



# Variable neighborhood search for solving the $k$ -domination problem

Milan Predojević  
M.P.

milan.predojevic@pmf.unibl.org  
Faculty of Sciences and Mathematics,  
University of Banja Luka  
Banja Luka, Serb Republic, Bosnia  
and Herzegovina

Aleksandar Kartelj  
A.K.

aleksandar.kartelj@matf.bg.ac.rs  
Faculty of Mathematics, University of  
Belgrade  
Belgrade, Serbia

Marko Djukanović  
M.Dj.

marko.djukanovic@pmf.unibl.org  
Faculty of Sciences and Mathematics,  
University of Banja Luka  
Banja Luka, Serb Republic, Bosnia  
and Herzegovina

## ABSTRACT

In this paper we are concerned with solving a generalized version of the well-known minimum dominating set problem, the so-called  $k$ -domination problem,  $k \in \mathbb{N}$ . This problem is about finding a minimal cardinality subset  $D$  of vertices of a graph  $G = (V, E)$  such that every  $v \in V$  belongs to  $D$  or has at least  $k$  neighbors from  $D$ . The  $k$ -domination problem has applications in distributed systems, biological networks etc. We propose a variable neighborhood search (VNS) metaheuristic for solving the  $k$ -domination problem. The VNS is equipped with an efficient fitness function that allows it to consider both feasible and infeasible solutions, while appropriately penalizing infeasible solutions. The control parameters of the VNS are tuned using a grid search approach. The method is compared to the best known heuristic approaches from the literature: the beam search and several greedy approaches. Experimental evaluations are performed on a real-world benchmark set whose instances represent the road networks of different cities. The VNS provided new state-of-the-art results for all considered problem instances with  $k \in \{1, 2, 4\}$ .

## CCS CONCEPTS

• Theory of computation  $\rightarrow$  Optimization with randomized search heuristics.

## KEYWORDS

Variable neighborhood search, graph domination, metaheuristics, combinatorial optimization

## ACM Reference Format:

Milan Predojević, M.P., Aleksandar Kartelj, A.K., Marko Djukanović, and M.Dj. 2023. Variable neighborhood search for solving the  $k$ -domination problem. In *Genetic and Evolutionary Computation Conference Companion (GECCO '23 Companion)*, July 15–19, 2023, Lisbon, Portugal. ACM, New York, NY, USA, 4 pages. <https://doi.org/10.1145/3583133.3590607>

Permission to make digital or hard copies of part or all of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for third-party components of this work must be honored. For all other uses, contact the owner/author(s).  
GECCO '23 Companion, July 15–19, 2023, Lisbon, Portugal  
© 2023 Copyright held by the owner/author(s).  
ACM ISBN 979-8-4007-0120-7/23/07.  
<https://doi.org/10.1145/3583133.3590607>

## 1 INTRODUCTION

One of the best known classes of problems studied from theoretical, computational and practical points of view are domination problems on graphs [10]. The basic problem of this class is the *minimum dominating set problem*. The subset  $D \subset V$  is called *dominating set* if each vertex  $v \in V$  belongs to  $D$  or there is at least one vertex  $w \in D$  such that  $uw \in E$ . The search for the smallest possible dominating set  $D$  of the graph  $G$  is called the *minimum dominating set problem* (MDSP) [9]. This problem has many applications, for example, in biological networks [18], document summarization [22], graph mining [2], etc. From an algorithmic point of view, this problem is NP-hard. It is solved by various exact approaches, such as branch-and-reduce algorithms [23], an approach that uses the fundamental cut-sets of the graph [15], etc. Heuristic approaches are more common in the literature, e.g., a genetic algorithm [11], simulated annealing [12], ant colony optimization [14], to name a few. Several generalizations of MDSP, arising from practical experience, are proposed in the literature: the minimum weight dominating set problem [21], the minimum total dominating set problem [25], the minimum connected dominating set problem [1], etc.

In this paper, we study the *minimum  $k$ -domination problem* (MkDP) [4], for a fixed  $k \in \mathbb{N}$ . The  $k$ -dominating set  $D$  of a graph  $G$  is such a subset of  $V$  that every vertex not belonging to  $D$  is adjacent to at least  $k$  vertices in  $D$  [16]. A minimum  $k$ -dominating set represents the optimal solution of MkDP. The NP-completeness of the  $k$ -domination decision problem is formally proved for the split graphs, see details in the above citation. (Note that the definition of this problem is not unique in the literature – there is a definition where the goal is to find a minimum cardinality vertex set  $D$  such that every vertex of  $G$  is within distance  $k$  of some vertex in  $D$  [3].)

