Homework 3: Recursion, Tree Recursion

hw03.zip (hw03.zip)

Due by 11:59pm on Thursday, September 26

Instructions

Download hw03.zip (hw03.zip). Inside the archive, you will find a file called hw03.py (hw03.py), along with a copy of the `ok` autograder.

Submission: When you are done, submit the assignment by uploading all code files you've edited to Gradescope. You may submit more than once before the deadline; only the final submission will be scored. Check that you have successfully submitted your code on Gradescope. See Lab 0 (../../lab/lab00#task-c-submitting-the-assignment) for more instructions on submitting assignments.

Using Ok: If you have any questions about using Ok, please refer to this guide. (../../articles/using-ok)

Readings: You might find the following references useful:

- Section 1.7 (https://www.composingprograms.com/pages/17-recursive-functions.html)

Grading: Homework is graded based on correctness. Each incorrect problem will decrease the total score by one point. **This homework is out of 2 points.**

Required Questions

Getting Started Videos

Q1: Num Eights

Write a recursive function `num_eights` that takes a positive integer `n` and returns the number of times the digit 8 appears in `n`.

Important: Use recursion; the tests will fail if you use any assignment statements or loops. (You can define new functions, but don't put assignment statements there either.)

```
def num_eights(n):
    """Returns the number of times 8 appears as a digit of n.

    >>> num_eights(3)
    0
    >>> num_eights(8)
    1
    >>> num_eights(88888888)
    8
    >>> num_eights(2638)
    1
    >>> num_eights(86380)
    2
    >>> num_eights(12345)
    0
    >>> num_eights(8782089)
    3
    >>> from construct_check import check
    >>> # ban all assignment statements
    >>> check(HW_SOURCE_FILE, 'num_eights',
    ...       ['Assign', 'AnnAssign', 'AugAssign', 'NamedExpr', 'For', 'While'])
    True
    """
    "*** YOUR CODE HERE ***"
```

Use Ok to test your code:

```
python3 ok -q num_eights
```

Q2: Digit Distance

For a given integer, the *digit distance* is the sum of the absolute differences between consecutive digits. For example:

- The digit distance of `61` is `5`, as the absolute value of `6 - 1` is `5`.
- The digit distance of `71253` is `12` (`abs(7-1) + abs(1-2) + abs(2-5) + abs(5-3) = 6 + 1 + 3 + 2`).
- The digit distance of `6` is `0` because there are no pairs of consecutive digits.

Write a function that determines the digit distance of a positive integer. You must use recursion or the tests will fail.

```
def digit_distance(n):
    """Determines the digit distance of n.

    >>> digit_distance(3)
    0
    >>> digit_distance(777) # 0 + 0
    0
    >>> digit_distance(314) # 2 + 3
    5
    >>> digit_distance(31415926535) # 2 + 3 + 3 + 4 + ... + 2
    32
    >>> digit_distance(3464660003)  # 1 + 2 + 2 + 2 + ... + 3
    16
    >>> from construct_check import check
    >>> # ban all loops
    >>> check(HW_SOURCE_FILE, 'digit_distance',
    ...       ['For', 'While'])
    True
    """
    "*** YOUR CODE HERE ***"
```

Use Ok to test your code:

```
python3 ok -q digit_distance
```

Q3: Interleaved Sum

Write a function `interleaved_sum`, which takes in a number `n` and two one-argument functions: `odd_func` and `even_func`. It applies `odd_func` to every odd number and `even_func` to every even number from 1 to `n` *inclusive* and returns the sum.

For example, executing `interleaved_sum(5, lambda x: x, lambda x: x * x)` returns `1 + 2*2 + 3 + 4*4 + 5 = 29`.

> **Important:** Implement this function without using any loops or directly testing if a number is odd or even (no using `%`). Instead of directly checking whether a number is even or odd, start with 1, which you know is an odd number.
>
> **Hint:** Introduce an inner helper function that takes an odd number `k` and computes an interleaved sum from `k` to `n` (including `n`).

```
def interleaved_sum(n, odd_func, even_func):
    """Compute the sum odd_func(1) + even_func(2) + odd_func(3) + ..., up
    to n.

    >>> identity = lambda x: x
    >>> square = lambda x: x * x
    >>> triple = lambda x: x * 3
    >>> interleaved_sum(5, identity, square) # 1   + 2*2 + 3   + 4*4 + 5
    29
    >>> interleaved_sum(5, square, identity) # 1*1 + 2   + 3*3 + 4   + 5*5
    41
    >>> interleaved_sum(4, triple, square)   # 1*3 + 2*2 + 3*3 + 4*4
    32
    >>> interleaved_sum(4, square, triple)   # 1*1 + 2*3 + 3*3 + 4*3
    28
    >>> from construct_check import check
    >>> check(HW_SOURCE_FILE, 'interleaved_sum', ['While', 'For', 'Mod']) # ban loops a
    True
    >>> check(HW_SOURCE_FILE, 'interleaved_sum', ['BitAnd', 'BitOr', 'BitXor']) # ban b
    True
    """
    "*** YOUR CODE HERE ***"
```

