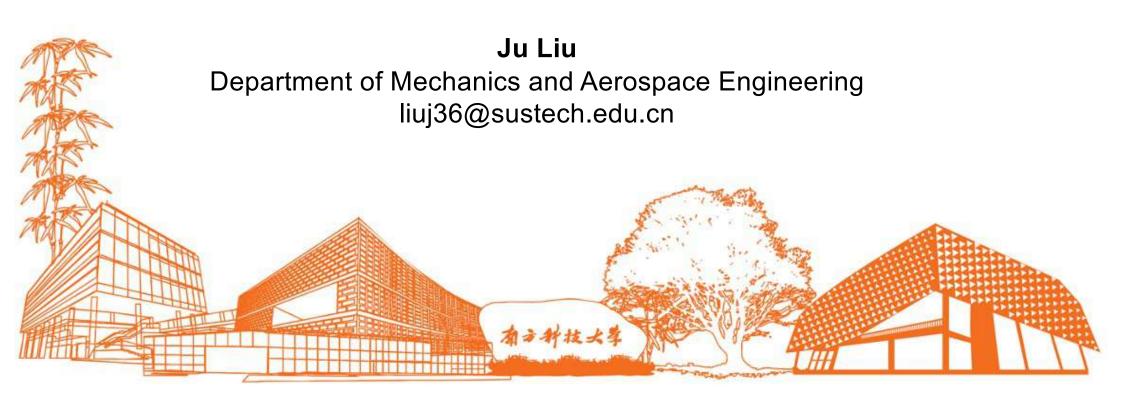
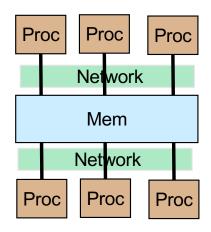
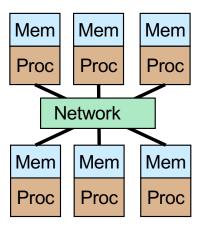
MAE 5032 High Performance Computing: Methods and Practices

Lecture 12: Message Passing Interface



Recall: shared vs distributed memory





Shared memory system: each core has access to a common memory

Distributed memory system: each core has access to a private memory

Why MPI





Sunway TaihuLight

Tianhe-2A

Allow solving large scientific/engineering problems

Recall: shared vs distributed memory

Shared memory parallelism:

- enabled by multi-threading
- easy to access data from any threads
- must handle cache coherence, NUMA architecture, etc.
- > looks simple, but there is a lot implicit complexity

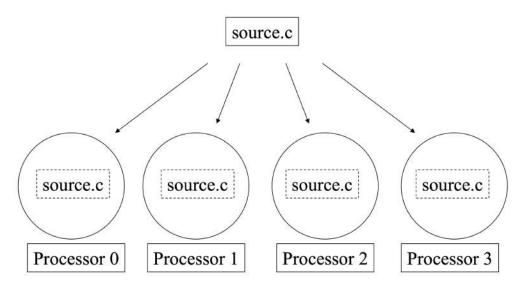
Distributed memory parallelism:

- only possible through multi-processing
- each process has its own private memory
- must explicitly exchange data across processes
- can scale beyond a single computer to solve much larger problems

SPMD programming model

SPMD: single program multiple data

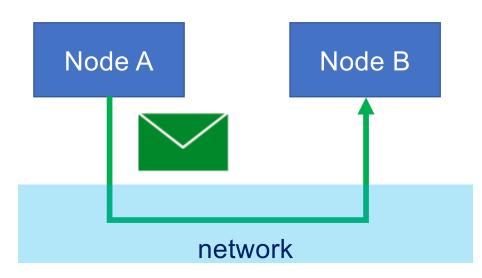
- A programming model for distributed memory parallelism
- Parallelism is achieved by creating multiple instances (processes) of the same program
- Each process operates on its own set of data
- Each process might follow a different execution path



SPMD programming model

SPMD: single program multiple data

- processes cannot communicate or synchronize implicitly through the shared memory
- we need explicit communication mechanism: message passing
- message can be classified in terms of participants:
 - point-to-point communication
 - collective communication

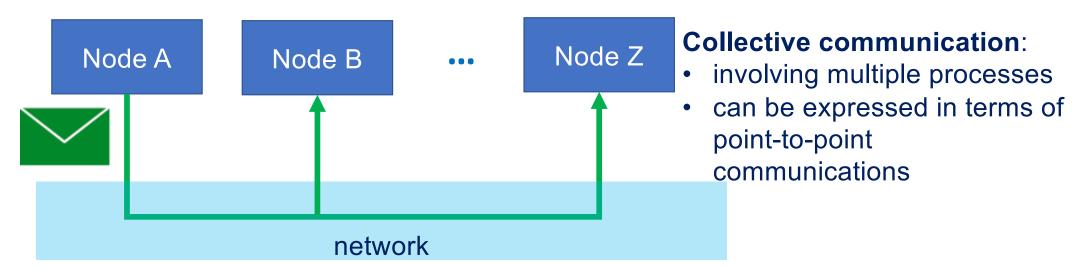


Point-to-point: processor A sends a message to another processor B

SPMD programming model

SPMD: single program multiple data

- processes cannot communicate or synchronize implicitly through the shared memory
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- message can be classified in terms of participants:
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MPI: Message Passing Interface

