Lab Report: *04*

Title: *Gibb's Phenomena of The Rectangular Pulse Train*

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Course Code: *CSE-260*

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Lab Report: Gibb's Phenomena of The Rectangular Pulse Train

Objective: To study and analyze Gibb's Phenomena associated with the Fourier series representation of a rectangular pulse train.

Introduction: Gibb's Phenomena occurs when a discontinuous periodic function is approximated by its Fourier series. It manifests as an overshoot near the discontinuities, which does not diminish as the number of terms in the series increases, although the width of the overshoot region decreases. The phenomenon is particularly evident in rectangular pulse trains, which are characterized by sharp transitions between high and low states.

Theory: A rectangular pulse train is a periodic signal that alternates between two levels (e.g, 0 and 1) within a single period. Mathematically, it can be expressed as a Fourier series:

where:

- is the average value of the signal over one period.
- and are the Fourier coefficients, determined by integrating the product of the function and the basis functions over one period.

For a rectangular pulse train, the Fourier coefficients decay as, but the abrupt transitions at the edges lead to overshoots in the reconstructed signal, highlighting Gibb's Phenomena.

Experimental Setup:

1. **Software Tools:** MATLAB or Python for numerical computation and signal visualization.
2. **Signal Parameters:** Define the rectangular pulse train with a period and pulse width.
3. **Procedure:**
   - Compute the Fourier coefficients for the rectangular pulse train.
   - Reconstruct the signal using a finite number of terms (e.g).
   - Plot the original signal and the reconstructed signal for comparison.
   - Analyze the overshoot and its behavior as increases.

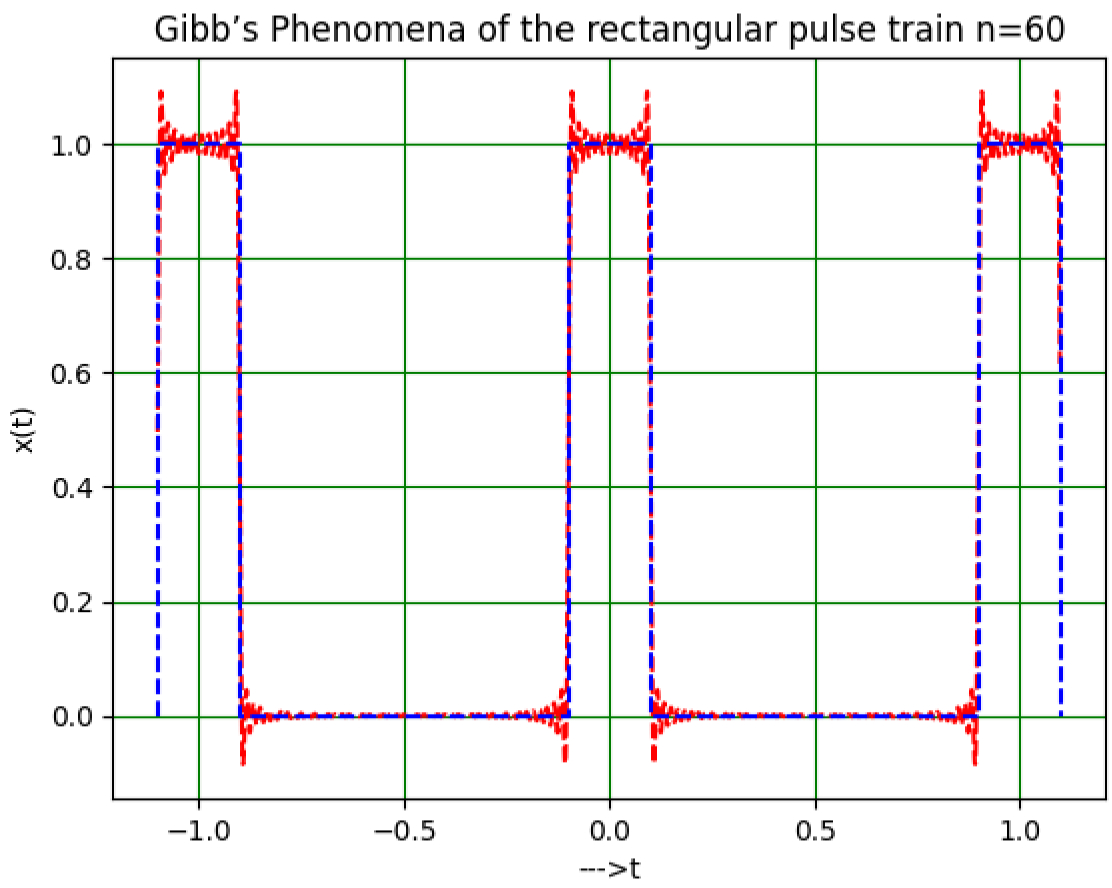
Gibb’s Phenomena of the rectangular pulse train

