CSE-233 : Section A

Summer 2020

Deterministic Turing Machine

Reference:

i) Book2 Chapter 3.1

ii) Neso Academy

Md. Saidul Hoque Anik
anik@cse.uiu.ac.bd

Overview

Consider the following languages-

1. a^*, a^*b^*
2. a^nb^n, ww^R
3. $a^nb^nc^n$, ww

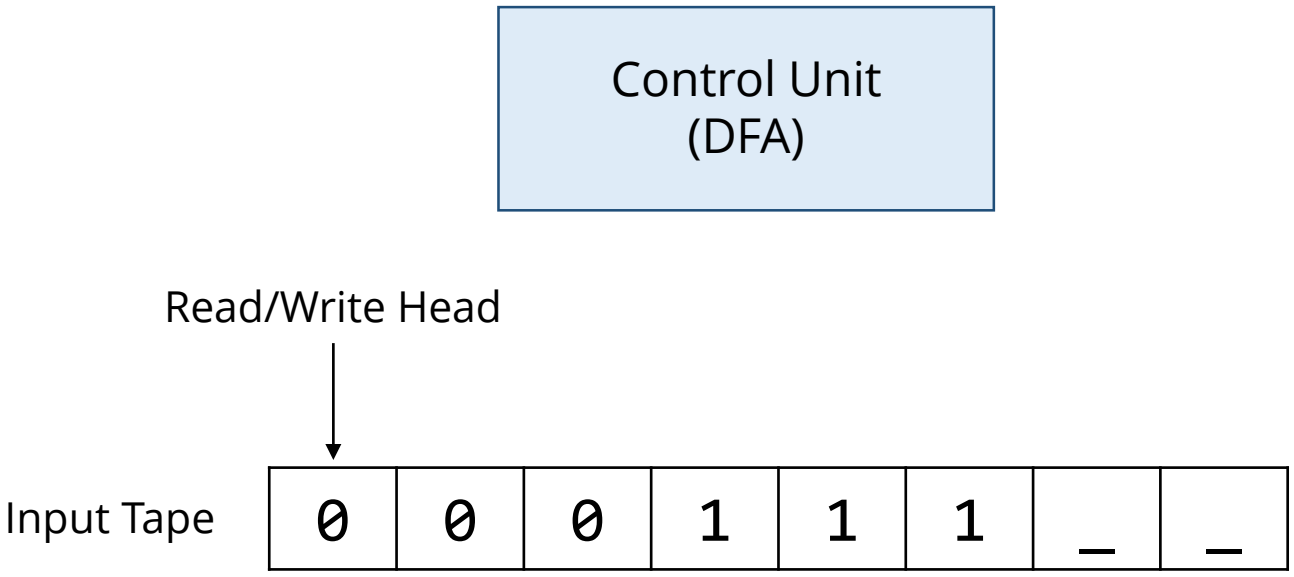
Overview

Consider the following languages-

1. a*, a*b* : can be detected using DFA/NFA
2. $a^n b^n$, ww^R : can be detected using PDA
3. $a^n b^n c^n$, ww : needs something more powerful

Overview

In a Turing machine, we have direct access to the input tape.



The head at each transition-

1. reads a symbol
2. writes a symbol
3. moves left/right

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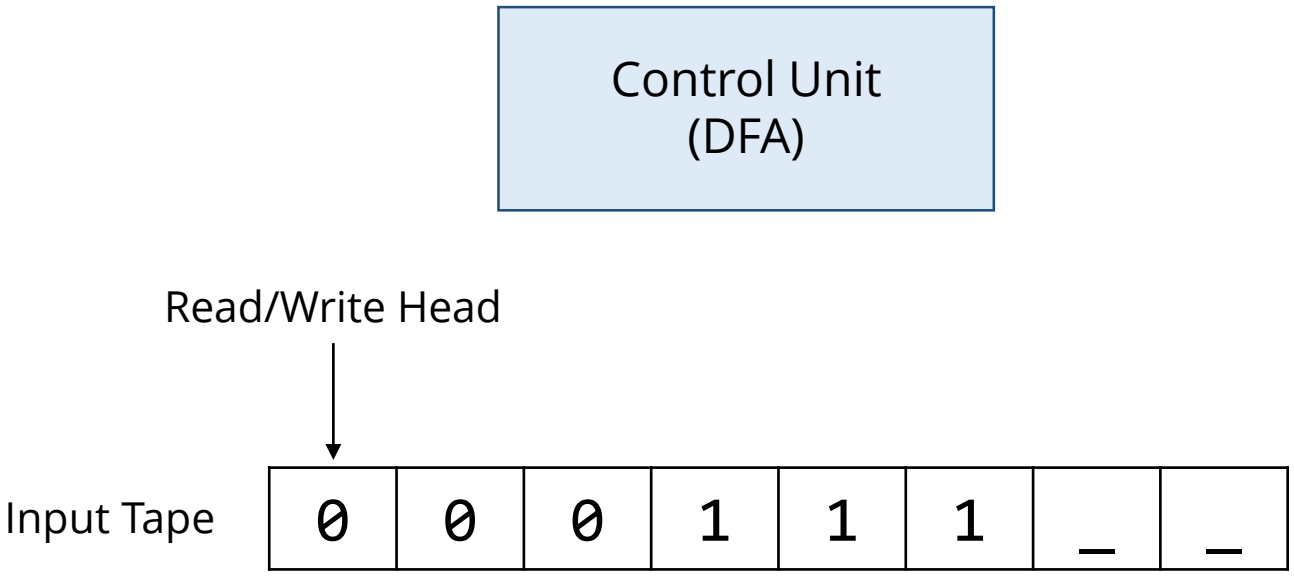

$0 \rightarrow x, R$

This means if the head is pointing at 0, we'll write x in its place,

and move the head to the right

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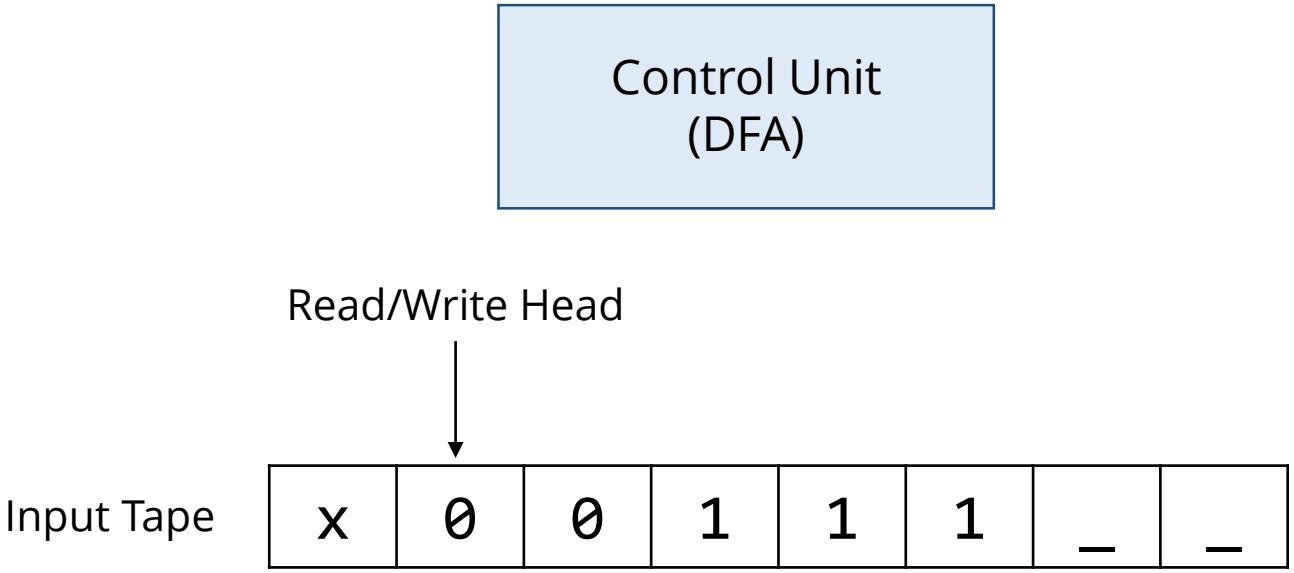

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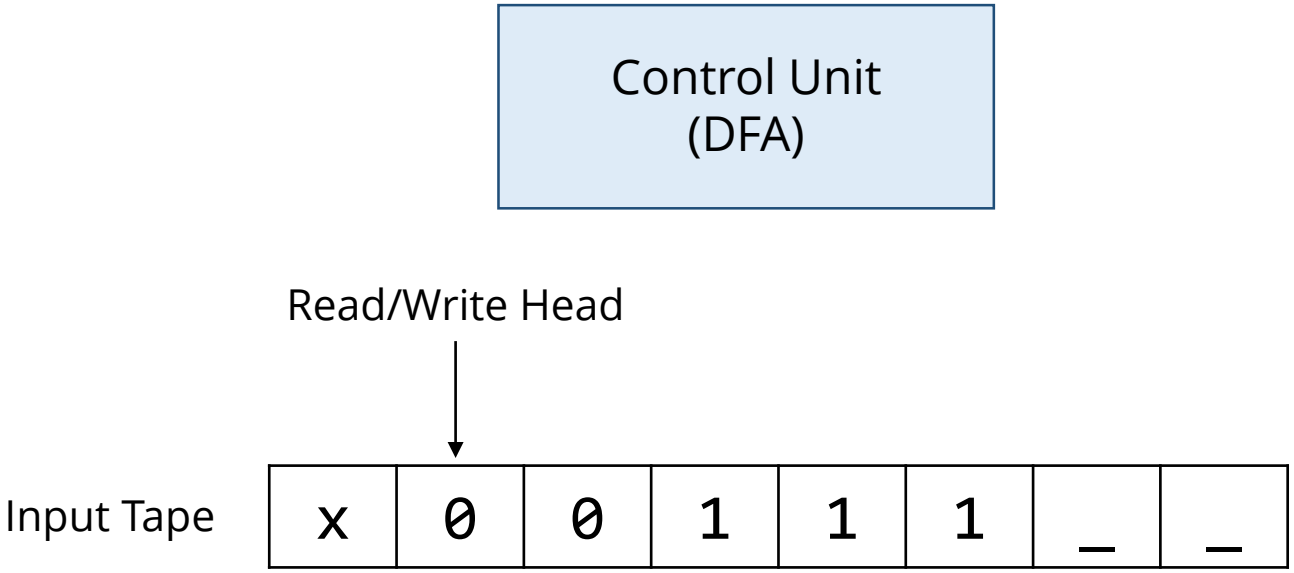