Regarding previous solutions to this problem, several greedy approaches have been proposed by Couture et al. [5], Gagarin et al. [8], and Gagarin and Corcoran [7]. Recently, Corcoran and Gagarin proposed a Beam search approach [4], which is currently the best-performing heuristic approach for small to medium sized real-world instances.

MkDP has applications in distributed systems [24], where a  $k$ -dominating set represents a set of processors such that each processor outside this set must have at least  $k$  neighbors in the set.

We propose a variable neighborhood search (VNS) metaheuristic for solving MkDP. The main contribution of our work is that the proposed VNS significantly improves the state-of-the-art results for all considered problem instances, in case of  $k \in \{1, 2, 4\}$ .

## 2 FORMAL PROBLEM DEFINITION

Let  $G = (V, E)$  be a simple undirected graph and  $k \in \mathbb{N}$  be fixed. For  $v \in V$ ,  $N(v)$  is a set of all adjacent vertices of  $v$  in the graph  $G$ , i.e.  $N(v) = \{w \mid vw \in E\}$ . A set  $D \subseteq V$  is called  $k$ -dominating set if for every  $v \in V \setminus D$  holds  $|N(v) \cap D| \geq k$ . The MkDP is an optimization problem whose objective is to find a  $k$ -dominating set with minimal cardinality:

$$\arg \min_{D \subseteq V} |D| \text{ s.t. } \forall v \in V \setminus D, |N(v) \cap D| \geq k. \quad (1)$$

As for the search space of the MkDP, we adapt it to the needs of our VNS. Not only feasible solutions are handled, but also infeasible ones – they are additionally penalized, see Section 3. In other words, every subset of  $V$  is a candidate solution in our VNS. Therefore, the size of the search space is  $2^{|V|}$ .

## 3 THE PROPOSED ALGORITHM

In this section, we first give an overview of the variable neighborhood search (VNS). Then we introduce the main components of Vns for solving MkDP: the fitness function, the shaking procedure and the efficient local search.

### 3.1 Variable neighborhood search

*Variable neighborhood search* is a single-based solution metaheuristic proposed by Mladenović and Hansen [17]. The basic idea of the approach is to systematically exchange neighborhoods of the current best solution (incumbent solution) to avoid getting stuck in a local optimum. Vns has proven to be one of the most powerful metaheuristic, achieving excellent results on diverse classes of problems, such as scheduling problems [6], vehicle routing problems [20], median problems [13], etc.

The Vns for solving the MkDP is given in Algorithm 1. Vns generally requires at least two control parameters  $d_{min}, d_{max} \in \mathbb{N}$ , which define the increasing sizes of the neighborhood structures  $N_{d_{min}}(D), \dots, N_{d_{max}}(D)$  around given solution  $D$ . A solution  $D'$  belongs to the neighborhood  $N_d(D)$  of the solution  $D$  if it can be obtained from  $D$  by removing  $\min(d, |D|)$  vertices from  $D$  and then adding  $d$  vertices from  $V \setminus D$  into  $D$ . The third parameter commonly used in Vns is  $p_{move} \in [0, 1]$ . This parameter corresponds to the probability of moving to a new solution if it has the same quality (fitness) as the incumbent solution. Finally, the fourth control parameter *penalty* is specific to MkDP. It is real-valued and is used to define the relative influence of solution feasibility and solution quality (size of  $k$ -dominating set) on the overall value of the fitness function (more details are given in Section 3.2). The return value of Vns is called  $D_{best}$  – it is the incumbent solution.

At the beginning, the neighborhood size  $d$  is set to the smallest, i.e. to  $d_{min}$ . The initial incumbent solution  $D_{best}$  is generated by performing local search on an empty set (local search is explained in Section 3.3). Then the algorithm enters the main loop (lines 5–18). The loop is run until at least one of the termination criteria is met. At each iteration of the loop, the following steps are executed:

- Shaking( $N_d(D_{best})$ ) – a solution  $D'$  is selected randomly from the set of solutions belonging to the  $d$ -th neighborhood structure around the solution  $D_{best}$ .