Harmonic Components N=60

In [ ]:

```
import numpy as np
from matplotlib import pyplot as plt
from matplotlib import pyplot as plt
import math
t = np.arange(-1.1, 1.1, 0.001)
T=1;     #period of rectangular pulse train
tau=0.2; #width of pulse
d=tau/T; xt=d;
n=60;  #number of harmonics components
for i in range(1, n):
  xt=xt+2*d* np.sinc(i*tau/T)*np.cos(2*math.pi*i*t/T)

t1  = np.array([-1.1, -1.1, -0.9, -0.9, -0.5, -0.1, -0.1, 0.1, 0.1, 0.5, 0.9, 0.
xt1 = np.array([0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0])
plt.plot(t, xt, 'r--',t1, xt1, 'b--')
plt.xlabel('--->t')
plt.ylabel('x(t)')
plt.title('Gibb’s Phenomena of the rectangular pulse train n=60')
plt.grid(True, which='both', color="g")
plt.show()
t1  = np.array([-1.1, -1.1, -0.9, -0.9, -0.5, -0.1, -0.1, 0.1, 0.1, 0.5, 0.9, 0.
xt1 = np.array([0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0])
```



Matlab Code:

```
>> t=-1.1:0.001:1.1;
T=1; %period of rectangular pulse train
tau=0.2; %width of pulse
a0=tau/T;
n=10; %number of harmonics components
ft=a0;
for i=1:n,
ft=ft+(2/T)*tau*sinc(i*tau/T)*cos(2*pi*i*t/T);
end
plot(t, ft); %plot the sum of the Fourier components
xlabel('Time--------');
ylabel('Amplitude');
title('Gibbs phenomenon for n=10');
hold on;
t=[-1.1 -1.1 -0.9 -0.9 -0.5 -0.1 -0.1 0.1 0.1 0.5 0.9 0.9 1.1 1.1];
ft=[0 1 1 0 0 0 1 1 0 0 0 1 1 0];
plot(t, ft, 'r'); %plots the original pulse wave
hold off;
grid on
```

Observations:

- **Fourier Approximation:** The red dashed curve (xt) represents the Fourier series approximation for 60 harmonics, which resembles the rectangular pulse train (blue dashed curve, xt1) except near the edges.
- **Gibb's Phenomena:** Overshoots are visible at the discontinuities of the pulse train, a characteristic artifact of Gibb's Phenomena.
- **Oscillations:** Small oscillations around the edges dampen away from the discontinuities but remain prominent near transitions.
- **Improvement with n=60:** Compared to fewer harmonics, the approximation is more accurate, especially in flat regions of the pulse.
- The grid helps visualize the alignment of the Fourier series with the ideal pulse train, especially highlighting discrepancies at edges.