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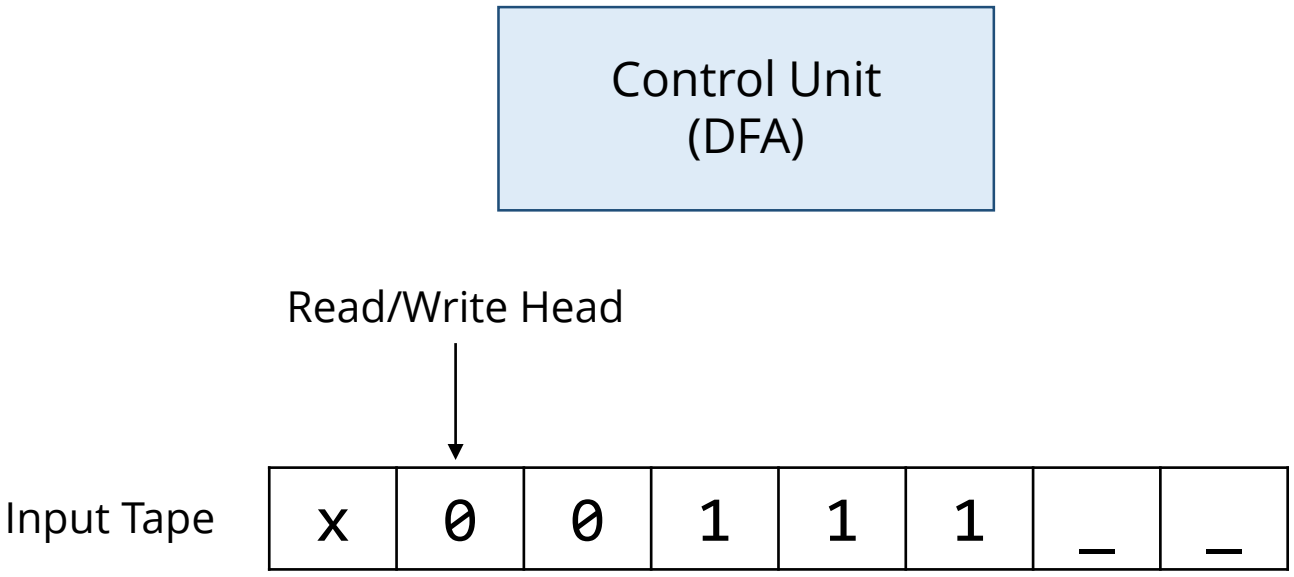

What if we write 0 → y, L?

This means if the head is now pointing at 0, we'll write y in its place, and move the head to the left

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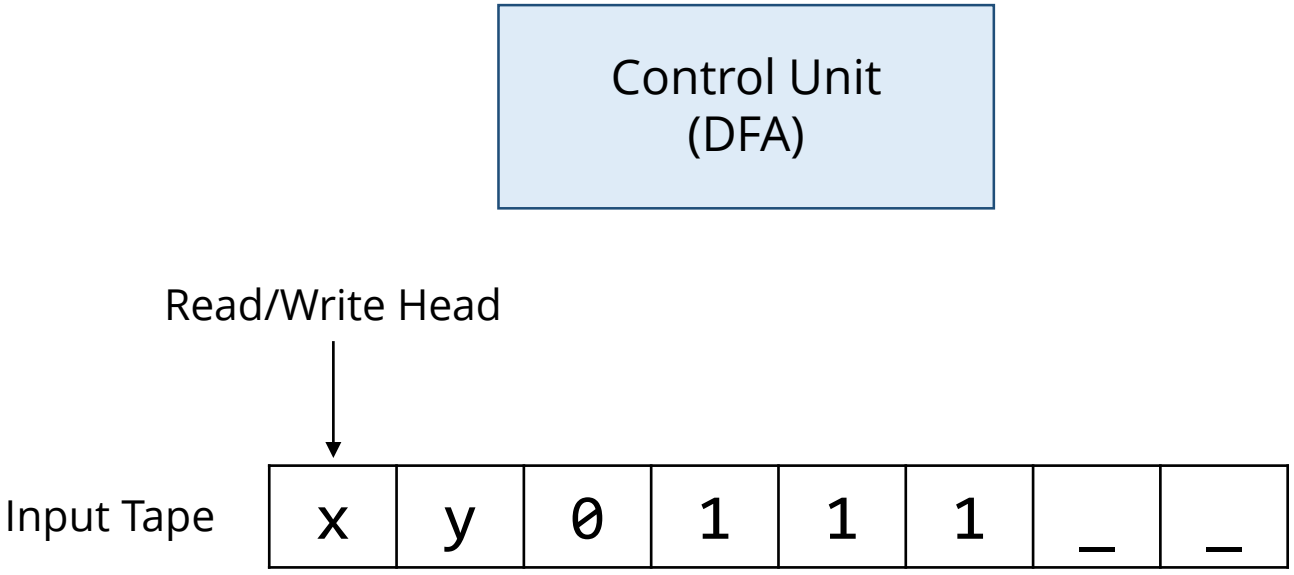

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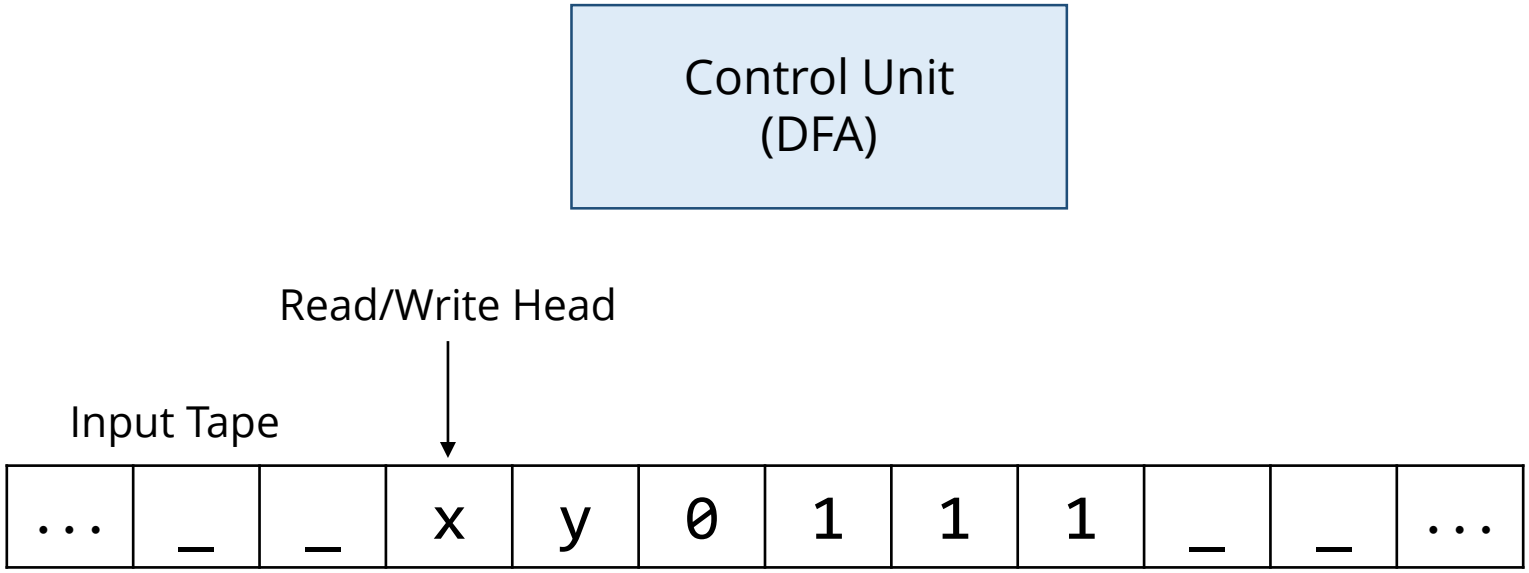
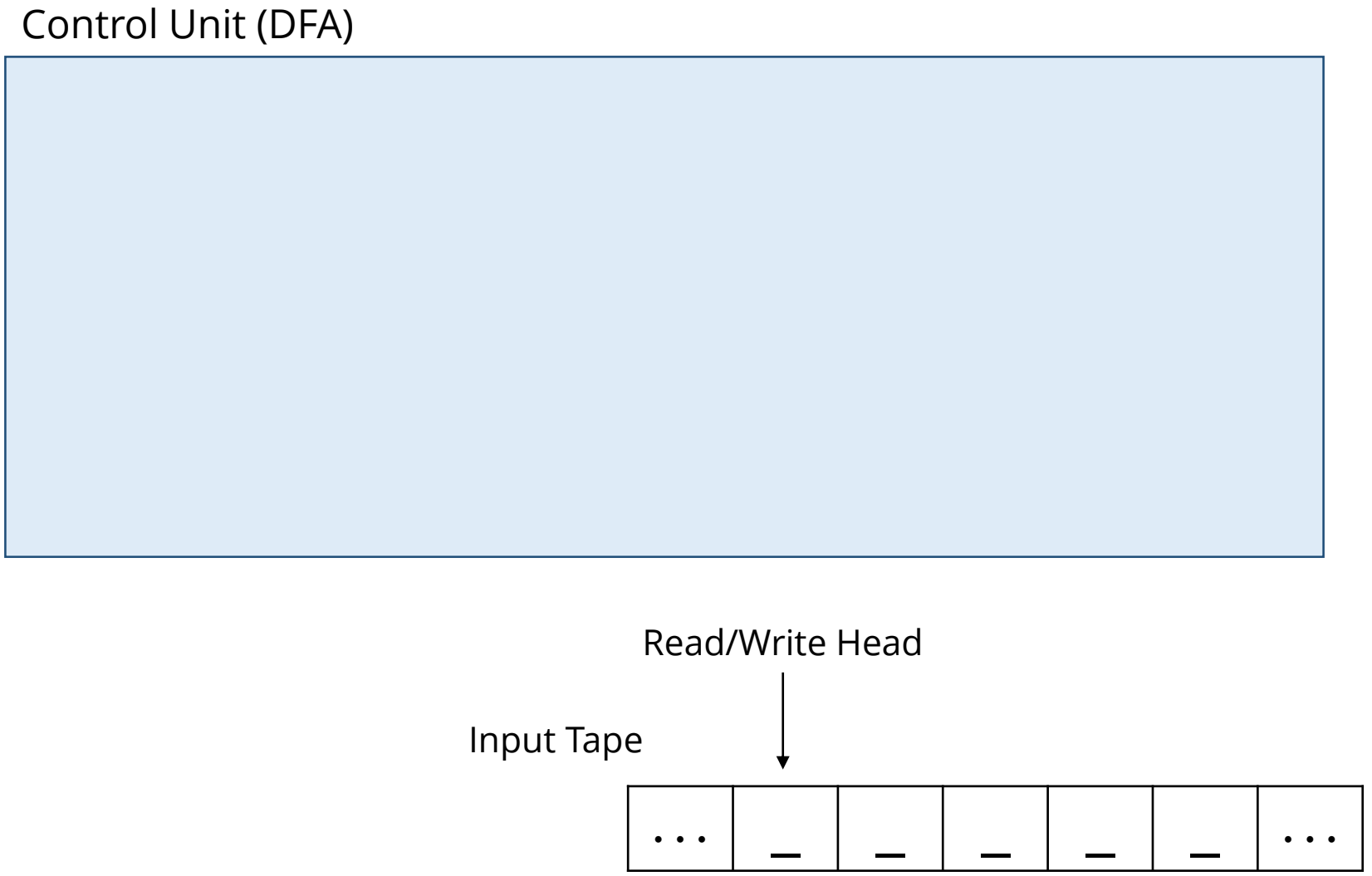