---

### Algorithm 1 VNS scheme for solving MkDP

---

```

1: Input:  $d_{min}, d_{max}, p_{move}, penalty$ 
2: Output: best found solution  $D_{best}$ 
3:  $d \leftarrow d_{min}, d_{max} \leftarrow d_{max\_init}$ 
4:  $D_{best} \leftarrow \text{LocalSearch}(\emptyset)$ 
5: while TerminationCriteriaNotMet() do
6:    $D' \leftarrow \text{Shaking}(N_d(D_{best}))$ 
7:    $D'' \leftarrow \text{LocalSearch}(D')$ 
8:   if  $\text{fitness}(D'') < \text{fitness}(D_{best}) \vee (\text{fitness}(D'') = \text{fitness}(D_{best}) \wedge r \in U_{[0,1]} < p_{move})$  then
9:      $D_{best} \leftarrow D''$ 
10:     $d \leftarrow d_{min}$ 
11:     $d_{max} \leftarrow \min(d_{max\_init}, |D_{best}|/2)$ 
12:   else
13:      $d \leftarrow d + 1$  // try with next neighborhood
14:     if  $d > d_{max}$  then
15:        $d \leftarrow d_{min}$ 
16:   end if
17: end if
18: end while
19: return  $D_{best}$ 

```

---

- LocalSearch – the selected solution  $D'$  may be improved by a local search procedure, as explained in Section 3.3.
- The solution  $D'$  becomes new incumbent if it has better fitness than the previous incumbent. Alternatively, it may become new incumbent with probability  $p_{move}$  if its fitness is the same. In both cases,  $d$  is reset to  $d_{min}$ . The parameter  $d_{max}$  is dynamically set to  $\min(d_{max\_init}, |D_{best}|/2)$  to prevent neighborhoods that are too large, i.e., neighborhoods larger than half the incumbent size.
- If the solution  $D'$  does not become new incumbent,  $d$  is increased – this further increases diversification. If this increase leads to  $d > d_{max}$ ,  $d$  is circularly reset to  $d_{min}$ .

### 3.2 Fitness function

To evaluate solution  $D$ , the following *fitness* function is used:

$$\text{fitness}(D) = (1 + \text{viols}(D)) \cdot (1 + \text{penalty} \cdot |D|) \quad (2)$$

with  $\text{viols}(D) = \sum_{v \in V \setminus D} k - C(D, v)$ ,  $C(D, v) = \min(k, |N(v) \cap D|)$ .

It can be seen that  $\text{viols}(D)$  quantifies the overall degree of solution  $D$  inadmissibility, i.e., for each vertex  $v$  that does not belong to a candidate dominating set  $D$ ,  $k - C(D, v)$  measures how strongly vertex  $v$  locally violates the  $k$ -domination condition. Thus,  $C(D, v)$  quantifies the opposite – how strongly the vertex  $v$  satisfies the  $k$ -domination condition. In particular, when vertex  $v$  has  $k$  or more vertex neighbors in  $D$ , the value of  $k - \min(k, |N(v) \cap D|)$  is zero. Otherwise,  $C(D, v)$  is positive and at most  $k$ . Therefore, the proposed fitness function evaluates both feasible and infeasible solutions. Since the fitness function is to be minimized, the following three observations can be made about the values of the fitness function:

- For sufficiently small values of the parameter *penalty*, the feasibility of the solution is relatively preferred over the cardinality of the solution. This means that when comparing feasible and infeasible solutions, the feasible solution is favored.
- When comparing two infeasible solutions of the same cardinality, the *less infeasible* solution is preferred, i.e. the one with the lower  $\text{viols}(\cdot)$  value.
- When comparing two feasible solutions, the one with the lower cardinality is preferred.

### 3.3 Local search

The goal of local search (LS) is to improve the solution  $D'$  obtained in the shaking phase by applying multiple local improvements to the structure of the solution.

Our LS procedure consists of two phases. In the first phase, the vertices are added to achieve feasibility – when the solution is not feasible. In the second phase, the vertices are removed to improve the objective function. The removal is done in a way that does not affect the feasibility previously achieved. In both phases, the best improvement strategy is used. This means that all eligible vertices are checked for inclusion/exclusion and then the best vertex is included/excluded. Each phase ends with the first iteration where no improvement in fitness is found.