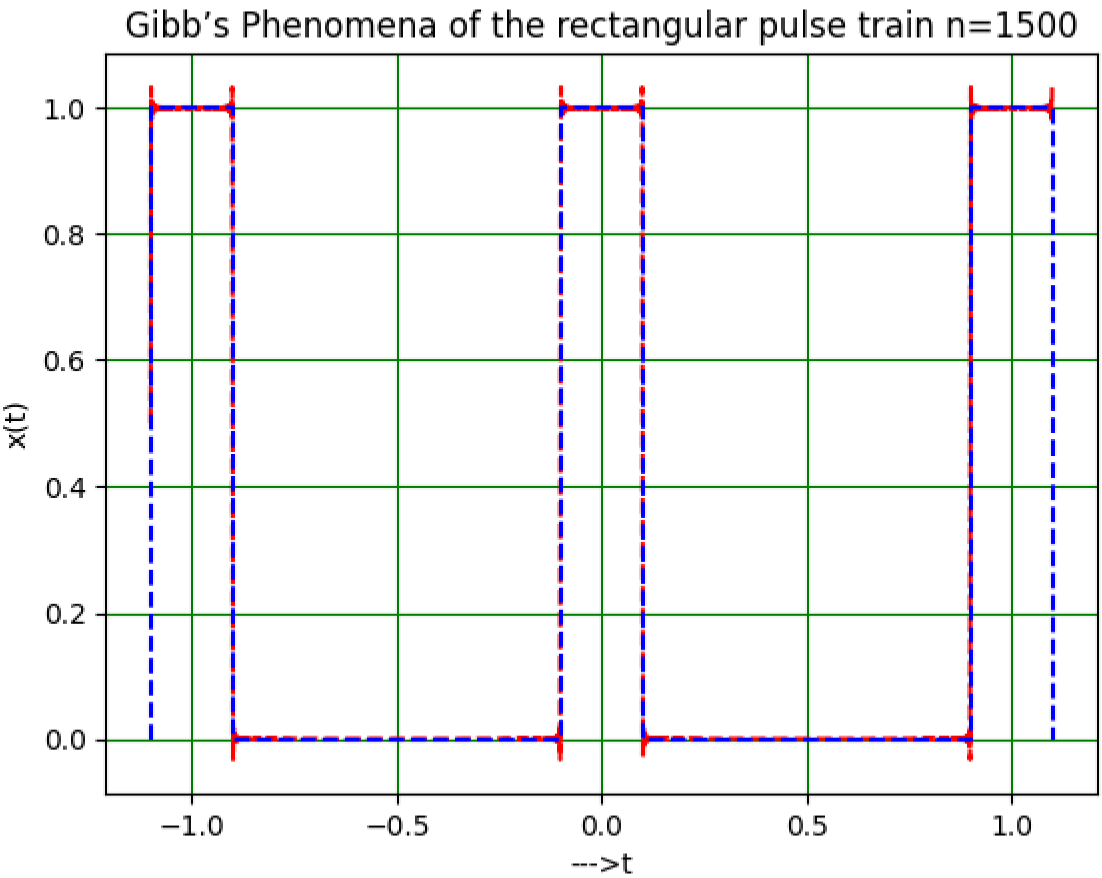
Harmonic Components N = 1900

In [2]:

```
import numpy as np
from matplotlib import pyplot as plt
from matplotlib import pyplot as plt
import math
t = np.arange(-1.1, 1.1, 0.001)
T=1;      #period of rectangular pulse train
tau=0.2; #width of pulse
```

```
d=tau/T; xt=d;
n=1500;  #number of harmonics components
for i in range(1, n):
  xt=xt+2*d* np.sinc(i*tau/T)*np.cos(2*math.pi*i*t/T)

t1  = np.array([-1.1, -1.1, -0.9, -0.9, -0.5, -0.1, -0.1, 0.1, 0.1, 0.5, 0.9, 0.
xt1 = np.array([0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0])
plt.plot(t, xt, 'r--',t1, xt1, 'b--')
plt.xlabel('--->t')
plt.ylabel('x(t)')
plt.title('Gibb's Phenomena of the rectangular pulse train n=1500')
plt.grid(True, which='both', color="g")
plt.show()
t1  = np.array([-1.1, -1.1, -0.9, -0.9, -0.5, -0.1, -0.1, 0.1, 0.1, 0.5, 0.9, 0.
xt1 = np.array([0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0])
```



Observations:

Enhanced Approximation with Higher Harmonics:

- The Fourier series approximation (red dashed curve, xt) includes n=1900 harmonics, significantly improving the accuracy compared to the n=60 case.
- The approximation closely resembles the original rectangular pulse train (blue dashed curve, xt1), particularly in regions away from the discontinuities.