The input tape is infinite. The head points at the first symbol of the input. The left and the right of the input string is filled with infinite blank symbols _.

Example 1

A Turing machine that writes 110 in the tape



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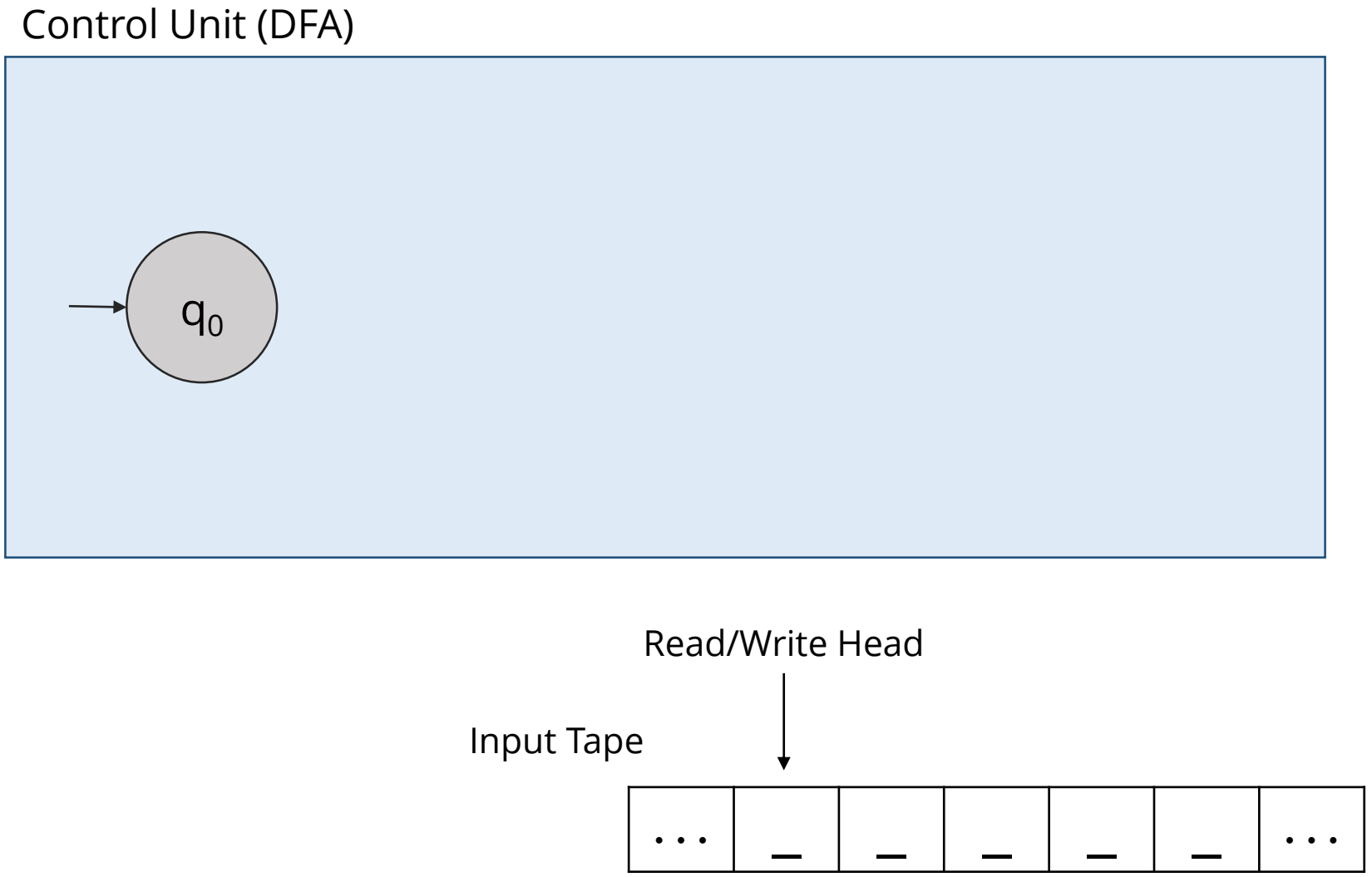

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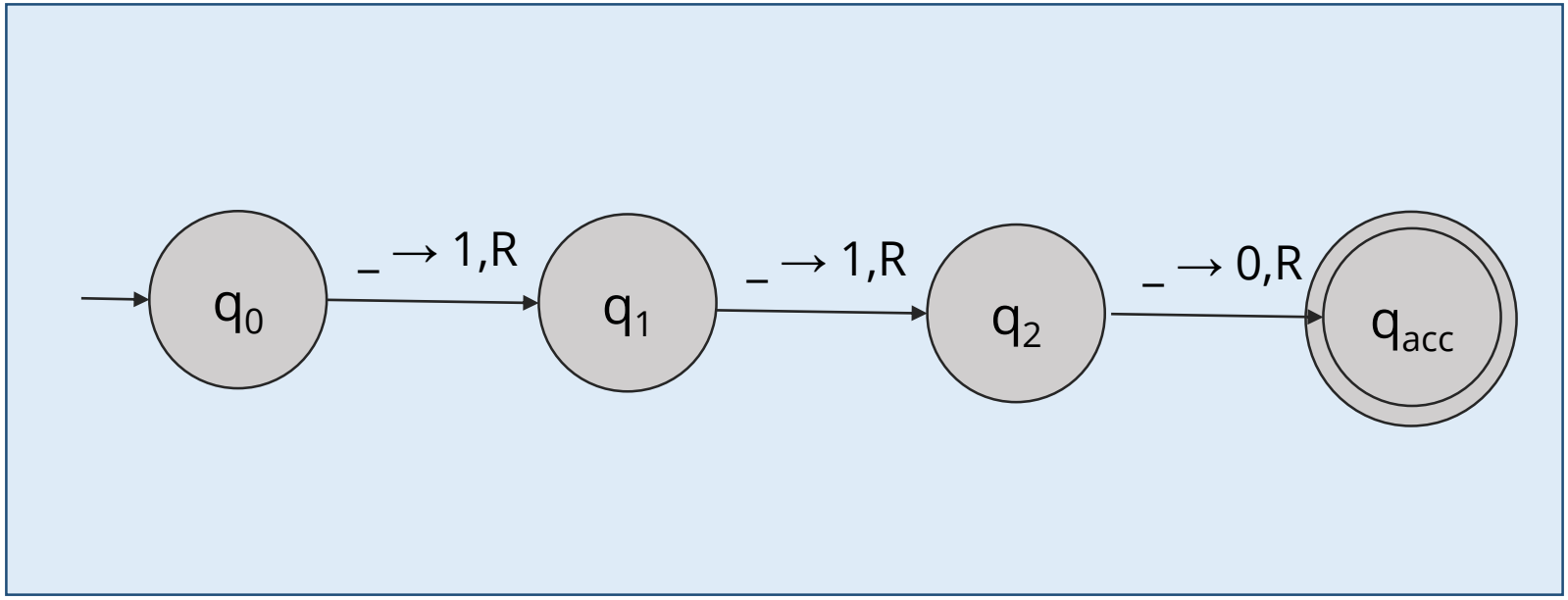