- MPI defines a standard API for message passing in SPMD applications
- MPI allows exploiting distributed memory systems
- There are several implementations
 - > MPICH https://www.mpich.org
 - OpenMP https://www.open-mpi.org
 - > MVAPICH http://mvapich.cse.ohio-state.edu
 - ➤ DeinoMPI https://mpi.deino.net
- Official documentation
 - https://www.mpi-forum.org/docs/mpi-3.1/mpi31-report.pdf

History of the MPI standard

- MPI-1
 - ➤ 1992 standardization of the message passing protocol into MPI. More than 60 people from 40 organizations involved. Point-to-point, block collective communications, one-sided communication, I/O, ...
- MPI-2
 - Fortran and C++ wrappers
- MPI-3
 - > Non-blocking collective communications
- MPI-4
 - > Support for using GPU for acceleration
- The MPI standard API goals:
 - > Work on distributed memory, shared memory, and hybrid systems
 - > Portable, efficient, easy to use

Compile and execute MPI programs

- MPI is simply a C library, we only need to link it to the main program
- MPI usually provide a wrapper mpicc/mpicxx/mpiifort

```
-> mpicc -show
gcc -Wl,-flat_namespace -Wl,-commons,use_dylibs -I/Users/juliu/lib/mpich-3.3.2/include
-L/Users/juliu/lib/mpich-3.3.2/lib -lmpi -lpmpi
```

Wrapper to launch multiple processes at once: mpiexec/mpirun

```
-> mpicc main.c -o hello
juliu::Kolmogorov {~/MAE5032}
-> mpirun -np 2 ./hello
Hello MAE5032!
Hello MAE5032!
```

- The number of processors does NOT need to be passed to mpirun in LSF/bsub
- Some clusters have their own wrapper of mpirun (e.g. srun in SLURM)

Hello world with MPI

```
#include <stdio.h>
#include <mpi.h>

int main( int argc, char **argv )
{
   MPI_Init(&argc, &argv);
   printf("Hello MAE5032!\n");

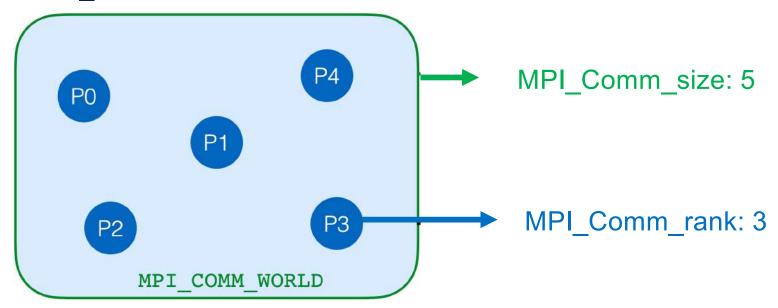
   MPI_Finalize();
   return 0;
}
```

- MPI is simply a C library, we only need to link it to the main program
- MPI usually provide a wrapper mpicc/mpicxx
- Wrapper to launch multiple processes at once: mpiexec, mpirun

Hello world with MPI

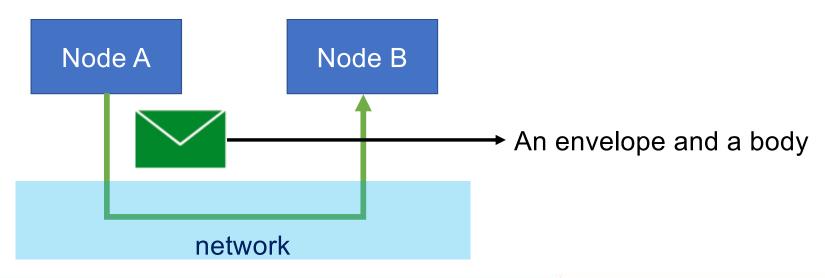
MPI communicators

- A communicator owns a group of processes that can communicate information among them. Each process has a unique rank within the communicator.
- MPI_COMM_WORLD is the communicator for all processes; defined after
 MPI Init is called
- All MPI routines involving communications require a communicator object of type MPI_Comm