Persistence of Gibb's Phenomena:

- Overshoots near the discontinuities are still clearly visible, indicating that Gibb's Phenomena persists even with a large number of harmonics.

- The overshoot magnitude remains constant, as it is a characteristic of the phenomenon, but the oscillations around the discontinuities have become finer.

Sharper Transitions:

- The Fourier series reconstructs sharper transitions at the edges of the pulses compared to the lower harmonics case. This is due to the inclusion of more harmonics, which increases the series' ability to capture high-frequency components.

Convergence to Ideal Signal:

- With n=1900, the Fourier series approximation converges much closer to the ideal signal (xt1) across the period.
- The main discrepancies remain confined to the regions near the sharp transitions, which is a limitation of Fourier series for discontinuous signals.

Visual Representation:

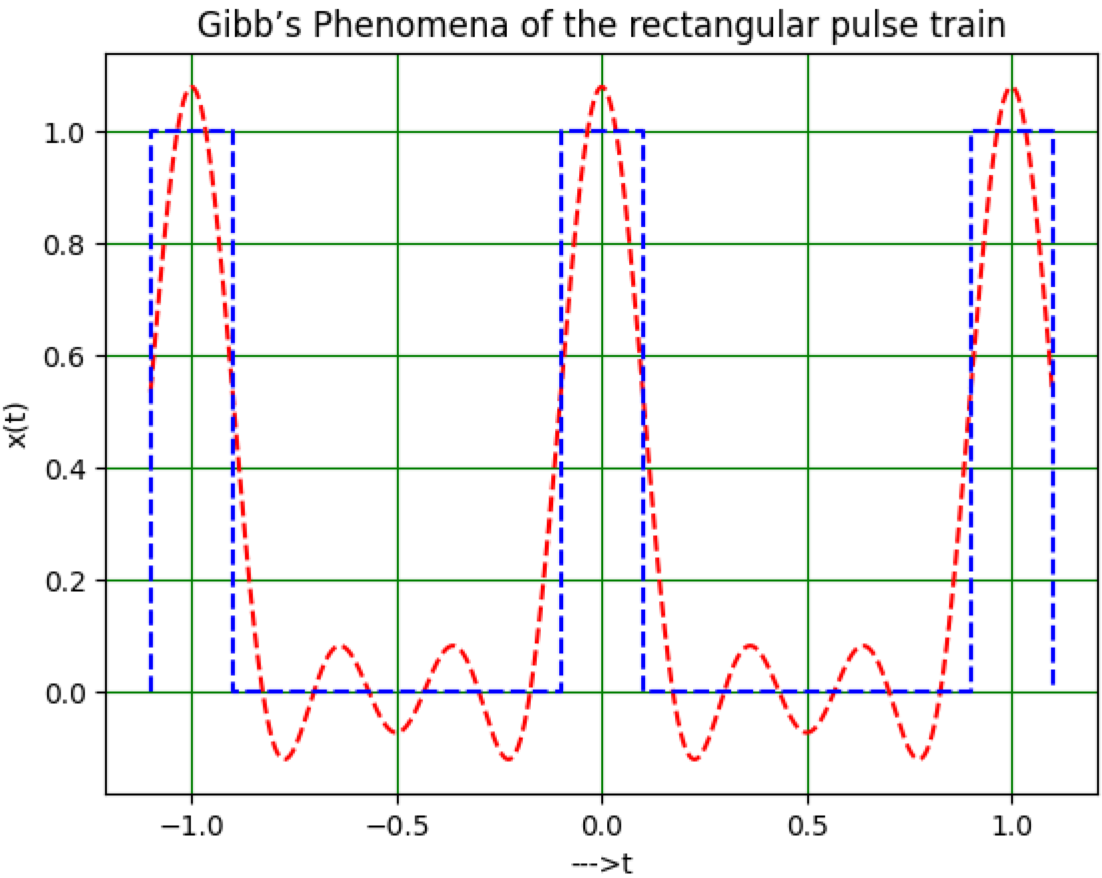
- The grid and axes labels effectively highlight the alignment between the original pulse train and the Fourier approximation.
- The red and blue curves visually demonstrate the differences and improvements compared to the n=60 case.