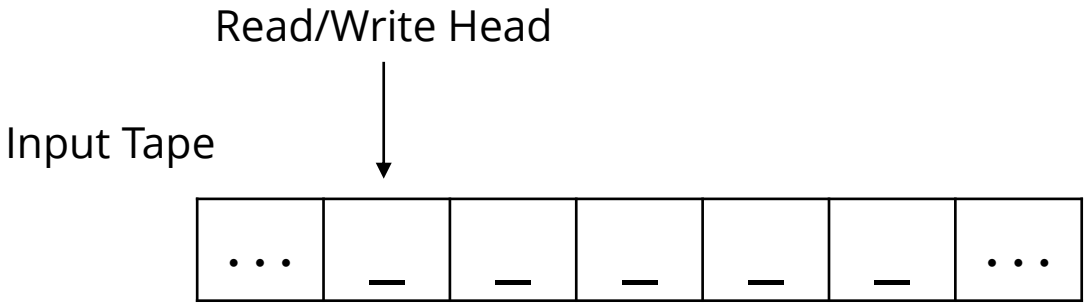

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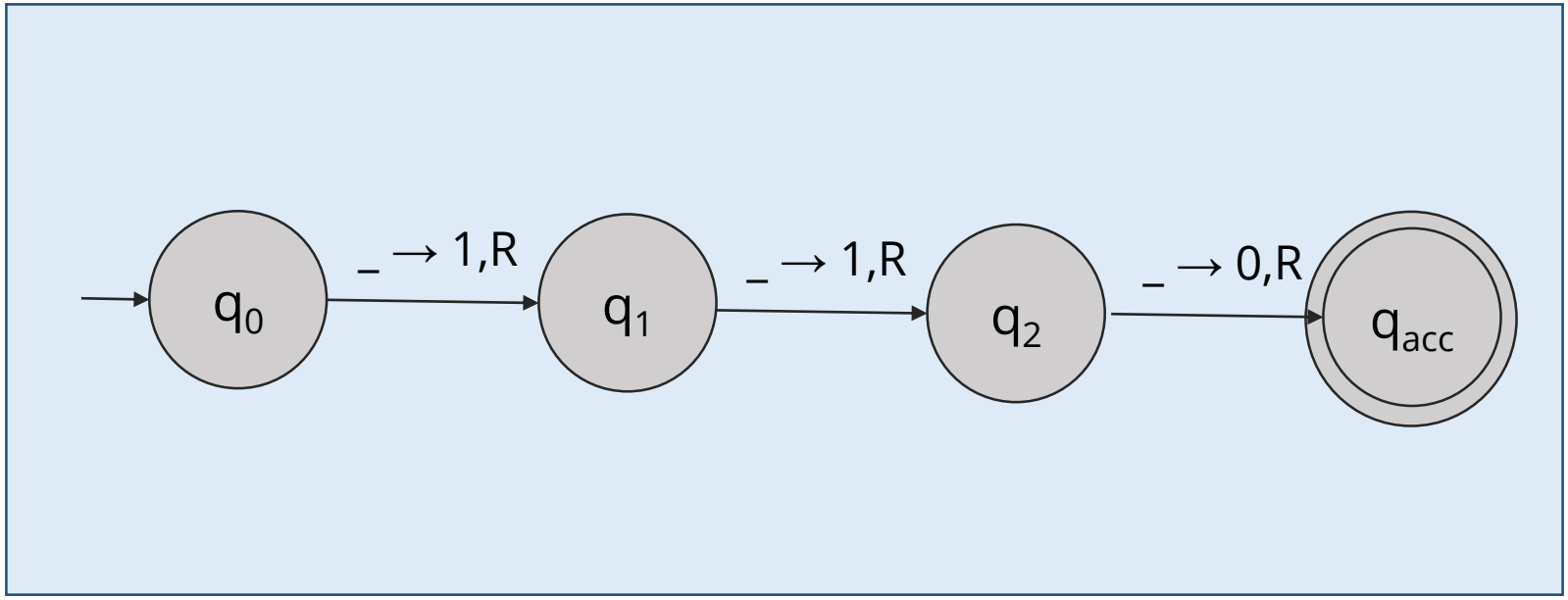

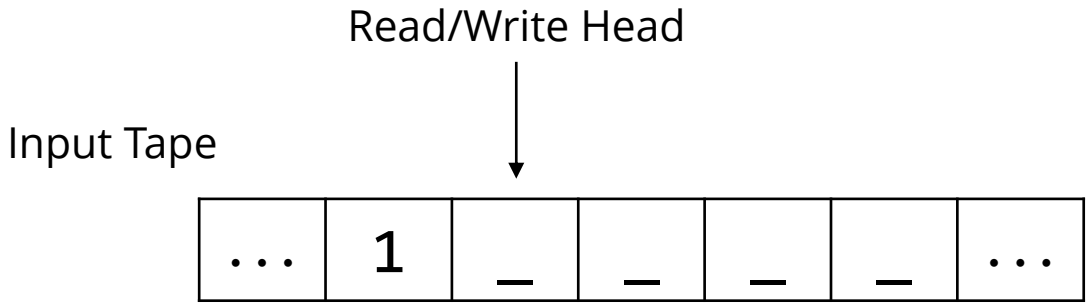

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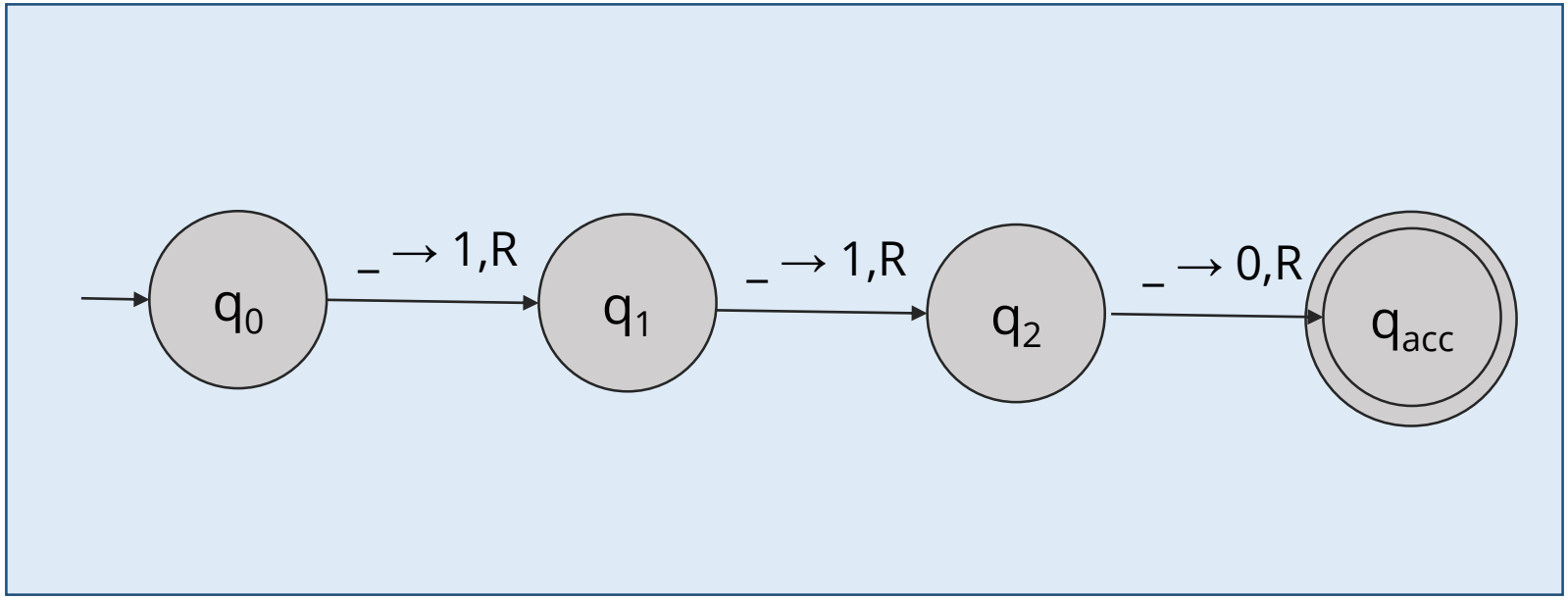