MPI rank and size

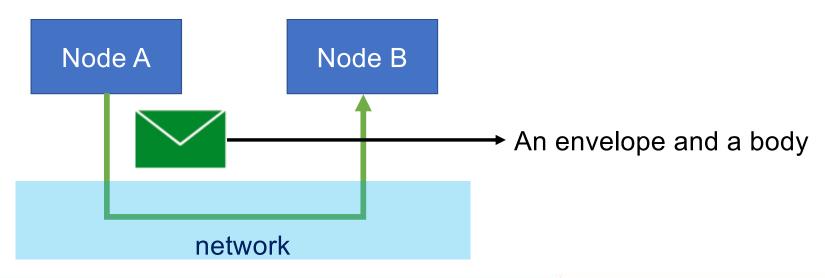
```
#include <stdio.h>
                                                      Communicator of ALL
#include <mpi.h>
                                                      processes
int main( int argc, char **argv )
                                                      Get process ID
 MPI_Init(&argc, &argv);
                                                      Get number of processes
 int rank, size;
 MPI_Comm_rank(MPI_COMM_WORLD, &rank);
                                                      Asynchronous execution:
 MPI Comm size(MPI COMM WORLD, &size);
                                                      order is not determinstic
 printf("Hello MAE5032 from rank %d out of %d!\n",
     rank, size);
                              -> mpirun -np 3 ./a.out
                              Hello MAE5032 from rank 2 out of 3!
 MPI_Finalize();
  return 0:
                              Hello MAE5032 from rank 0 out of 3!
                              Hello MAE5032 from rank 1 out of 3!
```



	Enve	lope	Body			
source	destination	communicator	tag	count	datatype	buffer

Rank 0 Rank 1 MPI_COMM_WORLD mytag 10 MPI_DOUBLE. mydata

Source & destination: rank IDs within the communicator Tag: user-defined message numeric label



	Enve	lope	Body			
source	destination	communicator	tag	count	datatype	buffer

count: number of elements

datatype: element data type

buffer: a pointer to the data itself

```
int MPI_Send(const void *buf, int count, MPI_Datatype
datatype, int dest, int tag, MPI_Comm comm);
int MPI_Recv(void *buf, int count, MPI_Datatype datatype,
int source, int tag, MPI_Comm comm, MPI_Status *status);
```

- MPI_Recv matches a message sent by MPI_Send only if comm, tag, source, and dest match.
- The count variable is the maximum size that can be received, message can be smaller.

int count;

int cancelled;
int MPI_SOURCE;

int MPI ERROR;

} MPI_Status, *PMPI_Status;

int MPI_TAG;

- The tag can be set to the wildcard MPI_ANY_TAG
- Source can be set to the wildcard MPI_ANY_SOURC
- MPI_STATUS_IGNORE can be used if the status is n
- MPI_STATUS is a struct containing at least MPI_SOU MPI_ERROR

MPI_Datatype	C type
MPI_CHAR	char
MPI_SHORT	short int
MPI_INT	int
MPI_LONG	long int
MPI_LONG_LONG_INT	long long int
MPI_LONG_LONG	long long int
MPI_SIGNED_CHAR	signed char
MPI_UNSIGNED_CHAR	unsigned char
MPI_UNSIGNED_SHORT	unsigned short int
MPI_UNSIGNED_INT	unsigned int
MPI_UNSIGNED_LONG	unsigned long int
MPI_UNSIGNED_LONG_LONG	unsigned long long
MPI_FLOAT	float
MPI_DOUBLE	double
MPI_LONG_DOUBLE	long double
MPI_BYTE	
MPI_PACKED	

```
int main(int argc, char **argv)
 int rank, size;
                                               -> mpirun -np 2 ./a.out
 MPI_Init(&argc, &argv);
                                               Rank 1 received the following integer: 7
 MPI_Comm_rank(MPI_COMM_WORLD, &rank);
 MPI_Comm_size(MPI_COMM_WORLD, &size);
 int tag = 42;
 int message;
                               Send one integer to rank 1
 if(rank == 0)
   message = 7;
   MPI_Send(&message, 1, MPI_INT, 1, tag, MPI_COMM_WORLD);
 else if(rank == 1)
   MPI_Recv(&message, 1, MPI_INT, 0, tag, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
   printf("Rank %d received the following integer: \%d\n", rank, message);
                             Receive one integer from rank 0
 MPI_Finalize();
 return 0;
```

```
int tag = 42;
                              -> mpirun -np 5 ./a.out
int message;
                              Rank 2 received the following integer: 4
if(rank == 0)
                              Rank 1 received the following integer: 1
                              Rank 3 received the following integer: 9
  for(int i=1; i<size; ++i)</pre>
                              Rank 4 received the following integer: 16
    message = i * i;
    MPI_Send(&message, 1, MPI_INT, i, tag, MPI_COMM_WORLD);
              Send one integer to all ranks != 0
else
  MPI_Recv(&message, 1, MPI_INT, 0, tag, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
  printf("Rank %d received the forlowing integer: %d\n", rank, message);
                   Receive one integer from rank 0
MPI_Finalize();
return 0;
```

Wildcards

```
int MPI_Send(const void *buf, int count, MPI_Datatype
datatype, int dest, int tag, MPI_Comm comm);
int MPI_Recv(void *buf, int count, MPI_Datatype datatype,
int source, int tag, MPI_Comm comm, MPI_Status *status);
```

wildcard enables programmers avoid having to specify a tag and/or source:

- MPI_ANY_SOURCE as source
- MPI_ANY_TAG as tag
- status structure is used to get wildcard values

