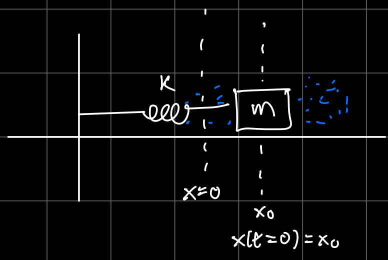


$F = ma$

$F_s = ma$

$F_s = -kx$ (Hooke's Law)

$$x(t) = x_0 \cos(\omega t) + \frac{v_0}{\omega} \sin(\omega t)$$

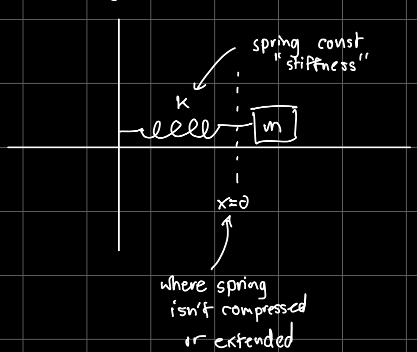
'omega'
angular
frequency

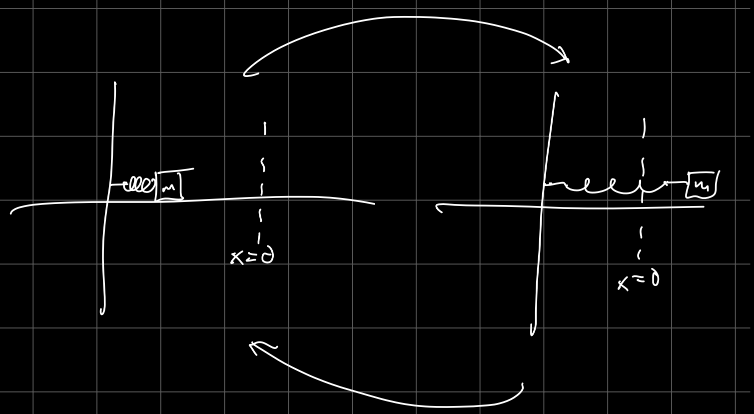
How can we make our model better?
think abt

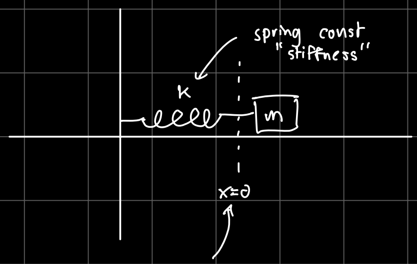
- friction
  - energy lost as heat
- is it possible $k(x)$? $k(t)$?
- sound → air drag as we move it
- assuming: pt. particle → rigid body $\not\to$ deformations

$$\left.\begin{aligned} &F_f \quad \text{force of friction} \\ &F_{drag} \quad \text{force of drag} \end{aligned}\right\} F_{damp}$$

Spring-mass system







DE

$F_{net} = ma = m\dfrac{d^2x}{dt^2}$

$F_s = m\dfrac{d^2x}{dt^2}$

$-kx = m\dfrac{d^2x}{dt^2}$ Last time

$\frac{d^2}{dt^2}$

$x(t) = x_0\cos(\omega t) + \frac{v_0}{\omega}\sin(\omega t)$

new!

$-kx + \underline{F_{damp}} = m\dfrac{d^2x}{dt^2}$

$F_k = \mu_k mg$

$F_s \leq \mu_s mg$ constant

Fdamp must change w/ velocity

$F_{damp} = -bv = -b\dfrac{dx}{dt}$

$-kx - b\dfrac{dx}{dt} = m\dfrac{d^2x}{dt^2}$

damping coeff

2 der

$\dfrac{d^2x}{dt^2} = \ddot{x}$

$\dfrac{dx}{dt} = \dot{x}$

$x = x(t)$

$-kx - b\dot{x} = m\ddot{x}$

$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$

$\omega = \sqrt{\frac{k}{m}} \rightarrow \frac{k}{m} = \omega^2$

$\Gamma = \frac{b}{m}$

'gamma'
damping term

$\ddot{x} + \Gamma\dot{x} + \omega^2 x = 0$

$\ddot{x} + \underline{\Gamma\dot{x}} + \omega^2 x = 0$

$\ddot{x} + \omega^2 x = 0$ (no damping) $\xrightarrow{\text{sol}^n?}$ $x(t) = x_0\cos(\omega t) + \frac{v_0}{\omega}\sin(\omega t)$