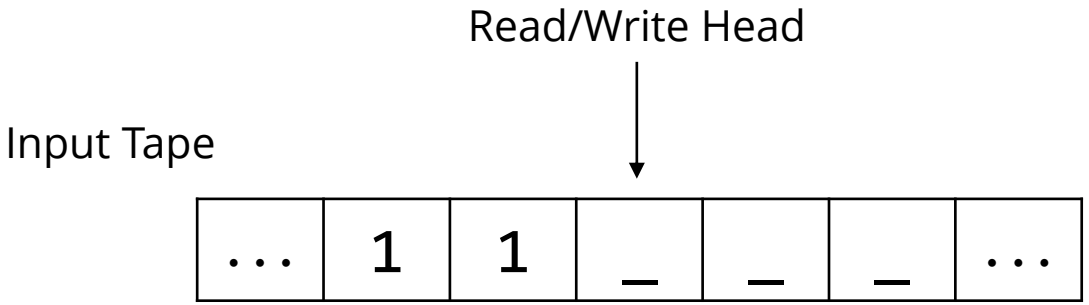

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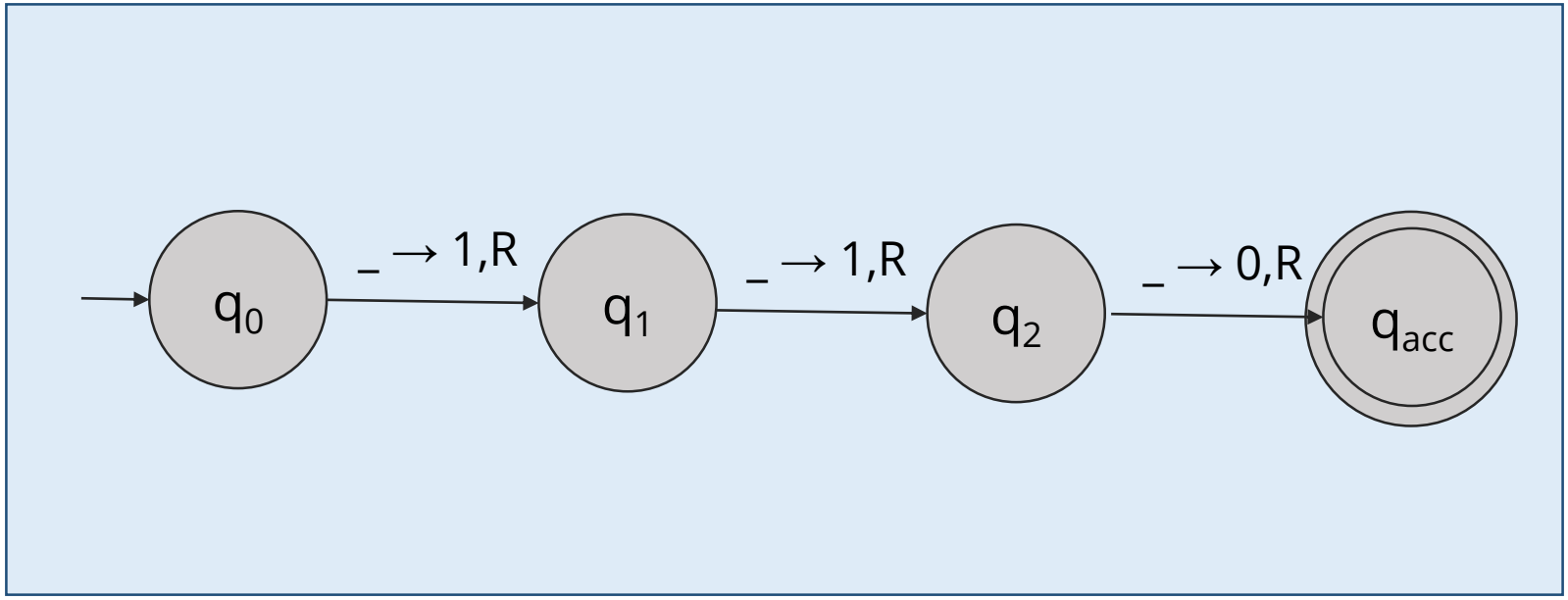
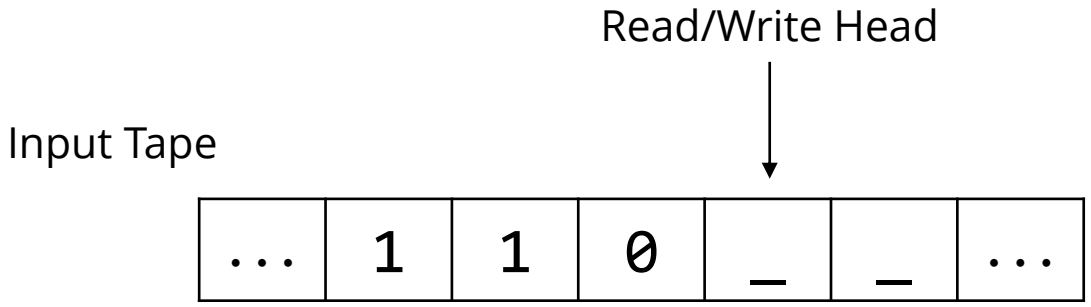
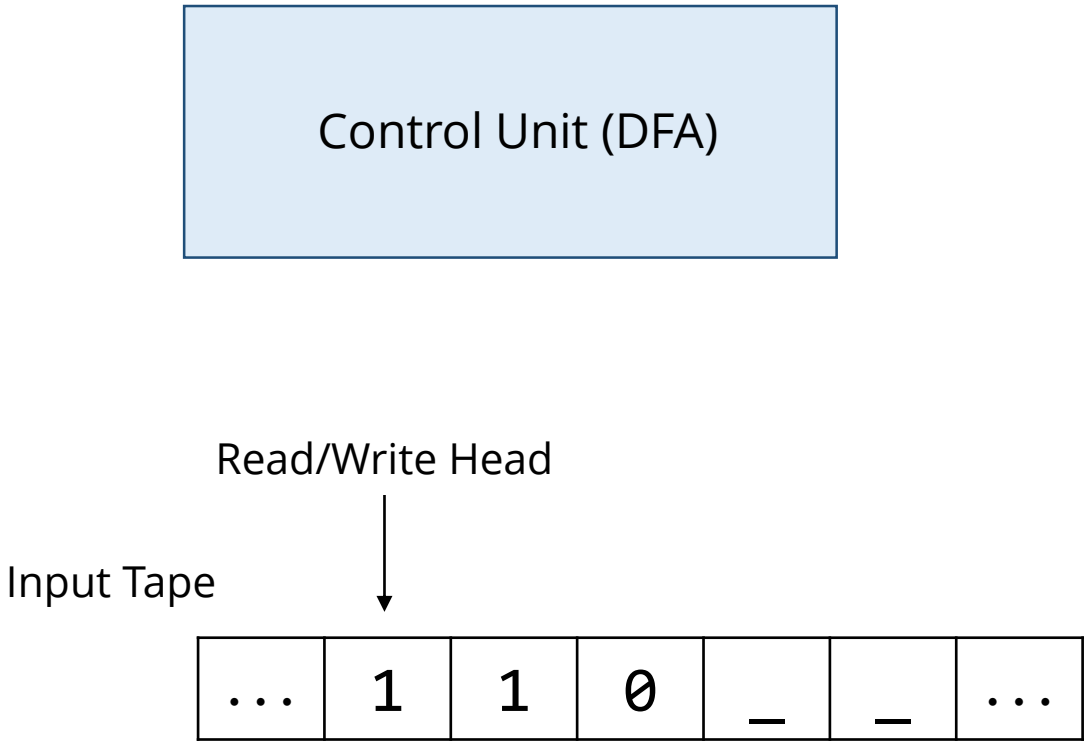




Task

A Turing machine that toggles the input in the tape.



Instantaneous Description

1011$q_7$01111 means

- the tape is 101101111,
- the current state is q_7
- the head is currently on the second 0.

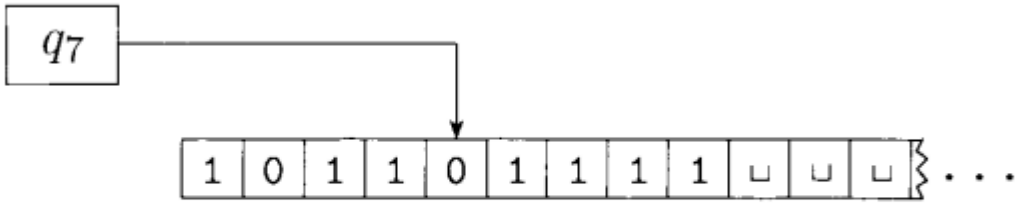


FIGURE 3.4
A Turing machine with configuration 1011$q_7$01111

Simulate Example 1

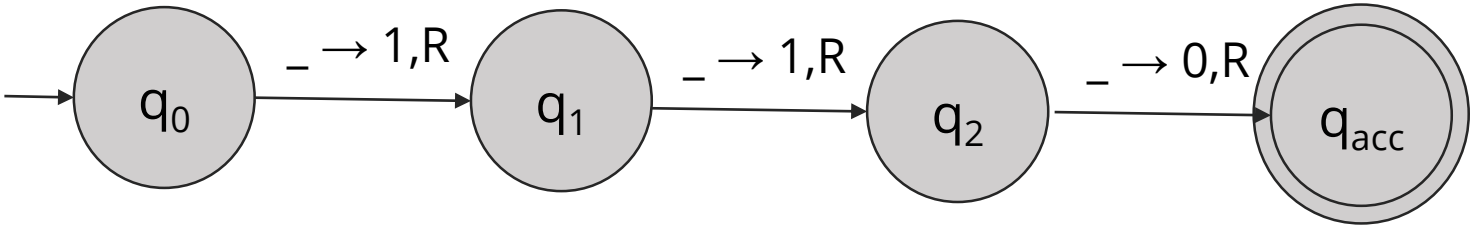
A Turing machine that writes 110.

q_0 _ _ _

1 q_1 _ _

11 q_2 _

110 q_{acc}



Simulate Task Solution on String 1011

A Turing machine that toggles the input in the tape.

