



United International University

Assignment 02

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Section: H

Course: Theory of Computation (CSE 2233)

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$$\begin{aligned}
 1. \quad & S \rightarrow YXZ \mid Y \\
 & Y \rightarrow 0Y1 \mid 01 \\
 & X \rightarrow aXb \mid \epsilon \\
 & Z \rightarrow BZ
 \end{aligned}$$

i) no need

ii) remove null

$$\underline{X \rightarrow \epsilon}$$

$$\begin{aligned}
 S &\rightarrow YXZ \mid Y \mid YZ \\
 Y &\rightarrow 0Y1 \mid 01 \\
 X &\rightarrow aXb \mid ab \\
 Z &\rightarrow BZ
 \end{aligned}$$

iii) remove unit production

$$\underline{S \rightarrow Y}$$

$$\begin{aligned}
 S &\rightarrow YXZ \mid 0Y1 \mid 01 \mid YZ \\
 Y &\rightarrow 0Y1 \mid 01 \\
 X &\rightarrow aXb \mid ab \\
 Z &\rightarrow BZ
 \end{aligned}$$

$$\begin{aligned}
 (iv, v) \quad & A_1 \rightarrow a & S_0 &\rightarrow 0 \\
 & B_1 \rightarrow b & S_1 &\rightarrow 1 \\
 & C_1 \rightarrow z \\
 & P \rightarrow YX & Q &\rightarrow S_0 Y \\
 & R &\rightarrow A_1 X_1
 \end{aligned}$$

$$\begin{aligned}
 S &\rightarrow PZ \mid QS_1 \mid S_0 S_1 \mid YZ \\
 Y &\rightarrow QS_1 \mid S_0 S_1 \\
 X &\rightarrow RB_1 \mid A_1 B_1 \\
 Z &\rightarrow BC_1
 \end{aligned}$$

(Pm).

2.

$$S \rightarrow ASB$$

$$A \rightarrow aAS \mid a \mid \epsilon$$

$$B \rightarrow Sbs \mid A \mid bb$$

1) $S' \rightarrow S$

$$S \rightarrow ASB$$

$$A \rightarrow aAS \mid a \mid \epsilon$$

$$B \rightarrow Sbs \mid A \mid bb$$

u) $A \rightarrow \epsilon$

$B \rightarrow \epsilon$

$$S' \rightarrow S$$

$$S \rightarrow ASB \mid SB$$

$$A \rightarrow aAS \mid a \mid aS$$

$$B \rightarrow Sbs \mid A \mid bb \mid \epsilon$$

$$S' \rightarrow S$$

$$S \rightarrow ASB \mid SB \mid AS \mid S$$

$$A \rightarrow aAS \mid a \mid aS$$

$$B \rightarrow Sbs \mid A \mid bb$$

w) for $S' \rightarrow S, S \rightarrow S, B \rightarrow A$

$$S' \rightarrow ASB \mid SB \mid AS$$

$$S \rightarrow ASB \mid SB \mid AS$$

$$A \rightarrow aAS \mid a \mid aS$$

$$B \rightarrow Sbs \mid aAS \mid a \mid aS \mid bb$$

[P.T.O.]

14, v)

$$A_1 \rightarrow a \quad Y \rightarrow SB_1$$

$$X \rightarrow AS$$

$$B_1 \rightarrow b$$

$$S' \rightarrow XB \mid SB \mid AS$$

$$S \rightarrow XB \mid SB \mid AS$$

$$A \rightarrow A_1 X \mid a \mid A_1 S$$

$$B \rightarrow YS \mid A_1 X \mid a \mid A_1 S \mid B, B_1$$

(Ans).

3.

$$S \rightarrow AS A \mid aB$$

$$A \rightarrow B \mid S$$

$$B \rightarrow b \mid \epsilon$$

1) $S' \rightarrow S$

$$S \rightarrow AS A \mid aB$$

$$A \rightarrow B \mid S$$

$$B \rightarrow b \mid \epsilon$$

u) null remove

$$\underline{B \rightarrow \epsilon:}$$

$$S' \rightarrow S$$

$$S \rightarrow AS A \mid aB \mid a$$

$$A \rightarrow B \mid S \mid \epsilon$$

$$B \rightarrow b$$

$$\underline{A \rightarrow \epsilon:}$$

$$S' \rightarrow S$$

$$S \rightarrow AS A \mid aB \mid a \mid SA \mid AS \mid S$$

$$A \rightarrow B \mid S$$

$$B \rightarrow b$$

iii) for $A \rightarrow B, S \rightarrow S$

$$S' \rightarrow ASA | aB | a | SA | AS$$

$$S \rightarrow ASA | aB | a | SA | AS$$

$$A \rightarrow b | ASA | aB | a | SA | AS$$

$$B \rightarrow b$$

iv, v)

$$Y_1 \rightarrow a$$

$$X \rightarrow AS$$

$$S' \rightarrow XA | Y_1 B | a | SA | AS$$

$$S \rightarrow XA | Y_1 B | a | SA | AS$$

$$A \rightarrow b | XA | Y_1 B | a | SA | AS$$

$$B \rightarrow b$$

(Ans) .