$\ddot{x} + \Gamma\dot{x} + \omega^2 x = 0$

Ansatz (german for approach)

→ guess

lambda

$x(t) = \underline{Ae^{\lambda t + \phi}}$

oscillations!

$$\ddot{x} + \Gamma\dot{x} + \omega^2 x = 0$$

2nd 1st

$$x(t) = \underline{Ae^{\lambda t + \phi}}$$

$$\dot{x} = A\lambda e^{\lambda t + \phi}$$

$$\ddot{x} = A\lambda^2 e^{\lambda t + \phi}$$

$$\underbrace{A\lambda^2 e^{\lambda t + \phi}}_{\times} + \Gamma \underbrace{A\lambda e^{\lambda t + \phi}}_{\times} + \omega^2 \underbrace{Ae^{\lambda t + \phi}}_{\times} = 0$$

$$\lambda^2 + \Gamma\lambda + \omega^2 = 0$$

what is λ?

$$\lambda_\pm = \frac{-\Gamma \pm \sqrt{\Gamma^2 - 4\omega^2}}{2}$$

$$\frac{1}{2} = \frac{1}{\sqrt{4}}$$

$$-\frac{\Gamma}{2} \pm \frac{1}{2}\sqrt{\Gamma^2 - 4\omega^2}$$

$$= -\frac{\Gamma}{2} \pm \sqrt{\frac{\Gamma^2}{4} - \omega^2}$$

$$= -\frac{\Gamma}{2} \pm \sqrt{-\left(\omega^2 - \frac{\Gamma^2}{4}\right)}$$

$$= -\frac{\Gamma}{2} \pm \sqrt{-1}\sqrt{\omega^2 - \frac{\Gamma^2}{4}}$$

$$\lambda_\pm = -\frac{\Gamma}{2} \pm i\underbrace{\sqrt{\omega^2 - \frac{\Gamma^2}{4}}}$$

3 scenario

oscillations damping

1) $\omega^2 > \frac{\Gamma^2}{4}$ underdamped

2) $\omega^2 < \frac{\Gamma^2}{4}$ overdamped

3) $\omega^2 = \frac{\Gamma^2}{4}$ critically damped

ω = angular freq, $\Gamma \equiv$ damping

$\omega = \frac{\sqrt{k}}{\sqrt{m}} \longleftrightarrow \Gamma = \frac{b}{m}$

$k \equiv$ spring constant 'stiffness'

$$x(t) = \underline{Ae^{\lambda t + \phi}} \quad \begin{matrix} Ae^{\lambda_+ t + \phi} \\ Ae^{\lambda_- t + \phi} \end{matrix}$$

superposition in DE

$$x(t) = Ae^{\lambda_+ t + \phi} + Ae^{\lambda_- t + \phi}$$

$$x(t) = Ae^{\overbrace{\left(-\frac{\Gamma}{2} + i\sqrt{\omega^2 - \frac{\Gamma^2}{4}}\right)}^{\lambda_+} t + \phi} + Ae^{\overbrace{\left(-\frac{\Gamma}{2} - i\sqrt{\omega^2 - \frac{\Gamma^2}{4}}\right)}^{\lambda_-} t + \phi}$$

damped freq $\longrightarrow$ $\omega_d = \sqrt{\omega^2 - \frac{\Gamma^2}{4}}$

$$Ae^{-\frac{\Gamma}{2}t}\left(e^{i\omega_d t+\phi} + e^{-i\omega_d t+\phi}\right)$$

