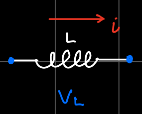
Goal: LC Circuit

- example of simple harmonic oscillator
  - what is it that's oscillating? Current and Voltage

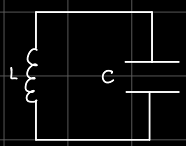
Inductors: also store energy, like capacitors, but they do so in the magnetic field

- usually cylindrical windings of wire
- 'inductance' L in Henries (H)
- device law:



$$V_L = L\frac{di}{dt}$$

Solving LC Circuit:



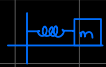
KVL: $V_L + V_C = 0$

$$L\frac{di}{dt} + V_C = 0$$

Recall $i = C\frac{dV_C}{dt}$ (device law, capacitors)

$$LC\frac{d^2V_C}{dt^2} + V_C = 0$$

$$\frac{d^2V_C}{dt^2} + \frac{1}{LC}V_C = 0 \quad \longleftrightarrow \quad \frac{d^2x}{dt^2} + \omega^2 x = 0$$

analogous to 

* See 'SHO Solution Derivation' in lesson 1 *

Sol^n form: $V_C(t) = A\cos(\omega t) + B\sin(\omega t)$

$$-A\omega^2\cos(\omega t) - B\omega^2\sin(\omega t) + \frac{1}{LC}\left[A\cos(\omega t) + B\sin(\omega t)\right] = 0$$

$$\frac{1}{LC}\left[A\cos(\omega t) + B\sin(\omega t)\right] = \omega^2\left[A\cos(\omega t) + B\sin(\omega t)\right]$$

$$\omega = \frac{1}{\sqrt{LC}}$$

- To solve for A & B, we use the initial conditions $V_C(t=0) = V_0$ and $i(t=0) = i_0$

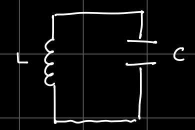
$V_C(t=0) = \underline{A = V_0}$

$$i(t) = C\frac{dV_C}{dt} = C\left[-A\omega\sin(\omega t) + B\omega\cos(\omega t)\right]$$

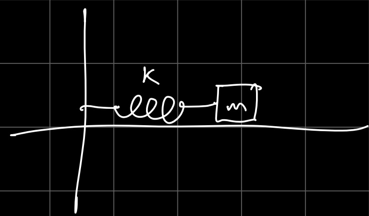
$$i(t=0) = CB\omega = i_0, \quad \underline{B = \frac{i_0}{\omega C}} = \frac{i_0\sqrt{LC}}{C} = i_0\sqrt{\frac{L}{C}}$$

$$\boxed{V_C(t) = V_0\cos\left(\frac{t}{\sqrt{LC}}\right) + i_0\sqrt{\frac{L}{C}}\sin\left(\frac{t}{\sqrt{LC}}\right)}$$

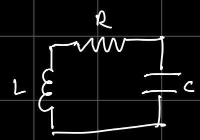
1) LC Circuit

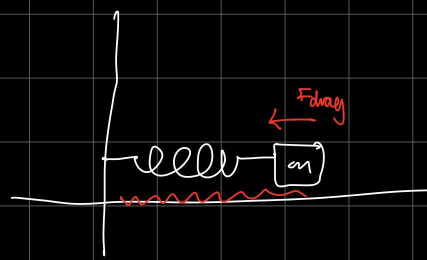


$v_c(t) = \cdots$

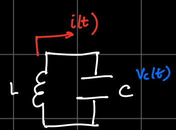


2) RLC Circuit

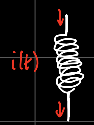




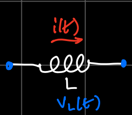
1) LC Circuit



- Inductor / solenoid



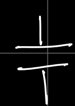
store energy (current) in the magnetic field



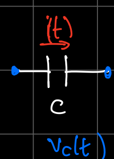
device law:

$V_L = -L \dfrac{di(t)}{dt}$

- capacitor

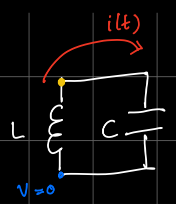


store energy electric field



device law

$i(t) = C \dfrac{dV_c(t)}{dt}$



$$V_L + V_C = 0$$

$$-L\frac{di(t)}{dt} - V_c = 0$$

Device cap: $i(t) = C\frac{dV_c}{dt}$

$$-L\frac{d}{dt}\left[C\frac{dV_c}{dt}\right] - V_c = 0$$

$$-LC\frac{d^2V_c}{dt^2} - V_c = 0$$

$$\downarrow$$

standard form

$$\frac{d^2V_c}{dt^2} + \frac{1}{LC}V_c = 0$$

$$\frac{d^2V_c}{dt^2} + \frac{1}{LC}V_c = 0 \longleftrightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

guess $x(t) = A\sin(\omega t) + B\cos(\omega t)$

$\frac{d^2}{dt^2}$

guess: $V_c(t) = A\sin(\omega t) + B\cos(\omega t)$

$$\frac{dV_c}{dt} = A\omega\cos(\omega t) - B\omega\sin(\omega t)$$

$$\frac{d^2V_c}{dt^2} = -A\omega^2\sin(\omega t) - B\omega^2\cos(\omega t)$$

$$-A\omega^2\sin(\omega t) - B\omega^2\cos(\omega t) + \frac{1}{LC}\left(A\sin(\omega t) + B\cos(\omega t)\right) = 0$$

