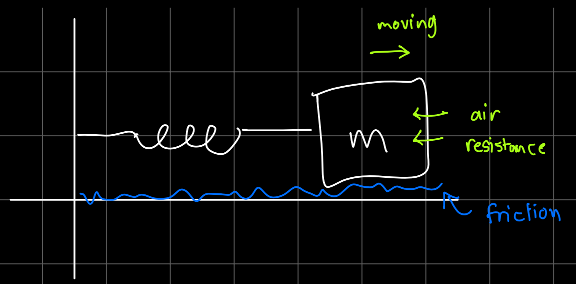


damping

$$x(t) = e^{-\frac{\Gamma}{2}t}\left(x_0\cos(\omega_d t) + b\sin(\omega_d t)\right)$$

$$b = \frac{v_0 \sim \frac{\Gamma}{2}x_0}{\omega_d}$$

$$\omega_d = \sqrt{\omega^2 - \frac{\Gamma^2}{4}}$$

$$\Gamma = \frac{b}{m}$$

no damping

$$x(t) = x_0\cos(\omega t) + \frac{v_0}{\omega}\sin(\omega t)$$

$$\omega = \sqrt{\frac{k}{m}}$$

dampening term

$$\omega_d = \sqrt{\omega^2 - \frac{\Gamma^2}{4}}$$

$$\omega$$

$$\lim_{\Gamma \to 0} \omega_d = \sqrt{\omega^2}$$

$$\omega_d = \omega$$

what happens $\Gamma \to 0$

Damped sol^n : $\Gamma \to 0$

$$x(t) = e^{-\frac{\Gamma}{2}t}\left(x_0\cos(\omega_d t) + b\sin(\omega_d t)\right)$$