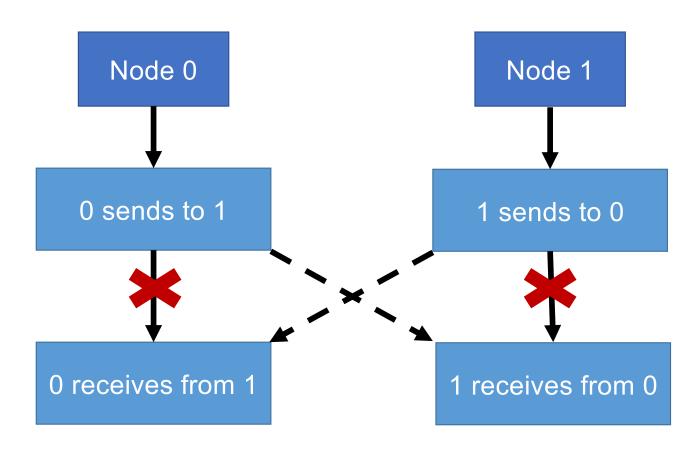
MPI_Send: communication modes

- MPI_Send: default send, usually blocked and may be buffered.
- MPI_Ssend: Synchronous send. Will be blocked until a matching receive has been posted and started receiving the message.
- MPI_Bsend: buffered send. Stores the message in a buffer and returns immediately. Users need to manually allocate the memory space.
- MPI_Rsend: Ready send. It assumes the matching recv function has already been posted. This function is <u>unsafe</u> as the behavior is undefined if the receiver is not yet ready.

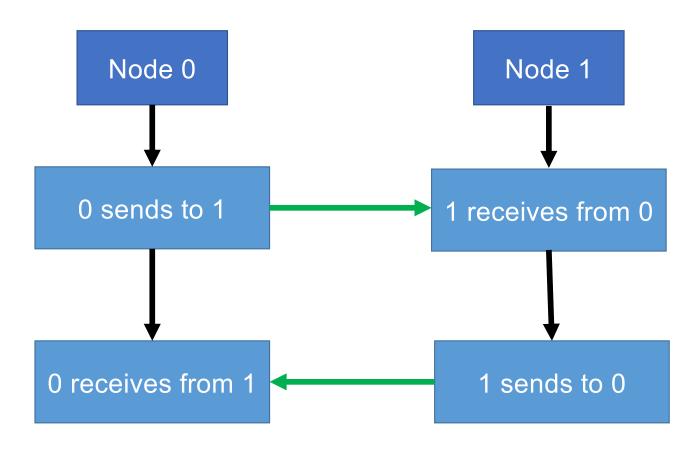
Deadlock

```
int tag1 = 42, tag2 = 43;
                                                -> mpirun -np 2 ./a.out
int sendMessage, recvMessage;
if(rank == 0)
 sendMessage = 7;
 MPI_Ssend(&sendMessage, 1, MPI_INT, 1, tag1, MPI_COMM_WORLD);
 MPI_Recv(&recvMessage, 1, MPI_INT, 1, tag2, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
else if(rank == 1)
 sendMessage = 14;
 MPI_Ssend(&sendMessage, 1, MPI_INT, 0, tag2, MPI_COMM_WORLD);
 MPI_Recv(&recvMessage, 1, MPI_INT, 0, tag1, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
printf("Rank %d received the following integer: %d\n", rank, recvMessage);
```

Deadlock



Deadlock fix 1: switch the order

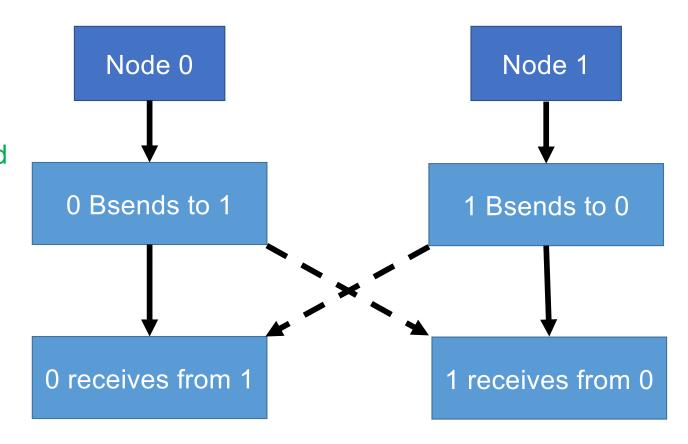


Deadlock fix 1: switch the order

```
int tag1 = 42, tag2 = 43;
                                      -> mpirun -np 2 ./a.out
int sendMessage, recvMessage;
                                      Rank 0 received the following integer: 14
if(rank == 0)
                                      Rank 1 received the following integer: 7
 sendMessage = 7;
 MPI Ssend(&sendMessage, 1, MPI INT, 1, tag1, MPI COMM WORLD);
 MPI_Recv(&recvMessage, 1, MPI_INT, 1, tag2, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
else if(rank == 1)
 sendMessage = 14;
 MPI_Recv(&recvMessage, 1, MPI_INT, 0, tag1, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
 MPI Ssend(&sendMessage, 1, MPI_INT, 0, tag2, MPI_COMM_WORLD);
printf("Rank %d received the following integer: %d\n", rank, recvMessage);
```

Deadlock fix 2: buffered send

Buffered send can return before the message is received.