## 4 EXPERIMENTAL EVALUATION

In this section we analyze the quality of the proposed Vns method.<sup>1</sup> For this purpose, we include the competing heuristic approaches from the literature. The following four methods for MkDP are compared: (i) The standard greedy method from [8, 19], denoted by SG; (ii) Greedy method from [7], denoted by PG; (iii) Beam search approach from [4], denoted by BS; (iv) Vns approach, as described in Section 3.

**Benchmark instances.** For comparison, we consider the benchmark introduced in [4]. It consists of 20 small to medium sized instances, where all instances represent road networks of different cities modeled through reachability graphs. In addition to these, the authors have provided the program that generate road networks for five large cities: Belgrade, Berlin, Boston, Dublin, and Minsk. Since the road networks change over time, the obtained graphs do not fully match those used in the [4]. Unlike the five large networks, these 20 instances are the same as in [4] – we obtained them directly from the authors.

**Testing environment.** Experiments were performed on a computer with Intel(R) Core(TM) i5-8265U CPU 1.80GHz and 16GB RAM, under Microsoft Windows 11 Pro OS. Vns is implemented in Python 3.9. The results of SG, PG and BS for small to medium sized instances are taken from [4], as reported in their experimental section. The results for the five large instances were obtained by running the original implementation of PG (acquired from the authors). According to the same authors, BS was inefficient for the large instances due to its high computational complexity. Therefore, we did not include it in the large instance comparison. PG and Vns algorithms are run ten times (using different random seeds) per each problem instance.

The termination criteria of the Vns are: (i) the maximum running time of 30 minutes, and (ii) the maximum number of 20000 iterations. The time limit of 30 minutes is not checked during initialization (line 4 in Algorithm 1). It means that Vns can take more than 30 minutes to finish for some very large instances, such as Dublin and Boston. More precisely, in these situations, after initialization, the most important steps of Vns (shaking and subsequent local search) are not even performed due to an expired time limit.

The algorithms SG, PG and BS terminate their execution as soon as the first feasible solution is reached. Therefore, we decided to use termination criteria which allow similar running times to those reported by comparison algorithms [4]. Namely, the best results were obtained with the BS configuration BS4, where 4 stands for the beam width. For this algorithm, in the case of  $k = 2$ , the average running times in seconds are 1736, 8834, 1257, 7156 and 3327 for the Bath, Belfast, Brighton, Bristol and Cardiff instances, respectively. As stated in [4], the comparison algorithms were also implemented in Python and executed on a desktop computer with somewhat better CPU: Intel Core i7-8700 @3.20GHz.

**Parameters tuning.** As mentioned earlier, Vns for MkDPA involves four control parameters:  $d_{min}$ ,  $d_{max}$ ,  $p_{move}$  and  $penalty$ . We chose a grid search method for tuning these parameters. As recommended in [17], we set the parameter  $d_{min}$  with the value 1. The possible values of parameters during tuning were:  $d_{max} \in \{5, 10, 15, 20, 25, 30, 40, 50, 100\}$ ,  $p_{move} \in \{0, 0.25, 0.5, 0.75, 1\}$ , and  $penalty \in \{0.005, 0.01, 0.015, 0.02\}$ . The parameter space thus consists of 180 ( $d_{max}, p_{move}, penalty$ ) parameter configurations. From 60 small to medium sized instances (20 cities for  $k \in \{1, 2, 4\}$ ), 20 instances were randomly selected for parameter tuning. For each of the 20 instances and each configuration, the Vns was run for 100 iterations. The best configuration, selected according to the best average rank over these 20 instances, is ( $d_{max}, p_{move}, penalty$ ) = (50, 0.5, 0.005).

### 4.1 Experimental results

Table 1 contains the overall results. The first column gives the name of the city road network. The next block contains network size information. The next three blocks give the Vns results and the best results from the literature so far, for  $k = 1, 2, 4$ , respectively. Each of these (three) blocks consists of 6 columns, where the first three columns show the results of Vns: average solution quality ( $\overline{|D|}$ ), standard deviation ( $\sigma(|D|)$ ) and average running time ( $\bar{t}$ ). The other three columns report the results of the best approach from the literature: name of the approach (Alg.), average solution quality over ten runs ( $\overline{|D|}$ ) and the corresponding standard deviation ( $\sigma(|D|)$ ). Note that labels BS1, BS2 or BS4 correspond to the BS approach with beam widths of 1, 2 and 4, respectively.