Use Ok to test your code:

```
python3 ok -q interleaved_sum
```

Q4: Count Dollars

Given a positive integer `total`, a set of dollar bills makes change for `total` if the sum of the values of the dollar bills is `total`. Here we will use standard US dollar bill values: 1, 5, 10, 20, 50, and 100. For example, the following sets make change for `15`:

- 15 1-dollar bills
- 10 1-dollar, 1 5-dollar bills
- 5 1-dollar, 2 5-dollar bills
- 5 1-dollar, 1 10-dollar bills
- 3 5-dollar bills
- 1 5-dollar, 1 10-dollar bills

Thus, there are 6 ways to make change for `15`. Write a **recursive** function `count_dollars` that takes a positive integer `total` and returns the number of ways to make change for `total` using 1, 5, 10, 20, 50, and 100 dollar bills.

Use `next_smaller_dollar` in your solution: `next_smaller_dollar` will return the next smaller dollar bill value from the input (e.g. `next_smaller_dollar(5)` is `1`). *The function will return* `None` *if the next dollar bill value does not exist.*

> **Important:** Use recursion; the tests will fail if you use loops.
>
> **Hint:** Refer to the implementation (https://www.composingprograms.com/pages/17-recursive-functions.html#example-partitions) of `count_partitions` for an example of how to count the ways to sum up to a final value with smaller parts. If you need to keep track of more than one value across recursive calls, consider writing a helper function.

```
def next_smaller_dollar(bill):
    """Returns the next smaller bill in order."""
    if bill == 100:
        return 50
    if bill == 50:
        return 20
    if bill == 20:
        return 10
    elif bill == 10:
        return 5
    elif bill == 5:
        return 1

def count_dollars(total):
    """Return the number of ways to make change.

    >>> count_dollars(15)  # 15 $1 bills, 10 $1 & 1 $5 bills, ... 1 $5 & 1 $10 bills
    6
    >>> count_dollars(10)  # 10 $1 bills, 5 $1 & 1 $5 bills, 2 $5 bills, 10 $1 bills
    4
    >>> count_dollars(20)  # 20 $1 bills, 15 $1 & $5 bills, ... 1 $20 bill
    10
    >>> count_dollars(45)  # How many ways to make change for 45 dollars?
    44
    >>> count_dollars(100) # How many ways to make change for 100 dollars?
    344
    >>> count_dollars(200) # How many ways to make change for 200 dollars?
    3274
    >>> from construct_check import check
    >>> # ban iteration
    >>> check(HW_SOURCE_FILE, 'count_dollars', ['While', 'For'])
    True
    """
    "*** YOUR CODE HERE ***"
```

Use Ok to test your code:

```
python3 ok -q count_dollars
```

Check Your Score Locally

You can locally check your score on each question of this assignment by running

```
python3 ok --score
```

This does NOT submit the assignment! When you are satisfied with your score, submit the assignment to Gradescope to receive credit for it.

Submit Assignment

Submit this assignment by uploading any files you've edited **to the appropriate Gradescope assignment.** Lab 00 (../../lab/lab00/#submit-with-gradescope) has detailed instructions.

Optional Questions

These questions are optional. If you don't complete them, you will still receive credit for this assignment. They are great practice, so do them anyway!