$$4. S \rightarrow S+S | S-S | (S) | T$$

$$T \rightarrow x | y | z | X$$

$$X \rightarrow X * X | X \% X | Y$$

$$Y \rightarrow 0 | 1$$

Upright side:

$$S' \rightarrow S$$

$$S \rightarrow S+S | S-S | (S) | T$$

$$T \rightarrow x | y | z | X$$

$$X \rightarrow X * X | X \% X | Y$$

$$Y \rightarrow 0 | 1$$

u) remove null

↳ none

w) for $X \rightarrow Y$, $T \rightarrow X$, $S \rightarrow T$ and $S' \rightarrow S$

$$S' \rightarrow S+S \mid S-S \mid (S) \mid x|y|z \mid X*Y \mid X\%Y \mid 0 \mid 1$$

$$S \rightarrow S+S \mid S-S \mid (S) \mid x|y|z \mid X*Y \mid X\%Y \mid 0 \mid 1$$

$$T \rightarrow x|y|z \mid X*Y \mid X\%Y \mid 0 \mid 1$$

$$X \rightarrow X*Y \mid X\%Y \mid 0 \mid 1$$

$$Y \rightarrow 0 \mid 1$$

iv, v)

$$A_1 \rightarrow +$$

$$B_1 \rightarrow (\quad S \rightarrow XA_3$$

$$A_2 \rightarrow -$$

$$B_2 \rightarrow) \quad T \rightarrow XA_4$$

$$A_3 \rightarrow *$$

$$X_1 \rightarrow x$$

$$A_4 \rightarrow \%$$

$$Y_1 \rightarrow y$$

$$P \rightarrow SA_1$$

$$Z_1 \rightarrow z$$

$$Q \rightarrow SA_2$$

$$R \rightarrow B_1 S$$

$$S' \rightarrow PS \mid QS \mid RB_2 \mid X_1 \mid Y_1 \mid Z_1 \mid SX \mid TX \mid 0 \mid 1$$

$$S \rightarrow PS \mid QS \mid RB_2 \mid X_1 \mid Y_1 \mid Z_1 \mid SX \mid TX \mid 0 \mid 1$$

$$T \rightarrow x|y|z \mid SX \mid TX \mid 0 \mid 1$$

$$X \rightarrow SX \mid TX \mid 0 \mid 1$$

$$Y \rightarrow 0 \mid 1$$

(dm).

$$\begin{aligned} 5. \quad S &\rightarrow ASB \\ A &\rightarrow aAS \mid a \mid \epsilon \\ B &\rightarrow Sbs \mid A \mid bb \end{aligned}$$

$$\begin{aligned} i) \quad S' &\rightarrow S \\ S &\rightarrow ASB \\ A &\rightarrow aAS \mid a \mid \epsilon \\ B &\rightarrow Sbs \mid A \mid bb \end{aligned}$$

$$\begin{aligned} ii) \quad A &\rightarrow \epsilon \\ S' &\rightarrow S \\ S &\rightarrow ASB \mid SB \\ A &\rightarrow aAS \mid a \mid aS \\ B &\rightarrow Sbs \mid A \mid bb \mid \epsilon \end{aligned}$$

$$\begin{aligned} B &\rightarrow \epsilon \\ S' &\rightarrow S \\ S &\rightarrow ASB \mid SB \mid AS \mid S \\ A &\rightarrow aAS \mid a \mid aS \\ B &\rightarrow Sbs \mid A \mid bb \end{aligned}$$

$$\begin{aligned} iii) \quad \text{for } S' &\rightarrow S, S \rightarrow S, B \rightarrow A \\ S' &\rightarrow ASB \mid SB \mid AS \\ S &\rightarrow ASB \mid SB \mid AS \\ A &\rightarrow aAS \mid a \mid aS \\ B &\rightarrow Sbs \mid aAS \mid a \mid aS \mid bb \end{aligned}$$

$$\begin{aligned} iv, v) \quad A_1 &\rightarrow a & X &\rightarrow AS \\ B_1 &\rightarrow b & Y &\rightarrow SB_1 \\ S' &\rightarrow XB \mid SB \mid AS \\ S &\rightarrow XB \mid SB \mid AS \\ A &\rightarrow A_1X \mid a \mid A_1S \\ B &\rightarrow YS \mid A_1X \mid a \mid A_1S \mid B_1B_1 \end{aligned}$$