Deadlock fix 3: non-blocking communication

Computation proceeds concurrently with send and receive

- Non-blocking send
 - Start send operation
 - Return before data copied into buffer
 - Must wait until data sent to overwrite the buffer

```
int MPI_Isend( void * buf, int count, MPI_Datatype datatype, int dest,
int tag, MPI Comm comm, MPI Request *request);
```

- Non-blocking receive
 - Start receive operation
 - Return before data received and copied into buffer
 - Must wait until data received to use the data in buffer

```
int MPI_Irecv( void * buf, int count, MPI_Datatype datatype, int dest,
int tag, MPI_Comm comm, MPI_Request *request );
```

Deadlock fix 3: non-blocking communication

Test whether request complete (non-blocking):

```
int MPI_Test( MPI_Request * request, int * flag, MPI_Status * status );
```

Block for request to complete:

```
int MPI_Wait( MPI_Request * request, MPI_Status * status );
```

- > request: handle on the non-blocking routine to wait on
- flag: the variable in which store the check result, true (1) if the underlying non-blocking is complete, false (0) otherwise
- > status: the struct containing the MPI_SOURCE, MPI_TAG, MPI_ERROR, etc.

Deadlock fix 3: non-blocking communication

```
int tag1 = 42, tag2 = 43;
int sendMessage, recvMessage;
MPI_Request sendReq;
MPI_Status status;
if(rank == 0)
  sendMessage = 7;
 MPI_Isend(&sendMessage, 1, MPI_INT, 1, tag1, MPI_COMM_WORLD, &sendReg);
 MPI_Recv(&recvMessage, 1, MPI_INT, 1, tag2, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
else if(rank == 1)
                              Non-blocking send returns immediately.
  sendMessage = 14;
 MPI_Isend(&sendMessage, 1, MPI_INT, 0, tag2, MPI_COMM_WORLD, &sendReg);
 MPI_Recv(&recvMessage, 1, MPI_INT, 0, tag1, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
MPI_Wait(&sendReg, &status);
                          Send is completed here and the buffer can be used now.
printf("Rank %d received the following integer: %d\n", rank, recvMessage);
```

Unknown message size

What if we don't know the size of the message on the receive side in advance?

Solution 1: send two messages, one for size first, then another for content.

Solution 2: Use MPI_Probe to obtain the message info

$$\frac{\pi}{4} = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$$

```
#include <stdio.h>
int main(int argc, char **argv)
{
  long nsteps = 10000000000;
  double sum = 0.0;

  for(long ii=0; ii<nsteps; ++ii)
    sum += (1.0 - 2.0 *(ii%2)) / (2.0 * ii + 1.0);

  printf("pi is %.12g \n", sum * 4.0);
  return 0;
}</pre>
```

How to parallelize this with MPI?

- Each rank performs its own chnk of the sum
- All ranks need to communicate and sum up its results to one rank
- This rank should report the result

$$\frac{\pi}{4} = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$$

```
#include <stdio.h>
int main(int argc, char **argv)
{
  long nsteps = 10000000000;
  double sum = 0.0;

  for(long ii=0; ii<nsteps; ++ii)
    sum += (1.0 - 2.0 *(ii%2)) / (2.0 * ii + 1.0);

  printf("pi is %.12g \n", sum * 4.0);
  return 0;
}</pre>
```

How to parallelize this with MPI?

- Each rank performs its own chnk of the sum
- All ranks need to communicate and sum up its results to one rank
- This rank should report the result

```
MPI_Init(&argc, &argv);
int size, rank;
MPI_Comm_rank(MPI_COMM_WORLD, &rank);
MPI Comm size(MPI COMM WORLD, &size);
                                                 Split the loop
long nsteps = 1000000000;
double sum = 0.0:
long chunk = (nsteps + size - 1) / size;
long start = chunk * rank;
long end = (chunk * (rank+1) > nsteps) ? nsteps : chunk*(rank+1);
double t1 = MPI_Wtime();
for(long ii=start; ii<end; ++ii)</pre>
  sum += (1.0 - 2.0 *(ii%2)) / (2.0 * ii + 1.0);
MPI Barrier(MPI COMM WORLD);
double t2 = MPI Wtime();
if(rank == 0) printf("Time taken is %f. \n", t2 - t1);
```

MPI_Wtime: return the wall-clock time in seconds.

- gives timing per rank
- can reduce it to get the max, mean, ...

MPI_Barrier: block untill all processes in the communicator has reached this routine

 all processes need to call it in the communicator

```
const int tag = 123;
if(rank == 0)
  double total_sum = sum;
  double other;
                                       Collect and addd the sum from other ranks
  for(int ii=1; ii<size; ++ii)</pre>
    MPI_Recv(&other, 1, MPI_DOUBLE, ii, tag, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
    total sum += other;
  printf("pi is %.12g \n", sum * 4.0);
}
else
  MPI_Ssend(&sum, 1, MPI_DOUBLE, 0, tag, MPI_COMM_WORLD);
                                  Send partial sums to rank 0
MPI_Finalize();
return 0;
```

Communication takes O(size) steps! Not ideal. HPC network is slower than main memory.