$$x(t) = x_0\cos(\omega t) + \frac{v_0}{\omega}\sin(\omega t)$$

$$b = \frac{v_0 \cancel{\sim \frac{\Gamma}{2}x_0}}{\omega_d}$$

$$b = \frac{v_0}{\omega_d} = \frac{v_0}{\omega}$$

$$F = m\ddot{x} \qquad \ddot{x} = \frac{d^2x}{dt^2}$$

$$-kx - b\dot{x} = m\ddot{x}$$

$$[N] \quad [N] \quad [N]$$

$$[N] = \left[\frac{kg\,m}{s^2}\right]$$

$F = ma$

$$[kg]\left[\frac{m}{s^2}\right]$$

$$F \to \left[\frac{kg\,m}{s^2}\right]$$

$b\dot{x}$ velocity $\frac{dx}{dt}$

$$[N]$$

$$\left[\frac{kg}{s}\right] \quad \left[\frac{m}{s}\right]$$

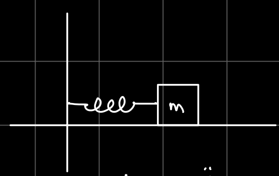
$$[kgm]$$

$b \to$ units of $\frac{kg}{s}$

$-\left[\frac{}{s^2}\right]$

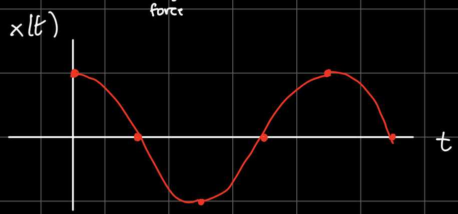
Mechanics

Simple Harmonic Motion

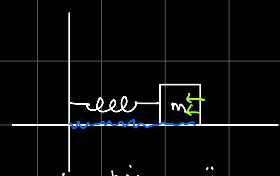


$-kx = m\ddot{x}$

spring force



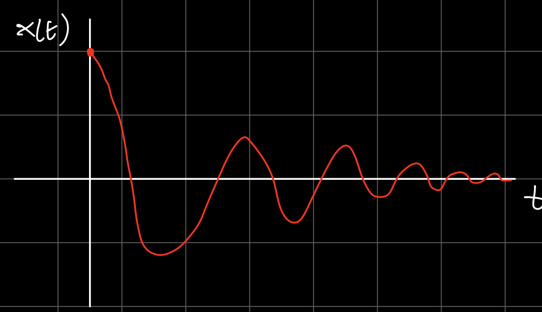
Damped Harmonic Motion



$-kx - b\dot{x} = m\ddot{x}$

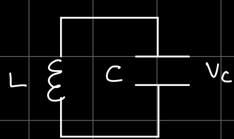
spring force

damping force

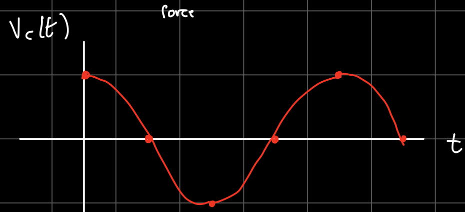


Circuits

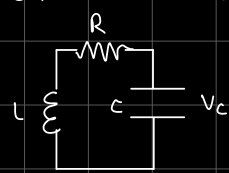
SHM : LC Circuit

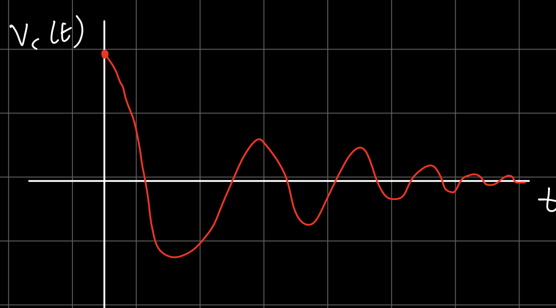


force

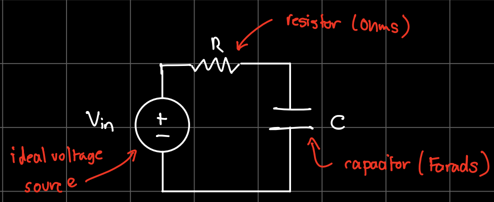


DHM : RLC Circuit





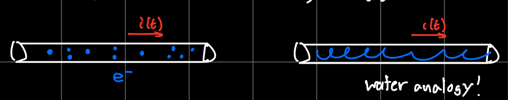
Goal: RC Circuit



<u>Voltage, Current, & Resistance</u>

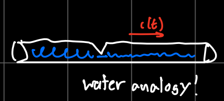
1) Current $i(t)$ units: amps

- flow of charge particles (in a wire) $\frac{dQ}{dt}$ ← electric charge (+, −)

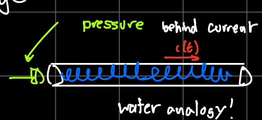


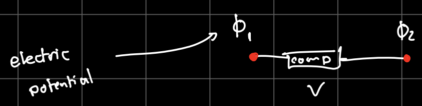
2) Resistance R (ohms)

- opposes current, dissipates energy



3) Voltage

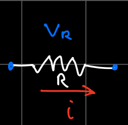




$V = \phi_2 - \phi_1$

Device law: how do we relate voltage across & current through

- Resistor



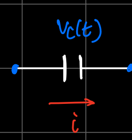
$V_R = iR$ (Ohm's law)

- Capacitor

↳ store electric charge → $v_c(t)$

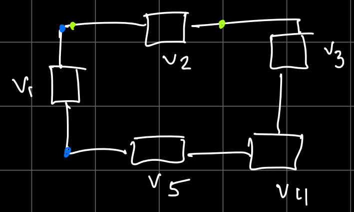
↳ capacitance $C = \frac{Q}{V}$ ← electric charge, ← voltage

$VC = Q$

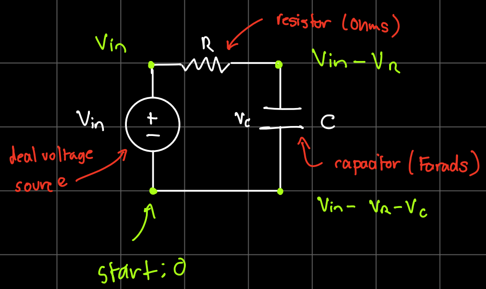


$i = C\,\frac{dV_C(t)}{dt}$

Kirchoff's Voltage Law (Loop Rule)



$V_1 + V_2 + V_3 + V_4 + V_5 = 0$



walk around circuit

$0 = V_{in} + V_R + V_c$

↑
positive

$0 = V_{in} - iR - V_c$

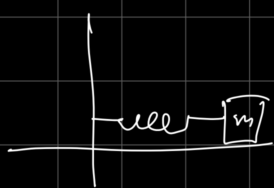
$i = C \frac{dV_c}{dt}$

$0 = V_{in} - RC \frac{dV_c}{dt} - V_c$

$V_{in} = V_c + RC \frac{dV_c}{dt}$ ← 1^{st} orde

Solve for V_c

inhomogenous DE



$0 = w^2 x + \ddot{x}$ ← 2^{nd} order

Solve for x

homogenous DE

$V_c(t) = V_{in} + C_1 e^{-\frac{t}{RC}}$

$V_c(t=0) = V_{in} + C_1 = V_0$ ← initial voltage in cap

$C_1 = V_0 - V_{in}$

$$\boxed{V_c(t) = V_{in} + (V_0 - V_{in})\, e^{-\frac{t}{RC}}}$$