A Primer in Quantum Computing
Lesson 1: Qubits, Quantum Mechanics

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1 Introduction

Toward the beginning of class, we motivated the need for quantum computers, but how exactly do we get there?

In quantum computing, *qubits* are the fundamental unit of computation, which means that all the stuff (calculations, algorithms) performed by a quantum computer boil-down, perhaps in a complicated way, to what the qubits are doing.

2 Qubits

Qubits are short for ”quantum-bits,” which to fully appreciate the name of, you need to somewhat know about regular, ”non-quantum” *bits.*

2.1 Bits and Classical Computing

Classical computers, like the one I'm using to write these notes up (just a Mac Book), use bits to do all the things you associate with your computer. We often think of bits as being in the ”on” or ”off”, like a switch; ”on” we label as 1, and ”off” we label as 0. **The state of a bit is always either 0 or 1!!!** Weirdly, we'll find that this isn't the case for *quantum* bits.

Bits by themselves don't appear very powerful, but when strung together, are simple ways to encode information. For example, ”01110001 01110101 01100001 01101110 01110100 01110101 01101101” encodes the word ”quantum.” The story will be the same for qubits: one qubit isn't particularly impressive, but many together could solve the problems we talked about at the beginning!

2.2 Qubits and Quantization

The textbook definition of a qubit is a ”two-state quantum system.” Sounds vague, and not very precise. Some questions that come to mind:

- Can anything (like a coin flip) be a qubit? If not, why?

- How does one *make* a qubit?
- What is a quantum system?

The answers to some of these questions are an active area of research! But let's lay the foundations for each of these briefly:

2.2.1 What is a quantum system?

Quantum systems are physical systems that obey the rules of *quantum mechanics*, the physics of the very small. So for example, atoms are quantum systems; their energy levels and motion is determined by quantum mechanics.

What's so different about quantum mechanics? The idea is that some parts of the system become *quantized*. An idea you might've seen in chemistry is that in an atom, the electrons have distinct shells they can be at around the nucleus. This is called the Bohr Model:

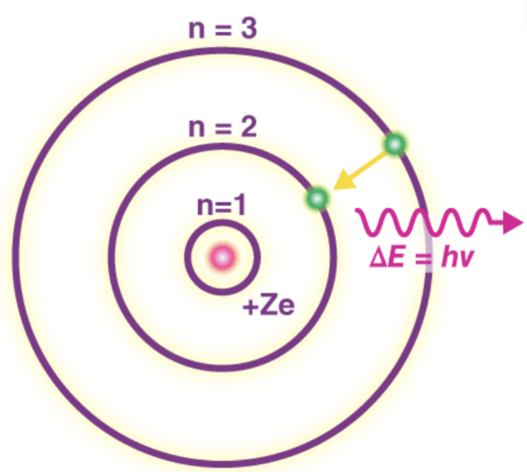

Figure 1: Credit: BYJU

It confused physicists at the time because the electron can **only** be at n=1, n=2, etc, **not** 1.5, 1.2, etc. If you're interested, this happens because the angular momentum of the electron around the nucleus is quantized. Ask me after class if you want to know more!

Another example of a quantized quantity is the *spin* of an electron, which can **only** be spin $\uparrow$ **or** spin $\downarrow$.

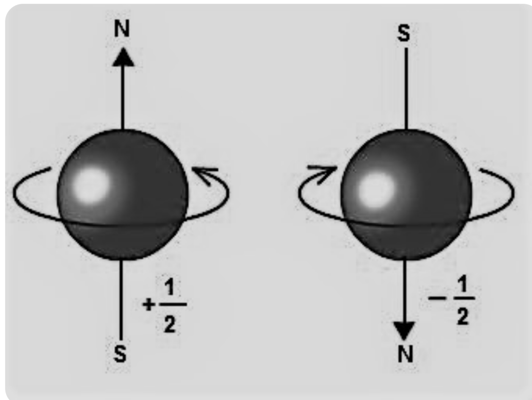

Figure 2: Credit: Unacademy

So there is no spin $\nearrow$ or spin $\nwarrow$, only spin $\uparrow$, spin $\downarrow$. Maybe by now you're getting the trend: it looks like quantum systems don't allow for "intermediate" value between outcomes. This doesn't seem very different from bits, which too could only by "on" or "off." To better understand the difference, it'll help to go to the math.

2.3 Mathematical Description of the Qubit

When we do the math, *we just assume* we somehow have a system that can only be in states $|0\rangle$ or $|1\rangle$. (This is why quantum computing theory is much further along than quantum computing experiment/engineering.)

Under this assumption, the state of this qubit, $|\psi\rangle$ can be written like this:

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle \tag{1}$$

Please ignore the slightly weird bracket-y notation ($|...\rangle$) for a second. What does Equation 1 *say*? It says we have a qubit, ψ, that **prior to measuring it**, has some probability (P_0) of being in state $|0\rangle$, and some probability (P_1) of being in state $|1\rangle$:

$$P_0 = |\alpha_0|^2 \tag{2}$$

$$P_1 = |\alpha_1|^2 \tag{3}$$

Now here's the kicker: $|\psi\rangle$ is in a *superposition*, or combination, of $|0\rangle$ and $|1\rangle$ **until measurement happens**. So before measuring it, the qubit is in a weird, quantum mixture of the two outcomes: **this is quantum uncertainty**.

So α_0, α_1 are **not** the probabilities, but they're closely related; they're called *probability amplitudes*. In principle, that's it! We'll learn that the whole of quantum computing is about manipulating these probability amplitudes to do what we want.

Now that we understand what the Equation physically means, let's go back and address the weird notation from earlier. The truth is that $|0\rangle$ and $|1\rangle$ are *vectors*:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{4}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{5}$$

Though you may not yet have taken a linear algebra class, you probably have a good idea of what a vector is. Here's what a 2D vector looks like:

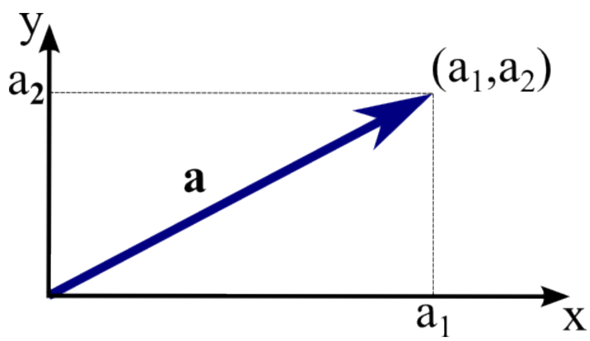

Figure 3: Credit: Math Insight

It has some *component* in x: a_1, and some *component* along y: a_2. Here, we're using X,Y as the *basis* vectors, which means we can write all other vectors in terms of X,Y. Similiarly, in this class we'll use $|0\rangle, |1\rangle$ as the basis vectors.

As such, there is a geometric interpretation of what a qubit looks like, which we'll explore in detail in future classes.

2.4 Quick Practice

I hand you the following qubits:

- $|\psi_a\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
- $|\psi_b\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$
- $|\psi_x\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle$

1. For each qubit, what are the probability amplitudes α_0, α_1? What about the probabilities P_0, P_1?
2. Is there anything funny about the probabilities you found?
3. Suggest a requirement that qubits should obey to fix this problem? This requirement is closely related to the *Born Rule* and the *Coppenhagen* interpretation of quantum mechanics.