$q_0$1011_

Simulate Task Solution on String 1011

A Turing machine that toggles the input in the tape.

$q_0 1011\_$

$0 q_0 011\_$

$01 q_0 11\_$

$010 q_0 1\_$

$0100 q_0\_$

$0100\_ q_{acc}$

Formal Description of a Turing Machine

A ***Turing machine*** is a 7-tuple, $(Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$, where Q, Σ, Γ are all finite sets and

1. Q is the set of states,
2. Σ is the input alphabet not containing the ***blank symbol*** $\sqcup$,
3. Γ is the tape alphabet, where $\sqcup \in \Gamma$ and $\Sigma \subseteq \Gamma$,
4. $\delta: Q \times \Gamma \longrightarrow Q \times \Gamma \times \{\text{L}, \text{R}\}$ is the transition function,
5. $q_0 \in Q$ is the start state,
6. $q_{\text{accept}} \in Q$ is the accept state, and
7. $q_{\text{reject}} \in Q$ is the reject state, where $q_{\text{reject}} \neq q_{\text{accept}}$.

What does this transition function mean?

$\delta(q_1, Z) = (q_2, Y, L)$

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If $\delta(q_1, Z) = (q_2, Y, L)$ then,

in state q_1, scanning Z under its tape head, the TM:

1. Changes the state to q_2.
2. Replaces Z by Y on the tape.
3. Moves the head one square in direction Left.

Note: L = move left; R = move right.

Example 2

Write the formal definition of a Turing machine, that

- When encounters a 1, changes it to 0 and reaches final state.
- If reaches a blank, changes it to 1 and move left.

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Tape Symbols = {}

States = {}

Transition Function: ?

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States = $\{q_0, q_{acc}\}$

Transition Function: ?

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States = $\{q_0, q_{acc}\}$

Transition Function:

- $\delta(q_0, 0) =$
- $\delta(q_0, 1) =$
- $\delta(q_0, \_) =$

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- $\delta(q_0, \_) = (q_0, 1, L)$

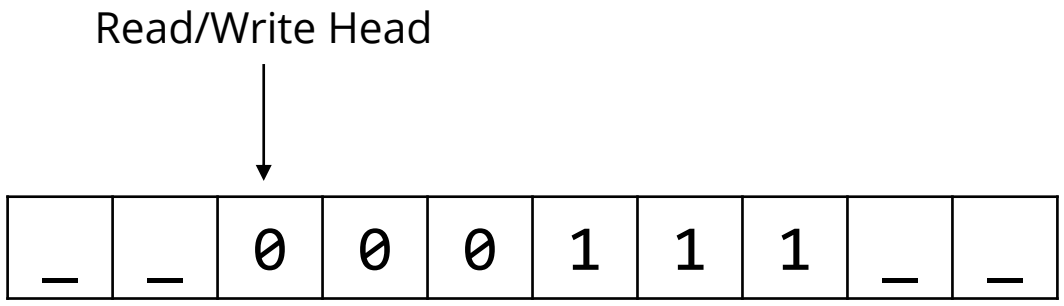
Task

Design a Turing Machine that recognizes the string 01*0

Simulate the string 01110 for the machine

Example 3

Design a Turing Machine that recognizes the Language L = $\{0^n1^n \mid n \geq 1\}$



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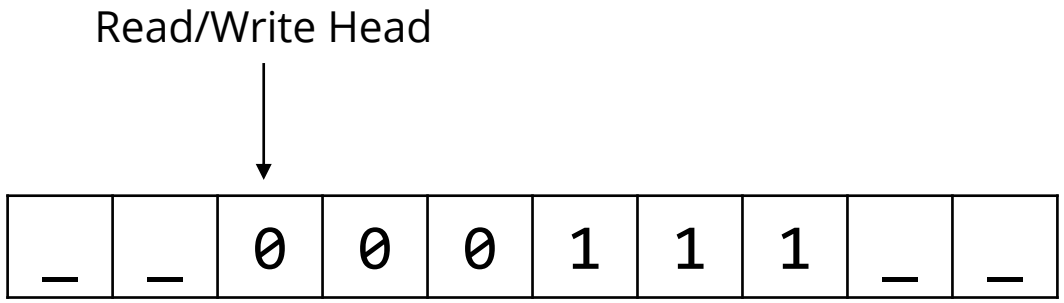

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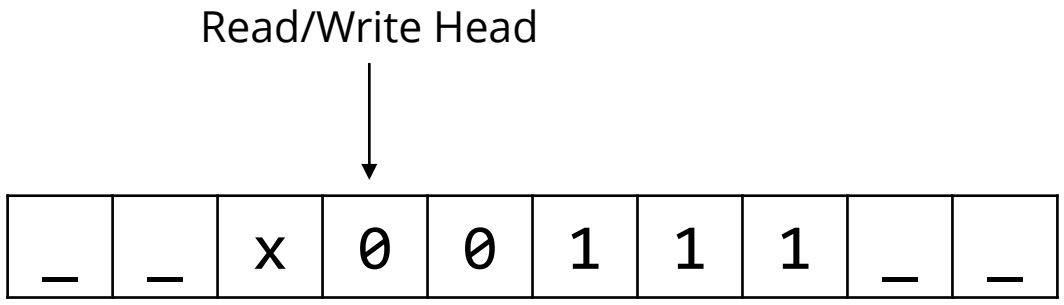

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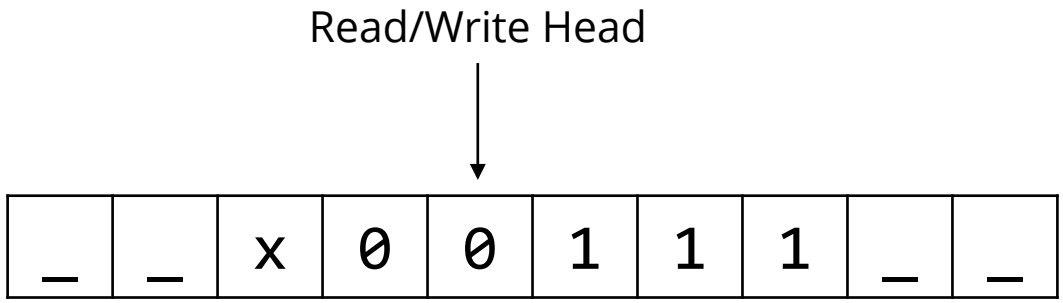

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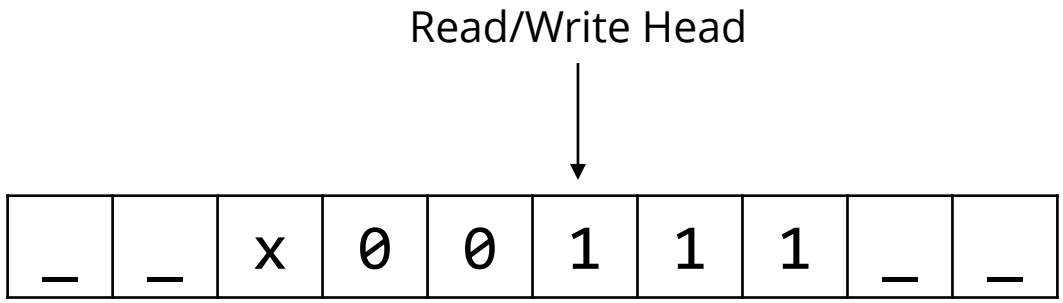

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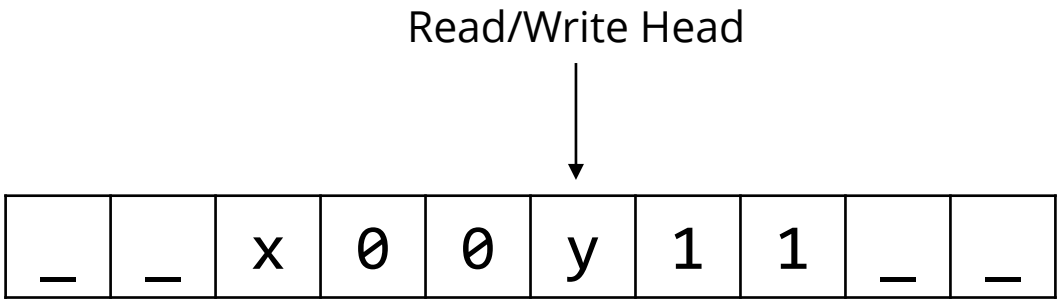

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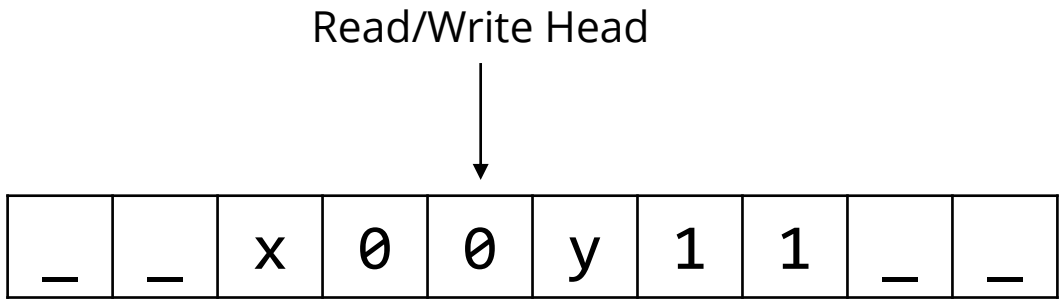