The following conclusions can be drawn from these results.

- For  $k = 1$ , Vns outperformed all competing approaches for small to medium sized instances. Vns also outperformed the PG approach for the large-sized instances.
- Concerning the results for  $k = 2$ , Vns outperformed all competing approaches by nearly 15% in terms of the average solution quality. Similar conclusions hold for the large instances: Vns produces  $\approx 5$ –16% improvement rate over the second best, PG approach.
- The similar conclusions hold for  $k = 4$ . Vns outperforms the second-best algorithm by more than 15% in some cases (see, for example, the instance Nottingham where Vns achieved a score 164.2 over BS4 which achieved a score 195.2). For the large instances, Vns outperforms PG in all cases.

<sup>1</sup>All instances, source codes of the Vns and grid search tuning algorithms, and detailed experimental results are publicly available on the GitHub page of this work: <https://github.com/mikiMilan/k-domination>.

Table 1: Results

City	Network size		Results for $k = 1$						Results for $k = 2$						Results for $k = 4$					
	$ V $	$ E $	$ D $	$\sigma( D )$	$\bar{t}$	Alg.	$ D $	$\sigma( D )$	$ D $	$\sigma( D )$	$\bar{t}$	Alg.	$ D $	$\sigma( D )$	$ D $	$\sigma( D )$	$\bar{t}$	Alg.	$ D $	$\sigma( D )$
Bath	910	18560	38	0	661.7	BS4	44.6	0.9	71.1	0.3	720.7	BS1	89	1.4	140.1	0.7	644.8	BS4	160	1.1
Belfast	1700	62617	39	0	1800.1	BS4	50.2	1.5	76.3	0.5	1800.3	BS4	97.6	1	148.3	0.7	1800.1	BS4	179.6	2
Brighton	976	35012	21	0	1334	BS4	28.2	0.6	40.1	0.3	1789.7	BS4	49.4	0.5	78	0.5	1800.1	BS4	94.8	1.9
Bristol	1569	47522	37	0	1800.1	BS2	47.4	1	73.8	0.4	1800.1	BS4	94	1.4	146.6	1.1	1797.8	BS4	176.4	0.8
Cardiff	1127	23155	39	0	968.2	BS4	50.6	1	78.3	0.5	900.5	BS4	95.6	1.6	157.5	0.8	660.8	BS4	183.2	1.4
Coventry	1175	26689	38	0	1098.1	BS4	44.8	0.4	73	0	1002.5	BS4	85.1	0.7	149.2	0.9	827.3	BS4	172.6	1.4