Blocking collective communications

Some very common collective operations have been implemented in MPI

These operations include:

- Reduce
- AllReduce
- Broadcast
- Gather
- AllGather
- Scatter
- Barrier
- AllToAll
- Scan
- ExScan
- Reduce_Scatter

All to One

One to All

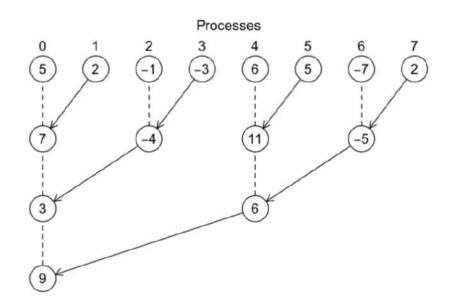
All to All

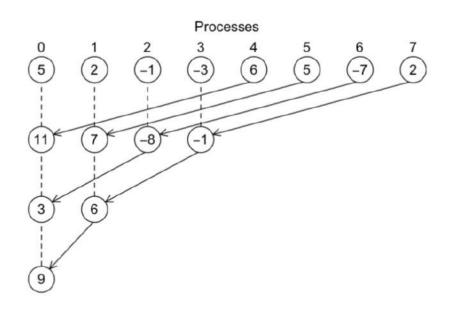
Other

They involve all ranks of a same communicator and therefore must be called by all the ranks of the concerned communicator

Blocking: the operation is completed for the process once the function returns.

There are better ways to do reduction, based on point-to-point communication





O(log(size)) rather than O(size)

More intermediate ranks are involved in the send and recv operations

```
int MPI_Reduce( const void *sendbuf, void *recvbuf, int
count, MPI_Datatype datatype, MPI_Op op, int root, MPI_Comm
comm);
```

- sendbuf: input data to reduce
- recvbuf: output result (only for root)
- count: number of elements (per process) to reduce
- datatype: type of element to reduce
- op: operation to operate
- root: the rank id to which the result is output
- comm: communicator

This is a **collective** operation, meaning **ALL** processes in the communicator must call it

Data can be reduced in place to save memory, in which case recvbuf is inputoutput and sendbuf should be set to **MPI_IN_PLACE**

MPI implements a few basic operations:

MPI_Op	Operation
MPI_MAX	maximum
MPI_MIN	minimum
MPI_SUM	sum
MPI_PROD	product
MPI_LAND	logical and
MPI_BAND	bit-wise and
MPI_LOR	logical or
MPI_BOR	bit-wise or
MPI_LXOR	logical exclusive or (xor)
MPI_BXOR	bit-wise exclusive or (xor)
MPI_MAXLOC	max value and location
MPI_MINLOC	min value and location

You can define a custom operation using MPI_OP_CREATE

The operation must be associative!

Reduction is called by all processes and send the result to rank 0

```
double total_sum = 0.0;

MPI_Reduce(&sum, &total_sum, 1, MPI_DOUBLE, MPI_SUM, 0, MPI_COMM_WORLD);

if(rank == 0)
    printf("pi is %.12g \n", sum * 4.0);

MPI_Finalize();
return 0;
```

only rank 0 has the result

In place reduction

```
double total_sum = 0.0;

MPI_Reduce(rank ? &sum : MPI_IN_PLACE, &sum, 1, MPI_DOUBLE, MPI_SUM, 0, MPI_COMM_WORLD);

if(rank == 0) printf("pi is %.12g \n", sum * 4.0);

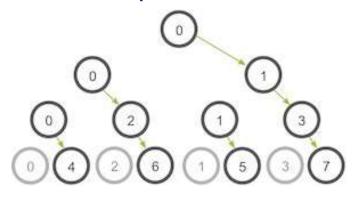
MPI_Finalize();
return 0;
```

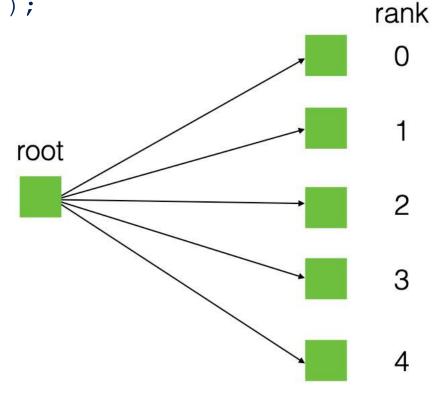
Broadcast

A broadcast sends data from the root rank to all ranks in the communicator

int MPI_Bcast(void *buffer, int count, MPI_Datatype
datatype, int root, MPI Comm comm);

- Can be naively implemented by send and recv
- MPI makes use of a tree structure to obtain better performance



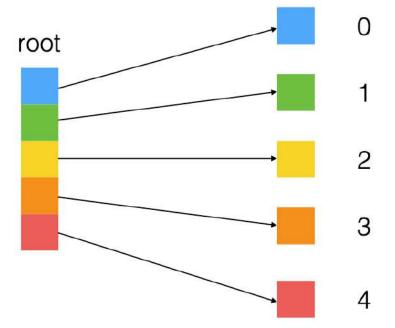


Scatter

Send data in separate chunks from the root rank to all ranks in the communicator

```
int MPI_Scatter( void *sendbuffer, int sendcount,
MPI_Datatype sendtype, void *recvbuf, int recvcount,
MPIDatatype recvtype, int root, MPI_Comm comm );
```