(Ans)

$$6. S \rightarrow aSBcD \mid BC$$

$$A \rightarrow AbCd \mid a$$

$$B \rightarrow CBA \mid \epsilon$$

$$C \rightarrow c \mid \epsilon$$

$$D \rightarrow d$$

$$D) S' \rightarrow S$$

$$S \rightarrow aSBcD \mid BC$$

$$A \rightarrow AbCd \mid a$$

$$B \rightarrow CBA \mid \epsilon$$

$$C \rightarrow c \mid \epsilon$$

$$D \rightarrow d$$

u) remove null:

$$\underline{B \rightarrow \epsilon}$$

$$S' \rightarrow S$$

$$S \rightarrow aSBcD \mid BC \mid aScD \mid C$$

$$A \rightarrow AbCd \mid a$$

$$B \rightarrow CBA \mid CA$$

$$C \rightarrow c \mid \epsilon$$

$$D \rightarrow d$$

$$\underline{C \rightarrow \epsilon}$$

$$S' \rightarrow S$$

$$S \rightarrow aSBcD \mid BC \mid aScD \mid c \mid B \mid \epsilon$$

$$A \rightarrow AbCd \mid a \mid Abd$$

$$B \rightarrow CBA \mid CA \mid BA \mid A$$

$$C \rightarrow c$$

$$D \rightarrow d$$

$$\underline{S \rightarrow \epsilon}$$

$$S' \rightarrow S$$

$$S \rightarrow aSBcD \mid BC \mid aScD \mid c \mid B \mid aBcD \mid acD$$

$$A \rightarrow AbCd \mid a \mid Abd$$

$$B \rightarrow CBA \mid CA \mid BA \mid A$$

$$C \rightarrow c \quad D \rightarrow d$$

iii) unid :

for $S' \rightarrow S, S \rightarrow B, S \rightarrow C, B \rightarrow C$

$S' \rightarrow a b c d \mid b c \mid a b c d \mid c \mid c b a \mid c a \mid b a \mid a b c d \mid a c d$

$S \rightarrow a b c d \mid b c \mid a b c d \mid c \mid c b a \mid c a \mid b a$

$A \rightarrow A b c d \mid a \mid A b c d$

$B \rightarrow c b a \mid c a \mid b a \mid a A$

$c \rightarrow c$

$d \rightarrow d$

iv, v)

$A_1 \rightarrow a$

$P \rightarrow A_1 S$

$T \rightarrow A B_1$

$B_1 \rightarrow b$

$Q \rightarrow c_1 D$

$U \rightarrow c D_1$

$c_1 \rightarrow c$

$R \rightarrow B Q$

$V \rightarrow A_1 B$

$D_1 \rightarrow d$

$S \rightarrow B A$

$S \rightarrow p r \mid b c \mid p q \mid c \mid c s \mid c a \mid b a \mid v q \mid A_1 q$

$A \rightarrow T U \mid a \mid T D_1$

$B \rightarrow c s \mid c a \mid b a \mid a A$

$c \rightarrow c$

$D \rightarrow d$

(Ans).

7.

$$S \rightarrow xP \mid yQ \mid y \mid RRz$$

$$P \rightarrow Qxx \mid nyR \mid \epsilon$$

$$Q \rightarrow yPPy \mid ny \mid zR$$

$$R \rightarrow x \mid y \mid PR \mid \epsilon$$

Onight side occurrence:
 ϵ not needed

u) remove null:

$$\underline{P \rightarrow \epsilon}$$

$$S \rightarrow xP \mid yQ \mid y \mid RRz \mid x$$

$$P \rightarrow Qxx \mid nyR$$

$$Q \rightarrow yPPy \mid ny \mid zR \mid yPy \mid yy$$

$$R \rightarrow x \mid y \mid PR \mid \epsilon \mid R$$

$$\underline{R \rightarrow \epsilon}$$

$$S \rightarrow xP \mid yQ \mid y \mid RRz \mid x \mid Rz \mid z$$

$$P \rightarrow Qxx \mid nyR \mid ny$$

$$Q \rightarrow yPPy \mid ny \mid zR \mid yPy \mid yy \mid z$$

$$R \rightarrow x \mid y \mid PR \mid R \mid P$$

iii) remove unit productions:

for $R \rightarrow R, R \rightarrow P$

$$S \rightarrow xP | yG | y | RRz | x | Rz | z$$

$$P \rightarrow Gxx | xyR | xy$$

$$G \rightarrow yPPy | xy | zR | yPy | yy | z$$

$$R \rightarrow x | y | PR | Gxx | xyR | xy$$

iv)

$$x_1 \rightarrow x \quad A \rightarrow RR$$

$$y_1 \rightarrow y \quad B \rightarrow Gx_1$$

$$z_1 \rightarrow z \quad C \rightarrow x_1y_1$$

$$D \rightarrow y_1P \quad E \rightarrow Py_1$$

$$S \rightarrow x_1P | y_1G | Az_1 | Rz_1 | z | x | y$$

$$P \rightarrow Bx_1 | CR | x_1y_1$$

$$G \rightarrow DE | x_1y_1 | z_1R | z | Dy_1 | y_1y_1$$

$$R \rightarrow x | y | PR | Bx_1 | CR | x_1y_1$$

(Ans).

8.

$$A \rightarrow 1 | B | CA | \epsilon$$

$$B \rightarrow 1B5 | 050B | \epsilon$$

$$C \rightarrow x | y | A$$

$$S \rightarrow 1A1 | 05 | 5 | A1$$

1) right side occurrence:

$$A' \rightarrow A$$

$$A \rightarrow 1 | B | CA | \epsilon$$

$$B \rightarrow 1B5 | 050B | \epsilon$$

$$C \rightarrow x | y | A$$

$$S \rightarrow 1A1 | 05 | 5 | A1$$

u) remove null:

1. $A \rightarrow \epsilon$

$$A' \rightarrow A | \epsilon$$

$$A \rightarrow 1 | B | CA | \epsilon$$

$$B \rightarrow 1B5 | 050B | \epsilon$$

$$C \rightarrow x | y | A | \epsilon$$

$$S \rightarrow 1A1 | 05 | 5 | A1 | 11 | 1$$

2. $B \rightarrow \epsilon$

$$A' \rightarrow A | \epsilon$$

$$A \rightarrow 1 | B | CA | \epsilon$$

$$B \rightarrow 1B5 | 050B | 15 | 050$$

$$C \rightarrow x | y | A | \epsilon$$

$$S \rightarrow 1A1 | 05 | 5 | A1 | 11 | 1$$

3. again $A \rightarrow \epsilon$

$$A' \rightarrow A | \epsilon$$

$$A \rightarrow 1 | B | CA | \epsilon$$

$$B \rightarrow 1B5 | 050B | 15 | 050$$

$$C \rightarrow x | y | A | \epsilon$$

$$S \rightarrow 1A1 | 05 | 5 | A1 | 11 | 1$$

4. $C \rightarrow \epsilon$

$$A' \rightarrow A | \epsilon$$

$$A \rightarrow 1 | B | CA | \epsilon$$

$$B \rightarrow 1B5 | 050B | 15 | 050$$

$$C \rightarrow x | y | A$$

$$S \rightarrow 1A1 | 05 | 5 | A1 | 11 | 1$$

5. again $A \rightarrow \epsilon$

$A' \rightarrow A | \epsilon$
 $A \rightarrow 1 | B | CA | C | A$
 $B \rightarrow 1BS | 0S0B | 1S | 0S0$
 $C \rightarrow x | y | A | \epsilon$
 $S \rightarrow 1A1 | 0S | S | A1 | 11 | 1$

after omitting $A \rightarrow \epsilon, C \rightarrow \epsilon$

$A' \rightarrow A | \epsilon$
 $A \rightarrow 1 | B | CA | C | A$
 $B \rightarrow 1BS | 0S0B | 1S | 0S0$
 $C \rightarrow x | y | A$
 $S \rightarrow 1A1 | 0S | S | A1 | 11 | 1$

here,
removal of $A \rightarrow \epsilon$ and $C \rightarrow \epsilon$ will be in loop and there are interconnected. Although removing these two epsilons don't affect the other variables as they are not changing anything. That's why those two epsilons are being omitted and we're proceeding to the next step.

ii) remove unit productions:

for $A' \rightarrow A, A \rightarrow C, A \rightarrow A, C \rightarrow A, S \rightarrow S$