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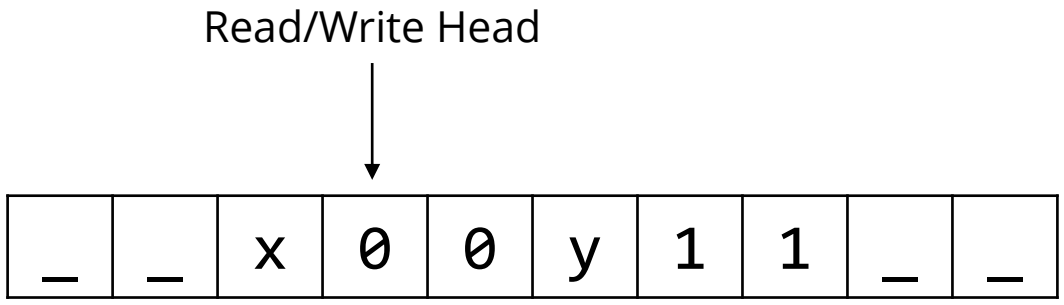

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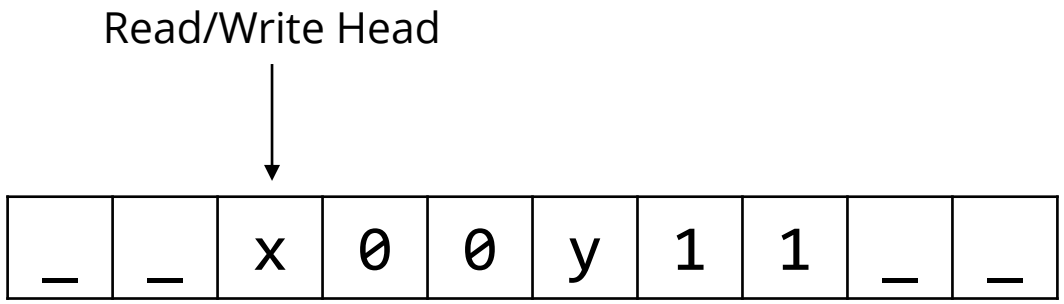

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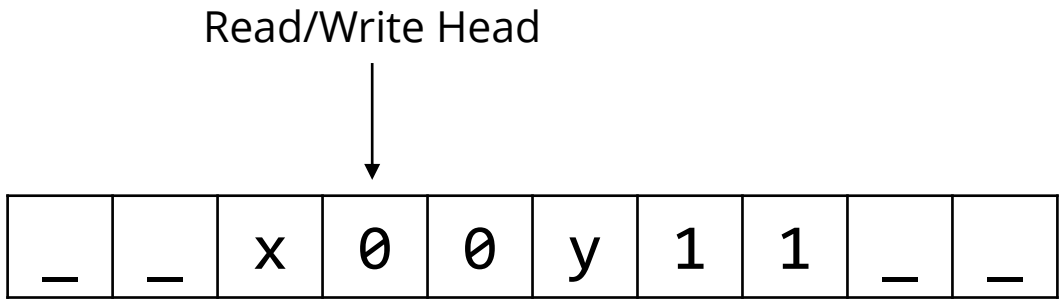

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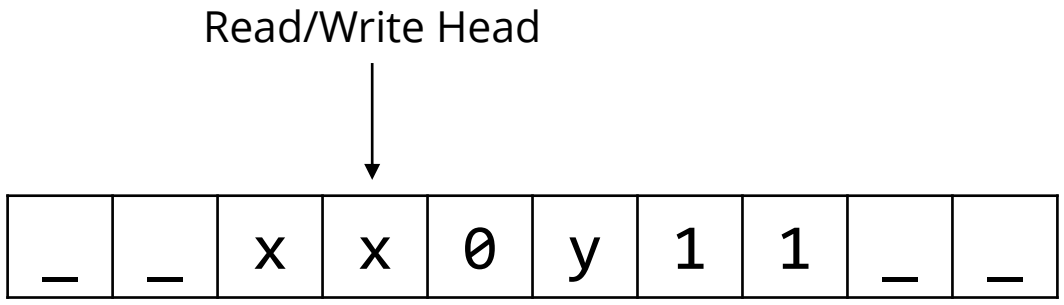

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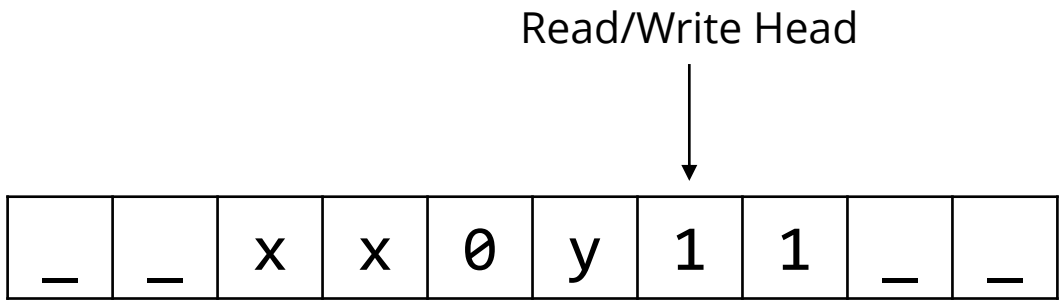

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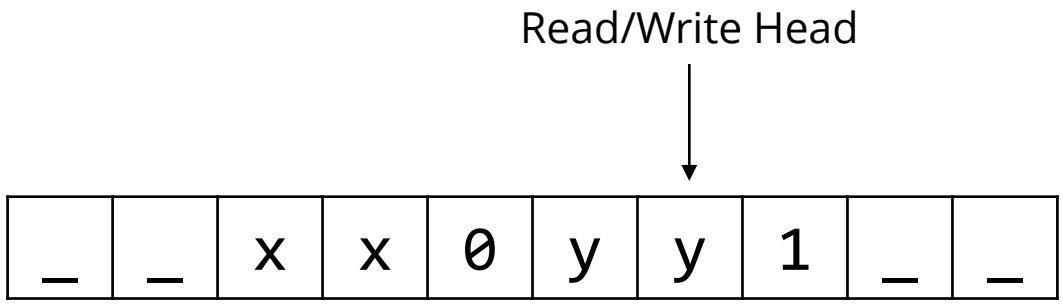

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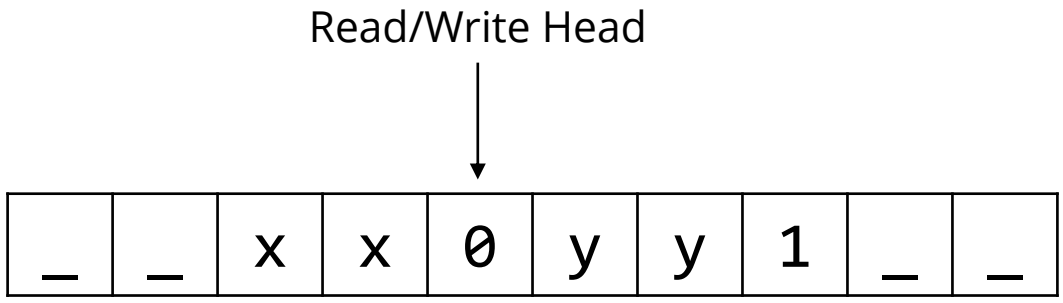
Example 3

Design a Turing Machine that recognizes the Language L = $\{0^n1^n \mid n \geq 1\}$



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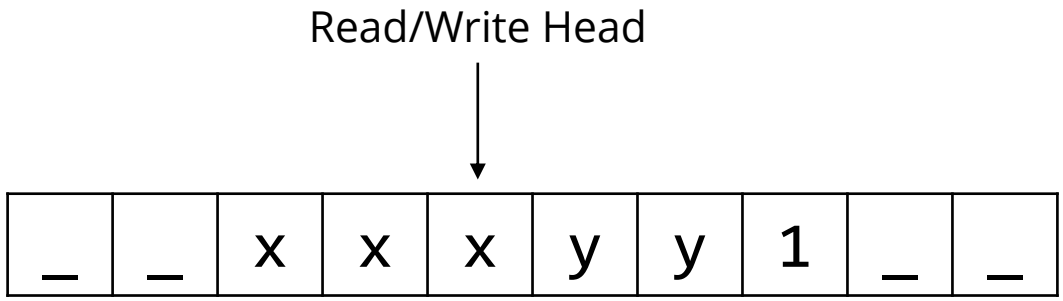

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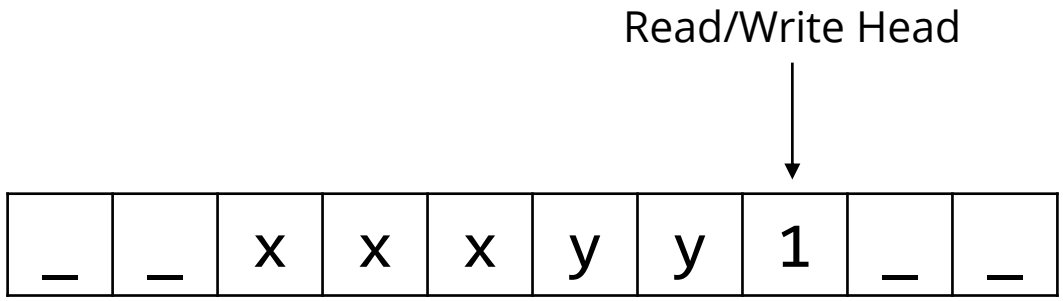