Euler's equation

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$Ae^{-\frac{\Gamma}{2}t}\left(\underbrace{\cos(\omega_d t+\phi) + i\sin(\omega_d t+\phi)}_{1^{st}\ \exp} + \underbrace{\cos(\omega_d t+\phi) - i\sin(\omega_d t+\phi)}_{2^{nd}\ \exp}\right)$$

$$x(t) = Ae^{-\frac{\Gamma}{2}t}\, 2\cos(\omega_d t+\phi)$$

$D = 2A$

$$e^{-\frac{\Gamma}{2}t} D\cos(\omega_d t+\phi)$$

amp phase form

trig identities

$$x(t) = e^{-\frac{\Gamma}{2}t}\left(a\cos(\omega_d t) + b\sin(\omega_d t)\right)$$

??

insert initial conditions

Last time $\underbrace{x_0}_{a}\cos(\omega t) + \underbrace{\frac{v_0}{\omega}}_{b}\sin(\omega t)$

$$x(t) = \underbrace{e^{-\frac{\Gamma}{2}t}}_{f(t)}\underbrace{\left(x_0\cos(\omega_d t) + b\sin(\omega_d t)\right)}_{g(t)}$$

$$x_0 = x(t=0) = a + 0 = a = x_0$$

$$v(t=0) = \dot{x}(t=0) = ?$$

$$v(t) = \dot{x}(t) = \dot{f}(t)\, g(t) + f(t)\, \dot{g}(t)$$

$$= \left(-\frac{\Gamma}{2}e^{-\frac{\Gamma}{2}t}\right)\left(x_0\cos(\omega_d t) + b\sin(\omega_d t)\right)$$

$$+ e^{-\frac{\Gamma}{2}t}\left[-\omega_d x_0\sin(\omega_d t) + \omega_d b\cos(\omega_d t)\right]$$

$$v_0 = v(t=0) = \dot{x}(t=0) = \left(-\frac{\Gamma}{2}e^{-\frac{\Gamma}{2}t}\right)\left(x_0\cos(\omega_d t) + b\sin(\omega_d t)\right)$$

$$+ e^{-\frac{\Gamma}{2}t}\left[-\omega_d x_0\sin(\omega_d t) + \omega_d b\cos(\omega_d t)\right]$$

$$-\frac{\Gamma}{2}x_0 + 1\left(0 + \omega_d b\right)$$

$$v_0 = -\frac{\Gamma}{2}x_0 + \omega_d b$$

$$b = \frac{v_0 - \frac{\Gamma}{2}x_0}{\omega_d}$$

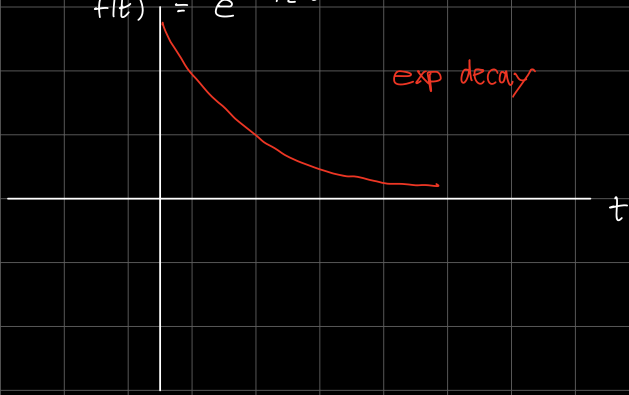
$$x(t) = e^{-\frac{\Gamma}{2}t}\left(x_0\cos(\omega_d t) + b\sin(\omega_d t)\right)$$

$$b = \frac{v_0 - \frac{\Gamma}{2}x_0}{\omega_d}$$

$$\omega_d = \sqrt{\omega^2 - \frac{\Gamma^2}{4}}$$

$$\Gamma = \frac{b}{m}$$

$$f(t) = e^{-\Gamma/2\, t}$$



$$f(t) = a\cos(\omega t) + b\sin(\omega t)$$