Exeter	1250	31997	38	0	1365.8	BS4	50.6	0.5	77	0	1544.2	BS4	95.7	1	158.1	0.7	943.2	BS4	182.3	0.6
Glasgow	1137	24323	50.1	0.3	920.6	BS4	59.2	0.7	94	0.5	1068.4	BS4	110.6	1.7	175.2	0.9	745.6	BS4	199.8	1.6
Leeds	1647	56511	40	0	1800.1	BS4	52.4	0.8	79.5	0.5	1800.1	BS4	99.6	1	152.8	0.8	1800.1	BS4	187.1	0.7
Leicester	1531	48219	38	0	1800.1	BS4	51.5	0.5	75	0	1800.1	BS4	94.1	0.8	149.3	0.7	1759	BS4	177.7	1.8
Liverpool	1273	42564	28	0	1800.1	BS4	38.4	0.5	57	0.5	1800.1	BS4	72	0.8	112.8	0.6	1800.1	BS4	133	0.8
Manchester	1991	77286	38.3	0.7	1800.2	BS4	45.9	0.5	77.9	0.3	1800.1	BS4	91.5	0.9	155.2	0.6	1800.1	BS4	178.5	1
Newcastle	1109	26614	44	0	1146.4	BS4	52.6	1.1	83.6	0.5	1020.5	BS4	95.4	1.1	152.4	0.5	951.3	BS2	171.5	1.2
Nottingham	1739	51595	44	0	1799.1	BS4	56.6	0.8	84.7	0.5	1800.2	BS4	103.3	0.8	164.2	0.8	1800.2	BS4	195.2	1.2
Oxford	479	8396	24	0	263.1	BS4	27.9	0.5	47	0	298	BS4	54.9	0.7	89	0	254.4	BS2	100.8	0.9
Plymouth	1122	35070	31	0	1398.8	BS4	40.3	0.8	61.3	0.5	1694.2	BS4	75	1.1	115.6	0.5	1688	BS4	137	1.2
Sheffield	1582	50534	42	0	1800.2	BS4	52.5	0.7	84.6	0.5	1747.7	BS4	98.9	1.3	161.4	0.8	1800	BS4	182.2	1.2
Southampton	796	19942	25	0	750.1	BS4	29.6	0.8	49.2	0.4	807.2	BS4	61.1	0.7	97.6	0.5	1129	BS4	113.2	1.4
Sunderland	1346	42013	36	0	1559.3	BS4	46.3	0.4	73	0	1049	BS4	89.1	1.1	141	0.5	1438.3	BS4	163.6	1
York	1044	23774	32	0	856.8	BS4	39.1	0.3	68	0	573	BS4	77.6	0.6	130.4	0.5	784.5	BS4	145.8	1.2
Belgrade	19586	7561185	86.5	1.5	1805.9	PG	103.4	0.5	171.1	2.4	1803.8	PG	197.3	0.9	341.9	2.2	1802.3	SG	374.5	1.8
Berlin	29461	9944851	102.1	1.9	1817.9	PG	125.9	0.5	204.9	1.9	1878.7	PG	240.1	1.2	396.4	3.1	1804.8	PG	446.2	1.4
Boston	44797	28164740	94.3	1.9	2391.9	PG	102.7	1.3	175.4	2	2007	PG	191.6	0.9	341	0	3819.2	PG	368.7	1.5
Dublin	37982	21630466	101.5	1.1	1819.8	PG	113.8	1.2	193.2	4.8	1815.9	PG	211.3	2.7	363	0	3002.4	PG	390.2	2
Minsk	10487	1375618	102.1	1.1	1801.5	PG	126	0.9	200	1.9	1800.5	PG	240.4	1.4	387.7	3.5	1801	PG	457.6	2.4