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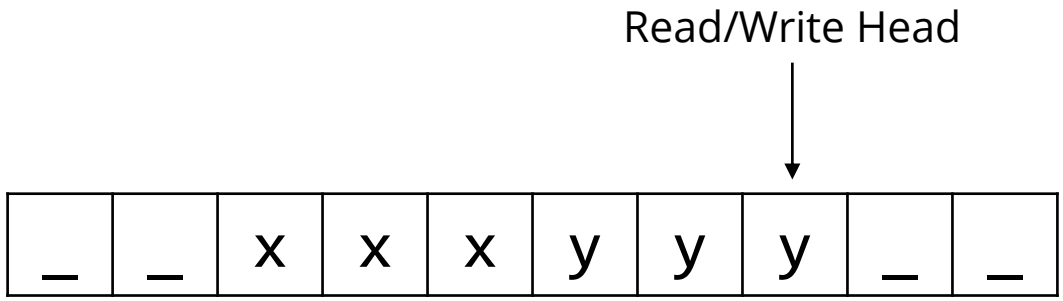

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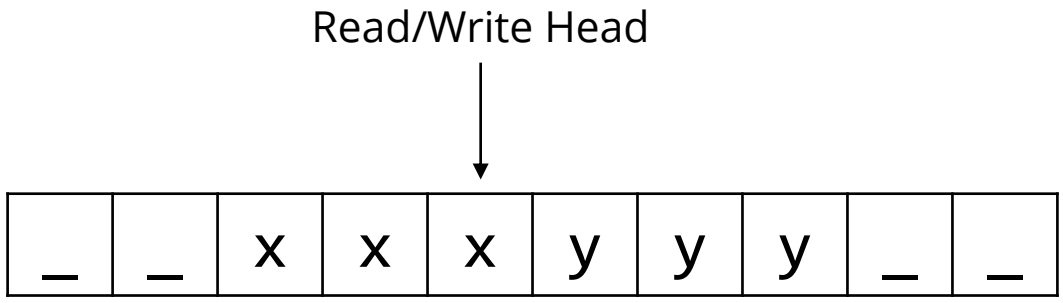

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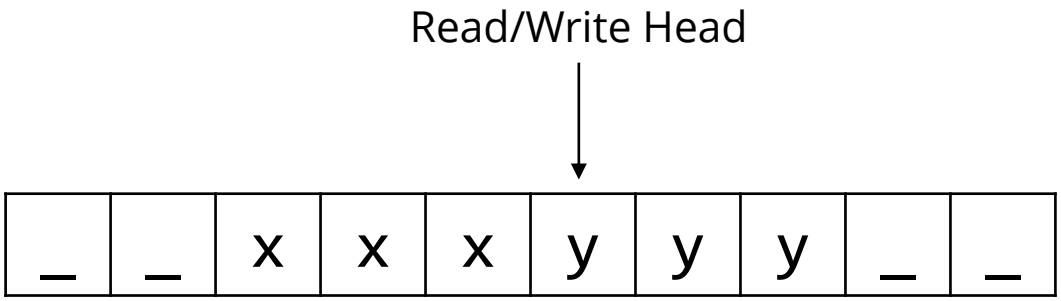

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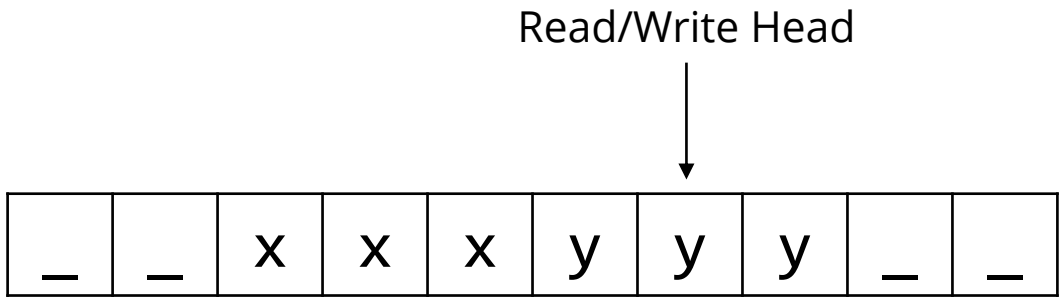

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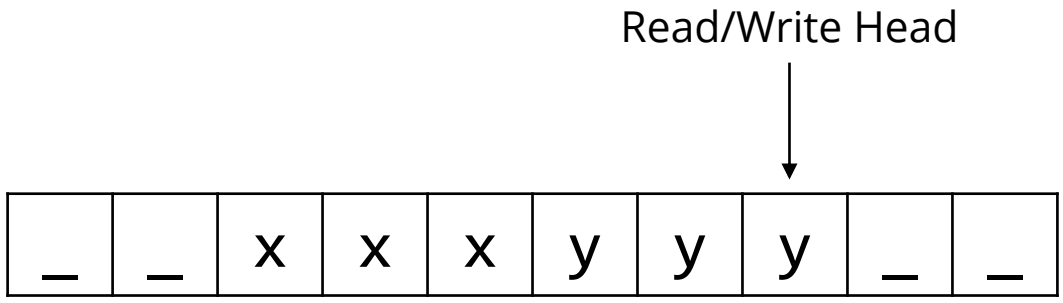

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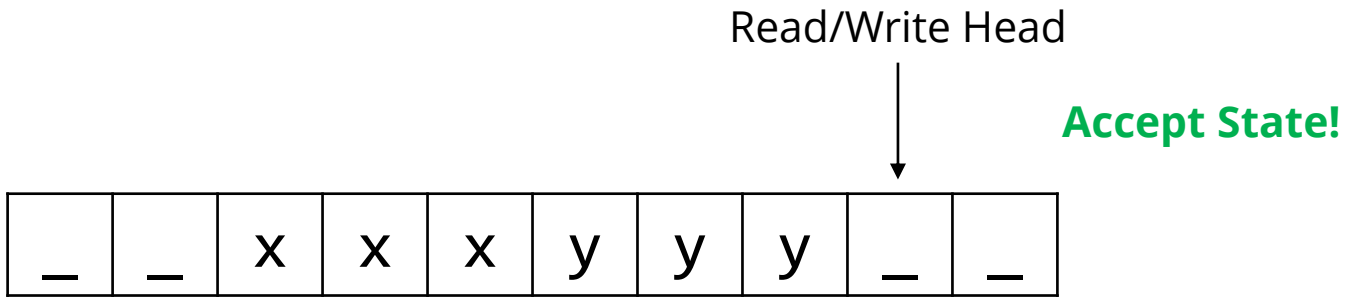

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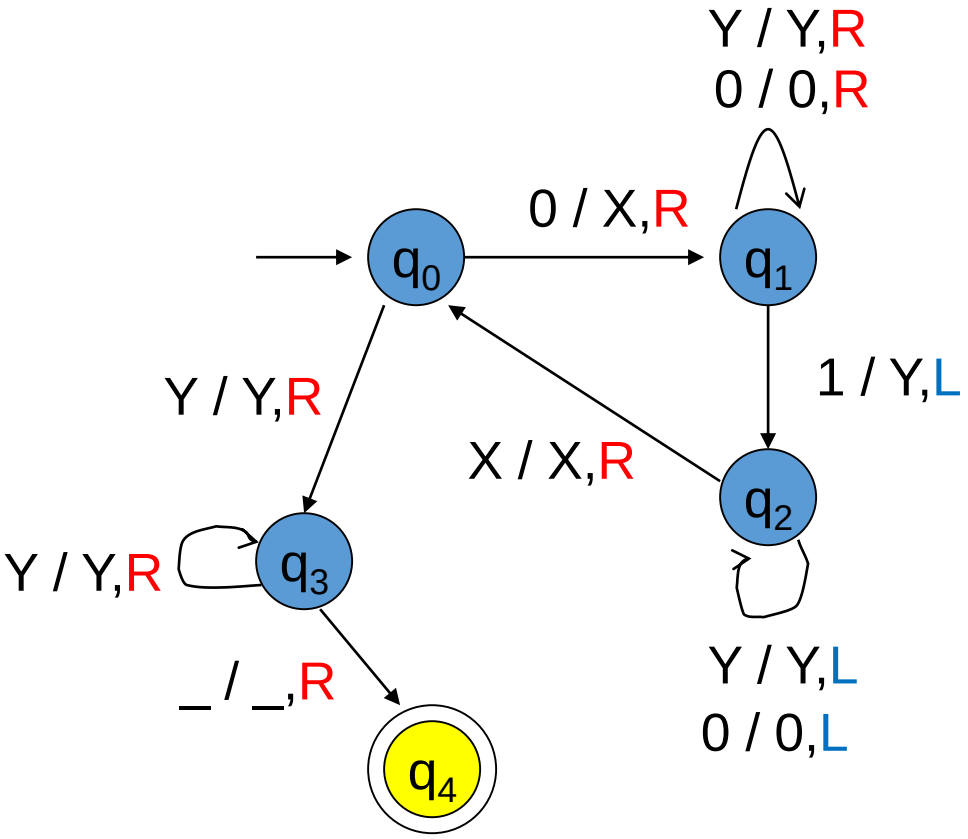
State Diagram of Control

Design a Turing Machine that recognizes the Language L = $\{0^n1^n \mid n \geq 1\}$



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Transition Function

Design a Turing Machine that recognizes the Language L = $\{0^n1^n \mid n \geq 1\}$

	Next Tape Symbol				
Curr. State	0	1	X	Y	B
→ q_0	(q_1,X,R)	-	-	(q_3,Y,R)	-
q_1	(q_1,0,R)	(q_2,Y,L)	-	(q_1,Y,R)	-
q_2	(q_2,0,L)	-	(q_0,X,R)	(q_2,Y,L)	-
q_3	-	-	-	(q_3,Y,R)	(q_4,_,R)
*q_4	-	--	-	-	-

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