$$\frac{1}{LC}\left(A\sin(\omega t) + B\cos(\omega t)\right) = A\omega^2\sin(\omega t) + B\omega^2\cos(\omega t)$$

$$\frac{1}{LC}\left(A\sin(\omega t) + B\cos(\omega t)\right) = \omega^2\left(A\sin(\omega t) + B\cos(\omega t)\right)$$

$$\frac{1}{LC} = \omega^2$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$v_c(t) = \underline{A}\sin(wt) + \underline{B}\cos(wt)$, $w = \frac{1}{\sqrt{LC}}$

free [m] $w = \sqrt{\frac{k}{m}}$

x_0
v_0

$v_0 = v_c(t=0) = ?$

$i_0 = i(t=0) = ?$

$v_c(t=0) = v_0 = B$

$v_c(t) = \underline{A\sin(wt)} + \underline{v_0\cos(wt)}$

device law $i(t) = C\frac{dv_c(t)}{dt}$

$$i(t) = C\left(\underline{Aw\cos(wt)} - v_0 w\sin(wt)\right)$$

$$i(t=0) = C\,(Aw + 0) = i_0$$

$$i_0 = CwA$$

$$A = \frac{i_0}{Cw}$$

master eq^{n}!

$$\boxed{v_c(t) = \frac{i_0}{Cw}\sin(wt) + v_0\cos(wt), \quad w = \frac{1}{\sqrt{LC}}}$$

$$i(t) = C\frac{dv_c(t)}{dt} = C\frac{d}{dt}\left[\frac{i_0}{Cw}\sin(wt) + v_0\cos(wt)\right]$$

$$= C\left[\frac{i_0}{C}\cos(wt) - v_0 w\sin(wt)\right]$$

$$\boxed{i(t) = i_0\cos(wt) - v_0 wC\sin(wt)}$$

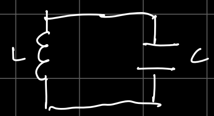
How is the <u>energy</u> $\underbrace{\text{moving around}}_{\text{oscillates}}$ in this circuit?

$E_{tot} = E_C + E_L$

$$\boxed{E_C = \frac{1}{2}CV_c(t)^2}$$

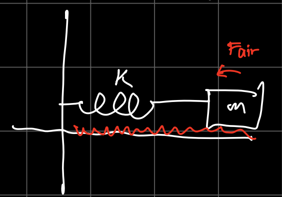
$$\boxed{E_L = \frac{1}{2}L\,i(t)^2}$$

LC Circuit

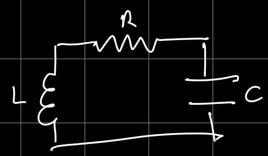


Spring-mass

Damped spring mass



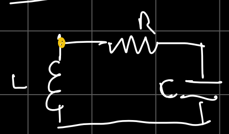
RLC Circuit



Can we use "Damped spring mass - sys" to solve RLC

RLC	Damped spring mass

RLC



KVL: $V_L + V_R + V_C = 0$

$$-L\frac{di(t)}{dt} - i(t)R - V_c = 0$$

$$i(t) = C\frac{dV_c}{dt}$$

$$-LC\frac{d^2V_c}{dt^2} - RC\frac{dV_c}{dt} - V_c = 0$$

$$LC\frac{d^2V_c}{dt^2} + RC\frac{dV_c}{dt} + V_c = 0$$

$$\frac{d^2V_c}{dt^2} + \underbrace{\frac{R}{L}}_{\Gamma}\frac{dV_c}{dt} + \frac{1}{LC}V_c = 0$$

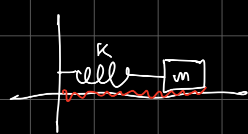
Sol^n

$$V_c(t) = e^{-\frac{R}{2L}t}\left(V_0\cos(\omega_d t) + \gamma\sin(\omega_d t)\right)$$

$$\omega_d = \sqrt{\omega^2 - \frac{1}{4}(R/L)^2}, \quad \omega = \frac{1}{\sqrt{LC}}$$

$$\gamma = \frac{i_0 - \frac{R}{2L}V_0}{\omega_d}$$

Damped spring mass



$F = ma$

$$F_S + F_{damp} = m\frac{d^2x}{dt^2}$$

damp coeff

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$\frac{d^2x}{dt^2} + \underbrace{\frac{b}{m}}_{\Gamma}\frac{dx}{dt} + \frac{k}{m}x = 0$$

Sol^n

$$x(t) = e^{-\frac{\Gamma}{2}t}\left(x_0\cos(\omega_d t) + \gamma\sin(\omega_d t)\right)$$

$$\gamma = \frac{V_0 - \frac{\Gamma}{2}x_0}{\omega_d}$$

$$\omega_d = \sqrt{\omega^2 - \frac{\Gamma^2}{4}}$$

$$\Gamma = \frac{b}{m}$$