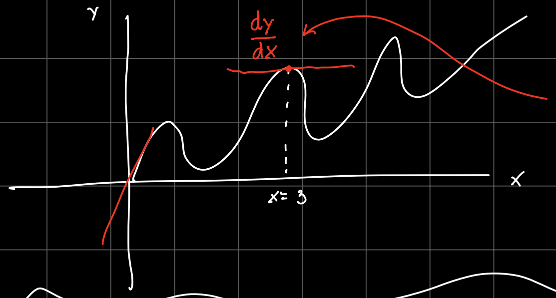
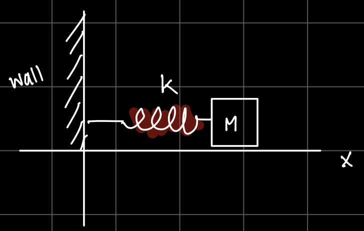
Derivative: instantaneous change in a $f^{\underline{n}}$



derivative of $f^{\underline{n}}$ y with respect to x

Simple Harmonic Oscillator



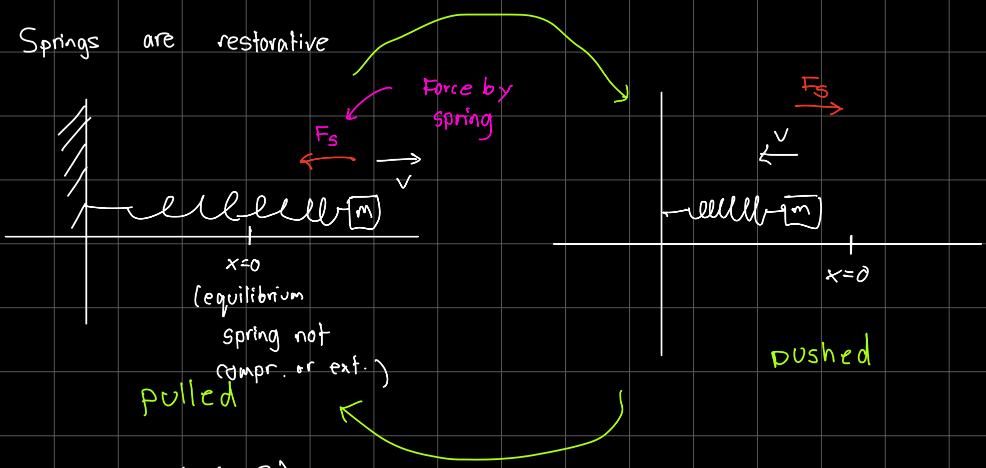
$x(t)$ ← pos in time of the mass

M – mass (kg)

k – spring constant $\left(\frac{N}{m}\right)$

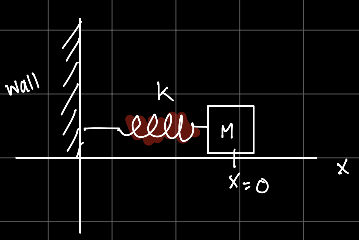
- stiffness of the spring

Springs are restorative



– Oscillator (Why?)

- Cycling betw. 2 extremes



sum of all the forces acting on mass m

N2L: $\vec{F}_{net} = m\vec{a}$ ← acceleration of mass

$F_{net,x} = m\,a_x$ ← accel along x

F_{net} along x

$F_s = Ma_x$

$-kx = Ma_x$

displacement

$-kx(t) = Ma_x(t)$

calculus $\longleftrightarrow$ physics

what is accel: $\frac{d^2x}{dt^2} = \frac{dv}{dt}$

accel: change in velocity over time

velocity: change in displacement (here also pos.) in time

accel: change in change in displacement (here also pos.) in time in time

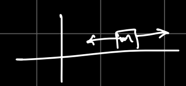
"tiny" "infinitessimal" change

avg vel: $\frac{\Delta x}{\Delta t}$ inst. vel = $\frac{dx}{dt}$

$$a_x(t) = \text{accel} = \frac{dv}{dt} = \frac{d}{dt}\left[\frac{dx}{dt}\right] = \frac{d^2x}{dt^2}$$

x x der

Master eqn: $\underbrace{-k\underline{x(t)} = M \frac{d^2x(t)}{\underline{dt^2}}}_{\text{solve for } x(t)}$



claim: $x(t) = A\cos(\omega t) + B\sin(\omega t)$

"omega"
angular freq.
"rate of our oscillations"

$$-\frac{k}{M}\, x(t) = \frac{d^2x(t)}{dt^2}$$

$$-\frac{K}{M}\left[A\cos(\omega t) + B\sin(\omega t)\right] = \frac{d^2}{dt^2}\left[A\cos(\omega t) + B\sin(\omega t)\right]$$

$$= \frac{d}{dt}\left[\underbrace{\frac{d}{dt}\left[\underline{A\cos(\omega t)} + B\sin(\omega t)\right]}_{1^{st}\text{ deriv}}\right]$$

$$= \frac{d}{dt}\left[\underbrace{-\omega A\sin(\omega t) + \omega B\cos(\omega t)}_{1^{st}\text{ deriv}}\right]$$

$$= -\omega^2 A\cos(\omega t) - \omega^2 B\sin(\omega t)$$

$$-\frac{K}{M}\left[A\cos(\omega t) + B\sin(\omega t)\right] = -\omega^2\left[A\cos(\omega t) + B\sin(\omega t)\right]$$

$$\frac{K}{m} = \omega^2$$

$$\omega = \sqrt{\frac{K}{M}}$$

$$x(t) = A\cos\left(\sqrt{\frac{K}{M}}\,t\right) + B\sin\left(\sqrt{\frac{K}{m}}\,t\right)$$

A & B come from initial conditions ICs of the system

2 unknowns A & B: 2 initial conditions

- initial position x_0
- initial velocity v_0

$$x(t) = A\cos\left(\sqrt{\frac{K}{M}}\,t\right) + B\sin\left(\sqrt{\frac{K}{m}}\,t\right)$$

$$x_0 = x(t=0) = A\cos\left(\sqrt{\frac{K}{M}}\cdot 0\right) + B\sin\left(\sqrt{\frac{K}{m}}\cdot 0\right)$$

$$x_0 = x(t=0) = A$$

$$x(t) = x_0\cos\left(\sqrt{\frac{K}{M}}\,t\right) + B\sin\left(\sqrt{\frac{K}{m}}\,t\right)$$

$$v(t) = -x_0\sqrt{\frac{K}{m}}\sin\left(\sqrt{\frac{K}{m}}\,t\right) + B\sqrt{\frac{K}{m}}\cos\left(\sqrt{\frac{K}{m}}\,t\right)$$

$$v_0 = v(t=0) = -x_0\sqrt{\frac{K}{m}}\sin\left(\sqrt{\frac{K}{m}}\cdot 0\right) + B\sqrt{\frac{K}{m}}\cos\left(\sqrt{\frac{K}{m}}\cdot 0\right)$$

$$0 + B\sqrt{\frac{K}{m}}$$

$$v_0 = B\sqrt{\frac{K}{m}}$$

$$B = \frac{v_0}{\sqrt{\frac{K}{m}}}$$

$$x(t) = x_0\cos\left(\sqrt{\frac{K}{M}}\,t\right) + \frac{v_0}{\sqrt{\frac{K}{m}}}\sin\left(\sqrt{\frac{K}{m}}\,t\right)$$

$$\boxed{\begin{gathered} x(t) = x_0 \cos(\omega t) + \frac{v_0}{\omega} \sin(\omega t) \\ \text{where } \omega = \sqrt{\frac{k}{m}} \end{gathered}}$$