Harmonic Components for small n=2

In [11]:

```
import numpy as np
from matplotlib import pyplot as plt
from matplotlib import pyplot as plt
import math
t = np.arange(-1.1, 1.1, 0.001)
T=1;     #period of rectangular pulse train
tau=0.2; #width of pulse
d=tau/T; xt=d;
n=2;  #number of harmonics components
for i in range(1, n):
  xt=xt+2*d* np.sinc(i*tau/T)*np.cos(2*math.pi*i*t/T)

t1  = np.array([-1.1, -1.1, -0.9, -0.9, -0.5, -0.1, -0.1, 0.1, 0.1, 0.5, 0.9, 0.
xt1 = np.array([0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0])
plt.plot(t, xt, 'r--',t1, xt1, 'b--')
plt.xlabel('--->t')
plt.ylabel('x(t)')
plt.title('Gibb’s Phenomena of the rectangular pulse train')
plt.grid(True, which='both', color="g")
plt.show()
t1  = np.array([-1.1, -1.1, -0.9, -0.9, -0.5, -0.1, -0.1, 0.1, 0.1, 0.5, 0.9, 0.
xt1 = np.array([0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0])
```



- With n=2, the waveform is smooth and far from the rectangular pulse shape.
- Minimal ripples appear due to fewer harmonics.
- The approximation lacks sharp transitions, showing poor accuracy.
- Increasing n will improve the waveform's resemblance to the rectangular pulse.

Overall Overview:

1. Verification of Gibb’s Phenomenon:

- The experiment successfully visualizes Gibb’s Phenomenon using the Fourier series approximation of a rectangular pulse train.
- Significant ripples (overshoots) are observed near the points of discontinuity in the pulse train when a finite number of harmonics is used.

2. Fourier Series Reconstruction:

- The Fourier series approximation provides a composition of the DC component, fundamental frequency, and harmonics of the rectangular pulse train.
- By summing these components, the approximation converges to the original rectangular wave except at discontinuities.

3. Effect of Limited Harmonics:

- With 60 harmonics, the approximation captures the general shape of the rectangular wave but shows pronounced ripples and deviations near discontinuities.

- With 1900 harmonics, the ripples reduce in width, and the reconstructed wave more closely resembles the original pulse train, indicating better convergence.

4. Persistence of Ripples:

- Despite increasing the number of harmonics, ripples near discontinuities do not disappear entirely. This is the essence of Gibb’s Phenomenon.
- The overshoot magnitude remains constant, but the ripple width decreases as more harmonics are added, improving the overall approximation.

5. Practical Implication:

- The inability to accumulate an infinite number of harmonics limits the exact reconstruction of a periodic wave. This highlights the practical limitations of Fourier series in representing signals with abrupt transitions.

6. Visualization of Results:

- The experiment effectively demonstrates how the addition of more harmonics improves the approximation of the rectangular pulse train, especially in regions away from discontinuities.
- The ripples at discontinuities diminish in width with more harmonics but remain a consistent feature.

Conclusion: This experiment verifies Gibb’s Phenomenon by showing that a Fourier series with finite harmonics converges to the original wave, except at discontinuities where ripples (overshoots) persist. Increasing the number of harmonics improves the approximation but does not eliminate the overshoots entirely, highlighting the trade-offs in practical signal reconstruction.