Q5: Count Dollars Upward

Write a **recursive** function `count_dollars_upward` that is just like `count_dollars` except it uses `next_larger_dollar` , which returns the next larger dollar bill value from the input (e.g. `next_larger_dollar(5)` is `10`). *The function will return `None` if the next dollar bill value does not exist.*

Important: Use recursion; the tests will fail if you use loops.

```
def next_larger_dollar(bill):
    """Returns the next larger bill in order."""
    if bill == 1:
        return 5
    elif bill == 5:
        return 10
    elif bill == 10:
        return 20
    elif bill == 20:
        return 50
    elif bill == 50:
        return 100

def count_dollars_upward(total):
    """Return the number of ways to make change using bills.

    >>> count_dollars_upward(15)  # 15 $1 bills, 10 $1 & 1 $5 bills, ... 1 $5 & 1 $10 b
    6
    >>> count_dollars_upward(10)  # 10 $1 bills, 5 $1 & 1 $5 bills, 2 $5 bills, 10 $1 b
    4
    >>> count_dollars_upward(20)  # 20 $1 bills, 15 $1 & $5 bills, ... 1 $20 bill
    10
    >>> count_dollars_upward(45)  # How many ways to make change for 45 dollars?
    44
    >>> count_dollars_upward(100) # How many ways to make change for 100 dollars?
    344
    >>> count_dollars_upward(200) # How many ways to make change for 200 dollars?
    3274
    >>> from construct_check import check
    >>> # ban iteration
    >>> check(HW_SOURCE_FILE, 'count_dollars_upward', ['While', 'For'])
    True
    """
    "*** YOUR CODE HERE ***"
```

Use Ok to test your code:

```
python3 ok -q count_dollars_upward
```

Exam Practice

Homework assignments will also contain prior exam-level questions for you to take a look at. These questions have no submission component; feel free to attempt them if you'd like a challenge!

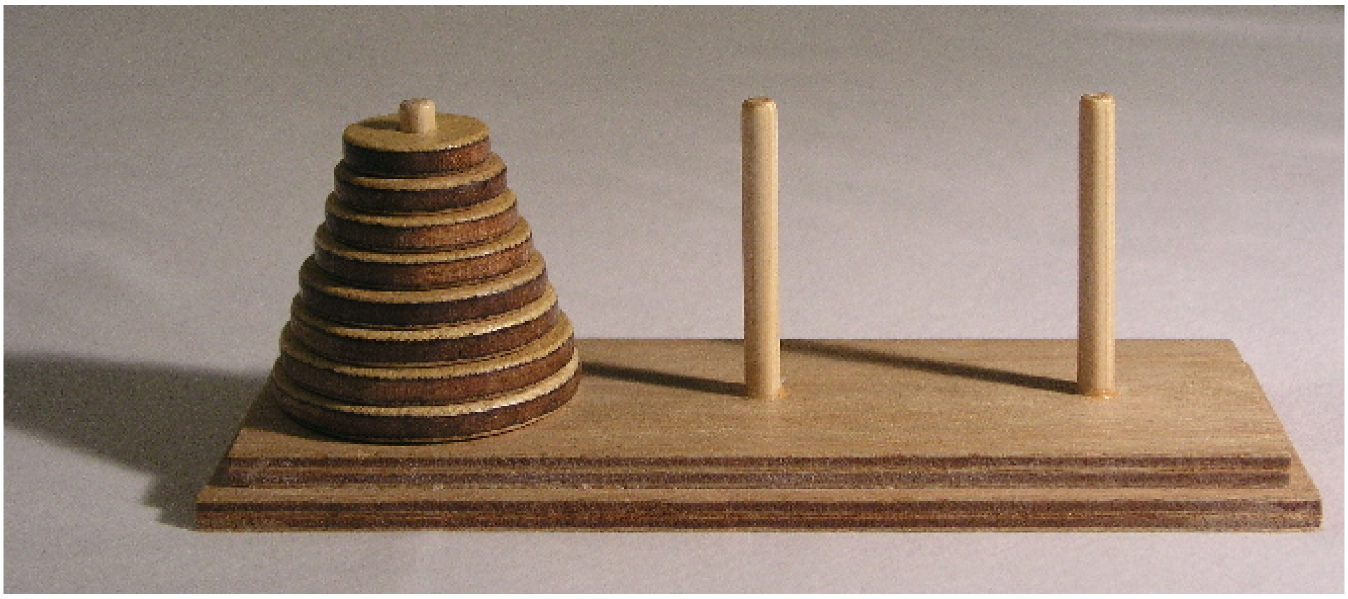
1. Fall 2017 MT1 Q4a: Digital (https://inst.eecs.berkeley.edu/~cs61a/fa21/exam/fa17/mt1/61a-fa17-mt1.pdf#page=5)
2. Fall 2019 Final Q6b: Palindromes (https://inst.eecs.berkeley.edu/~cs61a/sp21/exam/fa19/final/61a-fa19-final.pdf#page=6)

Just For Fun Questions

The questions below are out of scope for 61A. You can try them if you want an extra challenge, but they're just puzzles that are not required for the course. Almost all students will skip them, and that's fine. We will **not** be prioritizing support for these questions on Ed or during Office Hours.