- sendbuf, sendcount, sendtype only significant to root
- sendcount: number of elements sent to each process



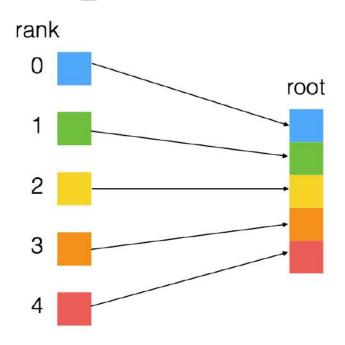
rank

Gather

Collect chunks of data from all ranks in the communicator to the root rank (inverse of scatter)

```
int MPI_Gather( void *sendbuffer, int sendcount,
MPI_Datatype sendtype, void *recvbuf, int recvcount,
MPIDatatype recvtype, int root, MPI_Comm comm );
```

- recvbuf, recvcount, recvtype: only significant to root
- sendcount: number of elements for a receive

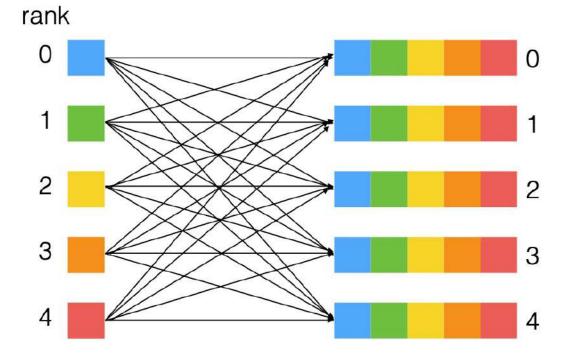


AllGather

Same as Gather but all ranks get the result

```
int MPI_AllGather( void *sendbuffer, int sendcount,
MPI_Datatype sendtype, void *recvbuf, int recvcount,
MPIDatatype recvtype, MPI_Comm comm );
```

 recvcount: number of elements for a single receive

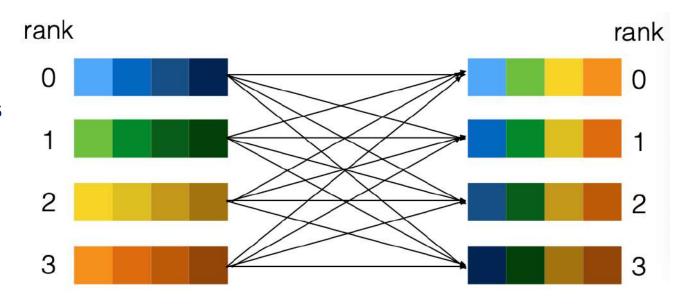


AIIToAII

Shuffle the data between ranks: acts like a transpose

```
int MPI_Alltoall( void *sendbuffer, int sendcount,
MPI_Datatype sendtype, void *recvbuf, int recvcount,
MPIDatatype recvtype, MPI_Comm comm );
```

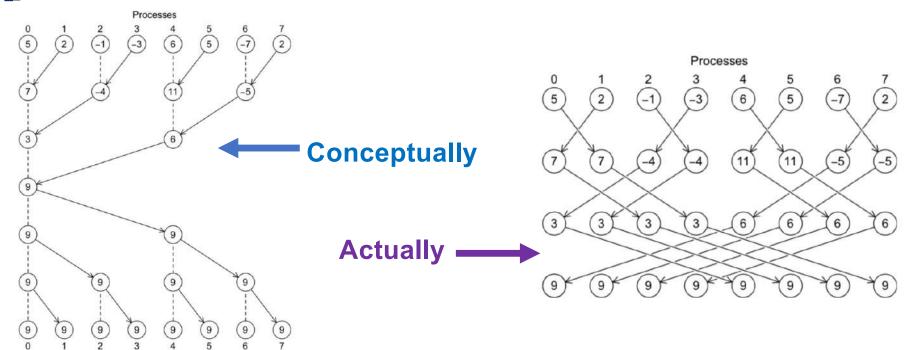
 Can be seen as an extension of AllGather, except that each rank has different data



AllReduce

Reduce the data, and broadcast the result to all ranks

int MPI_Allreduce(const void *sendbuffer, void *
recvbuffer, int count, MPI_Datatype datatype, MPI_Op op,
MPI Comm comm);



Non-blocking collective

Non-blocking collective:

- the data buffer cannot be used right after the function returns
- the call returns directly but creates a MPI_Request object
- MPI_Wait and MPI_Test can be used to wait or check the completion

```
int MPI_Ireduce(const void *sendbuf, void *recvbuf, int count,
MPI_Datatype datatype, MPI_Op op, int root, MPI_Comm comm, MPI_Request
*request);
int MPI_Iallreduce(const void *sendbuf, void *recvbuf, int count, MPI_Datatype
datatype, MPI_Op op, MPI_Comm comm, MPI_Request *request);
int MPI_Igather(const void *sendbuf, int sendcount, MPI_Datatype sendtype, void
*recvbuf, int recvcount, MPI_Datatype recvtype, int root, MPI_Comm comm,
MPI_Request *request);
.....
```

Same names as blocking with a "I" in front of the operation. Extra output argument of type MPI_Request at the end.

