to A), $C \to B$ (move the largest disk to B), $A \to C$, $C \to B$ (move the smallest disk to B).

From here, we can infer a solution in five steps:

3. Show that, in the process of transferring a tower under the restrictions of the preceding exercise, we will actually encounter every properly stacked arrangement of n disks on three pegs