Q6: Towers of Hanoi

A classic puzzle called the Towers of Hanoi is a game that consists of three rods, and a number of disks of different sizes which can slide onto any rod. The puzzle starts with `n` disks in a neat stack in ascending order of size on a `start` rod, the smallest at the top, forming a conical shape.



The objective of the puzzle is to move the entire stack to an `end` rod, obeying the following rules:

- Only one disk may be moved at a time.
- Each move consists of taking the top (smallest) disk from one of the rods and sliding it onto another rod, on top of the other disks that may already be present on that rod.
- No disk may be placed on top of a smaller disk.

Complete the definition of `move_stack`, which prints out the steps required to move `n` disks from the `start` rod to the `end` rod without violating the rules. The provided `print_move` function will print out the step to move a single disk from the given `origin` to the given `destination`.

Hint: Draw out a few games with various `n` on a piece of paper and try to find a pattern of disk movements that applies to any `n`. In your solution, take the recursive leap of faith whenever you need to move any amount of disks less than `n` from one rod to another. If you need more help, see the following hints.

Hint 1

Hint 2

The strategy used in Towers of Hanoi is to move all but the bottom disc to the second peg, then moving the bottom disc to the third peg, then moving all but the second disc from the second to the third peg.

Hint 3

One thing you don't need to worry about is collecting all the steps. `print` effectively "collects" all the results in the terminal as long as you make sure that the moves are printed in order.

```
def print_move(origin, destination):
    """Print instructions to move a disk."""
    print("Move the top disk from rod", origin, "to rod", destination)

def move_stack(n, start, end):
    """Print the moves required to move n disks on the start pole to the end
    pole without violating the rules of Towers of Hanoi.

    n -- number of disks
    start -- a pole position, either 1, 2, or 3
    end -- a pole position, either 1, 2, or 3

    There are exactly three poles, and start and end must be different. Assume
    that the start pole has at least n disks of increasing size, and the end
    pole is either empty or has a top disk larger than the top n start disks.

    >>> move_stack(1, 1, 3)
    Move the top disk from rod 1 to rod 3
    >>> move_stack(2, 1, 3)
    Move the top disk from rod 1 to rod 2
    Move the top disk from rod 1 to rod 3
    Move the top disk from rod 2 to rod 3
    >>> move_stack(3, 1, 3)
    Move the top disk from rod 1 to rod 3
    Move the top disk from rod 1 to rod 2
    Move the top disk from rod 3 to rod 2
    Move the top disk from rod 1 to rod 3
    Move the top disk from rod 2 to rod 1
    Move the top disk from rod 2 to rod 3
    Move the top disk from rod 1 to rod 3
    """
    assert 1 <= start <= 3 and 1 <= end <= 3 and start != end, "Bad start/end"
    "*** YOUR CODE HERE ***"
```

Use Ok to test your code:

```
python3 ok -q move_stack
```

Q7: Anonymous Factorial

This question demonstrates that it's possible to write recursive functions without assigning them a name in the global frame.

The recursive factorial function can be written as a single expression by using a conditional expression (http://docs.python.org/py3k/reference/expressions.html#conditional-expressions).

```
>>> fact = lambda n: 1 if n == 1 else mul(n, fact(sub(n, 1)))
>>> fact(5)
120
```

However, this implementation relies on the fact (no pun intended) that `fact` has a name, to which we refer in the body of `fact`. To write a recursive function, we have always given it a name using a `def` or assignment statement so that we can refer to the function within its own body. In this question, your job is to define `fact` recursively without giving it a name!

Write an expression that computes `n` factorial using only call expressions, conditional expressions, and `lambda` expressions (no assignment or `def` statements).

> **Note:** You are not allowed to use `make_anonymous_factorial` in your return expression.

The `sub` and `mul` functions from the `operator` module are the only built-in functions required to solve this problem.

```
from operator import sub, mul

def make_anonymous_factorial():
    """Return the value of an expression that computes factorial.

    >>> make_anonymous_factorial()(5)
    120
    >>> from construct_check import check
    >>> # ban any assignments or recursion
    >>> check(HW_SOURCE_FILE, 'make_anonymous_factorial',
    ...     ['Assign', 'AnnAssign', 'AugAssign', 'NamedExpr', 'FunctionDef', 'Recursior
    True
    """
    return 'YOUR_EXPRESSION_HERE'
```

Use Ok to test your code:

```
python3 ok -q make_anonymous_factorial
```