Non-blocking collective

```
double val = (double) (rank * rank);
double sumVals = 0;

MPI_Request request;
MPI_Iallreduce(&val, &sumVals, 1, MPI_DOUBLE, MPI_SUM, MPI_COMM_WORLD, &request);

doUnrelatedWork(); // this function overlaps with the reduction

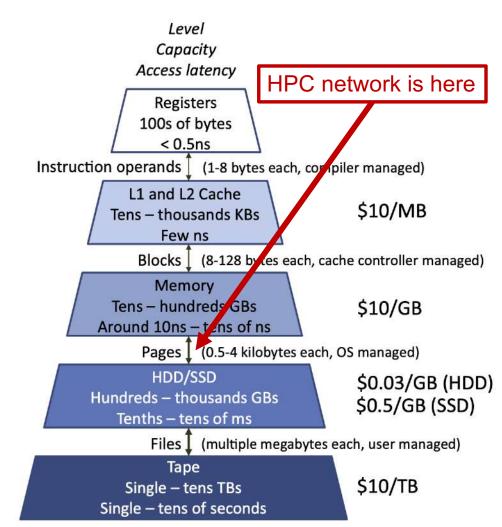
MPI_Wait(&request, MPI_STATUS_IGNORE); // Need to wait for the result if (rank == 0)
    printf("total sum is %g\n", sumVals);
```

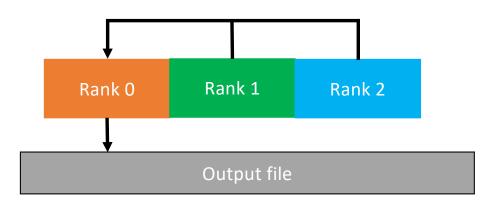
You will need to do Input and Output when

- postprocessing
- visualization
- checkpoint/restart

Regular I/O operations allow writing or reading files sequentially (one rank at a time)

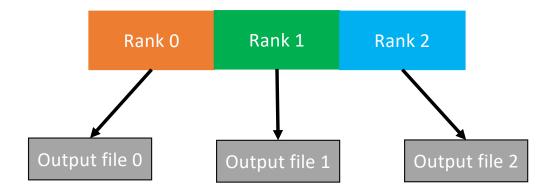
Larger problems require dumping possibly very large files
This can be a limiting factor for performance





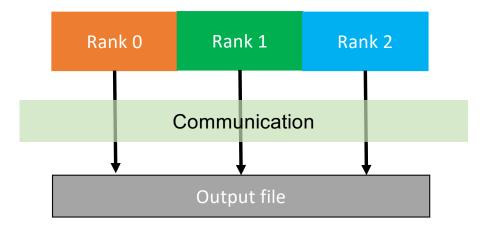
Dump the data to the root processor through the HPC network

- A single file
- Not scalable



Dump one file per rank

- Scalable
- Many small files, bad for HPC filesystem
- Difficult for postprocessing



Dump a common file from many ranks

- Scalable
- Single common file

MPI I/O is used in many mature libraries: HDF5, NETCDF, etc.

Logic is the same as the usual file management:

➤ Open/create a file handle

```
int MPI_File_open(MPI_Comm comm, const char *filename, int amode,
MPI_Info info, MPI_File *fh);
```

- Read/write data
- Close the file

```
int MPI_File_close(MPI_File *fh);
```

The opening and closing of the file are collective operations.

amode: access mode, possible values:

- MPI_MODE_APPEND Set initial position of all file pointers to end of file
- MPI_MODE_CREATE Create the file if it does not exist
- MPI_MODE_DELETE_ON_CLOSE Delete file on close
- MPI MODE EXCL Error creating a file that already exists
- MPI MODE RDONLY Read only
- MPI_MODE_RDWR Reading and writing
- MPI_MODE_SEQUENTIAL File will only be accessed sequentially
- MPI_MODE_WRONLY Write only
- MPI_MODE_UNIQUE_OPEN File will not be concurrently opened elsewhere

Summary

- MPI is used to create parallel programs based on message passing
- We use SPMD model for most of the time
- There are 6 basic calls: MPI_Init, MPI_Comm_rank, MPI_Comm_size, MPI Send, MPI Recv, MPI Finalize
- There are other point-to-point communication functions: buffered, non-blocking, etc.
- There are many collective communications that are handy and efficient
- There are MPI I/O functions, but we recommend using HDF5
- There are some topics not covered:
 - MPI communicator management by groups
 - one-sided communication
 - MPI derived data types
 - **>** ...

References: MPI standard document http://www.mpi-forum.org/docs/ Using MPI by Gropp, Lusk, and Skjellum, MIT Press, 1994