## 5 CONCLUSIONS AND FUTURE WORK

In this paper we have solved the  $k$ -domination problem, a generalized version of the prominent minimum dominating set problem, with the variable neighborhood search (Vns) metaheuristic. The efficiency of Vns has been validated on the real-world benchmark, where it has been shown that Vns outperforms all existing heuristic state-of-the-art approaches.

For future work, one could consider improving the Vns to work more efficiently with very large graphs, such as social networks. Vns could be compared to exact approaches such as integer linear programming models, solved by general-purpose solvers such as Cplex or Gurobi.

## ACKNOWLEDGMENTS

We thank Padraig Corcoran and Andrei Gagarin for providing problem instances and their algorithm implementations.

A. Kartelj was supported by grant 451-03-47/2023-01/200104 funded by Ministry of Science Technological Development and Innovations of the Republic of Serbia.

## REFERENCES

- [1] Sergiy Butenko, Xiuzhen Cheng, Carlos A Oliveira, and Panos M Pardalos. 2004. A new heuristic for the minimum connected dominating set problem on ad hoc wireless networks. In *Recent developments in cooperative control and optimization*. Springer, 61–73.
- [2] David Chalupa. 2018. An order-based algorithm for minimum dominating set with application in graph mining. *Information Sciences* 426 (2018), 101–116.
- [3] Gerard Jennhwa Chang. 1983. K-DOMINATION AND GRAPH COVERING PROBLEMS. (1983).
- [4] Padraig Corcoran and Andrei Gagarin. 2021. Heuristics for  $k$ -domination models of facility location problems in street networks. *Computers & Operations Research* 133 (2021), 105368.
- [5] Mathieu Couture, Michel Barbeau, Prosenjit Bose, and Evangelos Kranakis. 2006. Incremental construction of  $k$ -dominating sets in wireless sensor networks. In *International Conference On Principles Of Distributed Systems*. Springer, 202–214.
- [6] Krzysztof Fleszar and Khalil S Hindi. 2004. Solving the resource-constrained project scheduling problem by a variable neighbourhood search. *European Journal of Operational Research* 155, 2 (2004), 402–413.
- [7] Andrei Gagarin and Padraig Corcoran. 2018. Multiple domination models for placement of electric vehicle charging stations in road networks. *Computers & Operations Research* 96 (2018), 69–79.
- [8] Andrei Gagarin, Anush Poghosyan, and Vadim Zverovich. 2013. Randomized algorithms and upper bounds for multiple domination in graphs and networks. *Discrete Applied Mathematics* 161, 4–5 (2013), 604–611.
- [9] Fabrizio Grandoni. 2006. A note on the complexity of minimum dominating set. *Journal of Discrete Algorithms* 4, 2 (2006), 209–214.
- [10] Teresa W Haynes, Stephen Hedetniemi, and Peter Slater. 2013. *Fundamentals of domination in graphs*. CRC press.
- [11] Abdel-Rahman Hedar and Rashad Ismail. 2010. Hybrid genetic algorithm for minimum dominating set problem. In *International conference on computational science and its applications*. Springer, 457–467.
- [12] Abdel-Rahman Hedar and Rashad Ismail. 2012. Simulated annealing with stochastic local search for minimum dominating set problem. *International Journal of Machine Learning and Cybernetics* 3, 2 (2012), 97–109.
- [13] Alberto Herrán, J Manuel Colmenar, and Abraham Duarte. 2019. A variable neighborhood search approach for the Hamiltonian  $p$ -median problem. *Applied Soft Computing* 80 (2019), 603–616.
- [14] Chin Kuan Ho, Yashwant Prasad Singh, and Hong Tat Ewe. 2006. An enhanced ant colony optimization metaheuristic for the minimum dominating set problem. *Applied Artificial Intelligence* 20, 10 (2006), 881–903.
- [15] Ali KARCI. 2020. New algorithms for minimum dominating set in any graphs. *Computer Science* 5, 2 (2020), 62–70.
- [16] James K Lan and Gerard Jennhwa Chang. 2013. Algorithmic aspects of the  $k$ -domination problem in graphs. *Discrete Applied Mathematics* 161, 10–11 (2013), 1513–1520.
- [17] Nenad Mladenović and Pierre Hansen. 1997. Variable neighborhood search. *Computers & operations research* 24, 11 (1997), 1097–1100.
- [18] Jose C Nacher and Tatsuya Akutsu. 2016. Minimum dominating set-based methods for analyzing biological networks. *Methods* 102 (2016), 57–63.
- [19] Abhay K Parekh. 1991. Analysis of a greedy heuristic for finding small dominating sets in graphs. *Information processing letters* 39, 5 (1991), 237–240.
- [20] Dhekra Rezgui, Joughaina Chaouachi Siala, Wassila Aggoune-Mtalaa, and Hend Bouziri. 2019. Application of a variable neighborhood search algorithm to a fleet size and mix vehicle routing problem with electric modular vehicles. *Computers & Industrial Engineering* 130 (2019), 537–550.
- [21] QATAR SERBIA Romania. 2010. Ant colony optimization applied to minimum weight dominating set problem. In *Proceedings of the 12th WSEAS international conference on automatic control, modelling and simulation*. Catania, Italy, 29–31.
- [22] Chao Shen and Tao Li. 2010. Multi-document summarization via the minimum dominating set. In *Proceedings of the 23rd International Conference on Computational Linguistics (Coling 2010)*. 984–992.
- [23] Johan MM Van Rooij and Hans L Bodlaender. 2011. Exact algorithms for dominating set. *Discrete Applied Mathematics* 159, 17 (2011), 2147–2164.
- [24] Guangyuan Wang, Hua Wang, Xiaohui Tao, Ji Zhang, and Jinhua Zhang. 2013. Minimising  $k$ -dominating set in arbitrary network graphs. In *International Conference on Advanced Data Mining and Applications*. Springer, 120–132.
- [25] Fuyu Yuan, Chenxi Li, Xin Gao, Minghao Yin, and Yiyuan Wang. 2019. A novel hybrid algorithm for minimum total dominating set problem. *Mathematics* 7, 3 (2019), 222.