$A' \rightarrow 1 | 1BS | 0S0B | 1S | 0S0 | CA | x | y | \epsilon$
 $A \rightarrow 1 | 1BS | 0S0B | 1S | 0S0 | CA | x | y$
 $B \rightarrow 1BS | 0S0B | 1S | 0S0$
 $C \rightarrow x | y | 1 | 1BS | 0S0B | 1S | 0S0 | CA$
 $S \rightarrow 1A1 | 0S | A1 | 11 | 1$

(u, v)

$q_0 \rightarrow 0$

$P \rightarrow q, B$

$q_1 \rightarrow 1$

$Q \rightarrow q_0, s$

$x_1 \rightarrow x$

$R \rightarrow q_0, B$

$y_1 \rightarrow y$

$T \rightarrow q_1, A$

$A' \rightarrow 1 | Pq | QR | q, s | Qq_0 | CA | x | y | \epsilon$

$A \rightarrow 1 | Pq | QR | q, s | Qq_0 | CA | x | y$

$B \rightarrow Pq | QR | 1s | Qq_0$

$C \rightarrow x | y | 1 | Pq | QR | q, s | Qq_0 | CA$

$G \rightarrow Tq_1 | q_0, s | Aq_1 | q, q_1 | 1$

(Pm).

[P.T.O.]



$$\begin{aligned}
 &9. \quad \omega \rightarrow 2xy \mid 1\omega \mid 2y \\
 &\quad x \rightarrow 1x3 \mid 1\omega3 \mid \epsilon \\
 &\quad y \rightarrow \cancel{xy} \cdot y11 \mid 12y\omega3 \mid x \mid \epsilon
 \end{aligned}$$

$$\begin{aligned}
 &1) \quad \omega' \rightarrow \omega \\
 &\quad \omega \rightarrow 2xy \mid 1\omega \mid 2y \\
 &\quad x \rightarrow 1x3 \mid 1\omega3 \mid \epsilon \\
 &\quad y \rightarrow y11 \mid 12y\omega3 \mid x \mid \epsilon
 \end{aligned}$$

u) none null

$$\underline{x \rightarrow \epsilon:}$$

$$\begin{aligned}
 &\omega' \rightarrow \omega \\
 &\omega \rightarrow 2xy \mid 1\omega \mid 2y \\
 &x \rightarrow 1x3 \mid 1\omega3 \mid 13 \\
 &y \rightarrow \cancel{xy} y11 \mid 12y\omega3 \mid x \mid \epsilon
 \end{aligned}$$

$$\underline{y \rightarrow \epsilon:}$$

$$\begin{aligned}
 &\omega' \rightarrow \omega \\
 &\omega \rightarrow 2xy \mid 1\omega \mid 2y \mid \\
 &\quad \quad \quad 2x \mid 2 \\
 &x \rightarrow 1x3 \mid 1\omega3 \mid 13 \mid \\
 &y \rightarrow y11 \mid 12y\omega3 \mid x \mid 11 \mid \\
 &\quad \quad \quad 12\omega3
 \end{aligned}$$

w) for $\omega' \rightarrow \omega, y \rightarrow x$

$$\begin{aligned}
 &\omega' \rightarrow 2xy \mid 1\omega \mid 2y \mid 2x \mid 2 \\
 &\omega \rightarrow 2xy \mid 1\omega \mid 2y \mid 2x \mid 2 \\
 &x \rightarrow 1x3 \mid 1\omega3 \mid 13 \\
 &y \rightarrow y11 \mid 12y\omega3 \mid 1x3 \mid 1\omega3 \mid 13 \mid 11 \mid 12\omega3
 \end{aligned}$$

(iv,v)

$$P \rightarrow A_2 X$$

$$A_1 \rightarrow 1$$

$$Q \rightarrow A_1 X$$

$$A_2 \rightarrow 2$$

$$R \rightarrow W A_3$$

$$A_3 \rightarrow 3$$

$$S \rightarrow Y A_1$$

$$T \rightarrow A_2 Y$$

$$U \rightarrow T R$$

$$V \rightarrow A_1 A_2$$

$$W' \rightarrow P Y \mid A_1 W \mid A_2 Y \mid A_2 X \mid 2$$

$$W \rightarrow P Y \mid A_1 W \mid A_2 Y \mid A_2 X \mid 2$$

$$X \rightarrow Q A_3 \mid A_1 R \mid A_1 A_3$$

$$Y \rightarrow S A_1 \mid A_1 U \mid Q A_3 \mid A_1 R \mid A_1 A_3 \mid A_1 A_1 \mid V R$$

(Bm).

—X—

